Sensor Orientation - Exercise 25.3

April 2025

Part 1 - Tarmac

A plane is getting prepared for a terrain mapping mission in the Galapagos islands. The airplane is situated at zero latitude, longitude -90 deg and ~ 0 altitude.

Numerical data

- Earth radius $r_e = 6.38e6\ m$
- Earth rotation rate $\omega_e = 7.29\ rad.s^{-1}$
- Earth gravity $g = 9.81\ m/s^2$

a. What are the airplane coordinates in ECEF and ECI frames for the following three hour angles: midnight, noon, $\frac{\pi}{2}$?

Time	ECEF $[x, y, z]$	ECI $[x, y, z]$
midnight	$[0, -r_e, 0]$	$[0, -r_e, 0]$
noon	$[0, -r_e, 0]$	$[0, r_e, 0]$
$\frac{\pi}{2}$	$[0, -r_e, 0]$	$[r_e, 0, 0]$

b. The plane is currently static, facing East on the tarmac that we assume to be perfectly flat. Within the plane, an IMU is mounted, whose axes define the body frame with the following orientation:
- x-axis toward the plane's nose
- y-axis toward the right wing
- z-axis downward

Determine the IMU readings, i.e. acceleration $[f_x^b,\ f_y^b,\ f_z^b]$ and angular velocity $[\omega_x^b,\ \omega_y^b,\ \omega_z^b]$.

$f_b = [0,\ 0,\ -g]$ and $\omega_b = [0,\ -\omega_e,\ 0]$
(remember that normal forces are felt by accelerometers, not gravity directly)

c. Determine the roll, pitch and yaw as well as the direct cosine matrix $R_b^{l_{NED}}$.

$r = 0°$, $p = 0°$, $y = 90°$ thus $R_b^{l_{NED}} = R_3(y)^T = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

d. Determine the direct cosine matrix $R_{l_{NED}}^e$

$$R_{l_{NED}}^e = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

Part 2 - Take off !

The airplane takes off toward the San Cristobal island, 20km toward the East, with a flight attitude of 500 meters above ground.

a. What are the airplane coordinates above the San Cristobal Island with respect to the same local level NED defined at take-off ?

$$p^{l_{NED}} = \begin{bmatrix} 0 \\ 20'000 \\ -500 \end{bmatrix}$$

b.What are the coordinates of the airplane in ECEF ?

The local level NED frame's origin in ECEF was estimated in Part I.a:

$$l^e_{NED} = \begin{bmatrix} 0 \\ -r_e \\ 0 \end{bmatrix}$$

The position of the airplane in ECEF follows:

$$p^e = l^e_{NED} \; + \; R^e_{l_{NED}} \, p^{l_{NED}} = \begin{bmatrix} 0 \\ -r_e \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 20'000 \\ -500 \end{bmatrix} = \begin{bmatrix} 20'000 \\ -r_e - 500 \\ 0 \end{bmatrix}$$

c. After the aircraft maneuvers above the island, the instantaneous velocity of the plane is given by the navigation system in ECEF: $v^e = [50, \; 0, \; 50]$. What is the velocity in local NED ?

$$v^{l_{NED}} = {R^e_{l_{NED}}}^T \, v^e = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 50 \\ 0 \\ 50 \end{bmatrix} = \begin{bmatrix} 50 \\ 50 \\ 0 \end{bmatrix}$$

d. What is the heading of the airplane ?

Since we are in a local NED frame, the yaw and the heading are equivalent. Thus we have:

$$heading = yaw = arctan \frac{v_y^{l_{NED}}}{v_x^{l_{NED}}} = \frac{\pi}{4} rad$$