Optimal abatement – exercise 2

The damage caused by the total emissions of some pollutant by the sources in some area is equal to $D(E) = 2E + E^2$. Absent any abatement, the volume of emissions is $E_0 = 10$. A technical analysis has yielded this function for the marginal abatement cost (MAC) across all sources: $C'(\Delta) = 2 + 3\Delta$.

Calculate optimal abatement Δ^* and residual emissions E^*.

Calculate total damage, total abatement cost and total cost (damage + abatement cost) for no abatement ($\Delta=0$), full abatement ($\Delta=10$) and optimal abatement ($\Delta=\Delta^*$).

Answers:

Δ^* is given by $G'(\Delta^*) = C'(\Delta^*)$

We know C'(.) but not G'(.)

$G(\Delta) = D(E_0) - D(E_0-\Delta) \rightarrow G'(\Delta) = D'(E_0-\Delta) = 2 + 2(E_0-\Delta) = 22 - 2\Delta$, as $E_0 = 10$

$G'(\Delta^*) = C'(\Delta^*) \rightarrow 22 - 2\Delta^* = 2 + 3\Delta^* \rightarrow \Delta^* = 4$

Optimal residual emissions $E^* = E_0 - \Delta^* = 6$

<u>For no abatement (Δ=0)</u>

$E = 10 \rightarrow D(10) = 120$

$C(0) = 0$

$D + C = 120$

<u>For full abatement (Δ=10)</u>

$E = 0 \rightarrow D(0) = 0$

$C(10) = (32-2)\times10/2 + 2\times10 = 170$ using a graph and computing surfaces; or

$C'(\Delta) = 2 + 3\Delta \rightarrow C(\Delta) = 2\Delta + (3/2)\Delta^2 \rightarrow C(10) = 170$

$D + C = 170$

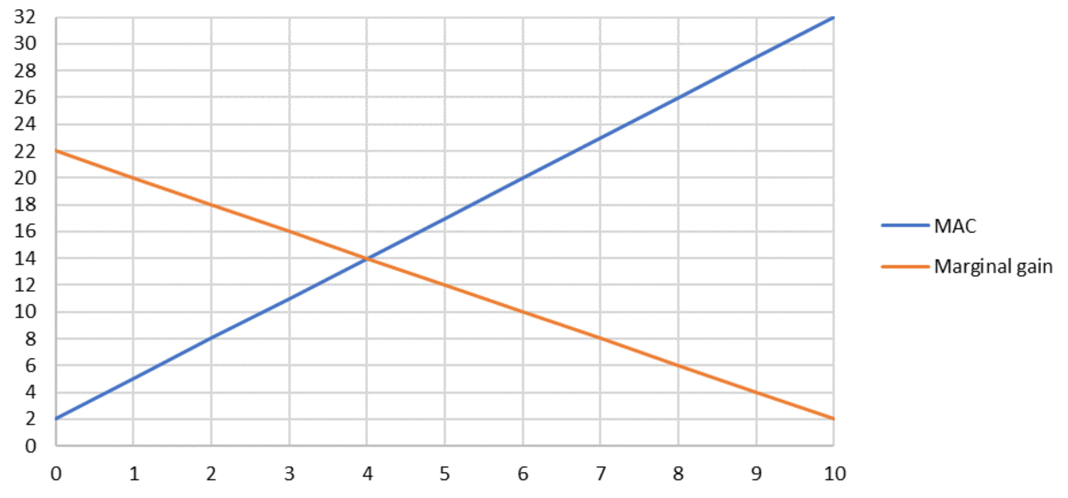
<u>For optimal abatement ($\Delta=\Delta^*$)</u>

$E^* = 6 \rightarrow D(6) = 48$

This could also be computed with the surfaces in the graph: $G(4) = (22-14)\times4/2 + 14\times4 = 72 \rightarrow D(6) = D(10) - G(4) = 120 - 72 = 48$

$C(6) = (14-2)\times4/2 + 2\times4 = 32$ computing surfaces; or $C(6) = 32$ using the cost function

$D + C = 80$



Optimal abatement – exercise 3

Cattle farming contributes to global warming because of the methane emissions of cows. Let us assume that the economic damage (D) caused by a herd of cows is proportional to the square of the number of "untreated" cows (V_U), that is, cows for which no methane avoidance measures have been taken:

$D(V_U) = 0.02 \times V_U^2$

There exist two solutions for reducing methane emissions. The cost of each abatement solution depends on the number of cows that are "treated" (V_T):

(a) a feed additive (FA), costing 4 per cow, or $4 \times V_T$ for all treated cows

(b) a methane capture system (MCS), which only works if the cows are kept in a barn; its cost is the sum of a fixed cost of 800 and a variable cost of 2 per cow, or $800 + 2 \times V_T$ for all cows treated with this system

1) Draw a figure that shows the treatment cost with each abatement solution depending on the number of cows treated for up to 600 cows (remember to name the axes of your chart). Calculate the number of cows at which one solution becomes more advantageous than the other. What does this imply for this livestock farm?

Let us consider a livestock farm with a herd of 200 cows. You are going to calculate the optimal number of cows to be treated and the best solution to do so.

2) Calculate total damage and total minimized abatement cost for two extremes: no abatement and 100% abatement. Which of these two extremes implies lower total cost?
3) Write the formula for the gain from abatement $G(V_T)$ in such a manner that you can easily calculate the marginal gain, and then write the formula for the marginal gain $G'(V_T)$.
4) Draw a figure that shows the marginal gain and the marginal cost of abatement, using for the latter your response to question 1 *(remember to name the axes of your chart)*. Try to draw the functions to scale and indicate values that characterize them on the axes.
5) Calculate the optimal level of abatement, i.e., the number of cows that should be treated on this farm with a view to maximizing abatement gain minus abatement cost.
6) Calculate total damage and total abatement cost for the optimal level of abatement. Compare with your answers for question 2).

Answers:

1) The cost of FA is a straight line of slope 4 from the origin. The cost of MCS is a straight line taking the value of 800 at the origin and a slope of 2. The two lines intersect at $V_T = 400$. For this number of treated cows, both technologies cost 1600. For $V_T < 400$, FA is cheaper; $V_T > 400$, MCS is cheaper. Given the size of the herd, only FA will be used.

2) No abatement: $D(200) = 0.02\times200^2 = 800$, $C(0) = 0$, total cost = 800
   100% abatement: $D(0) = 0$, $C(200) = 4\times200 = 800$, total cost = 800
   Same cost!

3) $G(V_T) = D(200) – D(200-V_T) = 800 – 0.02\times(200-V_T)^2$

   $G'(V_T) = 0.04\times(200-V_T) = 8 – 0.04\times V_T$

4) The marginal gain is a straight line from 8 on the vertical axis to V_T =200 on the horizontal axis.
   The marginal cost is equal to 4, so a horizontal line at this height, for all V_T up to the full stock of 200 (maximal abatement, i.e., number of cows than can be treated).
   Marginal gain and marginal cost intersect at $V_T = 100$ (to be calculated below).

5) The optimality condition is marginal gain = marginal cost
   $8 – 0.04\times V_T^* = 4$, or $V_T^* = 100$.

6) $D(100) = 0.02\times100^2 = 200$. $C(100) = 4\times100 = 400$. Total cost = 600 < 800.