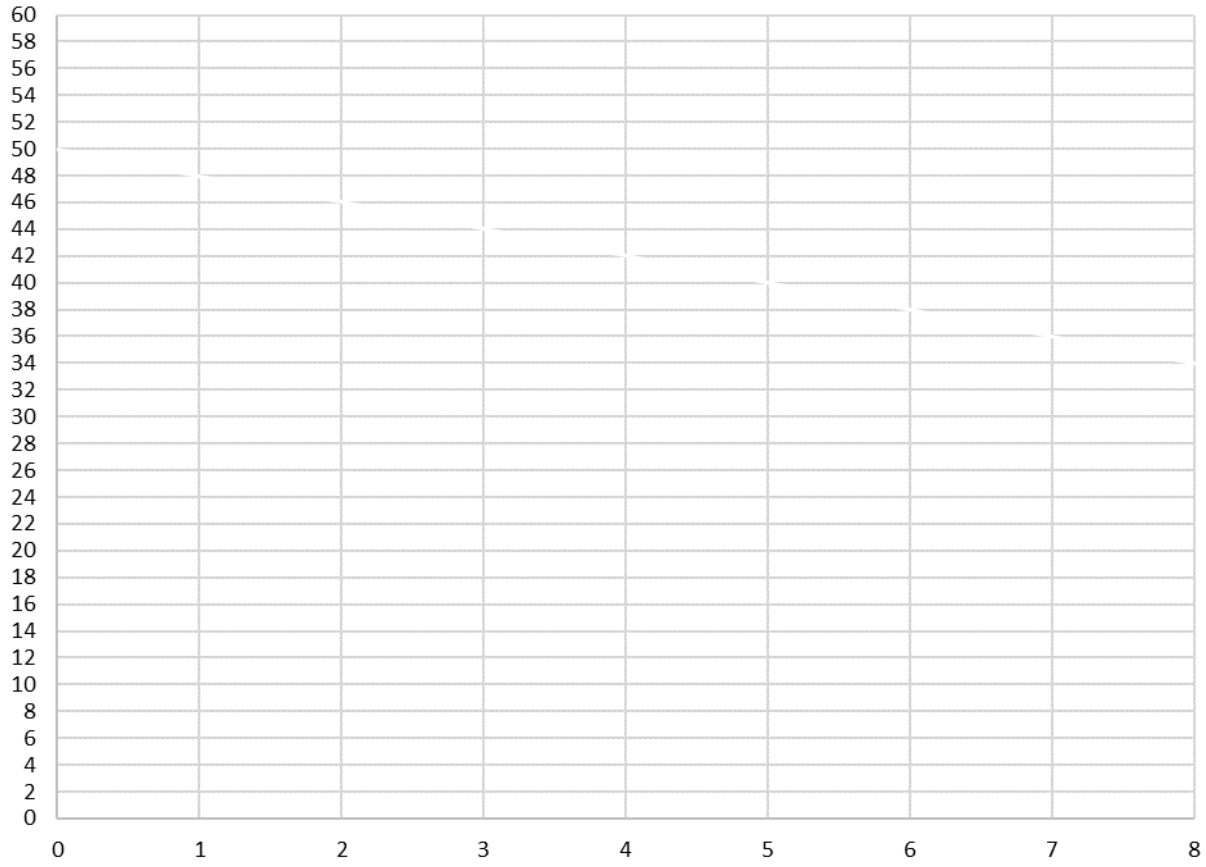
MARKET REGULATION - EXERCISE

Consider the competitive market for fertilizer, assumed to be a homogenous good. "Producers" make fertilizer and sell it to "users". Their aggregate demand is given by: $Q_D = 25 - p/2$. The producers' aggregate supply is given by $Q_S = p/4 - 2$ (which corresponds to a total cost of production $TC = 8q + 2q^2$).

1. Calculate the market equilibrium, i.e., the market price P_M and the quantity traded Q_M.

2. Represent this graphically. Warning: The supply curve does not start from (0,0).



3. Calculate the marginal willingness to pay (mWTP) function, i.e., the inverse of the demand function, and the marginal willingness to accept (mWTA) function, i.e., the inverse of the supply function.

4. For the market equilibrium, calculate the surpluses of the users and the producers, as well at the UP surplus, defined as the sum of these surpluses. Check you result for the UP surplus by recalculating it as total WTP minus total WTA (i.e., total cost).

Production causes external costs (environmental degradation, for example) as a function of the quantity produced: $EC = 2q + q^2$.

5. Calculate the marginal external cost function (= the external cost of an additional unit of production) and the social mWTA function. Add it to the graph above.

6. Calculate the socially optimal level of production Q*. Compare it with the market equilibrium and explain the differences.

7. Calculate the price P_H* at which the users would want to buy the quantity Q* of fertilizer, and the price P_L* at which the producers would want to sell the quantity Q* of fertilizer.

8. Calculate the UP surplus of for the social optimum Q*, under the assumption that the users with the highest WTP get Q* and that the producers with the lowest WTA (i.e., costs) produce it. To do this, calculate total WTP and total private WTA for Q*.

9. Calculate the total external costs for Q_M and Q* and recompute total surpluses corrected for external costs. What do your results show?

You are going to compare different possible interventions by the regulator:

10. The authority issues free production quotas to the sellers who produced Q_M, for a total amount equal to Q*. The trade of these quotas is organised, to make sure that it is the producers with the lowest production costs who produce Q*. What will be the price of fertilizer? Calculate the users' and producers' surpluses (all producers', whether they actually produce or not), the UP surplus and the social surplus corrected for external costs.

11. The authority introduces a unit tax on production, sufficient to move the market equilibrium to Q*. Calculate the rate of this tax and the new WTA and supply function for the producers when they have to pay this tax per unit of sales. Show that the new market equilibrium is the social optimum.

12. Compute the tax revenue. Calculate the distribution of this burden between users and producers. Compare the tax revenue with the external cost for Q* and explain the difference.

13. Calculate the users' and producers' surpluses, UP surplus and total surplus, where the latter includes tax revenue and external cost, following the introduction of this tax.

14. Instead of taxing the production of fertilizer, the authority decides to offer a subsidy to producers who reduce their production of fertilizer, proportional to the production reduction relative to Q_M. Calculate the rate of the subsidy sufficient to move the market equilibrium to Q* and show that if sellers get this subsidy per unit that they do not sell compared to Q_M, the new market equilibrium is the social optimum.

15. Compute the total amount of subsidy that will result, the user and producer surpluses, UP surplus and total surplus, where the latter includes subsidy cost and external cost.

16. Compare the outcomes of the equilibria you calculated: initial market equilibrium, equilibrium with a cap on production with free transferable production quotas, equilibrium with the incentive tax, equilibrium with incentive subsidy. Compare them in terms of surpluses and comment. Begin by filling your answers to the earlier questions into the table below.

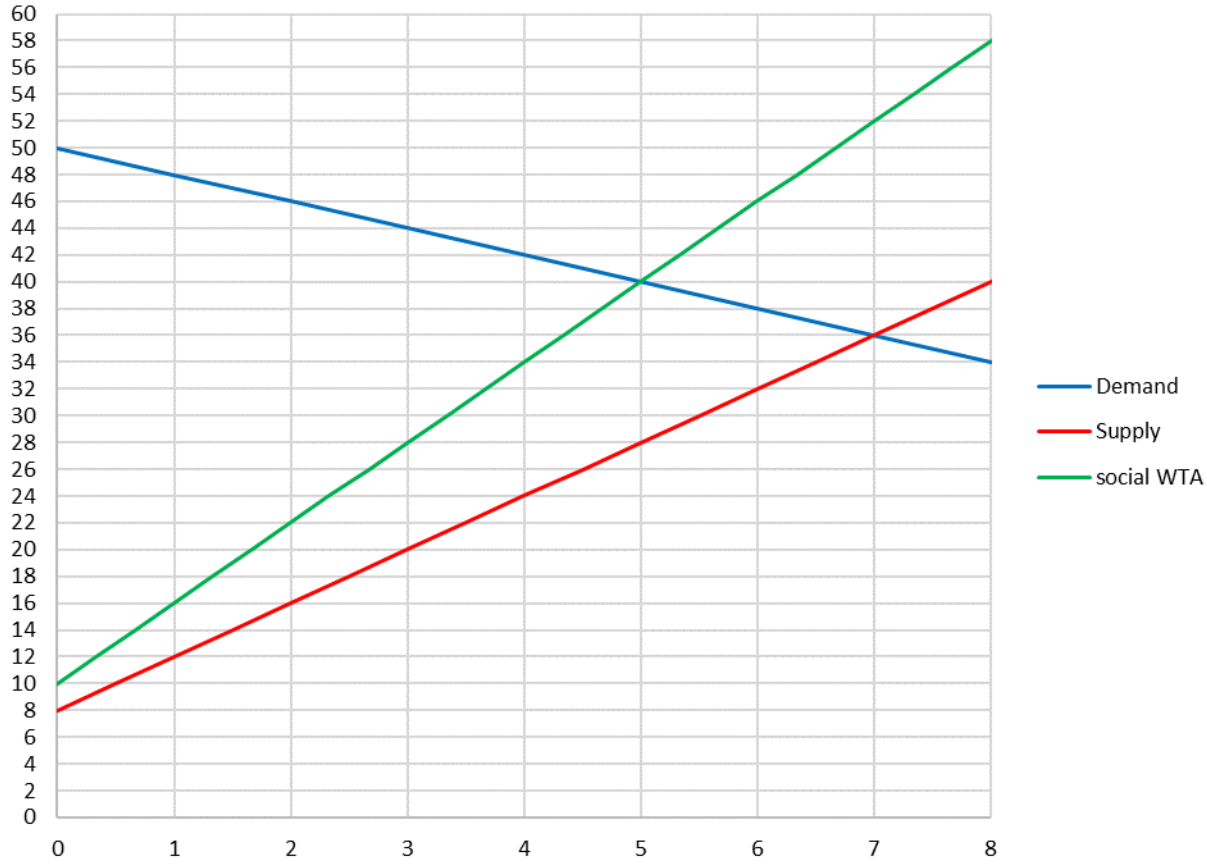
Equilibrium	Users' surplus	Producers' surplus	UP surplus	Transfer to 'social'	External cost	Social surplus
Q_M						
Cap						
Tax						
Subsidy						

Responses

1. Calculate the market equilibrium, i.e., the market price P_M and the quantity traded Q_M.

$Q_D(P_M) = Q_S(P_M) \rightarrow 25 - P_M/2 = P_M/4 - 2 \rightarrow P_M = \mathbf{36}$, $Q_M = \mathbf{7}$

2. Represent this market equilibrium graphically. *Warning: The supply curve does not start from (0,0).*



3. Calculate the marginal willingness to pay (mWTP) function, i.e., the inverse of the demand function, and the marginal willingness to accept (mWTA) function, i.e., the inverse of the supply function.

Inverting the demand function $Q_D(p) = 25 - p/2 \rightarrow$ **mWTP = 50 – 2q**

Inverting the supply function $Q_S(p) = p/4 – 2 \rightarrow$ **mWTA = 8 + 4q**

4. For the market equilibrium, calculate the surpluses of the users and the producers, as well at the UP surplus, defined as the sum of these surpluses. Check you result for the UP surplus by recalculating it as total WTP minus total WTA (i.e., total cost). *You can do this*

by calculating the surfaces on your graph, but it is recommended that you derive the total WTP and WTA functions from the mWTP and mWTA functions and use these for this question and later ones.

Users' surplus = (50−36)×7/2 = **49**; Producers' surplus = (36−8)×7/2 = **98**; UP surplus = (50−8)×7/2 = **147** = 49 + 98; total WTP = (50−36)×7/2 + 36×7 = **301**; total WTA = (36−8)×7/2 + 8×7 = **154**; UP surplus = 301 – 154 = 147

Alternative approach: integrate the mWTP and mWTA functions to obtain the WTP and WTA functions, which can then be used directly to compute surpluses, together with the amount paid/earned; for mWTP = 50 – 2q, WTP = 50q – q^2, so that WTP(Q_M) = 50×7 – 7^2 = 301; subtracting what users pay, i.e. 36×7 = 252, yields users' surplus = 301 – 252 = 49; for mWTA = 8 + 4q, WTA = 8q + $2q^2$, so that WTA(Q_M) = 8×7 + 2×7^2 = 154; subtracting this from producers' earnings, i.e., 252, yields producers' surplus = 252 – 154 = 98; UP surplus = WTP(Q_M) − WTA(Q_M) = 301 – 154 = 147

5. Calculate the marginal external cost function (= the external cost of an additional unit of production) and the social mWTA function.

Marginal external cost = **2 + 2q**; social mWTA = private mWTA + marginal external cost = **10 + 6q**

6. Calculate the socially optimal level of production Q*. Compare it with the market equilibrium and explain the differences.

Social optimum Q* such that mWTP(Q*) = social mWTA(Q*) → 50 – 2Q* = 10 + 6Q* → Q* = **5**

This could also be found by intersecting demand with a pseudo-supply curve for the social planner, i.e., supply curve incorporating external cost: Q_S = (p–10)/6 → 25 – P*/2 = (P*–10)/6 → P_H* = 40 for a quantity Q* = 5*

Q* < Q_M, because the full cost of producing this good is higher than what market actors consider

7. Calculate the price P_H* at which the users would want to buy the quantity Q* of fertilizer, and the price P_L* at which the producers would want to sell the quantity Q* of fertilizer.

For the users: mWTP = 50 – 2q becomes P_H* = 50 – 2×5 = **40**; for the producers: mWTA = 8 + 4q becomes P_L* = 8 + 4×5 = **28**

8. Calculate the UP surplus for the social optimum Q*, under the assumption that the users with the highest WTP get Q* and that the producers with the lowest WTA (i.e., costs) produce it. To do this, calculate total WTP and total private WTA for Q*.

Total WTP = (50−40)×5/2 + 40×5 = **225**; or, using the WTP function calculated above: WTP(5) = 50×5 – 5^2 = 225; total WTA = (28−8)×5/2 + 8×5 = **90**; or, using the WTA function calculated above: WTA(5) = 8×5 + 2×5^2 = 90; UP surplus = 225 – 90 = **135**

It would have been easier to calculate the surface of the UP surplus triangle for q = Q_M (as above) and to subtract the triangle between Q and Q_M: 147 − (40−28)×(7−5)/2 = 135*

9. Calculate the total external costs for Q_M and Q* and recompute total surpluses corrected for external costs. What do your results show?

External costs: EC = 2q + q^2; EC(Q_M) = 2×7 + 7^2 = **63**; EC(Q*) = 2×5 + 5^2 = **35**

Total surplus with external cost = UP surplus – external cost: for Q_M: 147 – 63 = **84**; for Q*: 135 – 35 = **100**

We see that the reduction of trade from Q_M to Q* reduces the UP surplus, but when the external cost is taken into account as it should, Q* leads to a greater surplus; this confirms that **Q_M is not optimal in the presence of external costs**

10. The authority issues free production quotas to the producers who had produced Q_M, for a total amount equal to Q*. The trade of these quotas is organised, to make sure that it is the producers with the lowest production costs who produce Q*. What will be the price of fertilizer? Calculate the users' and producers' surpluses, the UP surplus and the social surplus corrected for external costs.

Price of fertilizer = P_H* = **40**; users' surplus = (50−40)×5/2 = **25**; producers' surplus = (40−28)×5 + (28−8)×5/2 = **110**; UP surplus = 25 + 110 = **135**; social surplus = 135 – 35 = **100**

Same calculation with the WTP and WTA functions: WTP(5) = 50×5 – 5^2 = 225; WTA(5) = 8×5 + 2×5^2 = 90; UP surplus = 225 – 90 = 135; social surplus = 135 – 35 = 100; price of fertilizer = P_H* = 40; payment = 40×5 = 200; users' surplus = 225 – 200 = 25; producers' surplus = 200 – 90 = 110

Note that the producers' surplus is shared between producers who actually contribute to producing Q and those who do not, but were given quotas that they can sell to the former*

11. The authority introduces a unit tax on production, sufficient to move the market equilibrium to Q*. Calculate the rate of this tax and the new WTA and supply function for the producers when they have to pay this tax per unit of sales. Derive the new market equilibrium (Q_M*,P_M*) from the new demand and supply functions, and show that it corresponds indeed to the social optimum.

Tax rate = **12**, as the marginal external cost is equal to 2 + 2q, so for Q = Q* = 5 it is equal to 12; private mWTA with tax = private mWTA without tax + 12 = **20 + 4q**; the new supply function is Q_S*(p) = **p/4 – 5**; market equilibrium with tax: $Q_D(P_M^*) = Q_S(P_M^*) \rightarrow 25 - P_M^*/2 = P_M^*/4 - 5 \rightarrow P_M^* =$ **40**; $Q_M^* = Q^* =$ **5**

12. Compute the tax revenue. Calculate the distribution of this burden between users and producers. Compare the tax revenue with the external cost for Q* and explain the difference.

Tax revenue = 12×5 = **60**; for users, the tax raises the price from 36 to 40 for a total of 5 units purchased, so their contribution to this tax revenue is (40−36)×5 = **20**; for producers, the tax lowers the price from 36 for 28 units, so their contribution to this tax revenue is (36−28)×5 = **40**; two thirds of the tax revenue are borne by producers, one third by users

The external cost is EC(Q*) = 35, so, the **tax revenue exceeds it; this is because 12 is levied on all units traded, even though only the last unit has the highest marginal external cost of 12**

13. Calculate the users' and producers' surpluses, UP surplus and total surplus, where the latter includes tax revenue and external cost, following the introduction of this tax.

Users' surplus = (50−40)×5/2 = **25**; producers' surplus = (28−8)×5/2 = **50**; UP surplus = 25 + 50 = **75**; social surplus = UP surplus + tax revenue – external cost = 75 + 60 − 35 = **100**

Using the WTP and WTA functions; $WTP(5) = 50\times5 - 5^2 = 225$; $WTA(5) = 8\times5 + 2\times5^2 = 90$; users must pay 40×5 = 200, so their surplus is 225 – 200 = 25; producers earn net of tax 28×5 = 140, so their surplus is 140 – 90 = 50; UP surplus = 25 + 50 = 75; social surplus = UP surplus + tax revenue – external cost = 75 + 60 − 35 = 100

14. Instead of taxing the production of fertilizer, the authority decides to offer a subsidy to producers who reduce their production of fertilizer, per unit of fertilizer no longer produced relative to Q_M. Calculate the rate of the subsidy sufficient to move the market equilibrium

to Q* and show that if producers get this subsidy per unit that they do not produce compared to Q_M, the new market equilibrium is the social optimum.

Subsidy rate = 12, as the marginal external cost is equal to 2 + 2q, so for Q = Q* = 5 it is equal to 12; private mWTA with subsidy = private mWTA without subsidy + 12 = 20 + 4q; just like with the tax, as every additional unit sold deprives a producer of the subsidy; the new supply function is $Q_S^*(p) = p/4 – 5$; market equilibrium with subsidy: $Q_D(P_M^*) = Q_S(P_M^*) \rightarrow 25 - P_M^*/2 = P_M^*/4 – 5 \rightarrow P_M^* = 40$; $Q_M^* = Q^* = 5$; same result as with tax

15. Compute the total amount of subsidy that will result, the users' and producers' surpluses, UP surplus and total surplus, where the latter includes subsidy cost and external cost.

Cost of the subsidy: the authority must pay it on the reduction of sales, i.e., on 7 – 5; so, subsidy cost = (7−5)×12 = 24; users pay $P_M^* = 40$, so their users' surplus = (50−40)×5/2 = 25; producers get $P_M^* = 40$ for their sales of $Q_M^* = 5$, plus the subsidy, so their producers' surplus = (40−28)×5 + (28−8)×5/2 + 24 = 134; UP surplus = 25 + 134 = 159; social surplus = UP surplus – subsidy cost – external cost = 159 − 24 − 35 = 100

Same calculation with the WTP and WTA functions: $WTP(5) = 50\times5 – 5^2 = 225$; $WTA(5) = 8\times5 + 2\times5^2 = 90$; users pay $P_M^* = 40$ per unit, for a total of 40×5 = 200, so, their users' surplus is 225 – 200 = 25; producers get $P_M^* = 40$ per unit they produce, for a total of 40×5 = 200, plus subsidies on the units they no longer produce for a total of 12×(7−2) = 24; so, their producers' surplus is 200 +24 – 90 = 134. UP surplus = 25 + 134 = 159; social surplus = UP surplus – subsidy cost – external cost = 159 − 24 − 35 = 100

16. Compare the outcomes of the equilibria you calculated: initial market equilibrium, equilibrium with a cap on production with free transferable production quotas, equilibrium with the incentive tax, equilibrium with incentive subsidy. Compare them in terms of surpluses and comment. Begin by filling your answers to the earlier questions into the table below.

Equilibrium	Users' surplus	Producers' surplus	UP surplus	Transfer to 'social'	External cost	Social surplus
Q_M	49	98	147	0	63	84
Cap	25	110	135	0	35	100
Tax	25	50	75	60	35	100
Subsidy	25	134	159	-24	35	100

All interventions lead to the same social surplus, **higher** than that of the market equilibrium that did not consider the external cost; **the outcome is the same for the users**; **the outcomes**

differ substantially in terms of surplus for the producers: they fare much better when they get free production quotas than when they have to pay a tax on their production; the subsidy is even better for them; correspondingly, **the public treasury benefits from the tax revenue**, but must pay for the subsidy