

Normalization and weighting in multi-criteria assessment

Exercise 9
16.04.2025

Tasks

The purpose of this exercise is to test different methods of normalization and weighting by applying them to a toy data set. All of the necessary information to make the calculations are in the following slides.

Task 1 (normalization): Apply the four methods to normalize the data found in the impact matrix. Compare the results of the different methods. Get the spreadsheet from Moodle.

Q: Which normalization method would you use for this example?

Task 2 (weighting): Based on the normalized data, calculate the weighted-sum score for the three cities using both of the suggested weighting methods.

Q: What would be the arguments for and against of each of the two weighting methods?

1. Decision Matrix

IMPACT MATRIX					AMSTERDAM	FRANKFURT	STOCKHOLM
Dimensions	Indicators	Units	Obj				
Social	Average transport time	minutes / km	↓	3	3.2	3.05	
Social	Social segregation	subjective scale from 0 (very low) to 5 (very high)	↓	1	1	2	
Economic	Public transport accessibility	100 [monthly public transport fee (€) / median income (€)]	↓	2.37	2.11	2.29	
Economic	Cycling path density	number of cycle lanes (km) / km ²	↑	2.75	1.01	4.04	
Economic	Public transportation capital	100 M\$USD	↑	710.6	767.2	231.8	
Environmental	Particulate Matter (PM)	index annual mean value	↓	28	31	20	
Environmental	Azote dioxide (NO₂)	index annual mean value	↓	15	22	18	
Environmental	Plume Air Quality Index	index annual mean value	↑	33	38	31	
Environmental	CO₂ emitted by public transportation	kg / capita	↓	1000	1750	1500	

2. Normalization methods

METHOD 1: *Standard deviation from the mean*

(imposing a standard normal distribution):

$$\uparrow: \left(\frac{\text{actual value} - \text{mean value}}{\text{standard deviation}} \right)$$

$$\downarrow: -\left(\frac{\text{actual value} - \text{mean value}}{\text{standard deviation}} \right)$$

METHOD 3: *Distance from the mean* (the mean value is given 100, and alternatives receive scores depending on their distance from the mean):

$$\uparrow: 100 \left(\frac{\text{actual value}}{\text{mean value}} \right);$$

$$\downarrow: 100 \left(1 - \frac{\text{actual value} - \text{mean value}}{\text{mean value}} \right)$$

METHOD 2: *Distance from the group leader* (it assigns 100 to the leading alternative while other alternatives are ranked as percentage points from the leader):

$$\uparrow: 100 \left(\frac{\text{actual value}}{\text{best value}} \right); \downarrow: 100 \left(\frac{\text{best value}}{\text{actual value}} \right)$$

METHOD 4: *Distance from the best and worst performers* (positioning is in relation to the global maximum and minimum; the index takes values between 0 -*laggard*- and 100 -*leader*-):

$$100 \left(\frac{\text{actual value} - \text{worst value}}{\text{best value} - \text{worst value}} \right)$$

Note: The “best” and “worst” value can either be the maximum or minimum value, depending on the objective of the indicator.

Note: For methods 2,3 and 4 the equation varies depending on whether the objective of the indicator at hand is to increase or to decrease.

3. Weighting

IMPACT MATRIX

Dimensions	Indicators	Units	Obj	Weight [%]	AMSTERDAM	FRANKFURT STOCKHOLM
Social	Average transport time	minutes / km	↓	?		
Social	Social segregation	subjective scale from 0 (very low) to 5 (very high)	↓	?		
Economic	Public transport accessibility	100 [monthly public transport fee (€) / median income (€)]	↓	?		
Economic	Cycling path density	number of cycle lanes (km) / km ²	↑	?		
Economic	Public transportation capital	100 M\$USD	↑	?		
Environmental	Particulate Matter (PM)	index annual mean value	↓	?		
Environmental	Azote dioxide (NO₂)	index annual mean value	↓	?		
Environmental	Plume Air Quality Index	index annual mean value	↑	?		
Environmental	CO₂ emitted by public transportation	kg / capita	↓	?		

METHOD 1: Give equal weight to each *dimension*

METHOD 2: Give equal weight to each *indicator*