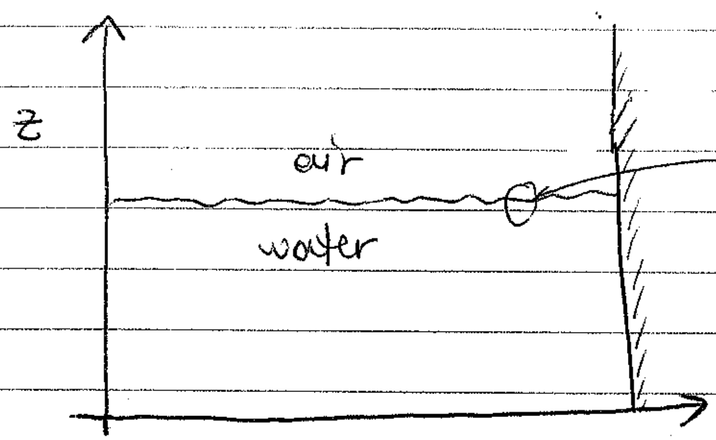
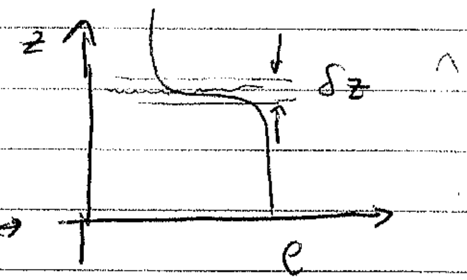
what is the impact of this density "jump" at properties like pressure, etc.

Surface tension

a force that develops over an interface (discontinuity).





δz is the "transition" regime where density gradient is large. $\underline{\underline{\delta z \sim \lambda}}$

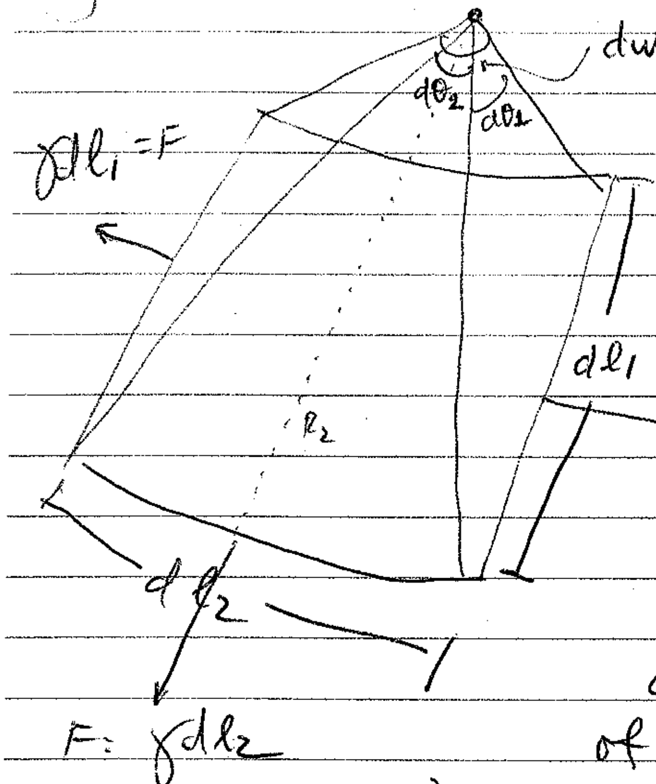
There is a net attraction force between molecules of the same kind. Because of the interface, there is a force imbalance (different molecules across the interface attract each other differently than the "similar" molecules do to each other far away from the interface). This force imbalance leads to a rise of a net force between two phases, and is called "surface tension":

$\gamma :=$ coefficient of interfacial tension

$$\gamma := \frac{\text{force}}{\text{unit length}} \quad \text{or} \quad \frac{\text{energy}}{\text{surface}} \qquad \left(\text{Note: Force} = \frac{\text{energy}}{\text{length}}\right)$$

Q: How does surface tension affect the pressure field across the interface?

Force balance across an interface



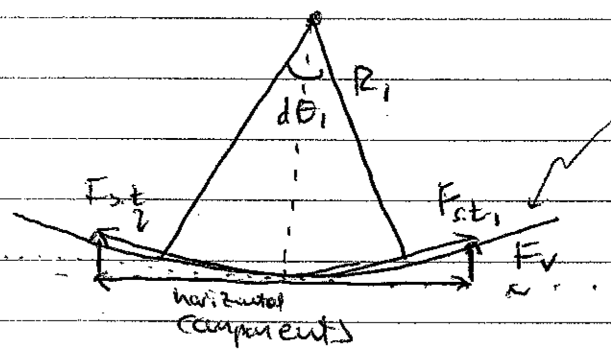
Solid angle ω

$$d\omega = d\theta_1\, d\theta_2$$

force from surface tension: $F = \gamma dl_i$

$\gamma dl_1 = F$ ($\gamma = \frac{F}{\text{length}}$, which length? The "perpendicular" one) (*)

Each direction has its own curvature defined by it's radius of curvature. Let's take what is going on one projection:



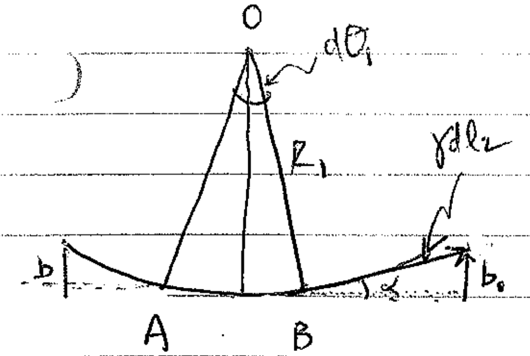
$$\mathbf{F}_{st_1} = \mathbf{F}_{v_1} + \mathbf{F}_{hor_1}$$

$$\mathbf{F}_{st_2} = \mathbf{F}_{v_2} + \mathbf{F}_{hor_2}$$

$\mathbf{F}_{hor_1} = -\mathbf{F}_{hor_2}$ so they cancel out (equilibrium).

$\mathbf{F}_{v_1}, \mathbf{F}_{v_2}$ are not cancelled out by each other, so we should calculate them.

(*) much like Pressure $= \frac{\text{Force}}{\text{Area}} \rightarrow$ Force = Pressure × Area (perpendicular to $\mathbf{F}$)



definition

$$\sin\alpha = \frac{b}{\gamma dl_2} \Rightarrow b = \sin\alpha\, \gamma dl_2$$

but from similarity, $\alpha = \frac{d\theta_1}{2}$ $\Rightarrow$

$$\Rightarrow b = \sin\left(\frac{d\theta_1}{2}\right)\gamma dl_2 \simeq \frac{\gamma}{2}\, dl_2\, d\theta_1 \quad ①$$

Taylor's expansion, for small θ, $\sin\theta \simeq \theta$

by definition, $R_1 d\theta_1 = dl_1$ ②

Substituting ② into ① gives: $b = \frac{\gamma}{2}\frac{dl_1 dl_2}{R_1}$

Because you have two components of force (one on each side of the triangle OAB, then the total force across the interface in the cross-section we examined is:

$$F_{d\theta_1} = b + b = 2b = \gamma\frac{dl_1 dl_2}{R_1}$$

There is also another force component, along the projection $\partial\theta_2$, which as before can be shown to be equal to:

$$F_{d\theta_2} = \gamma\frac{dl_1 dl_2}{R_2}$$

All together the total force across the interforce for the area $dl_1 dl_2$ is:

$$\underset{\sim}{F} = \underset{\sim}{F}_{d\theta_1} + \underset{\sim}{F}_{d\theta_2} = \gamma\, dl_1 dl_2 \left[\frac{1}{R_1} + \frac{1}{R_2}\right]$$

now $\left(\dfrac{F}{dl_1 dl_2}\right) := \Delta P$

has units of pressure.

Pressure "jump" across the interface

so $\Delta P = \gamma \left[\dfrac{1}{R_1} + \dfrac{1}{R_2}\right]$ Young-Laplace eqn.

Special cases

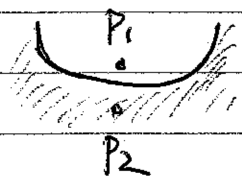
a) Flat surface, $R_1 \to \infty$, $R_2 \to \infty$ $\Delta P \to 0$

no pressure jump across an interface

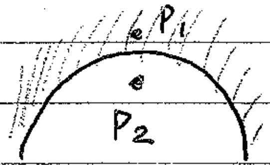
b) cylinder, $R_1 \to \infty$, $R_2 = \alpha$ $\Delta P = \dfrac{\gamma}{\alpha}$

c) sphere $R_1 = \alpha$, $R_2 = \alpha$ $\Delta P = \dfrac{2\gamma}{\alpha}$

So, what does this physically mean?



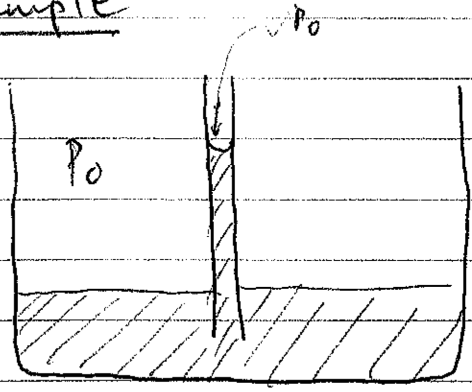
$P_1 > P_2$



$P_1 < P_2$

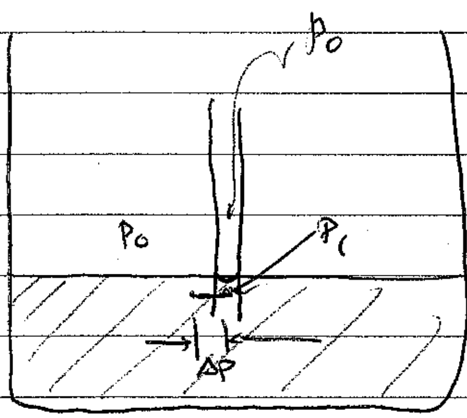
Pressure is higher on the "positive" curvature side

<u>example</u>



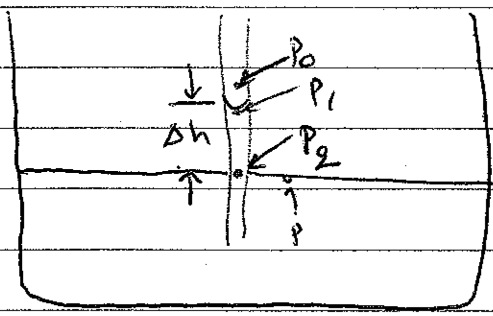
Capilary rise

when curvature is high enough, the pressure across the interface is lower enough to generate a large pressure imbalance:



$t = 0$

$P_1 < P_0$, so there is a ΔP between the curved interface in capillary and flat air-water interface. As a result, water is "pushed" through the capillary, until the ΔP becomes zero



$t = t_{final}$
(when the system equilibrates)

At equilibrium, the force balance is:

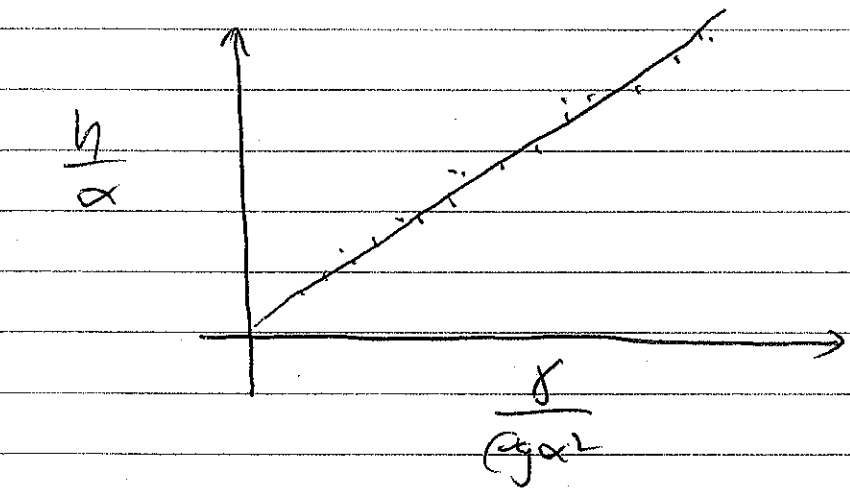
$$P_2 = P_1 + \Delta h \rho g$$

P_2 must be equal to P_0

So $P_0 = P_1 + \Delta h \rho g$. But $P_0 - P_1 = \Delta P_{interface}$

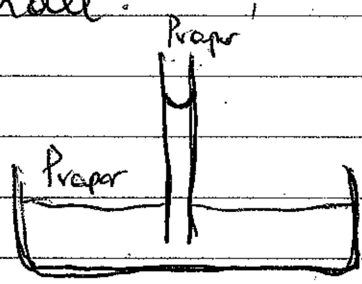
So: $h = \dfrac{2\gamma}{\rho g \alpha}$. To make dimensionless, we divide by α

$$\frac{h}{\alpha} = \frac{2\gamma}{\rho g \alpha^2} \rightarrow \text{Bond \#} = \frac{\text{Surf. t. forces}}{\text{gravity}}.$$



experiments for any type of geometry: they fall on "line"!

Q: If you measure the evaporation rate using a tube, what happens if the tube is too small?



think about it.

$$P_{sat}^{(flat)} + \Delta P = P_{sat}^{(curve)}$$

Since $P_{sat}^{(curve)}$ is lower than $P_{sat}^{(flat)}$, that means that the evaporation rate will be underestimated (decreased) and diffusion coef will be underestimated.