

First Law :

$$\Delta U = Q + W$$

$\nearrow$ internal energy
 $\nwarrow$ net heat

work, (-) if system did work for us.
 (+) if we did work to the system.

internal energy
(average velocity, rotation of molecules, etc).

for an ideal gas: $\Delta U = C_v \Delta T$, or $dU = C_v dT$
 we deal with intensive, or per kg mass, so $C_v = \frac{C_v}{m_{\text{mol}}}$

in differential form:

$$dU = dQ + dW \quad (\text{internal energy 1st law})$$

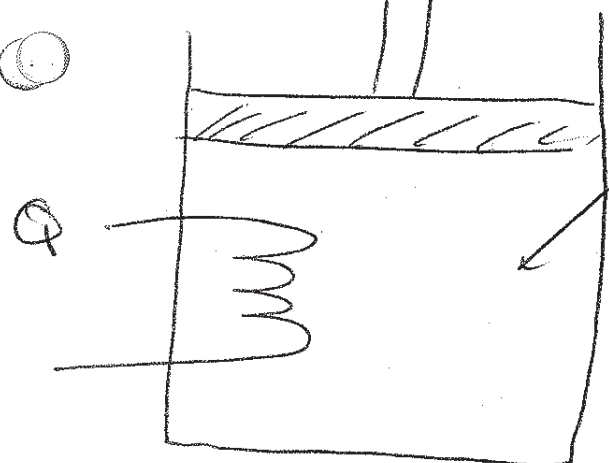
$\uparrow \quad \uparrow$
 each of these terms are not perfect differentials,
 (because $\oint dQ \neq 0$, $\oint dW \neq 0$), but the
 sum of both are (i.e., $\oint (dQ + dW) = 0$)
 because the function U does not depend
 on how you reach the state, only on the
 state itself, i.e. $\oint dU = 0$.

In general, $U(V, T) \leftarrow$ function of V and T .
 Since dU is a perfect differential, it can be
 written as :

$$dU = \left(\frac{\partial U}{\partial V} \right)_T dV + \left(\frac{\partial U}{\partial T} \right)_V dT$$

So:

①

 $U_{\text{initial}} \rightarrow U_{\text{final}}$

Let's take the general case where

a gas is put in a cylinder

and gas do/get p-v work and supply same heat Q .

② In general conservation of energy would say

 ΔU

↑
change in (internal)
energy of system

 $= Q + W$

↑
net
heat

↑
net work

This is known as the
"first law" of thermo

heat convention:

+ when Q is added to
system,

- when Q is removed from
system

U : measure of "internal energy" associated with molecules

○ N molecules of temperature T
volume

Kinetic Theory says: for an ideal gas:

$$\underbrace{\frac{1}{2} m v^2}_{\text{K.E. of each molecule}} = \underbrace{\frac{3}{2} k T}_{\text{monatomic molecule (3 d.o.f. freedom)}}$$

K.E. of each molecule

monatomic molecule
(3 d.o.f. freedom)

○ So: for whole gas:

$$\underbrace{\frac{1}{2} N m v^2}_{\text{internal energy}} = \frac{3}{2} N k T$$

internal energy

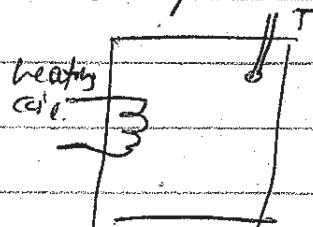
$$U = \underbrace{\frac{3}{2} N k T}_{\uparrow}$$

constant, if V const (b/c N depends on volume).

○ So $du = \left(\frac{3}{2} N k \right) dT$ ← call that "heat capacity" C_v

Now, if we apply the first law assuming that we only heat the system, but also keep the volume constant:

$$du = \left(\frac{\partial u}{\partial T} \right)_v dT + \left(\frac{\partial u}{\partial v} \right)_T dv = dq + dw$$

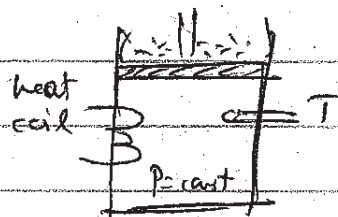


so: $du = dq = \left(\frac{\partial u}{\partial T} \right)_v dT$
 by definition c_v

Similarly, for $p = \text{const}$ process; if we define a function $h = u + pr$, then

$$dh = du + d(pr) = du + pdv + vdp \Rightarrow$$

$$\Rightarrow du = dh - pdv - vdp = dh - pdv$$



and the first law becomes:

$$du = dq \Rightarrow dh = dq + pdv$$

but, $dh = \left(\frac{\partial h}{\partial T} \right)_p dT + \left(\frac{\partial h}{\partial p} \right)_T dp$ (assume $h = h(p, T)$)

so: $dh = dq + pdv = \left(\frac{\partial h}{\partial T} \right)_p dT$

by definition c_p

enthalpy form of first law.

Q: Is C_v larger or smaller than C_p ?

A: $C_v < C_p$ (for monatomic ideal gases)
 $C_p = C_v + R$

Q: Why?

A:

Take a given amount of heat, δQ . No work (i.e. systems from previous page). Let's see what the increase in T can be for $V = \text{const}$ (process 1)

$$\delta Q = \Delta U = C_v \Delta T_1$$

$p = \text{const}$ (process 2)

$$\delta Q = \Delta H = C_p \Delta T_2$$

(but $H = U + pV$)

$\delta Q = \text{same}$

$$\Rightarrow C_v \Delta T_1 = C_p \Delta T_2 \quad (1)$$

$$\text{but } C_p \Delta T_2 = \Delta (U + pV)_2 = \Delta U_2 + p \Delta V \quad \Rightarrow$$

$\downarrow$
 $C_v \Delta T_2$

$\uparrow$
 const

$$\Rightarrow C_v \Delta T_1 = C_v \Delta T_2 + p \Delta V \Rightarrow C_v (\Delta T_1 - \Delta T_2) = p \Delta V$$

since $C_v, p \Delta V > 0$ then $\Delta T_1 - \Delta T_2 > 0 \Rightarrow$

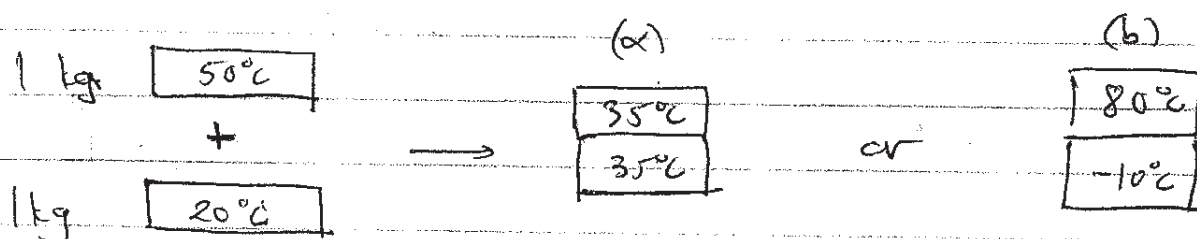
$\Delta T_1 > \Delta T_2$, but from equation ①,

$$\frac{C_v}{C_p} = \frac{\Delta T_2}{\Delta T_1}, \text{ so that means } \boxed{\frac{C_v}{C_p} < 1.}$$

i.e. the amount of heat you need to supply to raise the T of a closed system is smaller than for the cast system, because the heat goes towards work done by the system to raise the cylinder.

Second Law - Introduction.

Conservation of energy (1st law) is a necessary but not sufficient condition for a process to be feasible. There is a second law that tells us whether this is possible. Ex:



Both satisfy 1st law!
($\Delta U = \Delta Q$)

But only (a) makes sense.

We don't need an explicit law to figure out (b) won't happen, but for more complicated processes, we do. This is the 2nd law.