Thermodynamics of Earth systems

Lecture 2

Material covered in Lecture 2

Conclusion of Part 1: Introduction

- Hydrostatic equation: application to ocean and hypothetical constant density atmosphere; solid earth
- Hypsometric equation (atmosphere)

In-class worksheets

Part 2: Framework

First Law of thermodynamics

- Basic concepts & processes
- Work; expansion work
- Heat: heat capacity, basics of heat transfer mechanisms

Hydrostatic Balance

Although the horizontal atmosphere is in a constant state of motion, vertical velocities are fairly small (especially averaged over the large scale). Consequently, to understand the vertical structure of the atmosphere, we can approximate it to be largely steady.

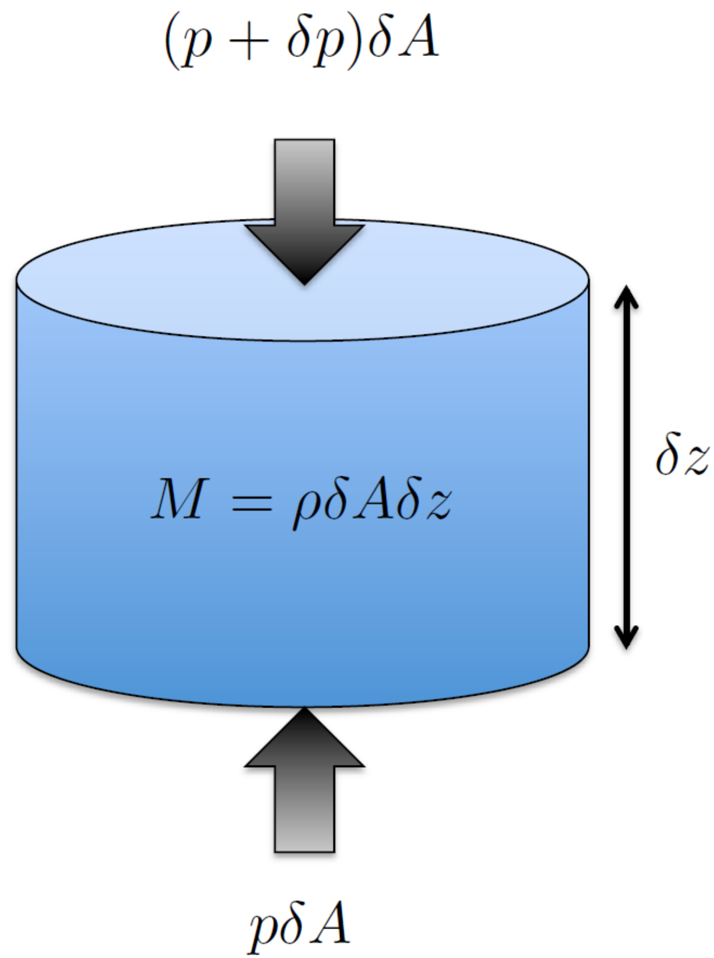


Figure: A vertical column of air of density ρ, horizontal cross-section δA, height δz and mass $M = \varrho\ \delta A\ \delta z$. The pressure at the lower surface is p, the pressure at the upper surface is $p + \delta p$.

Slides courtesy of Paul Ullrich (UC Davis)

Hydrostatic Balance

If the cylinder of air is not accelerating, it must be subject to zero net force. The vertical forces are:

Pressure acting on bottom face

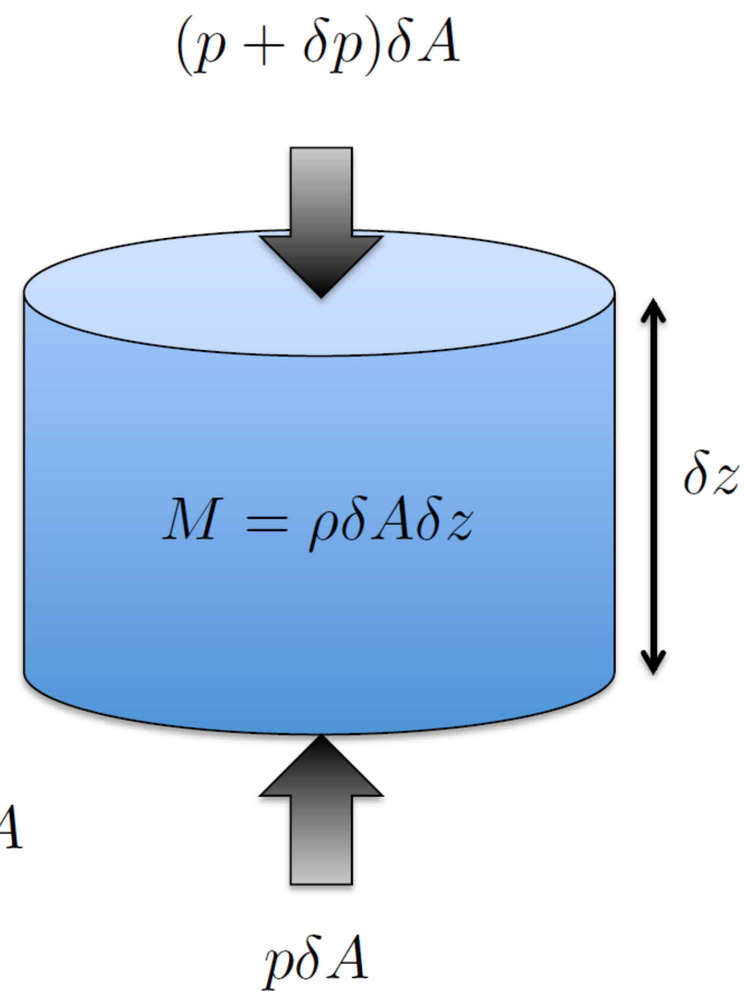
$$F_B = p\delta A$$

Gravity

$$F_g = -gM$$
$$= -g\rho\delta A\delta z$$

Pressure acting on top face

$$F_T = -(p + \delta p)\delta A$$



Slides courtesy of Paul Ullrich (UC Davis)

Hydrostatic Balance

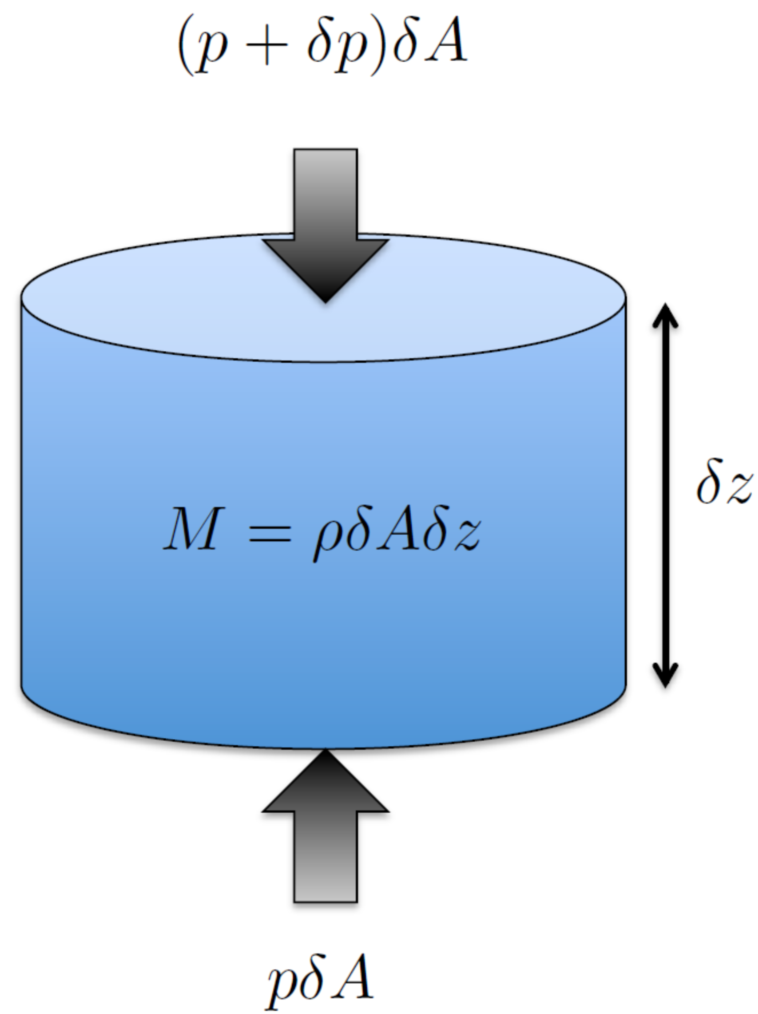
$$F_g + F_T + F_B = 0$$

$$\delta p + g\rho\delta z = 0$$

Taylor Series $\delta p \approx \dfrac{\partial p}{\partial z}\delta z$

Hydrostatic Balance

$$\frac{1}{\rho}\frac{\partial p}{\partial z} = -g$$



Slides courtesy of Paul Ullrich (UC Davis)

Aside: Consider the special case of an isothermal (constant temperature) atmosphere:

$$\frac{1}{\rho}\frac{\partial p}{\partial z} = -g$$

Hydrostatic Balance

This equation does not give pressure explicitly in terms of height, since the density of air is not known.

Ideal gas law

$$\rho = \frac{p}{R_d T}$$

$$\Rightarrow \quad \frac{\partial p}{\partial z} + \frac{pg}{RT} = 0$$

For an isothermal atmosphere $(T = T_0)$ this equation can be exactly solved:

$$\Rightarrow \quad p(z) = p_s \exp\left(-\frac{zg}{R_d T}\right)$$

← Exponential decay

Slides courtesy of Paul Ullrich (UC Davis)

Aside: Consider the special case of an isothermal (constant temperature) atmosphere:

$$\Rightarrow \quad p(z) = p_s \exp\left(-\frac{gz}{R_d T_0}\right)$$

Definition: The **scale height** of an isothermal atmosphere is given by:

$$H = \frac{R_d T_0}{g}$$

$$\Rightarrow \quad p(z) = p_s \exp\left(-\frac{z}{H}\right)$$

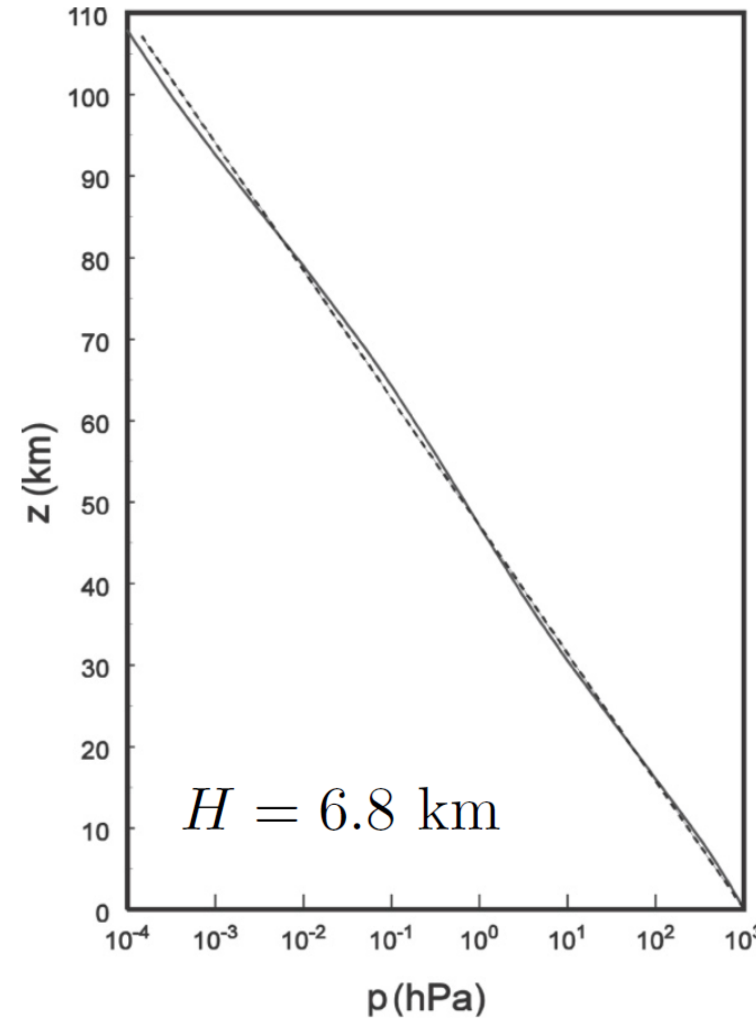
The **scale height** is an example of a quantity which imparts a notion of a "natural measuring stick" for an idealized atmosphere. This notion will generalize to more realistic atmospheric flows as well.

Aside: Consider the special case of an isothermal (constant temperature) atmosphere:

For an isothermal atmosphere $T = T_0$

$$p(z) = p_s \exp\left(-\frac{z}{H}\right)$$

Figure: Observed profile of pressure (solid) plotted against isothermal profile. Observed temperature variations only lead to small variations in the pressure from an exponential profile.



Slides courtesy of Paul Ullrich (UC Davis)

Geopotential

$$\nabla \Phi = g\mathbf{k}$$

In height coordinates, geopotential is purely a function of z

$$\Rightarrow \quad \frac{d\Phi}{dz} = g$$

Integrate $\Rightarrow$ $$\Phi(z) - \Phi(0) = \int_0^z g dz$$

Define $\Phi(0) = 0$ $\Rightarrow$ $$\Phi(z) = \int_0^z g dz = gz$$

Slides courtesy of Paul Ullrich (UC Davis)

The use of **geopotential on constant pressure surfaces** is analogous to the use of **pressure on constant height surfaces**.

Question: How are geopotential and pressure connected?

From $\frac{d\Phi}{dz} = g$ ⇨ $g dz = d\Phi$

From $\frac{dp}{dz} = -\rho g$ ⇨ $g dz = -\frac{dp}{\rho}$

↑ Hydrostatic balance

Ideal gas law $\rho = \frac{p}{R_d T}$

$$d\Phi = -\frac{R_d T dp}{p}$$

Slides courtesy of Paul Ullrich (UC Davis)

The use of **geopotential on constant pressure surfaces** is analogous to the use of **pressure on constant height surfaces**.

Question: How are geopotential and pressure connected?

From $\frac{d\Phi}{dz} = g$ → $g\,dz = d\Phi$

From $\frac{dp}{dz} = -\rho g$ → $g\,dz = -\frac{dp}{\rho}$

Hydrostatic balance

Ideal gas law $\rho = \frac{p}{R_d T}$

$$d\Phi = -\frac{R_d T\,dp}{p}$$

Slides courtesy of Paul Ullrich (UC Davis)

$$d\Phi = -\frac{R_d T dp}{p}$$

Recall: From elementary calculus, $\frac{dp}{p} = d \ln p$

$$d\Phi = -R_d T d \ln p$$

Integrate over a layer

$$\Phi(z_2) - \Phi(z_1) = -R_d \int_{p_1}^{p_2} T d \ln p$$

Use geopotential height

$$Z_2 - Z_1 = -\frac{R_d}{g} \int_{p_1}^{p_2} T d \ln p$$

$Z_2 - Z_1$ is the thickness of the layer bounded above by p_2 and below by p_1
This thickness is proportional to the temperature of the layer.

Slides courtesy of Paul Ullrich (UC Davis)

Hypsometric Equation

From hydrostatic balance and the ideal gas law we have

$$Z_2 - Z_1 = -\frac{R_d}{g}\int_{p_1}^{p_2} T\,d\ln p$$

If the temperature in a layer is constant then

$$h = Z_2 - Z_1 = \frac{R_d T}{g}\ln\left(\frac{p_1}{p_2}\right)$$

Hypsometric Equation

This is the relationship between layer thickness and temperature.

Worksheet time!

Download Worksheet 2 from the Moodle page and work on it.

You might need the PDFs of reading material from the Moodle page as well