ENV-413: Thermodynamics of the Earth systems

Hydrostaticsics and hypsometric equation

Compressibility and Expansion Coefficients (section 1.9)

6. The coefficient of thermal expansion is defined as

$$\alpha = -\rho^{-1}\left(\frac{\partial \rho}{\partial T}\right)_p$$

Evaluate the coefficient of thermal expansion for an ideal gas.

Hydrostatic balance

7. Hydrostatic balance is a balance between which two forces?

8. The apparent acceleration of gravity do the earth varies slightly with latitude and altitude. However, an average value of g is ___________

9. For the ocean, we can assume that density is constant with depth. Use the hydrostatic equation (1.37a) to estimate the ocean pressure (in bars) at 1 km depth (note: assume that the atmospheric pressure is 1 bar)

10. Consider a submarine with a gage pressure reading of 2×10^7 Pa. Using an ocean density value of 1025 kg m^{-3}, calculate the depth of the submarine below the surface (for g, use a value of 9.8 m s^{-2}. How many atmospheres does this ocean pressure correspond to?

Derivation of the scale height of the atmosphere, H:

1. Write the hydrostatic equation (in differential form)

2. (refer to p 28) In the real atmosphere, with density decreasing with height, there is no defined top of the atmosphere. However, if the atmosphere had constant density with height, then the atmosphere would have a finite depth, H.

a) Assuming a constant density for the atmosphere, integrate the hydrostatic equation from the surface to the top of the atmosphere.

Write an expression for H in terms of

b) p_o, the surface pressure

c) T_o, the surface temperature

3. Derive an expression for the lapse rate of the constant density atmosphere (by differentiating the ideal gas law with respect to z). Calculate this lapse rate. Compare it with the average lapse rate observed in the atmosphere.

6. The "thickness" of an atmospheric layer is z_2-z_1. Evaluate the thickness of a layer of atmosphere between 800 and 900 hPa with average temperature 300K and specific humidity 20 g kg^{-1}. Compare the thickness determined with the virtual temperature versus that determined without the virtual temperature correction.

Problem

Derive a formula for the dependence of density upon height in a hydrostatic atmosphere of constant lapse rate of temperature, Γ $(T=T_0 - \Gamma z)$

Given:

$$Hydrostatic\ Equation: \frac{\partial p}{\partial z} = -\rho g$$

$$Ideal\ Gas\ Law: p = \rho R T$$

$$Temp.\ Structure: T(z) = T_o - \Gamma z$$