

ENV-413: Thermodynamics of the Earth systems

Exercise session for Lecture 9

1. Colligative properties depend on the total number of particles dissolved in a solvent: sometimes this means keeping alert.... Estimate the freezing point of 100g of water containing 2 g of NaCl. (remember, K_b for water is 0.51 kg mol^{-1})

Handwritten notes on a chalkboard and digital screen for Exercise 1:

On the digital screen (left):

$$\Delta T = K_b \cdot X_{\text{salt}} = \dots$$

Below the screen:

$$X_{\text{salt}} = \frac{\text{mole salt}}{\text{mole salt} + \text{mole H}_2\text{O}}$$

$$X_{\text{salt}} = \frac{0.034}{5.034}$$

On the chalkboard (right):

$(\text{NaCl}) = (2 + 35.5) \text{ g/mole} = 57.5$

$$n_{\text{NaCl}} = \frac{2}{57.5} = 0.034 \text{ moles}$$

$$n_{\text{H}_2\text{O}} = \frac{100}{18} \approx 5$$

Moles of dissolved salt $\Rightarrow 2 \times n_{\text{NaCl}} \rightarrow \text{Na}^+ \rightarrow \text{Cl}^-$

concentrated solutions: dissociation is incomplete

dilute solutions: dissociation complete

$\text{NaCl} \rightarrow \text{max ions generated for them that}$

\hookrightarrow if dilute, \hookleftarrow concentrated.

Van't Hoff Factor

2. Common salt (NaCl) is spread on roads to prevent icing in the wintertime. The cost of salt is \$1/kg. A rich new source of CaCl_2 was discovered and mined at \$1.4/kg. Which salt is more cost-effective?

Handwritten notes on a chalkboard and digital screen for Exercise 2:

On the digital screen (left):

$$\Delta T_{\text{same}} \Rightarrow X_{\text{NaCl}} ; X_{\text{CaCl}_2}$$

$$\Delta T_{\text{NaCl}} = X_{\text{NaCl}} \cdot 2 \Rightarrow \frac{n_{\text{NaCl}} \cdot 2}{n_{\text{NaCl}} \cdot 2 + n_{\text{H}_2\text{O}}} = \frac{n_{\text{CaCl}_2} \cdot 3}{n_{\text{CaCl}_2} \cdot 3 + n_{\text{H}_2\text{O}}}$$

$$\Delta T_{\text{CaCl}_2} = X_{\text{CaCl}_2} \cdot 3$$

of ions when dissolved

On the chalkboard (right):

$$\frac{1 \text{ kg NaCl}}{1 \text{ kg H}_2\text{O}} \rightarrow \frac{n_{\text{NaCl}}}{n_{\text{H}_2\text{O}}} \xrightarrow{1} n_{\text{CaCl}_2} \text{ we need}$$

(n_{CaCl_2} that we get) (n_{NaCl} we have)

NaCl is cheaper still

1. From the following data, roughly sketch the p-T phase diagram for carbon dioxide: a) critical point at 31°C and 73 atm; b) triple point at -57°C and 5.3 atm; c) solid is denser than liquid at the triple point. Identify all regions and lines drawn. Mark on the diagram points which are examples of the following (use the letters A, B, C)

A. unsaturated vapor

B. a parcel containing liquid and solid in equilibrium

C. a parcel of saturated vapor and liquid in metastable equilibrium at $T < -57^\circ\text{C}$

2. Which phases of water are possible on Earth?

a) vapor b) liquid c) solid

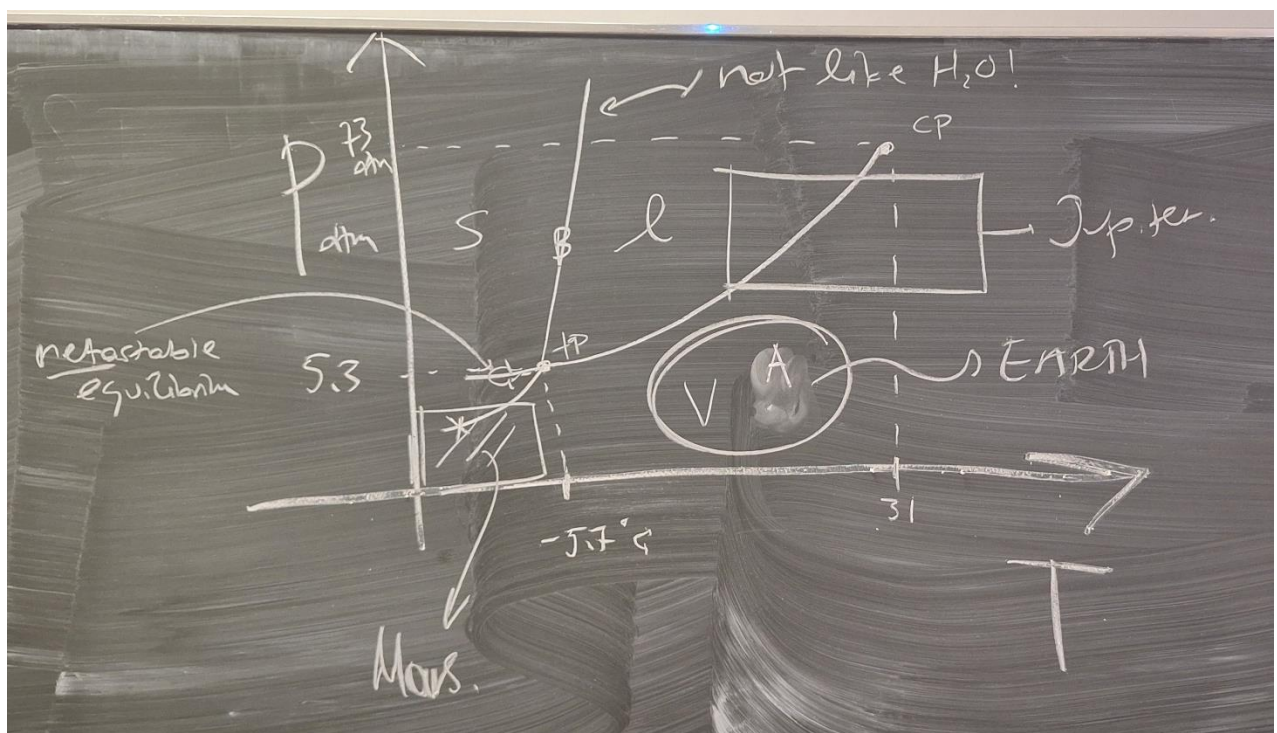
Which phases of CO_2 are possible on Earth?

a) vapor b) liquid c) solid

Which phases of CO_2 are possible on Mars (where $p \sim 1\text{ mbar}$)?

a) vapor b) liquid c) solid

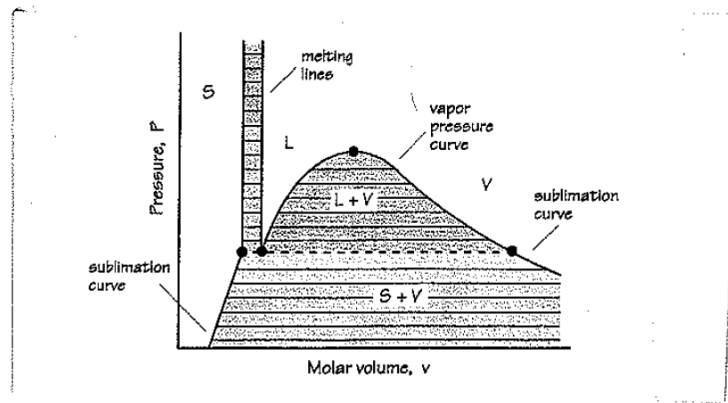
Solutions for 1,2



3. A phase diagram is given below in the p-v plane. Label the following on the diagram:
- Number of thermodynamic degrees of freedom associated with each region and line
 - label the triple line and the critical point

Mark on the diagram examples of each of the following

- unsaturated vapor
- liquid and vapor in equilibrium
- path of isothermal compression, starting from point X until system is completely liquid



4. Consider a two-component system consistent of H_2O and NaCl . Write the Gibbs phase rule for this two-component system.

5. To conveniently represent the phase diagram on a graph, we can eliminate one degree of freedom if we examine the system only at constant pressure. For the two-component system at constant pressure, what is the maximum number of thermodynamic degrees of freedom for this system?

