ENV-413: Thermodynamics of the Earth systems

Exercises 2: Introduction, Composition

1. NO_2 cannot be obtained in pure form in the gas phase because it exists as a mixture of NO_2 and N_2O_4. At 25°C and 0.98 atm, the density of this gas mixture is 2.7 g/L. What is the partial pressure of each gas?

$T = 25°C + 273.15 = 298.15\,K$

$R = 0.08206\ L\cdot atm/K\cdot mol$

$PM = \rho RT \rightarrow M = \frac{\rho RT}{P}$

$$M_T = \frac{(2.7\ g/L)(0.08206\ L\cdot atm/K\cdot mol)(298.15\ K)}{0.98\ atm} = 67.4\ g/mol$$

χ_{NO_2} ... mole fraction of NO_2 $\qquad M_{NO_2} = 46.01\ g/mol$

$\chi_{N_2O_4}$ ... mole fraction of N_2O_4 $\qquad M_{N_2O_4} = 92.01\ g/mol$

$$(\chi_{NO_2})M_{NO_2} + \underbrace{(1-\chi_{NO_2})}_{\chi_{N_2O_4}}M_{N_2O_4} = 67.4$$

$$\chi_{NO_2}(46.01) + (1-\chi_{NO_2})92.01 = 67.4$$

$$46.01\cdot\chi_{NO_2} + 92.01 - 92.01\cdot\chi_{NO_2} = 67.4$$

$$-46\cdot\chi_{NO_2} = -24.61$$

$$\chi_{NO_2} = 0.535$$

$$\therefore \chi_{N_2O_4} = 1 - 0.535 = \boxed{0.465}$$

$P_{NO_2} = \chi_{NO_2}\cdot P_T = (0.535)(0.98) = 0.52\ atm$

$P_{N_2O_4} = \chi_{N_2O_4}\cdot P_T = (0.465)(0.98) = 0.46\ atm.$

2. On Mars the atmosphee is mainly CO_2, the temperature is $220°K$. and the acceleration of gravity is $3.7\ m/s^2$. What is the scale height of the Martian atmosphere? Compare the Scale Height to the Earth's atmosphere and explain why their scale heights are different.

$$H_{Mars} = \frac{P_o}{\rho g} = \frac{R_d T_o}{g}$$

$= 11.234m$

$= \boxed{11.23\ km}$

$\therefore R_d = \frac{8.314\ kg \cdot m^2/s^2 \cdot mol \cdot K}{0.04401\ kg/mol} = 188.95\ \frac{m^2}{s^2 K}$

$M_{CO_2} \rightarrow 0.04401\ kg/mol$

$\therefore T = 220°C$

$\therefore g = 3.7\ m/s^2$

The Scale height of the E is approximately 8.5 km. This number is smaller compared to that of Mars, which is 11.23 km. The variables that determine H (scale height) are molar mass of the atmosphere, temperature and the gravity. For the temperature, the Earth's temperature and the gravity are higher than those of Mars. However, the average molar mass of the atmosphere differs due to the different atmospheric composition; Earth ~78 N_2 1.9% O_2 + Ar... Mars ~ 100% CO_2.

3. At what temperature do He atoms have to be cooled (or warmed) so that they have the same root mean square as O_2 @ 25°C?

$T = 25°C + 273.15 \approx 298K$.

O_2 $$V_{rms} = \sqrt{\frac{3RT}{M_{O_2}}} = \sqrt{\frac{3 \cdot (8.3145 \frac{kg \cdot m^2}{s^2 K \cdot mol}) (298\ K)}{32 \times 10^{-3}\ kg/mol}} = 482\ m/s$$

$T = ?$

He $$V_{rms} = \sqrt{\frac{3(8.314\ kgm^2/s^2 kmol)\ x}{4 \times 10^{-3}\ kg/mol}} = \sqrt{6235.5x} = 482$$

$6235.5x = 232324$

$x = 37.2\ K.$

4. Calculate the pressure in a column of sea water in atmosphere at depths below the surface of 10 and 1000 m. You can assume that the pressure @ the surface is 1 atm, and the density of sea water is 1.0 g/cm³.

$P = P_{atmosphere} + P_{fluid}$

10m

$P_{fluid}^{10m} = \rho g h = 1000\ kg/m^3 \times 9.81\ m/s^2 \times 10\ m = 98100\ kg/ms^2$

$= \boxed{98100\ Pa}$

$\rho = 1.0\ g/cm^3 \times \frac{1\ kg}{1000\ g} \times \frac{(100\ cm)^3}{m^3} = 1000\ kg/m^3$

$P_{10m} = (101325\ pa + 98100\ pa) \times \frac{1\ atm}{101325\ pa} = \boxed{1.9682\ atm}$

~~100m~~

$P_{fluid}^{100m} = \rho g h = 1000\ kg/m^3 \times 9.81\ m/s^2 \times 1000\ m = 9810000\ kg/ms^2$

$= 9810000\ Pa$

$P_{1000m} = (101325\ pa + 9810000\ pa) \times \frac{1\ atm}{101325\ pa} = \boxed{97.8172\ atm}$

5. A barometer - cross section area of 1.00 cm² @ sea level, 76 cm Hg, $\rho_{Hg} = 13.6\ g/cm^3$

$P_A = \rho_{Hg} g h = 13600\ \frac{kg}{m^3} \times 9.81\ m/s^2 \times 0.76\ m = 1.014 \times 10^5\ pa$

more accurate answer →

$= 101325\ pa$

$= 101325\ kg/ms^2$

surface area of the Earth $= 4\pi r^2$

$= 4\pi(6371000\ m)^2$

$= 5.1 \times 10^{14}\ m^2$

Total pressure $=$ (surface area) $\times$ (P_A) $= 5.1 \times 10^{14}\ m^2 \times 101325\ kg/ms^2$

$= 5.168 \times 10^{19}\ kg\,m/s^2$

mass $= \text{Pressure}/g = \frac{5.168 \times 10^{19}\ kg\,m/s^2}{9.81\ m/s^2} = \boxed{5.268 \times 10^{18}\ kg}$