

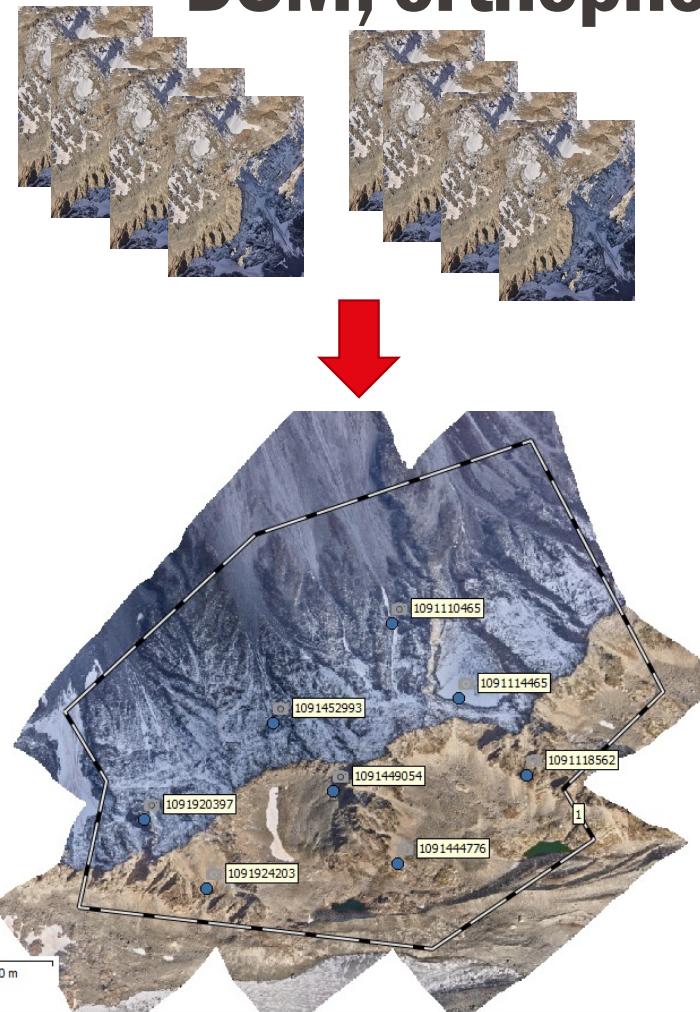
Lecture 5

Optimization & Mapping

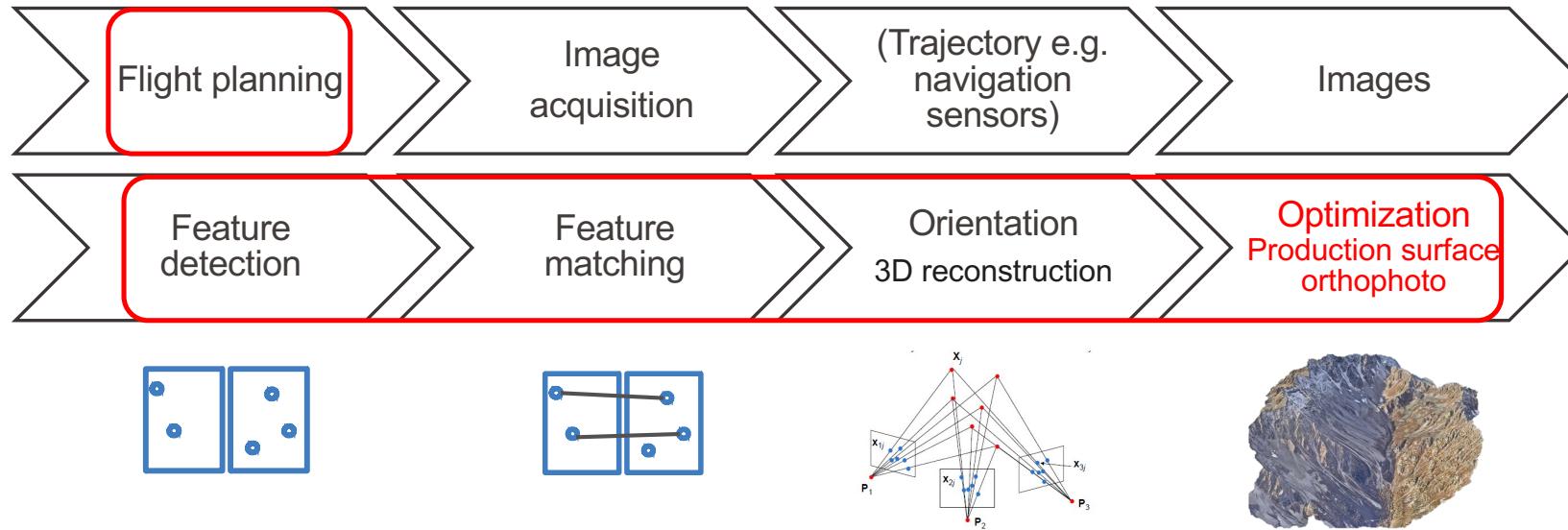
ENV408: Optical Sensing & Modeling for Earth Observations

Jan Skaloud

Ex 5 – Optimization, DSM, orthophoto



- Tomorrow: use **8** (possibly 45) neighboring photographs (to those used in Ex. 1,2,3,4) together with navigation-derived poses to obtain (via a professional software) a digital surface model with orthorectified image.
- Today: understand the concepts behind this process and the influencing parameters.



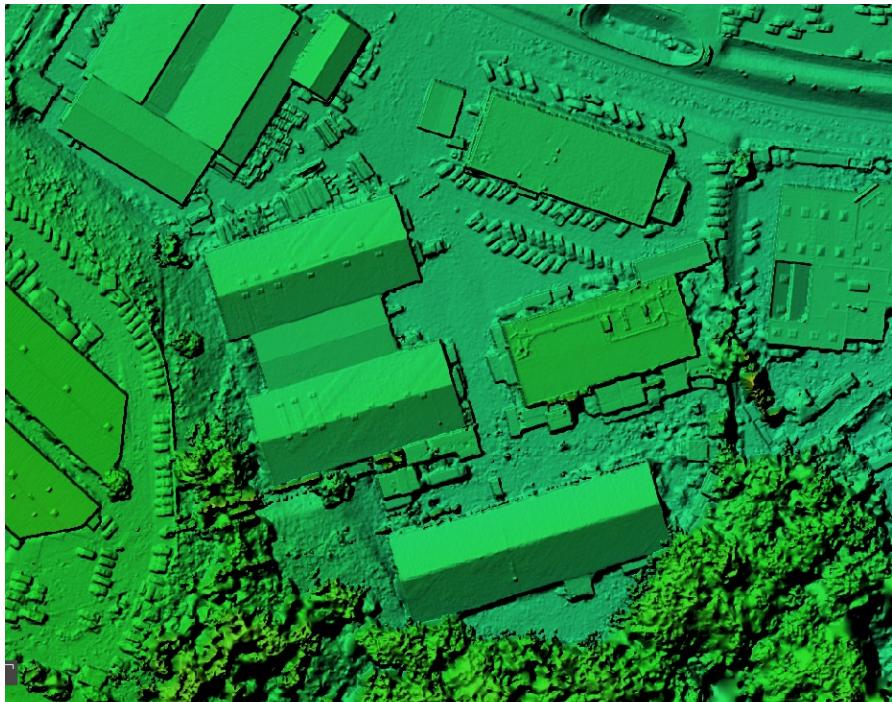
Lectures

- Image primes (L1)
- Salient *features* (L2)
- Image *orientation* (L3)
- Stereo vision (L4)
- **Optimization & Mapping (L5)**

Exercises

- Image ‘corrections’ (Lab01)
- Detection & matching (Lab02)
- Approx. absolute orientation (Lab03)
- Approx. relative orientation (Lab04)
- **Calibration, DEM, ortho-photo (Lab05)**

Main mapping products

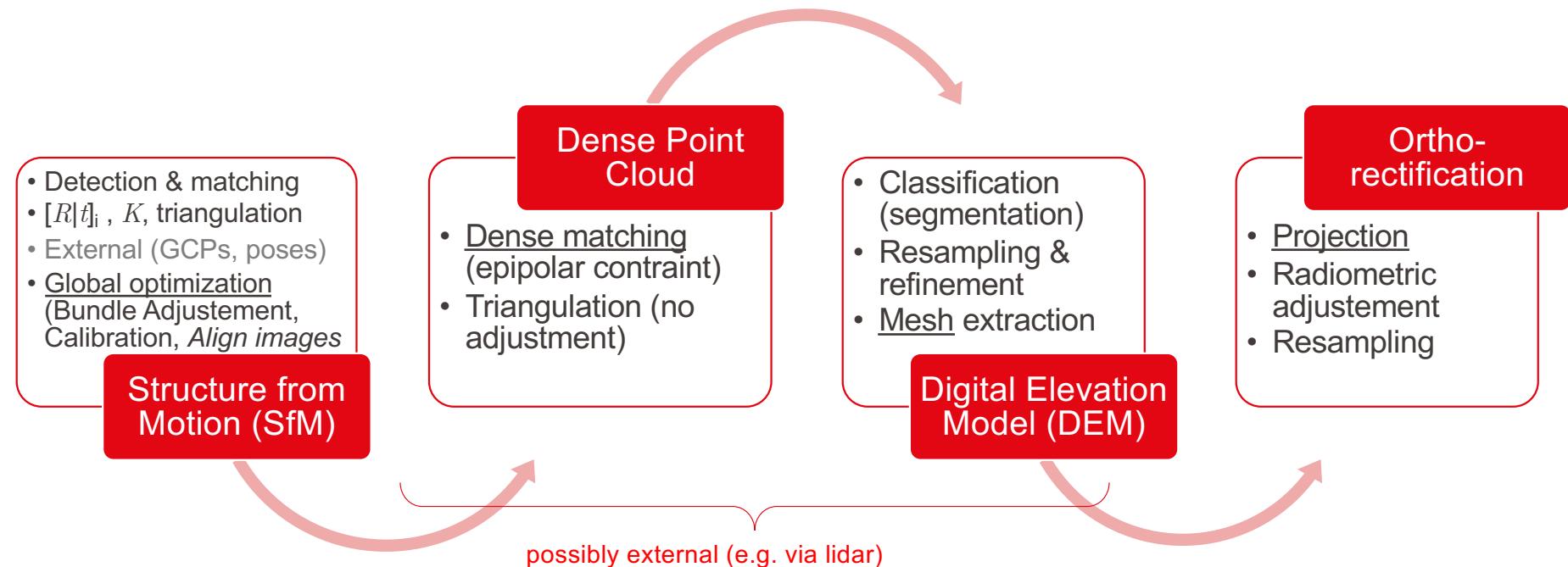


Digital elevation model (DEM)
colour represents a height above reference surface



Ortho-rectified image (needs DEM)
pixels correspond to Δx , Δy (m) on a projected surface

Main stages





Optimization

Dense matching / 3D point cloud

3D meshes and elevation models

Orthorectification

Appendix

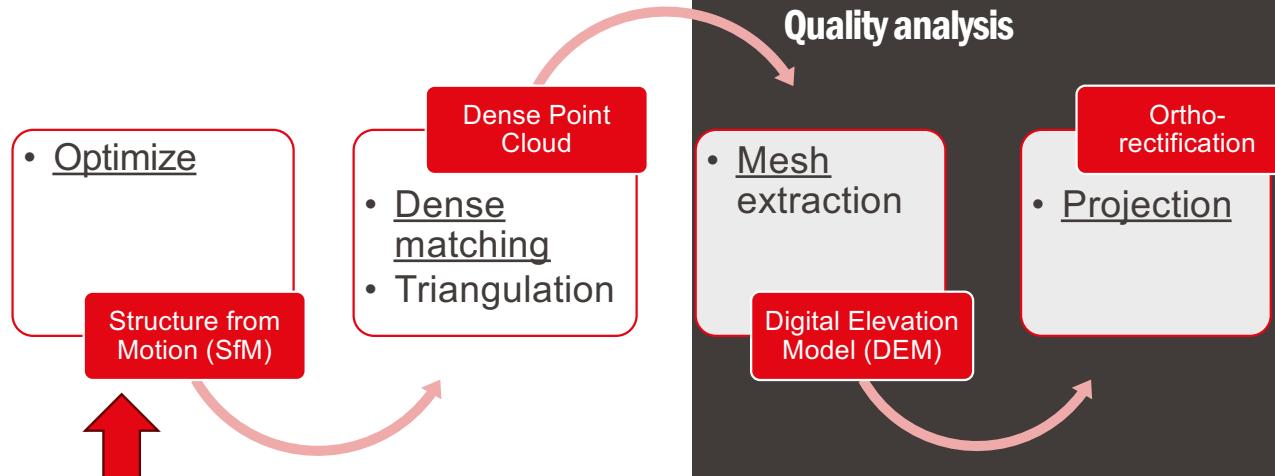
Global optimization

Incremental SfM

Effect of random errors

Additional observations

Quality analysis



Incremental SfM (Structure from Motion)

- I. Describe & match images (SIFT, Lab02), deduce connectivity matrix (which image matches which image)
- II. Initialize 3D model with suitable image pair (coplanarity, Lab04)
- III. Expand model to all images, iterate steps below:
 - a. Add connected image
 - b. Compute its pose from previously triangulated points (e.g. via DLT, Lab03)
 - c. Triangulate new points (coplanarity, Lab04)
 - d. Optimize (minimize reprojection error) via collinearity for all images

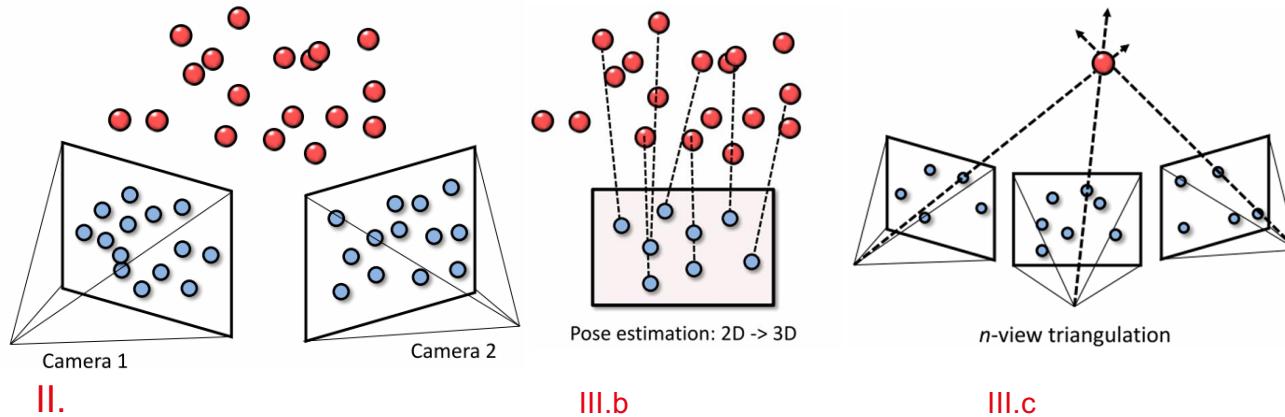


Image: P. Moulon

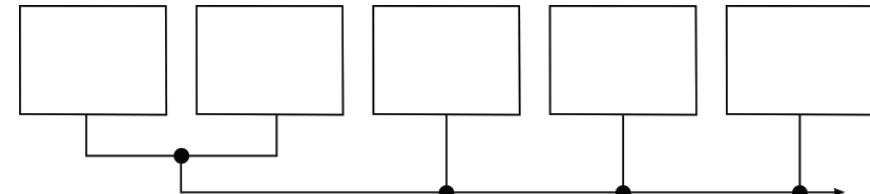
Scene reconstruction without scale*

Estimate essential matrix E (>4 points)

- Filter wrong matches (RANSAC, lecture 1.2) + extract $[R|t]$

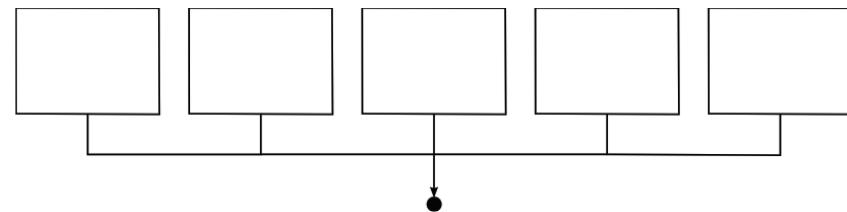
Sequentially (one image pair at a time)

- Good for unknown camera param, pose, poor overlap, etc.
- Initial step/approximation



Global (all)

- Good for scenes with known image distortions, large overlap, sufficient texture, etc.
- Final step



*Scale reconstruction: several possibilities: a. GCPs, b. pose via GPS(/INS), c. a+b (details later)

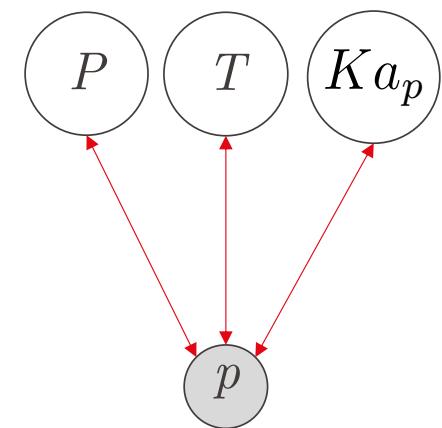
Scene reconstruction – global optimization

Goal:

increase reconstruction **accuracy**

- Refine: exterior $T = (R \mid t)$ & interior $(K \mid a_p)$ parameters + image coordinates p_{ij}
- How: using collinearity conditions involving all images j in a set J
- Better identification of extreme (faulty) observations

$(P_i, T_j = [R_j \mid t_j] \text{ in local-frame, no scale!})$



Optimization – starting point

With *minimum* inputs:

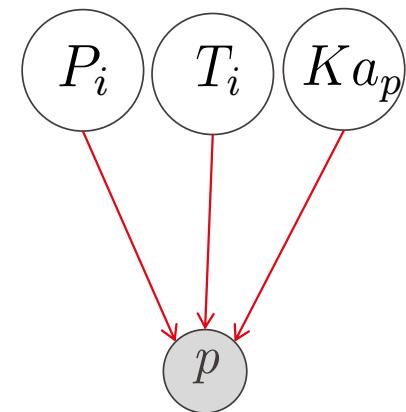
- Observed image coordinates p_i
- Keypoints 3D coordinates P_i
- Camera poses $T_j = [R|t]$
- Camera additional parameters (calibration) a_p

Via following observation model (collinearity equations)

$$p_{i,j} + v_{i,j} = \Pi(P_i, T_j, K, a_p)$$

With *optional* additional inputs:

- (possibly) observed point coordinates in object space (GCP) – later
- (possibly) observed pose (position and/or attitude) – later



Optimization – general

Minimize errors via maximum likelihood estimate of

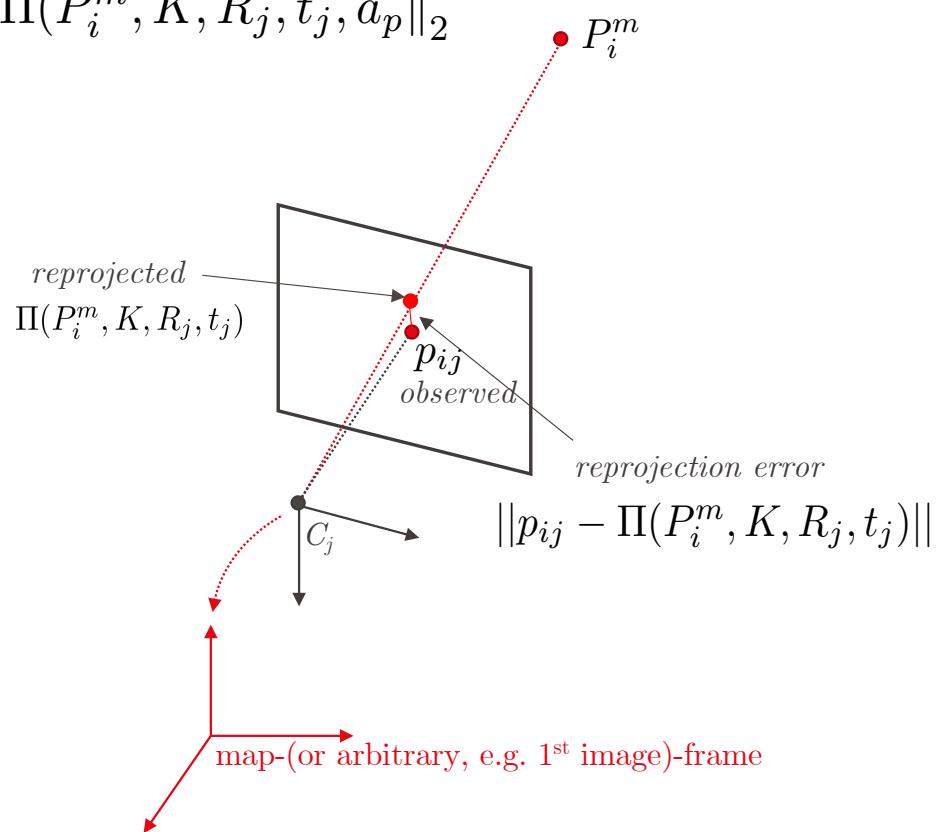
$$\operatorname{argmin}_{K, R, t, a_p} v = \operatorname{argmin}_{K, R, t, a_p} \sum_{i=1}^n \|p_{ij} - \Pi(P_i^m, K, R_j, t_j, a_p)\|_2$$

With:

- Parameters P_i^m, K, R_j, t_j, a_p
- Joint covariance matrix $C_{\ell\ell}$

How to minimize errors: least-squares

- For errors following (multivariate) normal distribution
- Choose P_i^m, K, R_j, t_j, a_p that minimize the sum of squared errors v weighted by the inverse of their uncertainty (covariance matrix) $C_{\ell\ell}^{-1}$



Optimization – input dependency

- Input
 - Image coordinates (key-points)
 - :
 - (possibly) observed points in object space (later)
 - (possibly) observed camera poses (partial or full - later)
 - :
 - approximated values of exterior or interior orientation & 3D point coordinates!
- Output
 - Follows the fundamental I/O principal

GARBAGE IN → GARBAGE OUT

- - Adjusted values of all observations + exterior and interior orientation & 3D points

Effect of random errors – precision of points

Image block (forward+side overlap)

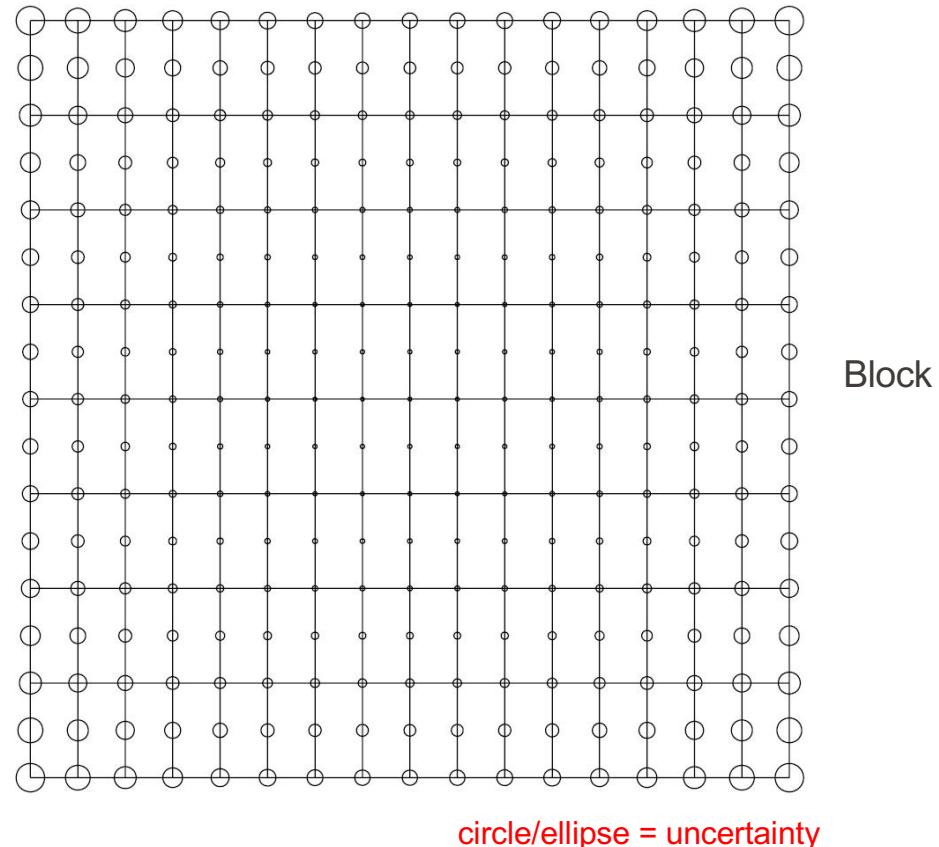
- Only image coordinates (p_i)
- Ideal (hypothetical) case

Prediction / covariance of points \mathbf{P}_i

P_i are part of the parameter space

$$C_{\hat{x}\hat{x}} = (A^T C_{\ell\ell}^{-1} A)^{-1}$$

$$A = \left(\frac{\partial \Pi(\cdot)}{\partial \mathbf{x}} \right)_{|(\ell, \mathbf{x}_0)}$$



after W. Förstner, UAV-g 2017

Effect of random errors in image observations

Image overlap forward + side (if in block*)

When no accurate prior knowledge on camera pose



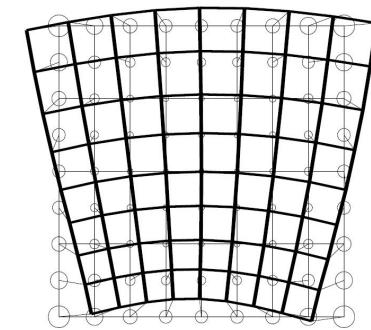
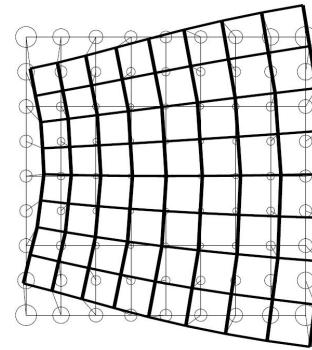
Possible deformation caused by accumulation of random errors



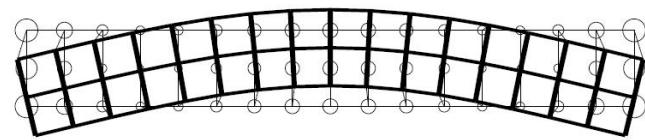
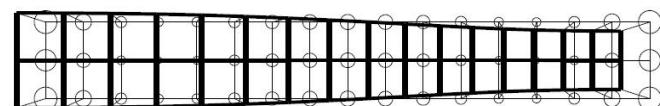
control is needed!

circle/ellipse = uncertainty

BLOCK



STRIP/LINE

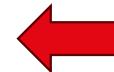
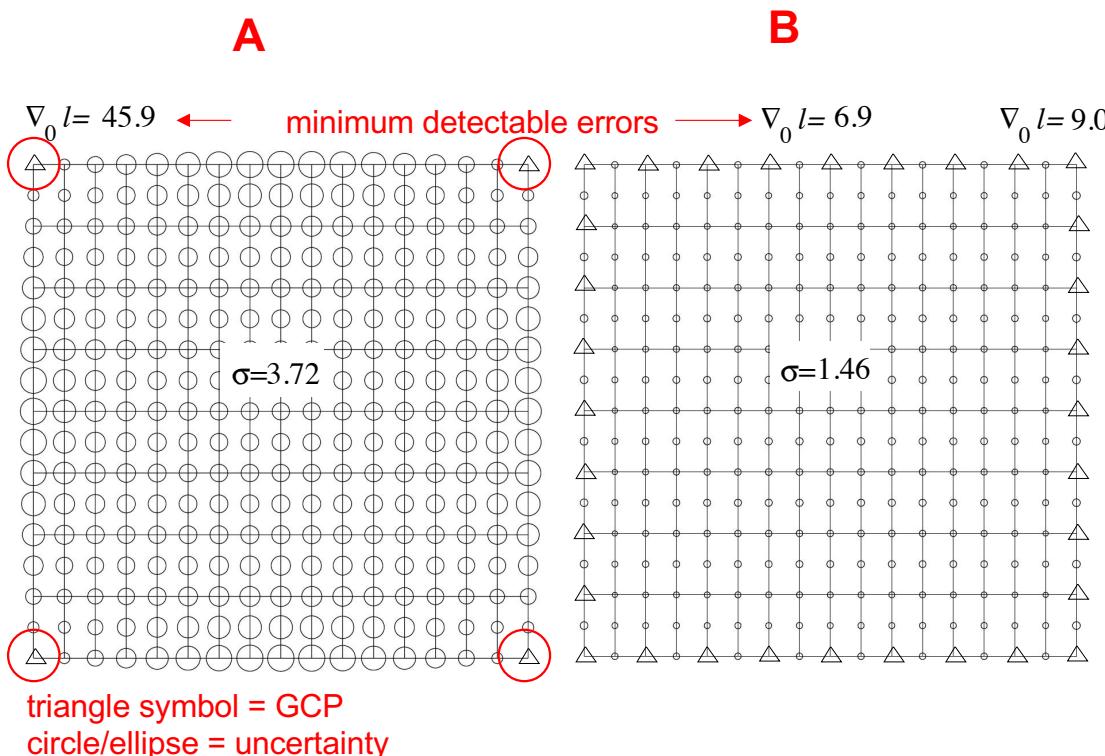


after W. Förstner, UAV-g 2017

*block is from several strips (e.g. flight lines) where images overlap also laterally/sideways

Improving quality

e.g. via ground control points (GCPs)



GCP spatial distribution:

- A. Only 4 GCPs at corners**
 - Unstable boundaries, 6 sigma
 - GCP outliers (large errors) are not detectable

- B. GCPs at boundaries**
 - Homogenous precision, < 2 sigma
 - Small GCP outliers not detectable

- Note: a better setup is with precise aerial position (+ attitude)
 - Camera poses observed
 - Fewer or no GCPs are needed! (Lab05)

- After W. Förstner, UAV-g 2017

Optimization – additional observations

Ground control points (GCPs)

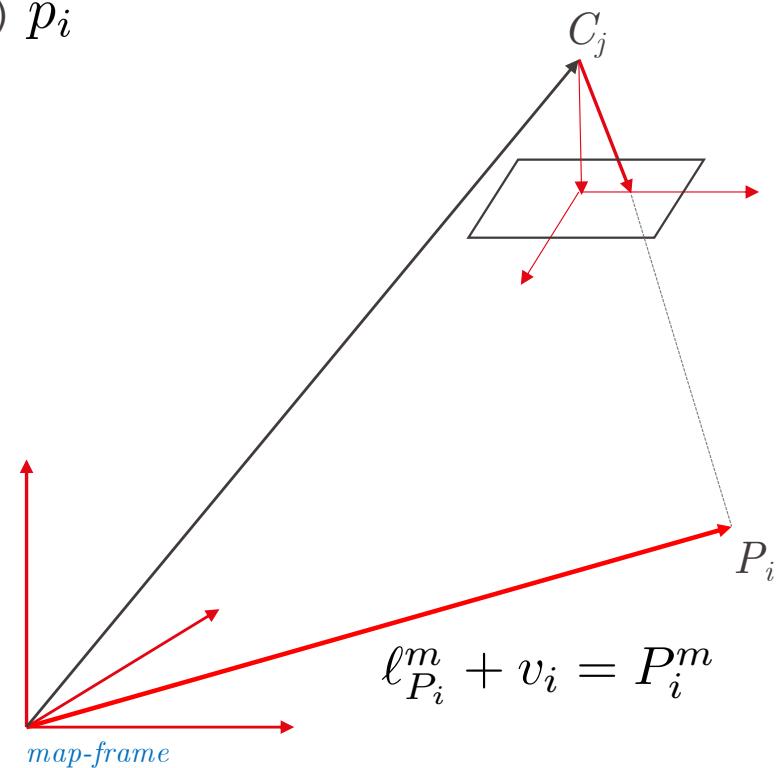
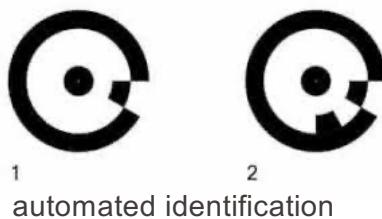
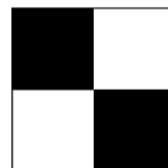
Points with known coordinates in mapping frame, measured on site P_i^m and known image coordinates (measured manually or semi-automatically) p_i

Using GPS/GNSS - RTK/PPK:

- 1.5-2 cm horizontal uncertainty
- 2.5-3 cm vertical uncertainty

Signalization

- Uncoded
- Coded



Optimization – additional observations

Sensor aerial position + orientation



GPS / GNSS receiver + inertial measurement unit (IMU)

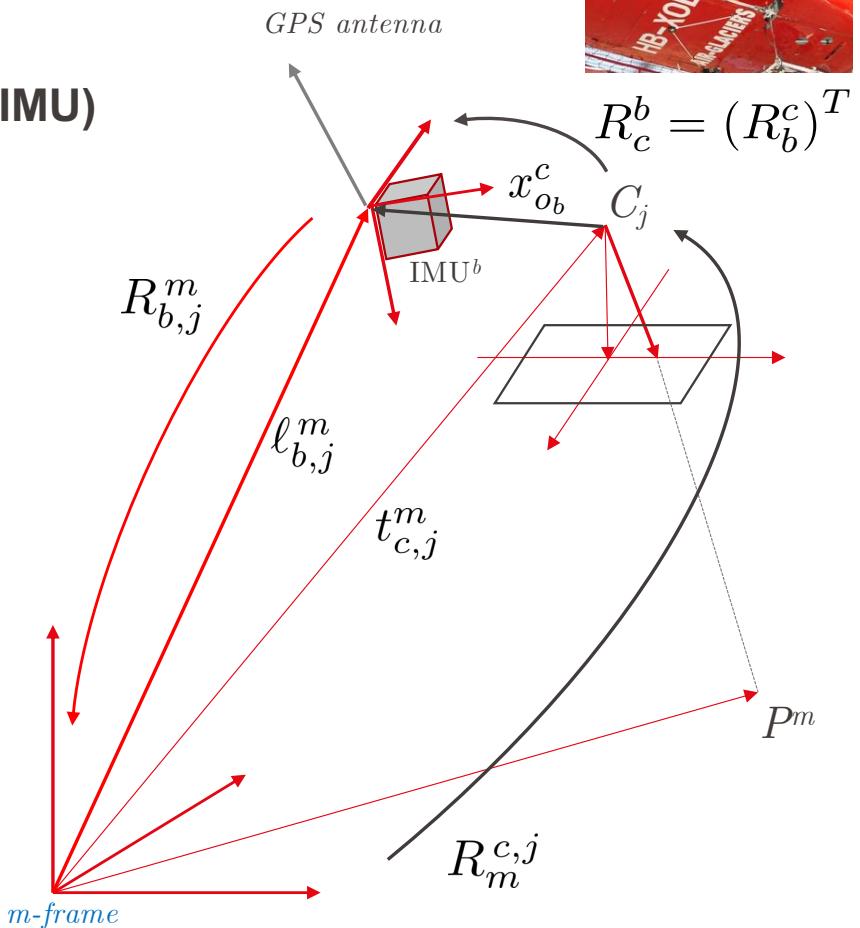
- IMU orientation, interpolated for $C_j : R_{b,j}^m$
- orientation offset (bore-sight): R_c^b
- IMU position, interpolated for $C_j : \ell_{b,j}^m$
- spatial offset to IMU origin (lever-arm): x_{ob}^c

Position observation (GPS only or GPS/IMU)

$$\ell_{b,j}^m + v_{b,j} = t_{c,j}^m + R_{c,j}^m x_{o_b}^c$$

Attitude observation (requires IMU)

$$\ell_{R_m^{b,j}} + v_{R_m^{b,j}} \equiv \left(R_{b,j}^m R_{v,j} \right)^T = \left(R_{c,j}^m R_b^c \right)^T$$



Quantify effect of random errors

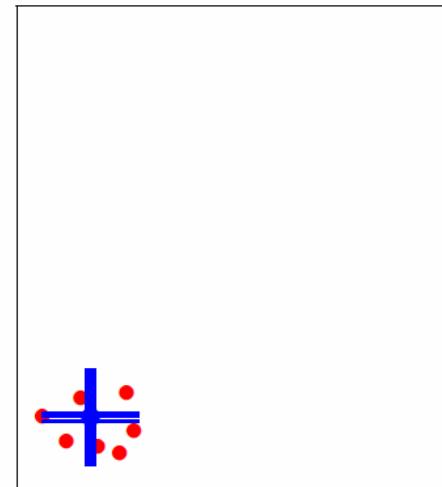
Internal measures

- Control **correctness**
- Needs to be checked

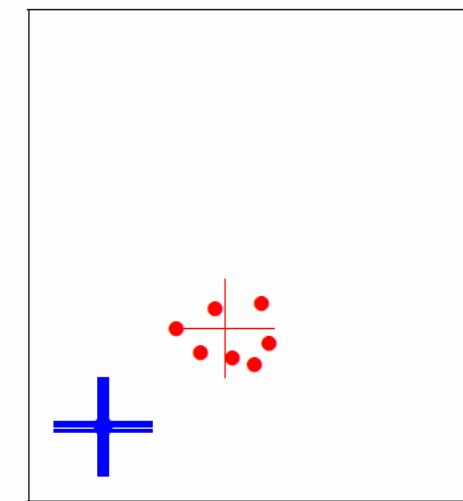
External measures

- Control **sufficiency**
- Needs to be sufficient

noise



noise + bias



Quality metrics example

for a drone flight

A. Internal

- Discordance between observations
- Size & distribution of residuals vs. estimated precision

B. External

- comparison with ground truth, e.g. $< 2 \times \text{GSD}$ (ground sampling density)
- e.g. using reference values, points (**check points** = coordinates in object space observed, but not used in optimization)

Metric - Mean square error (MSE) of:

- reprojection error (all image observations)
- aerial pose (position & attitude)
- check points (in object space, metric)
- check points (in image space, pixels)

Example – summary

Number of images:	440
Flying altitude:	155 m
Ground resolution:	2.66 cm/pix
Coverage area:	0.547 km ²
Camera stations:	440
Tie points:	381,168
Projections:	1,698,383
Reprojection error:	0.445 pix

Quality report example

Camera calibration

Type **Frame** Resolution 7920 x 6004 Focal Length 35 mm eq. Pixel Size 4.6 10E-6 m

	Value	Error	F	Cx	Cy	B1	B2	K1	K2	K3	P1	P2
C	5607.25	0.018	1.00	0.00	0.04	-0.13	0.01	-0.35	0.32	-0.29	0.01	-0.00
Cx	30.2154	0.021		1.00	-0.01	0.02	-0.09	-0.00	-0.00	0.00	0.82	-0.01
Cy	52.4225	0.018			1.00	0.14	0.04	0.01	-0.00	0.00	-0.00	0.66
B1	-0.0246013	0.003				1.00	-0.00	0.04	-0.06	0.07	-0.02	-0.08
B2	0.120007	0.003					1.00	-0.00	0.00	-0.00	0.08	0.00
K1	0.0161593	3e-005						1.00	-0.97	0.92	0.00	0.02
K2	-0.0621043	0.00022							1.00	-0.99	-0.00	-0.01
K3	0.063624	0.00048								1.00	0.01	0.01
P1	-0.000934223	1.2e-006									1.00	0.01
P2	0.00104344	9e-007										1.00

Table 2. Calibration coefficients and correlation matrix.

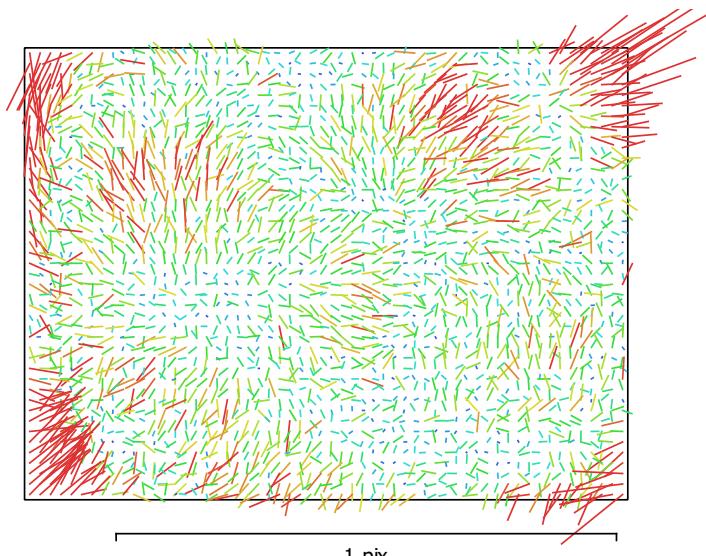


Fig. 2. Image residuals for unknown.

Quality report example

Camera location

internal metric - prediction

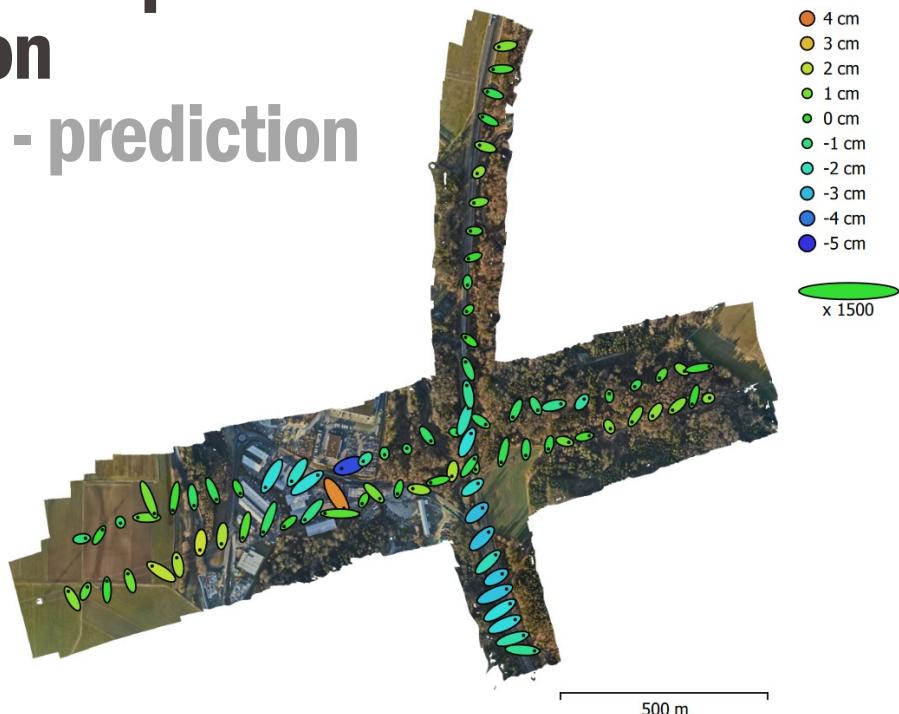


Fig. 3. Camera locations and error estimates.

X error (cm)	Y error (cm)	Z error (cm)	XY error (cm)	Total error (cm)
1.7	1.8	1.4	2.5	2.9

Table 3. Average camera location error.

X - Easting, Y - Northing, Z - Altitude.

Quality report example

Check* points

external metric evaluation

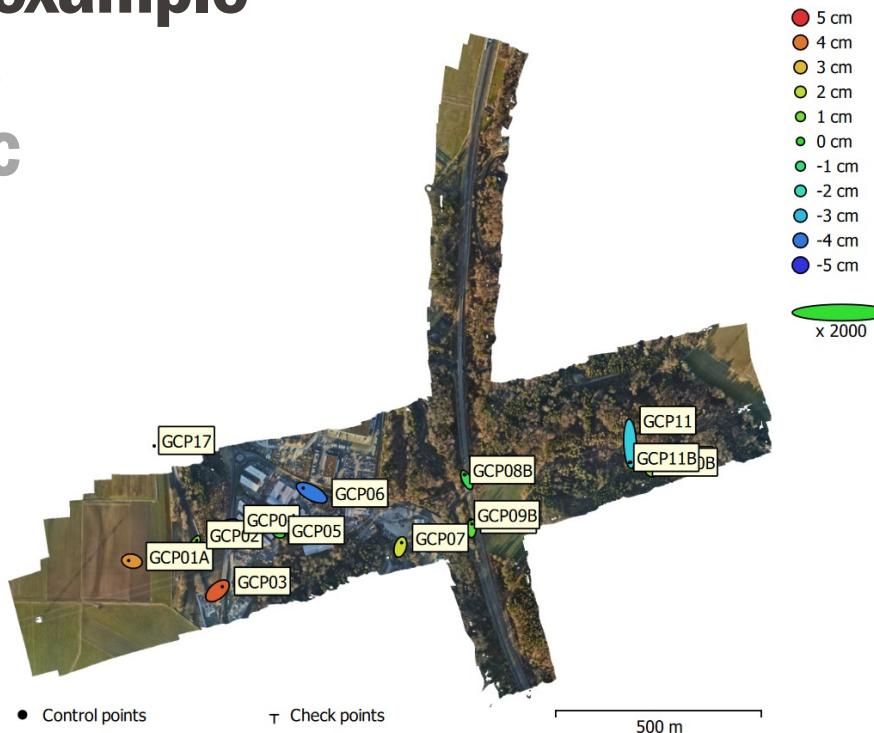


Fig. 5. GCP locations and error estimates.

Count	X error (cm)	Y error (cm)	Z error (cm)	XY error (cm)	Total (cm)
11	0.9	1.7	2.5	1.9	3.18

Table 5. Control points RMSE.
X - Easting, Y - Northing, Z - Altitude.

*check points

are GCP that are used as tie-points, i) position of which is determined via photos & evaluated against known position.

(i.e. their known object coordinates are NOT used to constrain optimizations)

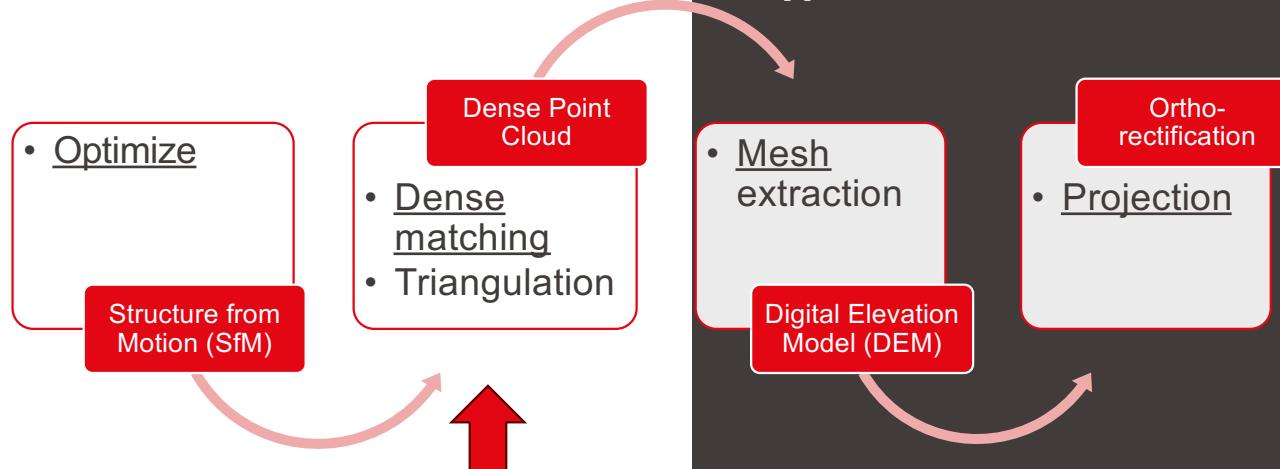
What are the principal factors influencing precision/accuracy?
→ Appendix II.

Dense matching / 3D point cloud

3D meshes and elevation models

Orthorectification

Appendix



3D point cloud from multi-view stereo example



Potential accuracy of 3D point cloud 0.5-2 pixels (at [image scale](#)) so 0.5- 2x GSD.
Challenges: occlusion (see Appendix IV.), semi-transparent structures, homogenous texture.

High-density image matching stages

Required

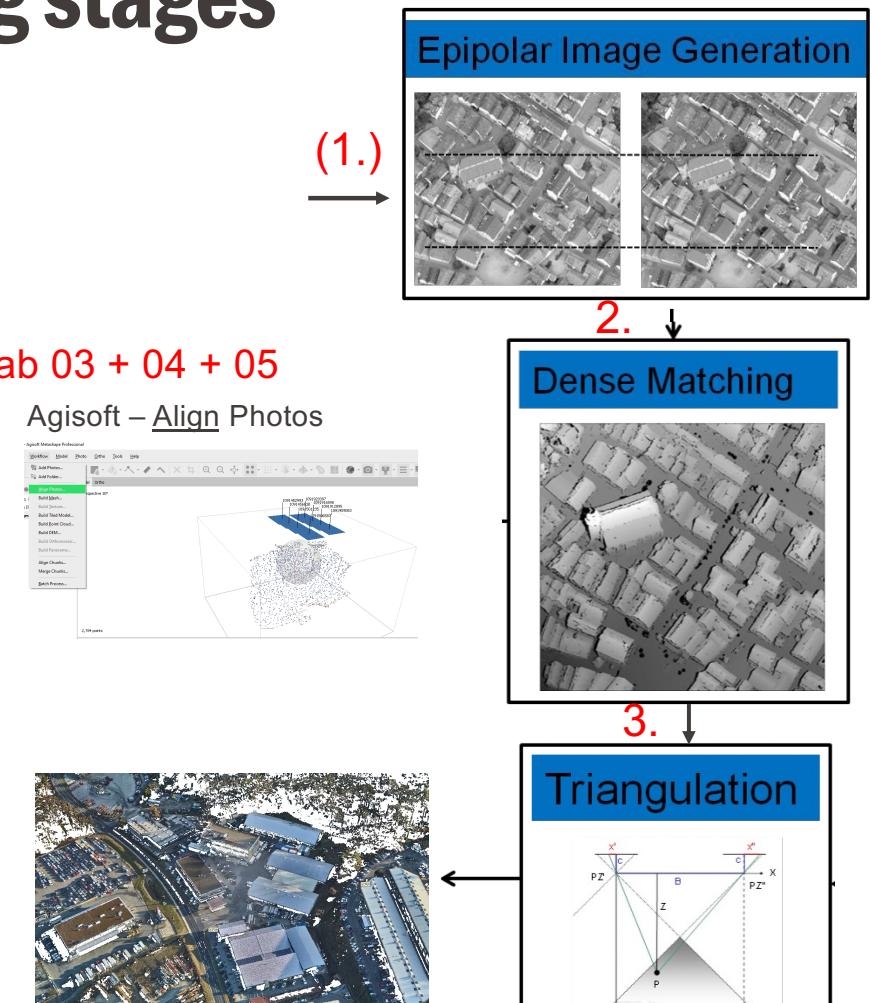
Image orientation parameters (+global optimization), **Lab 03 + 04 + 05**

Steps

1. Epipolar image generation (optional, appendix III.)
i.e. image transformation for simplified matching
2. Dense matching
disparity (per pixel) along epipolar lines
3. Triangulation
we know how, Lab 04

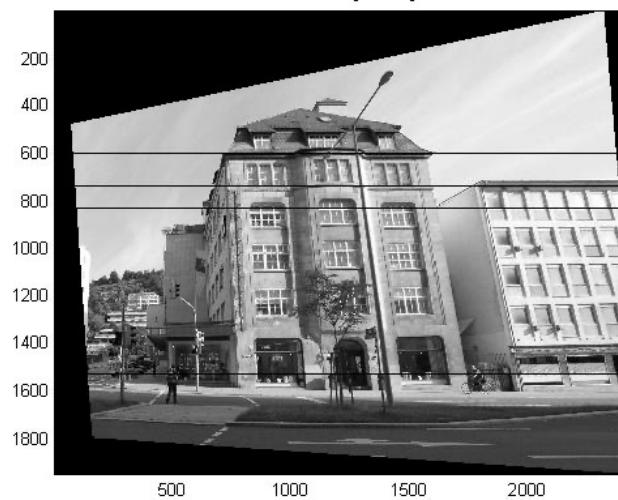
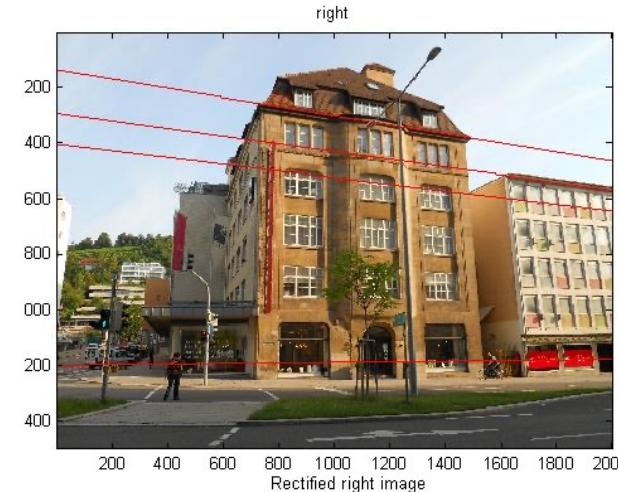
Further process

Digital elevation model – see below



Source: N. Haala, EuroSDR 2019

Epipolar lines = **rows** in the epipolar images



What it does ?

- 2D Transformation
- Epipolar lines = rows

Why doing that ?

- Simplification for dense matching
- Region algorithm algo

How ?

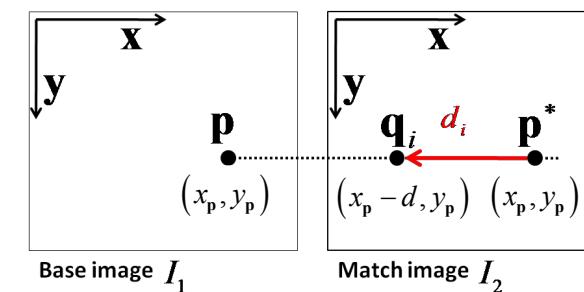
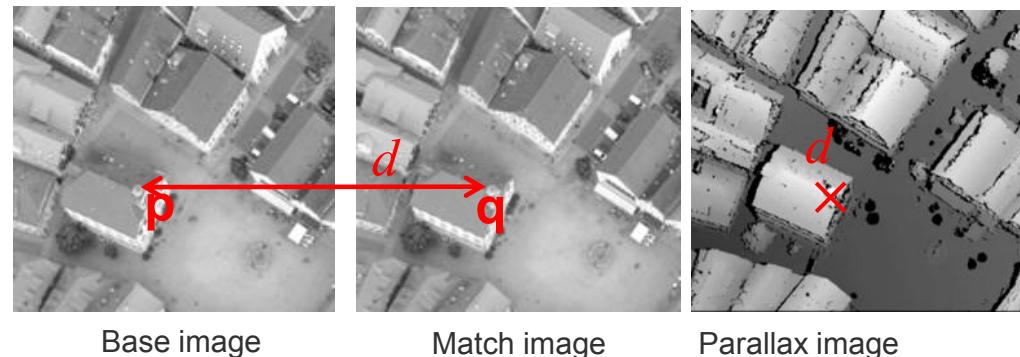
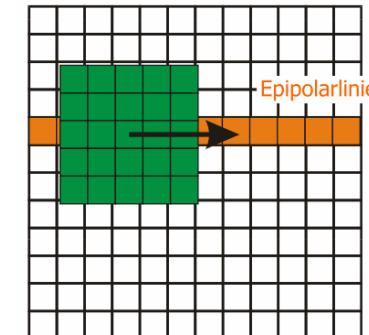
- 5 slides on “epipolar image generation” are in Appendix III.

Source: N. Haala, EuroSDR 2019

Dense pixel matching

Epipolar images: pairs of epipolar lines are parallel to rows

- This simplifies the matching to 1D search
- Correspondences per each pixel can be stored in disparity (parallax) images (shade = distance)



Source: N. Haala, EuroSDR 2019

Window based matching

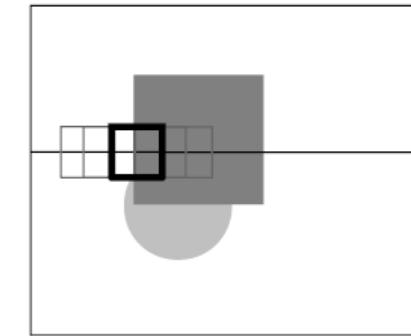
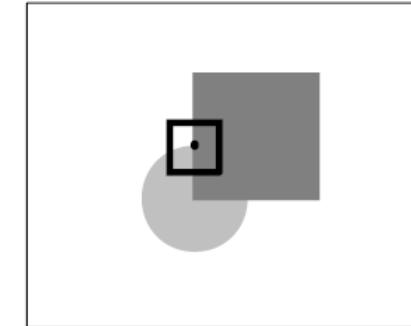
Challenge: Homogenous texture results in **matching ambiguity** (same gray values for many *different* object points)

Goal: Increase robustness of similarity measure by comparing matching windows (correlation).

Condition: implies constant disparity (depth) within the mask

Side effects: Blurred edges, loss of resolution (small objects)

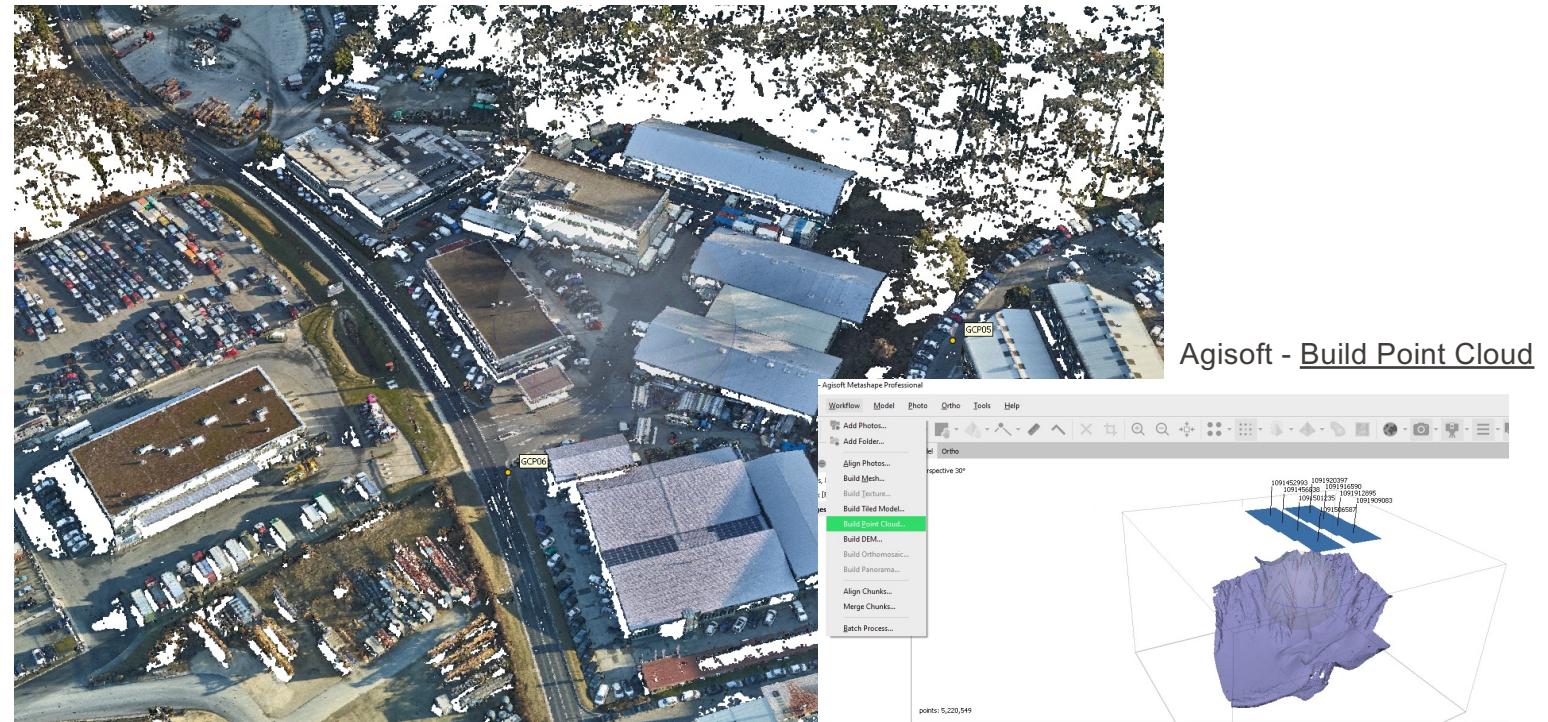
Other approaches: hierarchical stereo matching, segmentation by Markov fields, graph cut labeling, multi-stereo (across several images) matching, deep learning ...



- Source: Hirschmuller, Heiko 2005, Accurate and efficient stereo processing by semi global matching and mutual information.

Dense matching + triangulation = 3D cloud / image pair

Union of stereo pairs: combined 3D cloud (in mapping frame)



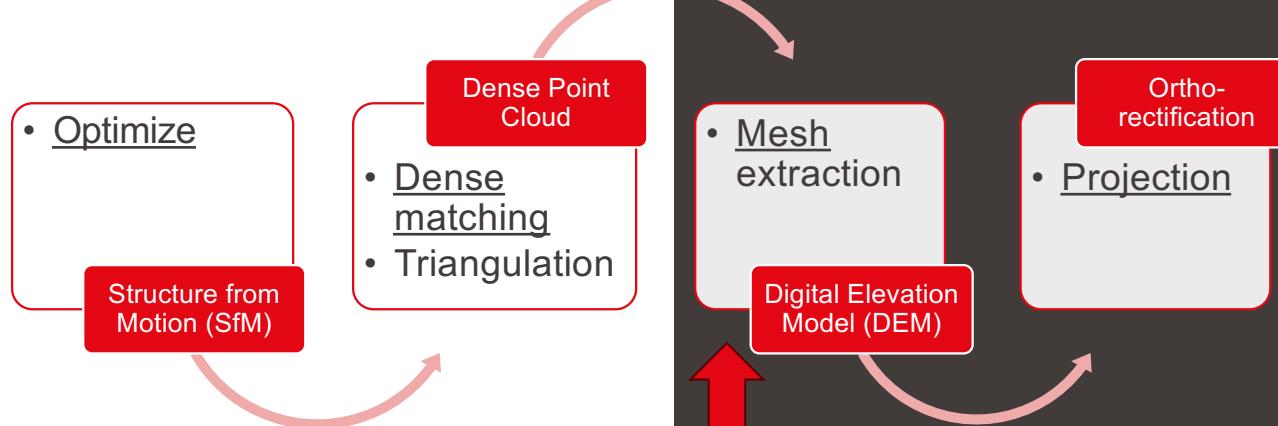
Optimization

Dense matching / 3D point cloud

3D meshes and elevation models

Orthorectification

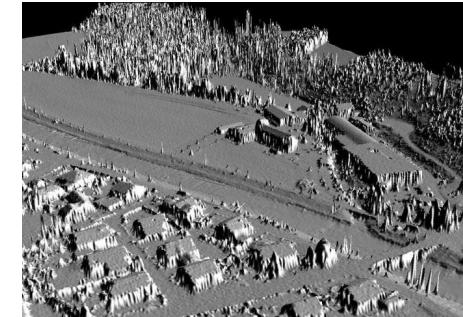
Appendix



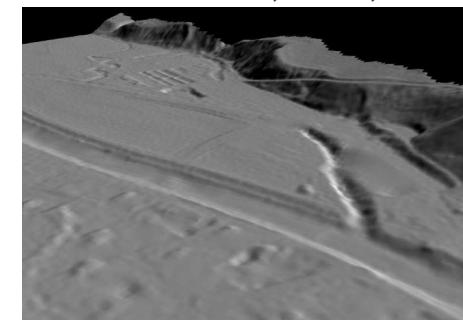
Types of digital elevation models (DEM)

Digital **surface** model (DSM)

Representation of Earths surface **including** buildings and vegetation



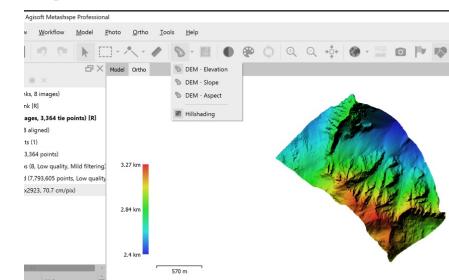
© Ressl, TU Wien, 2007



Digital **terrain** model (DTM)

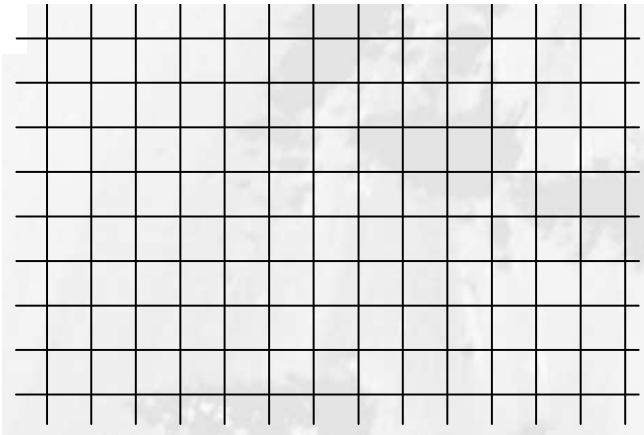
Representation of Earth surface **without** buildings and vegetation

Agisoft – Build DEM



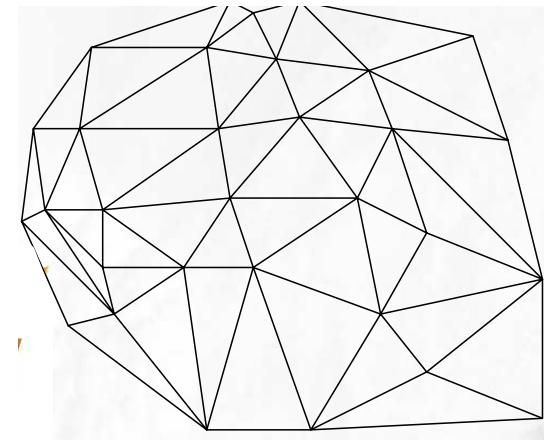
Raster (2D vector)

- ✓ easy to use (format)
- ✓ Once upper-left and lower-right corners only defined with spacing, only sequence of height needed (as an image)
- ✓ Choice for national models at different resolutions



Triangulated irregular network (TIN)

- ✓ More efficient in representing varying levels of details
- ✓ Preserves break-lines
- ✓ Can be extended to 3D



Point cloud organisation

explicit surface representation priors

Properties

- Massive point clouds ($>10^9$ points)
- Not all can be in computer memory
- Not all needs to be treated together

Strategies

- Sub-sample
- Divide into blocks (**Tiles**)
- Efficient spatial data structure (**K-d Tree**)

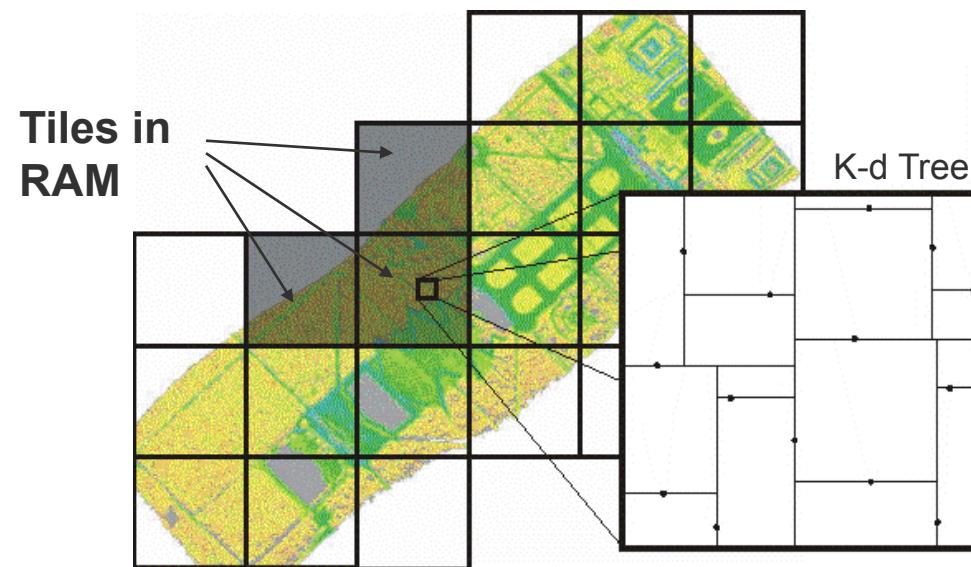


Image credit: Haala, 2019

3D Meshing usage

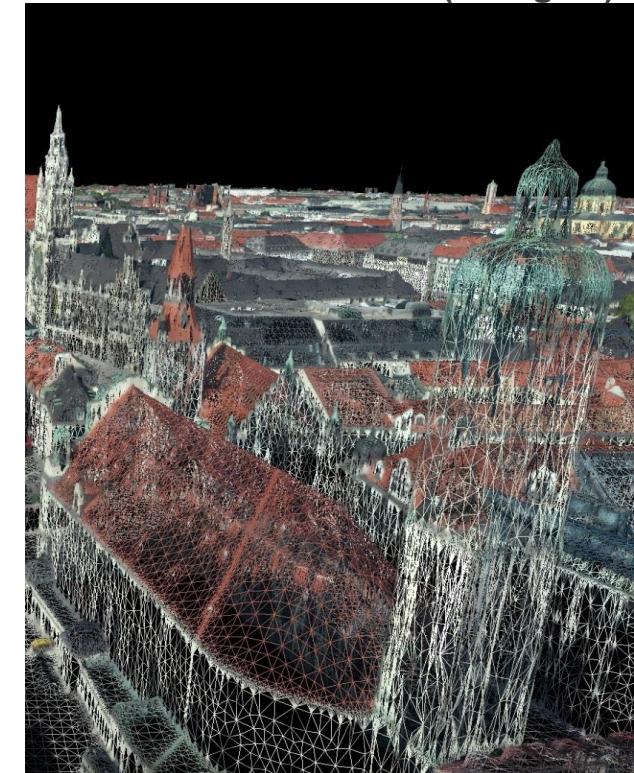
3D model



3D model with texture

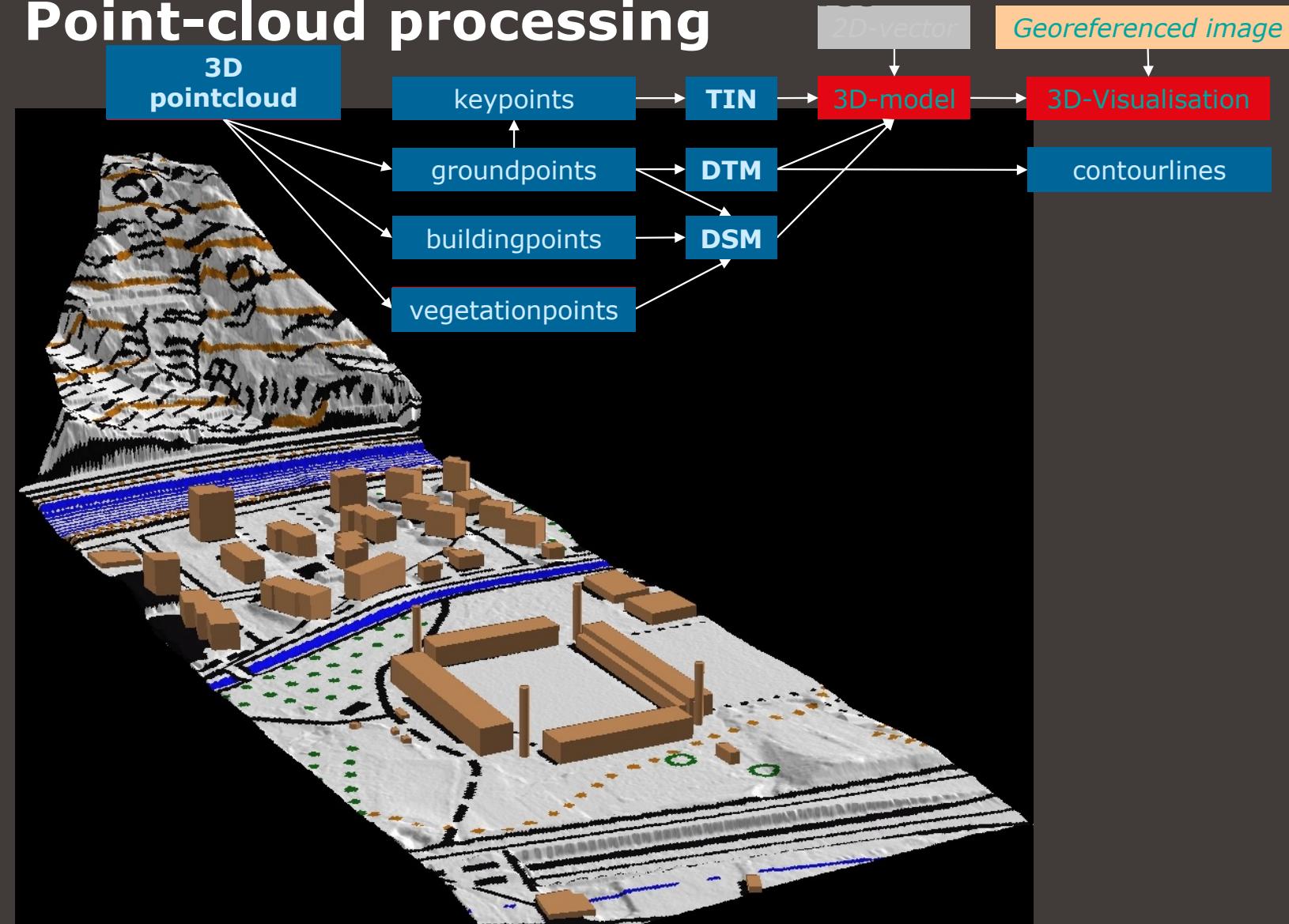


3D model as a 3D mesh (triangles)



Source: N. Haala, EuroSDR 2019

Point-cloud processing



Triangle meshes

explicit surface representation

Consists of geometric and topological component

- Set of vertices
- Set of triangular faces connecting them (often represented as *edges of a graph*)

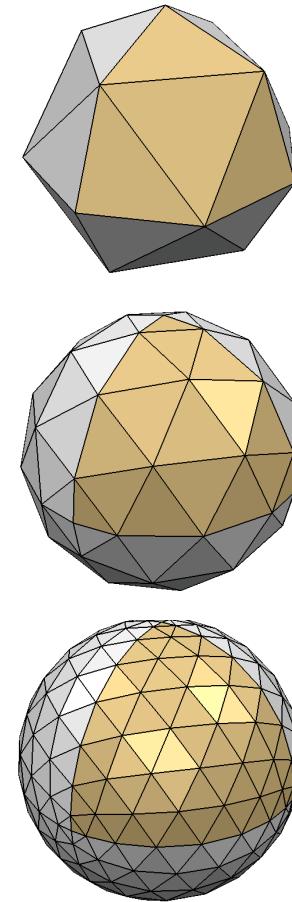
Continuous piecewise linear surface

- Approximation error of order $O(h^2)$, $h = \text{max. edge length}$
- Error reduced by $\frac{1}{4}$ for $\frac{1}{2} h$

Properties

- No. triangles = 2x No. of vertices
- No. edges = 3 x No. vertices
- Average No. of incident edges* = 6

*incident edges are sharing common vertex



Botsch, et al. 2006

Data structures

- Face Vertex List - direct edges (straight forward implementation, complex processing)¹
- Doubly Connected Edge List (DCEL) – half edges (processing efficient, enforces manifold structure, implementation complex)¹

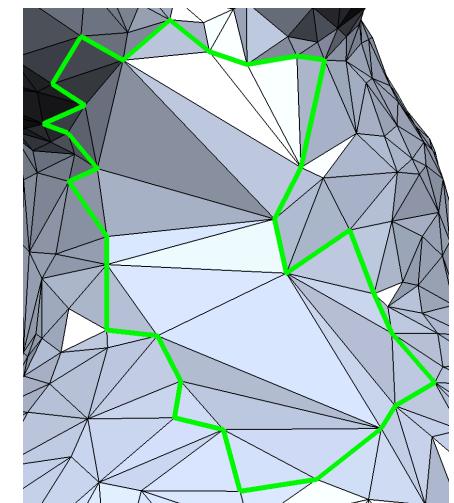
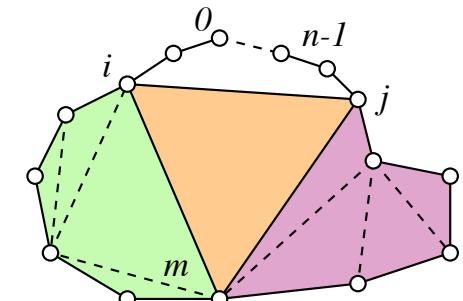
Mesh libraries

- [CGAL](#) (Computational Geometry Algorithms Library) – robust, efficient, scalable
- [OpenMesh](#) – efficient for processing based on halfedge data structures

Computation

- Purpose: visualisation, 2.5D models, 3D models, quality, breaklines, smoothness, level of detail, topology (clustering)
- Implies filtering, editing, corrections ...

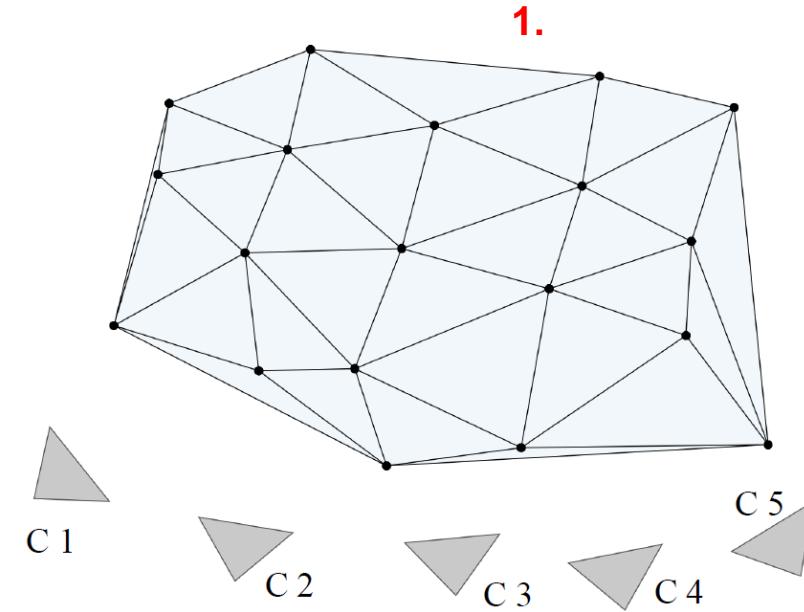
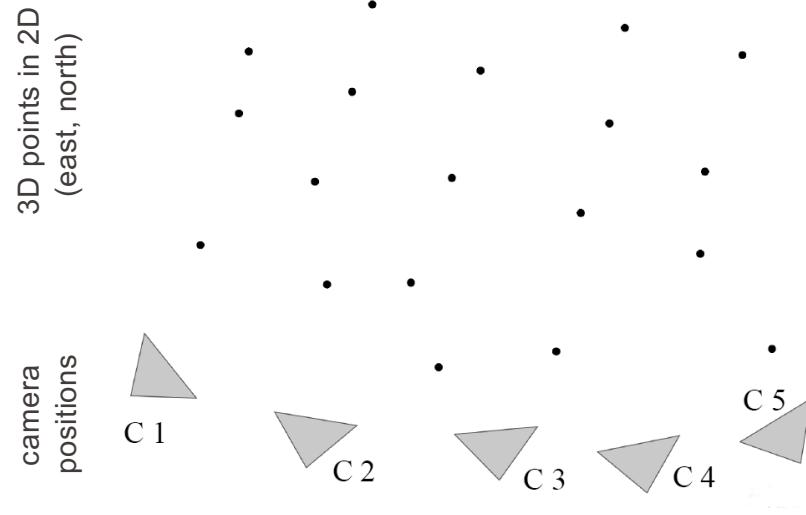
[1] – <https://observablehq.com/@2talltim/mesh-data-structures-traversal>



Goal: minimizes normal variation and total area.

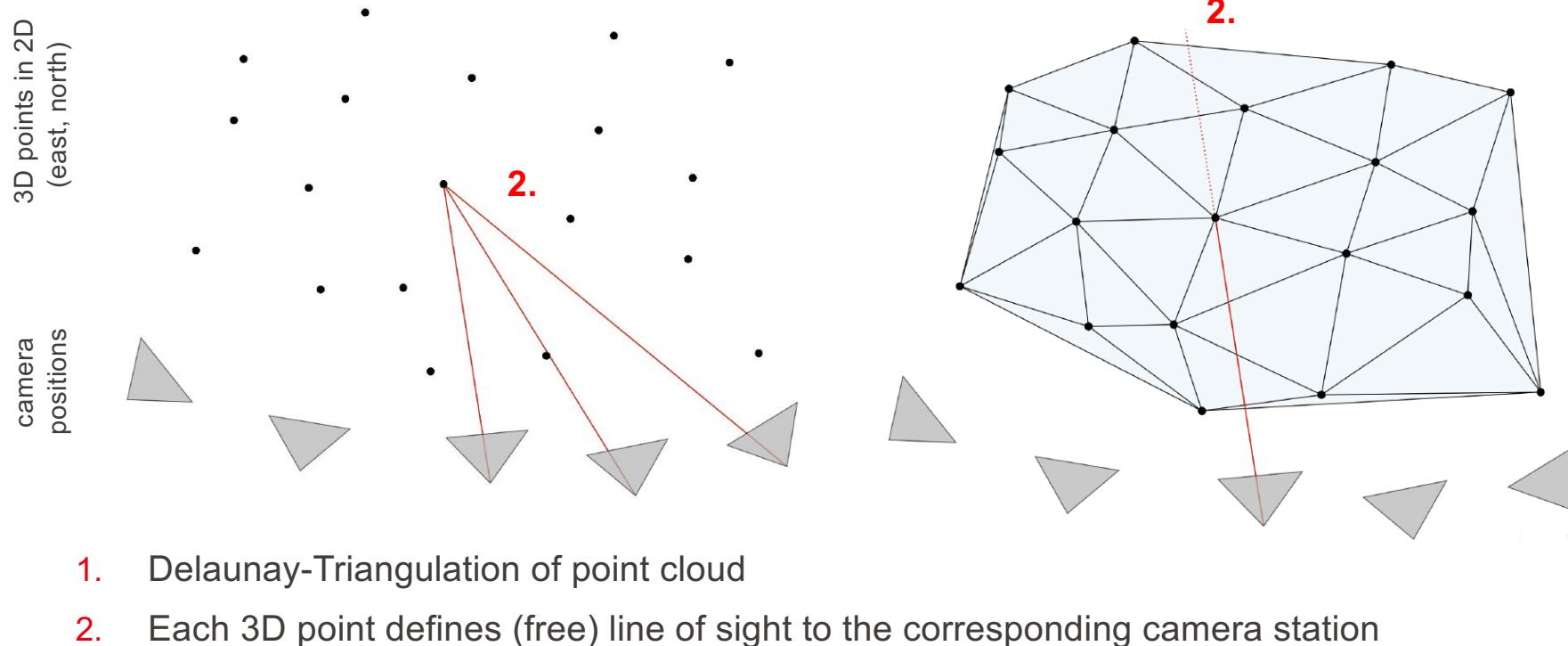
Source: Botsch et al. 2006

Example of 2D triangulation

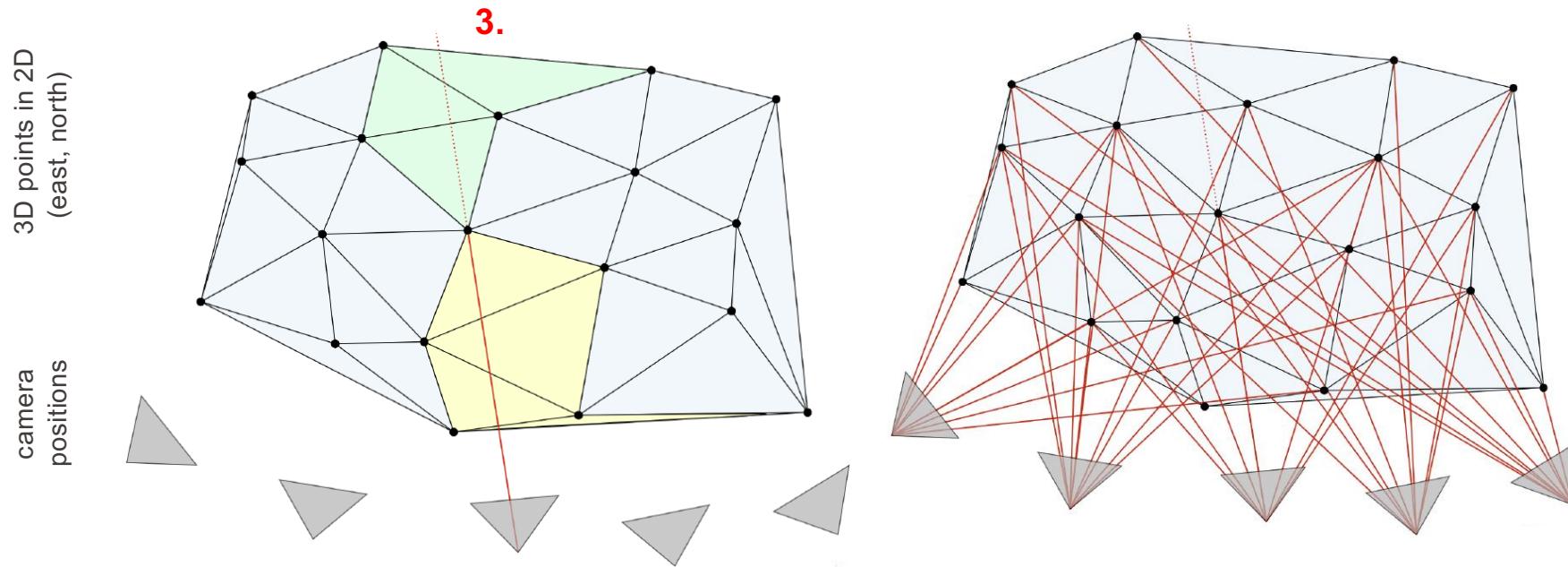


1. Delaunay-Triangulation of point cloud (in 2D / horizontal plain)

Example of 2D triangulation



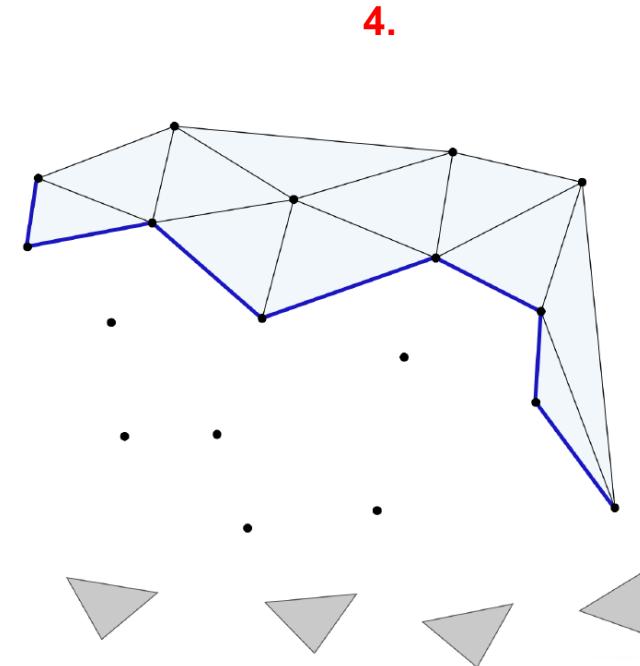
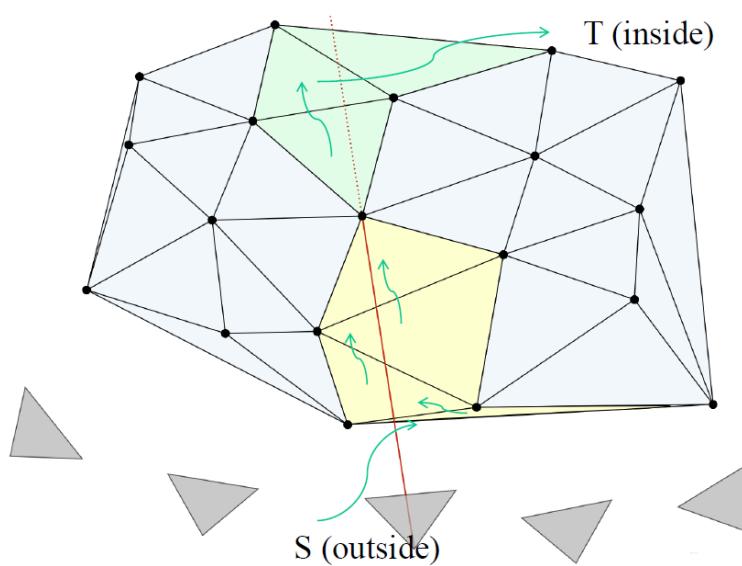
Example of 2D triangulation



1. Delaunay-Triangulation of point cloud
2. Each 3D point defines (free) line of sight to the corresponding camera station
3. Label each Delaunay cell as outside or inside of the observed object (while considering visibility and geometric quality)

Example of 2D triangulation

3D points in 2D
(east, north)



1. Delaunay-Triangulation of point cloud
2. Each 3D point defines (free) line of sight to the corresponding camera station
3. Label each Delaunay cell as outside or inside of the observed object (while considering visibility and geometric quality) – optimization by Graph-cut
4. Extract surface (ok for 2.5D representation)

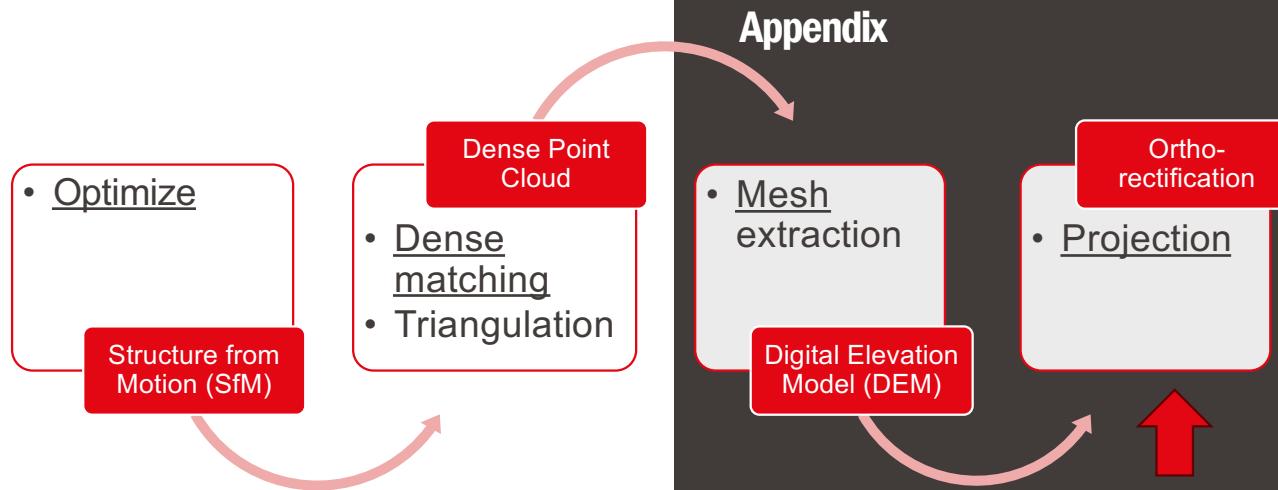
Optimization

Dense matching / 3D point cloud

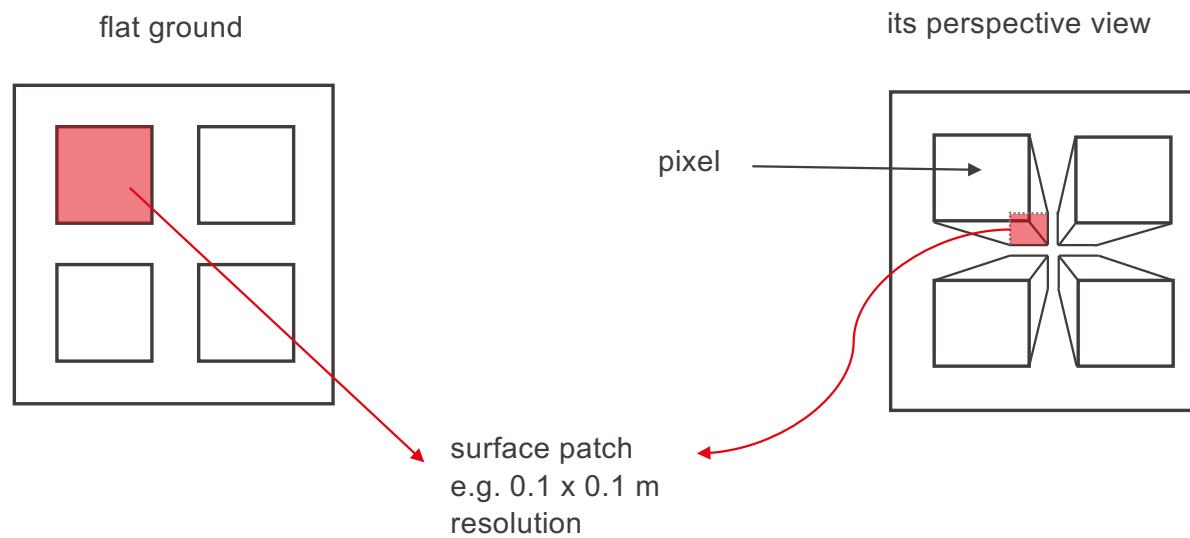
3D meshes and elevation models

Orthorectification

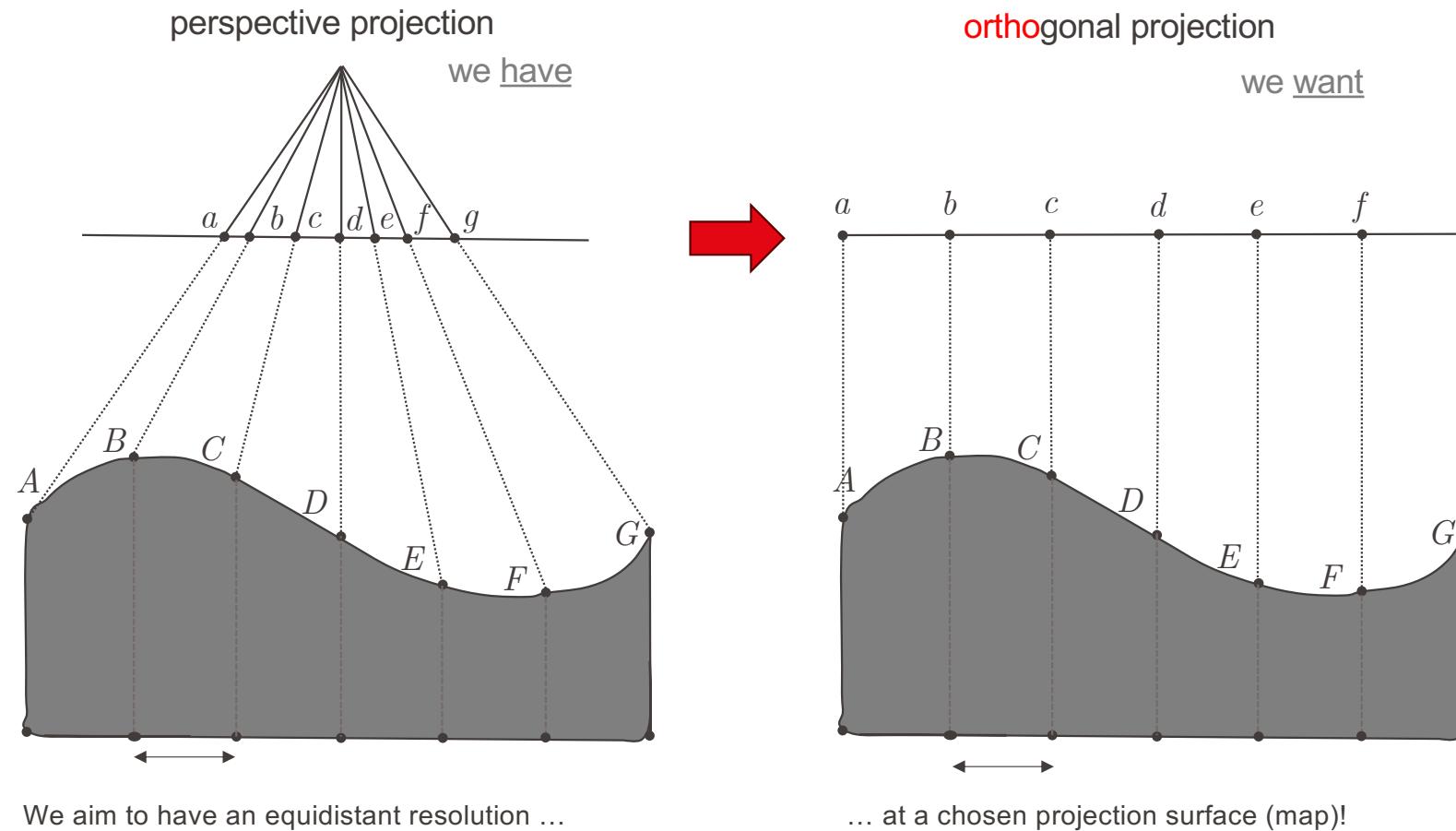
Appendix



Why ortho-rectification?



Why ortho-rectification?



Perspective projection

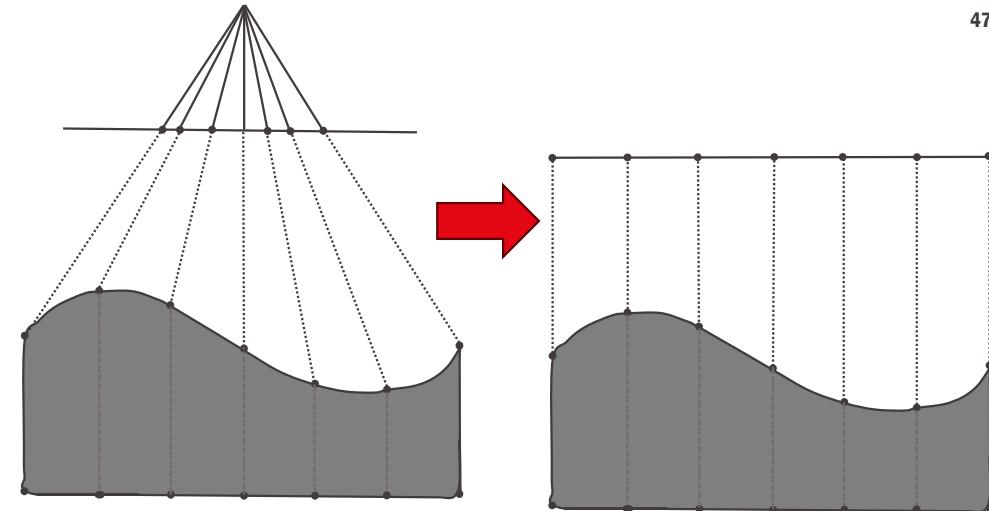
- Causes displacement of reality (radially) per each pixel (also due to relief).
- Does not account for Earth curvature.

Orthographic projection

- Removes this displacement and produces planimetric photo from which features can be mapped.

Prerequisites

- *Orientation* (external & internal) known.
- Existance of a digital elevation model *DEM* – external or via photo-derived point cloud. (resolution, accuracy, format)



Ortho-photo

Is an image where each point is derived from an *orthogonal projection* on a *planar surface*.

Characteristics

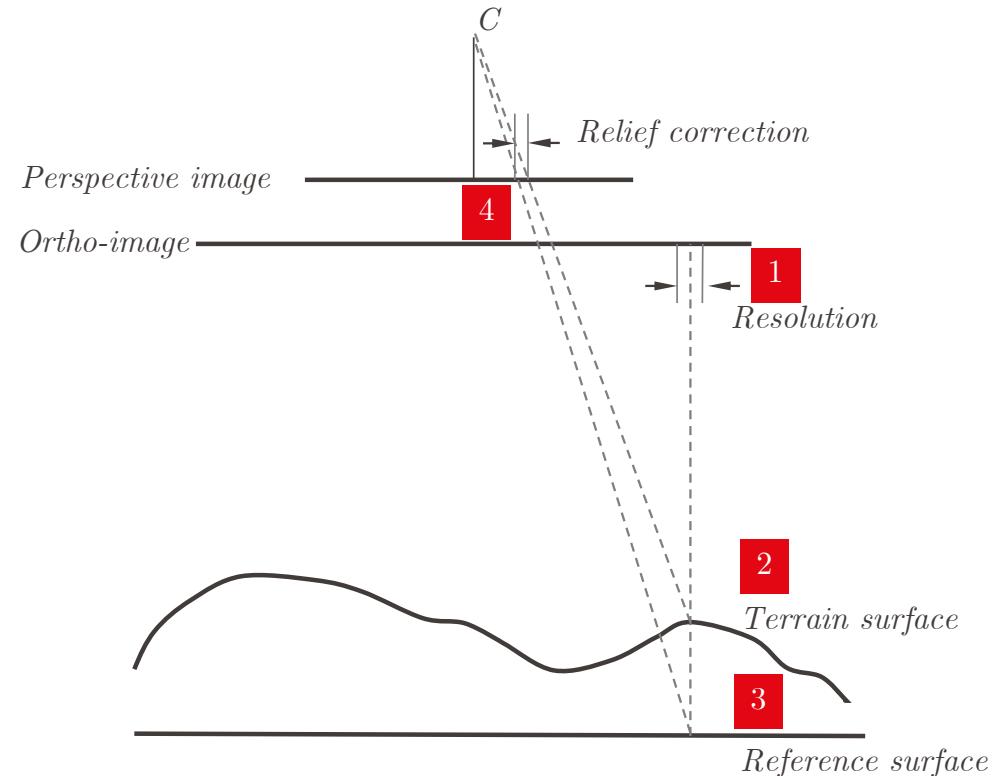
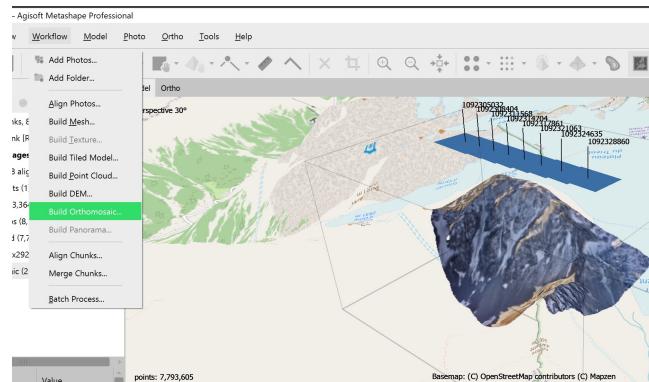
- Constant **scale** (as in a map)
- Inferring on **distances (or angles)** is possible
- Overlay of the photo on a **map** is possible
- ~~Stereo-viewing is NOT possible~~

Process of ortho-rectification

For each X, Y

1. on ortho-photo (grid) coordinates
2. Project to DEM
3. Project to image coordinates (x, y)
4. Correction due to terrain height
5. Interpolate (radiometrically) from surrounding pixels (and photos)

Agisoft – Build Orthomosaic



Process of ortho-rectification

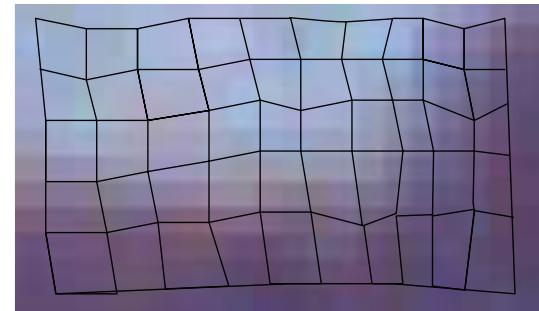
Characteristics per stage

- A. Re-projection of pixel coordinates is uneven.
- B. New RGB values obtained via interpolation (e.g. bilinear, bi-cubic, nearest-neighbour).
- C. Position is redressed on the grid of resolution at a reference height (GSD - ground sampling density):

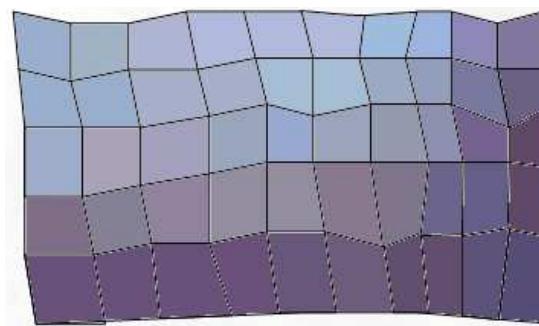
Characteristics – general

- The distance between points on orthophoto = (accounting for photo scale)
= the horizontal distance between the corresponding points on the ground.

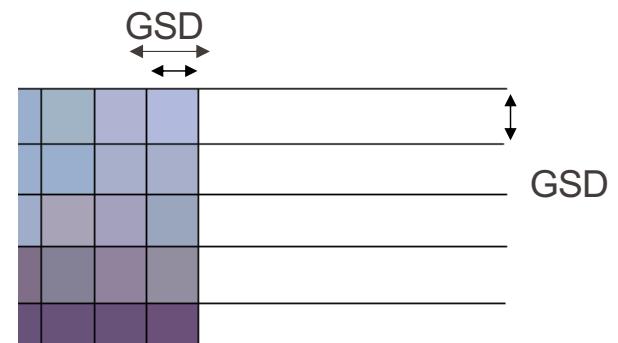
A. x, y



B.
RGB



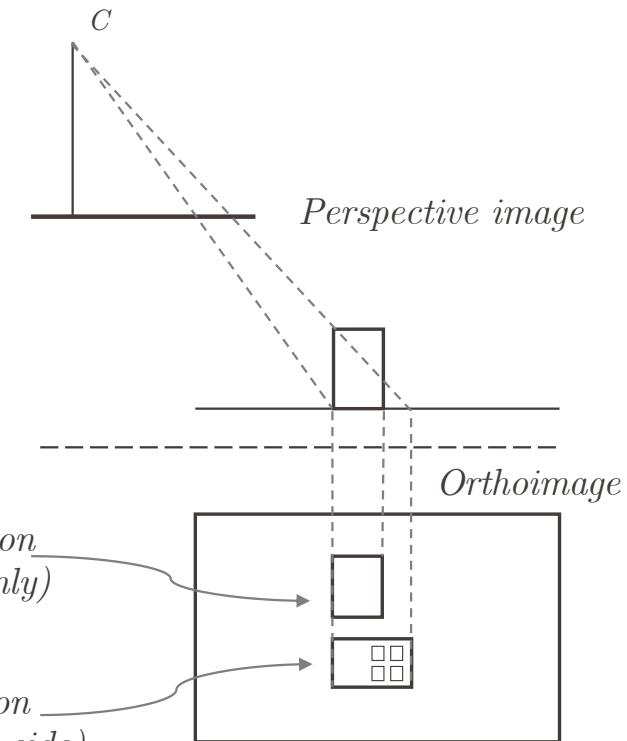
C.



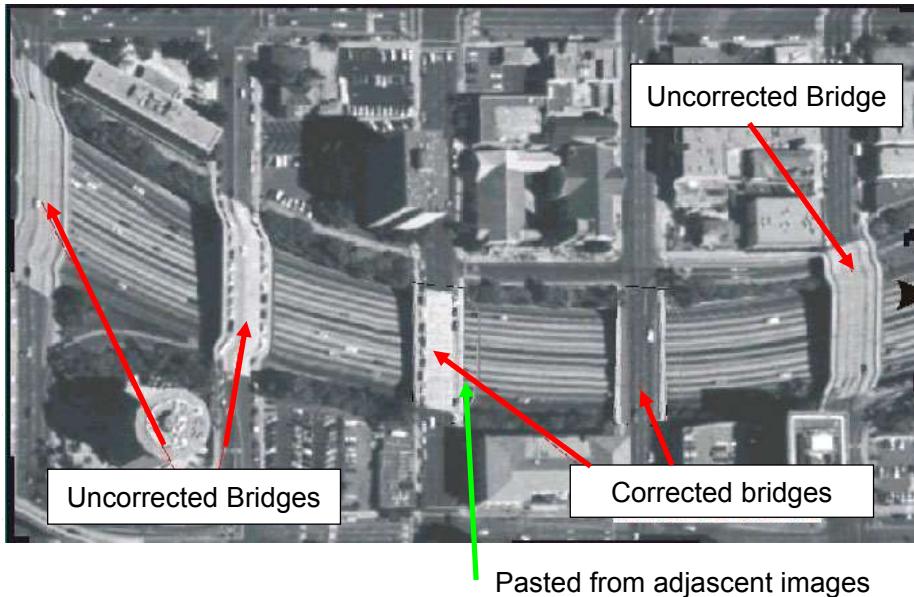
Building correction

- If buildings are not included in the DEM (normal case of DTM), they are not orthorectified and remain “leaning”
- Use of DSM is recommended

Same building in different perspective projections



Idem for 'bridge correction' ...



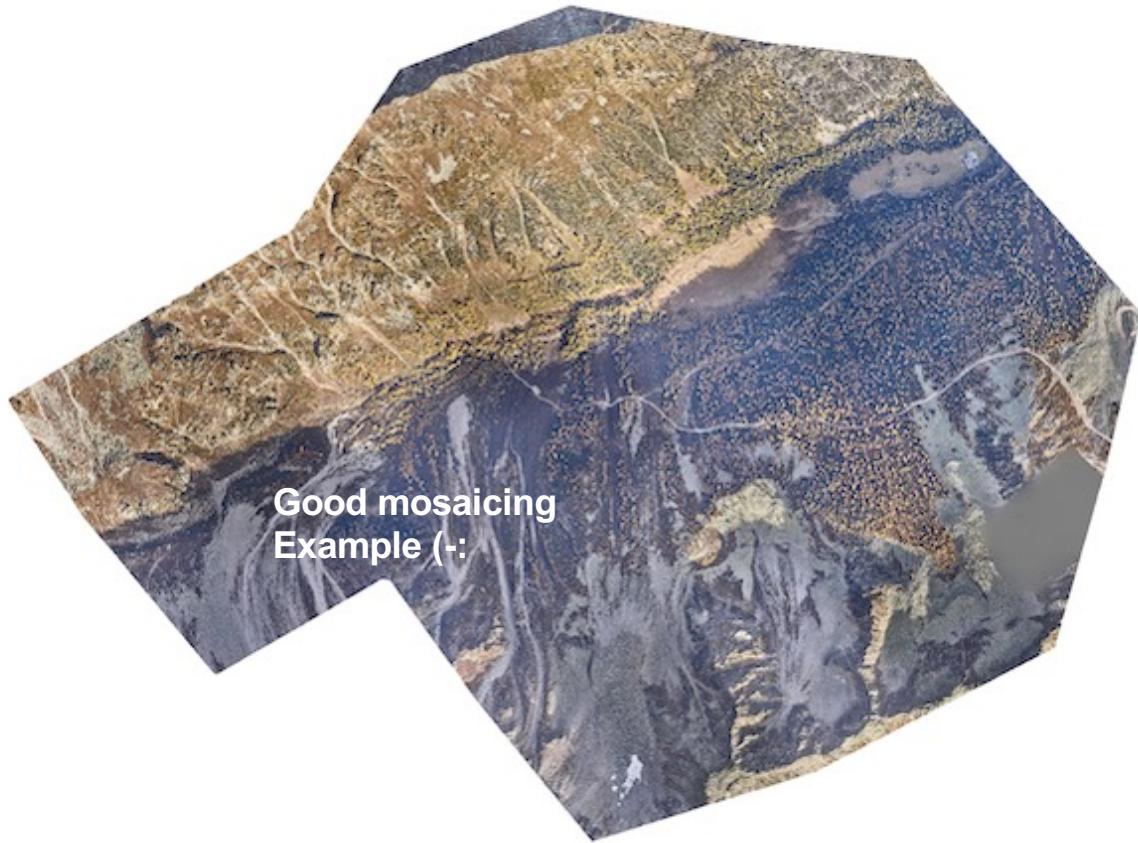
True ortho-photo

- Detailed DSM
- All buildings & bridges

Challenges

- Visibility affects DSM quality
(Appendix IV: image vs lidar)
- Along-track 80% ...
- Cross-track 80% ...
- Productivity!

Orthophoto – mosaicing



[1] <https://www.sciencedirect.com/science/article/pii/S0924271615002774>,
<http://web.eecs.umich.edu/~pettie/matching/Edmonds-Karp-network-flow.pdf>



Goal

- From image collection – all geometric *distortions corrected*, and imagery *colour balanced* to produce seamless image

Pixel

- can have several representations – choice!

Strategy

- radiometric differences are less visible on borders: seamlines or cutlines
- Urban: geometry (building unity)
- Rural: radiometry (field unity)

Algorithm

- Graph-cut ¹

Understanding - self assessment

- What is main idea of “incremental” structure from motion (SfM)?
- Why is “incremental” SfM performed in steps?
- What are the pre-requisites before global optimization can be performed?
- What is the objective function to minimize in the global optimization?
- How are random errors influencing feature coordinates?
- What are the possible approaches to mitigate the influence of random errors on points?
- What a should and optimization quality report contain?
- What should the criteria of acceptance be?
- What type of analysis could be done prior to a mission to determine the chance that its objectives can be fulfilled? (What are the influencing factors?)

Understanding - self assessment

- What are the processing stages for creating 3D point clouds from image pixels?
- What is necessary to known before dense-matching can be started and how the pixel-to-pixel correspondence search can be simplified?
- What are the main challenges in performing dense-matching for 3D point cloud and which can be overcome by using a different technology?
- How DTM is created from a 3D point cloud?
- What are the differences between DSM and DTM?
- What are the respective pros & cons for DEM in raster and TIN representations?
- What is the ortho-rectification of images and what is needed for its creation?
- What is needed that building's facades are not visible on an ortho-photo?
- What is the true orthophoto?

Dense matching / 3D point cloud

3D meshes and elevation models

Orthorectification

Appendix I. planning exercise 4 exam!

II. optimization factors

III. epipolar image generation

IV. image vs. lidar point-cloud

