

# Sensing and Spatial Modeling for Earth Observation

## Part A - From 2D to 3D, final exam - 22 June 2023

SCIPER number / numéro SCIPER :

No document allowed except the 2 A4 cheat sheets. Attention, the exam is 4 PAGES!  
Duration = 1h. 100% = 23 out of 24 points (pts). Please use a pen, not a pencil.

**Additional instructions:** Unless stated otherwise 1 problem = 1.5 pts. Most of the tasks below include multiple-choice and open-ended questions. In case of the multiple-choice questions, *any number* of selectable answers may be correct or incorrect (except when explicitly stated otherwise). Please mark *correct* answers by *crossing* them.

### Image Formation (6 pts)

1. Given focal distance,  $f$ , and camera constant,  $c$ , find out the distance,  $d$ , between an object and the camera lens satisfying the thin lens relation:
2. The following is true in relation to the focal point (only one answer is correct):
  - ☐ It is used to define the distance between the optical center and the image plane.
  - ☐ It is a point where an object appears on an image.
  - ☐ It is a point into which the rays parallel to optical axis are redirected by the lens.
  - ☐ It is a point where an object is always in a focus.
3. The following can be said true about the pinhole approximation:
  - ☐ It is also called perspective projection.
  - ☐ It is a situation in which focal plane and focal point practically coincides.
  - ☐ It is a situation in which lens distortions are not considered.
  - ☐ It is a base for formulating colinearity equation.
4. The following can be said true about the principal point of symmetry (only one answer is correct):
  - ☐ It is a point where optical axis intersect the image plane.
  - ☐ It is always situated in the middle of a digital image.
  - ☐ It defines an origin of rows and columns counter.
  - ☐ It is situated in the top-left corner of an image

### Image Orientation (6 pts)

5. (3 pts) Write down an equation that relates pixel coordinates  $(u, v)$  on an image to its 3D coordinates  $(X, Y, Z)$  in object space while defining all the terms:
6. Which of the following parameters are part of the interior orientation?
- ☐ Image pose
  - ☐ Camera constant
  - ☐ Image coordinates of an object
  - ☐ Image coordinates of a perspective point (of collimation)
  - ☐ Image coordinates of the epipolar point
  - ☐ Radial distortion of the lens
  - ☐ Other parameters needed to satisfy perspective projection
  - ☐ Baseline of image stereo-pair
7. DLT is useful for:
- ☐ Finding camera exterior orientation (pose) without a need of approximation.
  - ☐ Finding approximate value of camera constant.
  - ☐ Solving collinearity condition for unknown feature coordinates in object space.
  - ☐ Finding distance to object in stereo-vision in a linear approximation.
  - ☐ Rapidly localizing camera from several known points on the image.

### Stereo and Multiple View (6 pts)

8. The following can be said true about the *coplanarity* constraint:
- ☐ It is the primary constraint used in global optimisation.
  - ☐ It is mainly used to orient image with respect to object space.
  - ☐ It is mainly useful to find relative orientation between image pairs.
  - ☐ It is useful for reconstructing relative depth with respect to a baseline.
  - ☐ It makes use of epipolar geometry.

9. Triangulation in stereo vision requires:

- ☐ That two images are captured simultaneously.
- ☐ That the captured scene on them is partially the same (i.e. overlapping).
- ☐ That the absolute pose (position and attitude) of each image is known.
- ☐ That the baseline is different from zero.
- ☐ That disparity is different from zero.
- ☐ That disparity is larger than a half of the image width.

10. The following can be said true about global optimization:

- ☐ It minimizes the reprojection errors using coplanarity conditions between all points and images.
- ☐ It handles linear or linearised observations equations.
- ☐ It is also known as *bundle adjustment*.
- ☐ It requires *approximate* knowledge of camera constant.
- ☐ It requires *approximate* knowledge of camera coordinates in object space.
- ☐ It requires *approximate* knowledge of key-features in object space.
- ☐ It requires *approximate* knowledge of key-features in image space.
- ☐ Given the observation models it optimizes simultaneously the camera exterior and interior orientation.
- ☐ It can include observation of pose from a navigation system.
- ☐ It is performed in iteration.
- ☐ It *always* provides the best results if (*please complete below*) ...

11. What is the minimum number of correspondences to find the Essential matrix and why?  
**Justify your answer:**

### Simple Image Acquisition (6 pts)

12. **Situation.** In the design of a UAV-mapping scenario you are asked to consider the following camera parameters specified below together with a legal restriction of maximum flying altitude (height) above the terrain of **120 m**. Further, note that the camera is mounted in the UAV with its *larger* size oriented *across* (i.e. perpendicular) to the flying direction.

Parameter	Value
Sensor size	$13.3 \times 8.76$ mm
No. pixels	$5472 \times 3648$
Camera const.	10.5 mm
UAV speed	10 m/s

**Task.** Answer the following questions:

- (a) (3 pts) What is the expected *ground sampling distance*-GSD (size of the projected pixel on ground) on a flat terrain?
- (b) (2 pts) What will be the *distance*,  $d$ , between two consecutive images in the same flight line if the planned image overlap is 75 % ?
- (c) (1 pts) What is the *time interval*,  $\Delta t$ , between the two consecutive images?

# Sensing and Spatial Modeling for Earth Observation

## Final exam - 22 June 2023

### Part C - Keypoints and machine learning

SCIPER number / numéro SCIPER :

No document allowed except the 2 A4 cheat sheets. Attention, the exam is 4 PAGES!  
Duration = 1h. 100% = 16 out of 17 points (pts). Please use a pen, not a pencil.

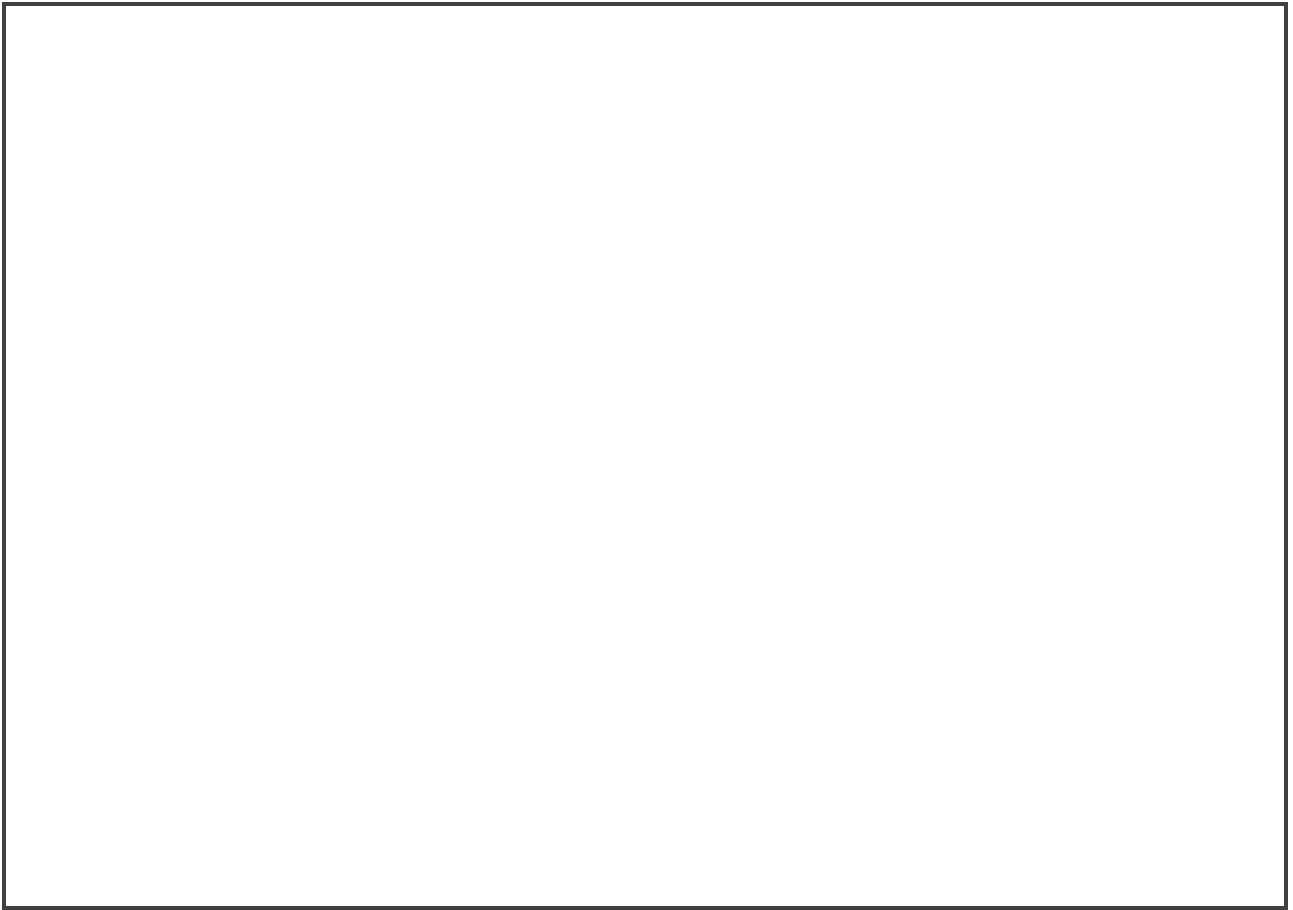
**Explain your answers and detail your calculations.**

#### Question C1: Keypoints (5pts)

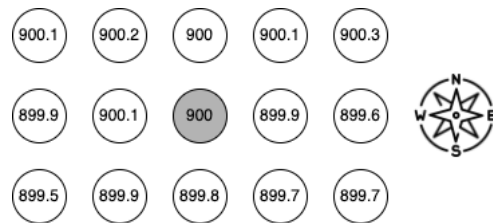


While creating a new panoramic image of the Château de Tourbillon for the city of Sion, unexpected wind resulted in your series of drone images being affected by rotation. To stitch these images together accurately, you decided to use the Scale-Invariant Feature Transform (SIFT) to identify matching points between two images such that you can compute the homography between the images. However, you find that SIFT alone isn't able to accurately match the images and there are some incorrect correspondences between the matches.

You remember that the RANdom SAMple Consensus (RANSAC) algorithm can help remove these incorrect matches. Using pseudocode (or a short paragraph), describe the main steps of the RANSAC algorithm. Detail briefly each step with your own words (5pts).




### Question C2: Features (4 pts)



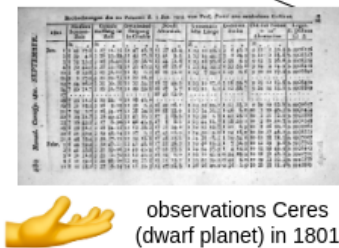
Given the DEM grid above at resolution  $\lambda = 0.5$  meters compute using the steepest drop method the descending gradient in percent and the aspect [N, NE, E, SE, S, SW, W, NW] of the slope at the grey center point of measure. The slope gradient in percent should be computed with one decimal. All measures in the DEM grid are in meters (4pts).

### Question C3: Linear Regression (8 pts)


Imagine it is 1801. You are Karl Friedrich Gauss and invented in 1795 the ordinary least squares method to fit a regression model to data observations. You are approached by Giuseppe Piazzi (Astronomer) to fit a model through his observation data:



**Giuseppe Piazzi**  
(1746 - 1826)  
Astronomer



observations Ceres  
(dwarf planet) in 1801



**Karl Friedrich Gauss**  
(1777 - 1855)  
Mathematician

I observed this data.  
but I don't know further!

let me fit a model with  
Ordinary Least Squares!

**Observation data:** you have four observations, with inputs  $x$  and corresponding outputs  $y$

$x$	2	2	1	1
$y$	4	2	2	0

1. calculate the optimal regression coefficients using the ordinary least squares method (6pts).

2. with your estimated  $\hat{\beta}_0$  and  $\hat{\beta}_1$  predict values for new test data  $x_1 = -1$ ,  $x_2 = 3$  (2pts)



# Sensing and Spatial Modeling for Earth Observation

## Final exam, part C - 22 June 2023

SCIPER number / numéro SCIPER :

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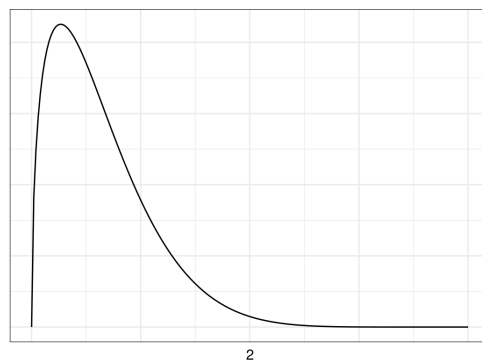
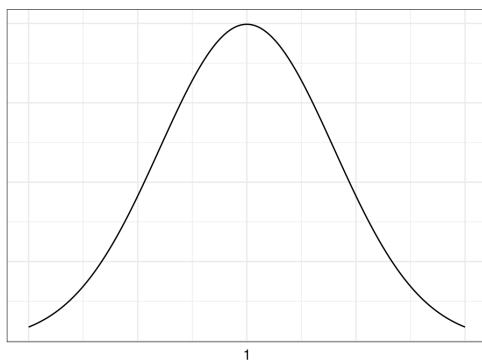
Please use a pen, not a pencil. **Explain your answers.**

1. (a) We are interested in different variables over a given (finite) domain of interest, listed below:

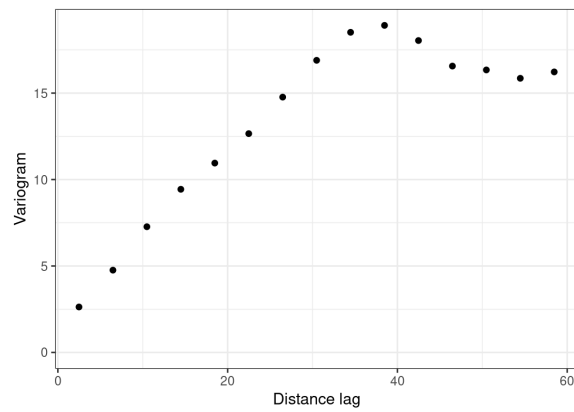
- (1) the number of (human) inhabitants in each community within the domain;
- (2) the type (or class) of land surface provided by an atlas;
- (3) a quantity  $V$  where  $V$  is real and  $\in ]-1, +1[$ .

Among those variables, which one(s) is(are) suitable for Geostatistics?

- (b) We consider two variables characterized by the PDF below. Which variable (1 or 2) is better suited for kriging?



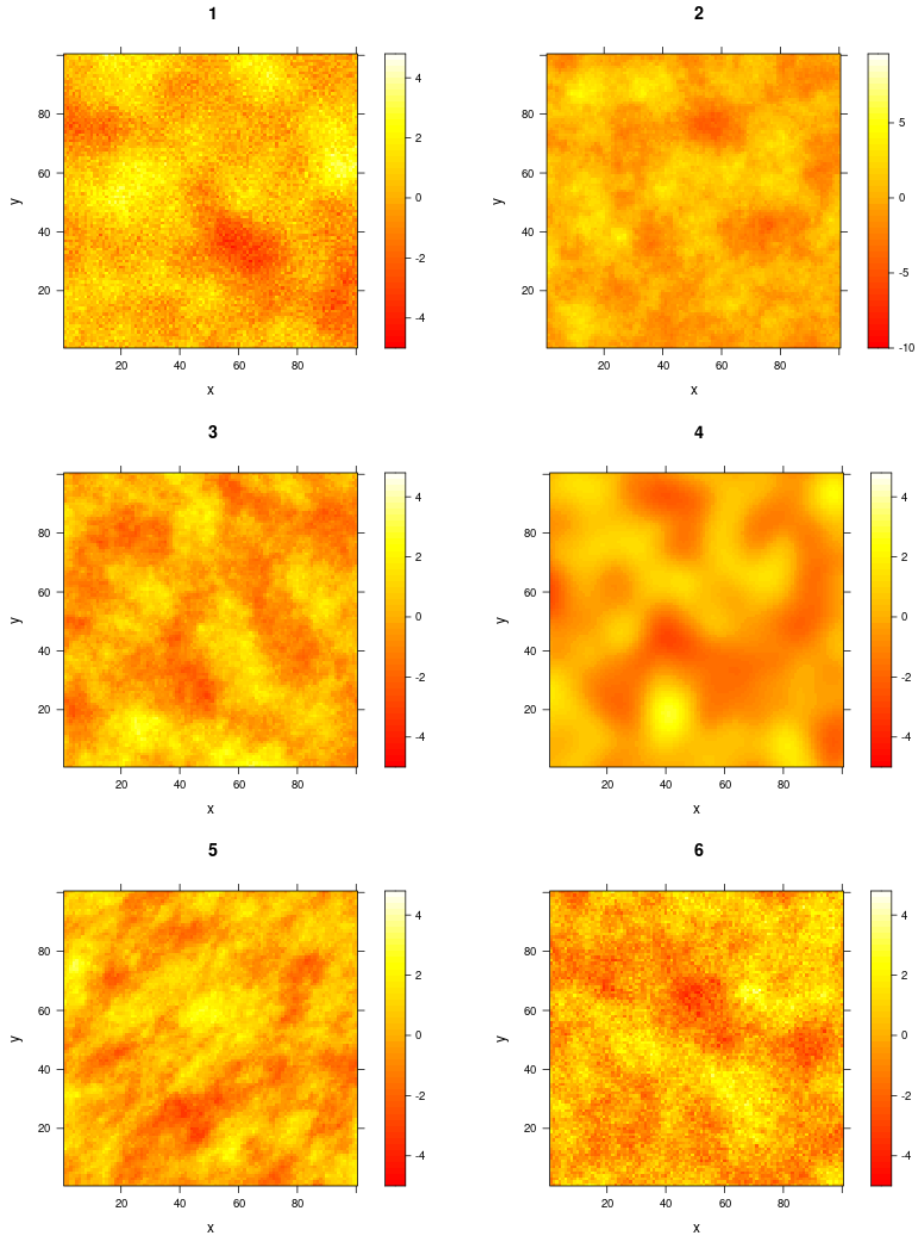
2. (a) We have obtained the following sample variogram values. Draw by hand on this plot the variogram model you would fit to this sample variogram.



- (b) What are the variogram parameter values you can estimate from this plot?

- (c) To what type of random function does this variogram correspond?

3. (a) Which graph (1, 2, 3, 4, 5 and 6) corresponds to which of the following cases:
- vario. sph. isotropic, range = 20, sill = 1, nugget = 0.
  - vario. Gau. isotropic, range = 11, sill = 1, nugget = 0.3
  - vario. Gau, isotropic, range = 11, sill = 1, nugget = 0.
  - vario. sph. isotropic, range = 20, sill = 2, nugget = 0.
  - vario. sph. isotropic, range = 20, sill = 1, nugget = 0.3
  - vario. sph. anisotropic (ratio=0.6), range = 20, sill = 1, nugget = 0.



4. (a) We consider some observations of a given quantity  $V$ , provided below.

Location	1	2	3	4	5
$x$ [km]	1	2	4	6	8
$V$ [-]	2	3	2	2	4

Compute the sample variogram of  $V$  (Matheron estimator) for a distance lag  $h$  of  $2 \pm 1$  km, i.e.  $h \in [1, 3)$ , and  $4 \pm 1$  km, i.e.  $h \in [3, 5)$ .

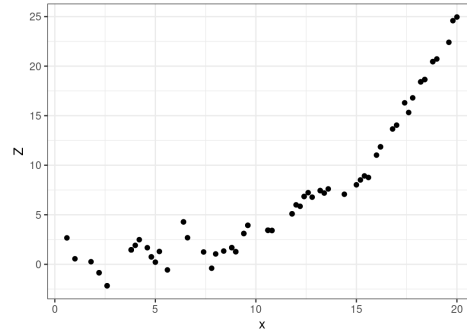
- (b) Let's assume that the covariance of  $V$  is given by  $\text{Cov}(h) = c_1 \exp(-h/c_2)$ , with  $c_1 = 2$  and  $c_2 = 2$ . We want to estimate  $V_0^*$  at  $x_0 = 10\text{m}$  by interpolating  $V_1 = 5$  at  $x_1 = 9\text{m}$  and  $V_2 = 2$  at  $x_2 = 12\text{m}$ , using ordinary kriging.

- i. Write the system of equations to solve to estimate the kriging weights  $\lambda_1$  and  $\lambda_2$ .

ii. Derive the weights  $\lambda_1$  and  $\lambda_2$  and estimate  $V_0^*$ .

iii. Compute the associated estimation variance and provide the 80% confidence interval around  $V_0^*$ .

(c) We now consider the RF  $Z$  with the measurements plotted below.



- i. We want to estimate  $Z$  at the location  $x_0 = 10$  using the measured values  $Z_1$  at  $x_1 = 9$  and  $Z_2$  at  $x_2 = 11$ . What type of kriging do you recommend?
  
  
  
  
  
  
  
  
  
  
- ii. Write the corresponding system of equations to solve (specifying as many values as you can).