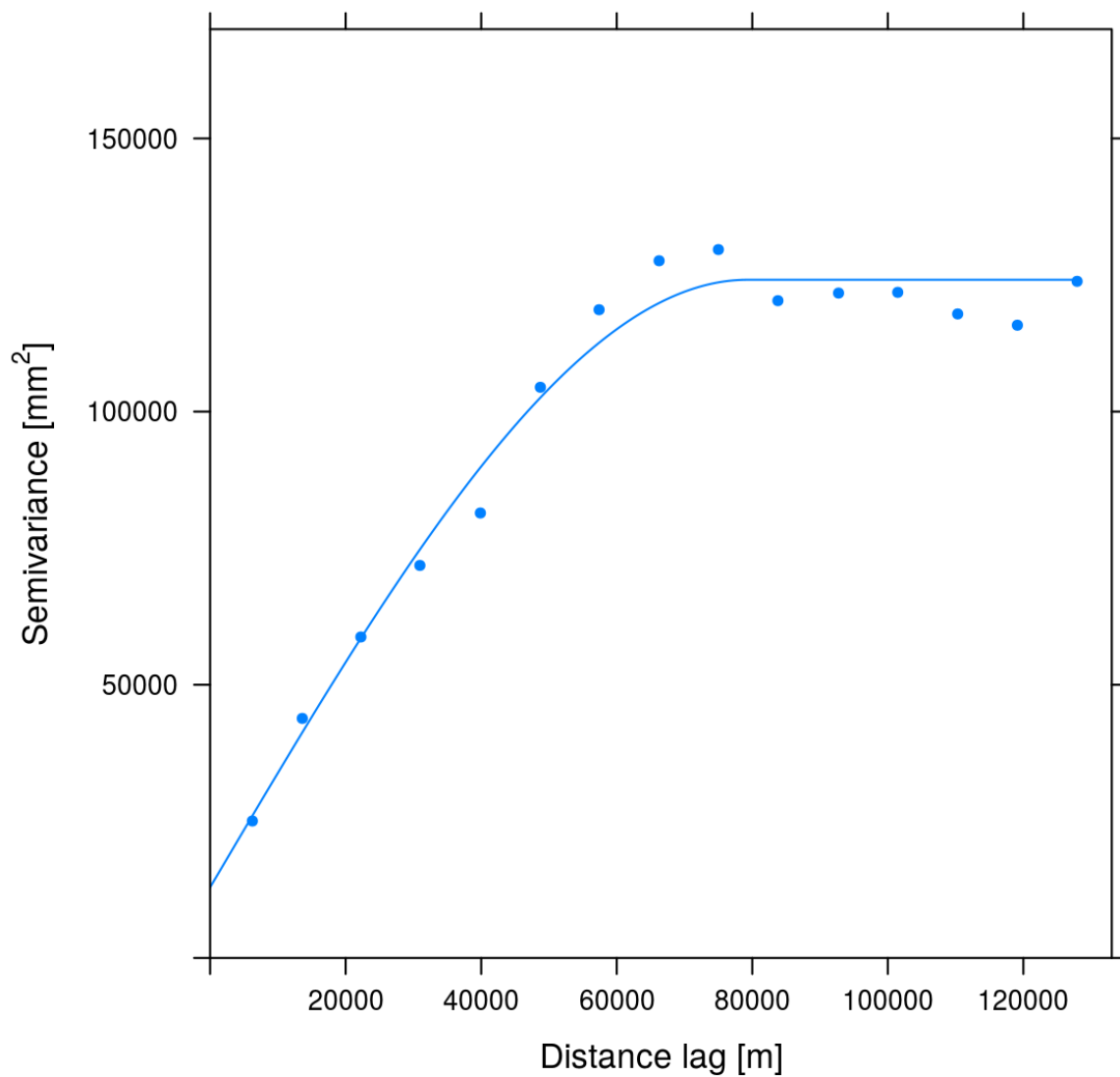


Exercise 4 - Simple and ordinary kriging - Solutions

Office hours: Friday 09:00-12:00

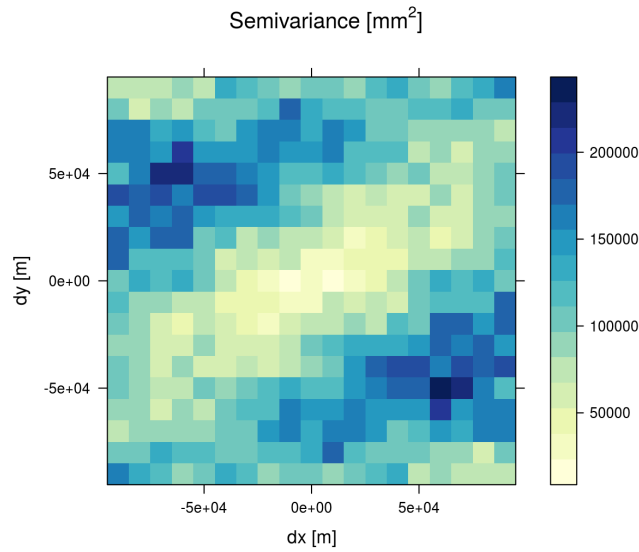
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1. Read the prediction dataset and compute the isotropic sample variogram of the precipitation values. Fit a spherical variogram model. What are the values of the nugget, the sill and the range of the fitted model?

Isotropic sample variogram of precipitation

The fitted values of the nugget, the sill and the range are 12'892 mm², 124'106 mm² and 79.2 km respectively.

2. Compute the variogram map (2D variogram) of the precipitation values. What are the directions of minimum and maximum variability?

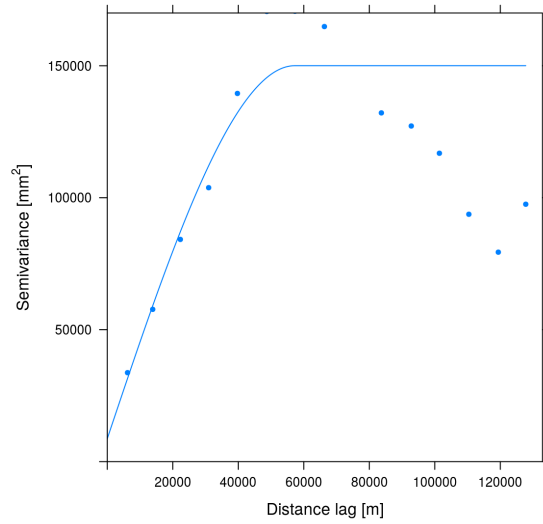
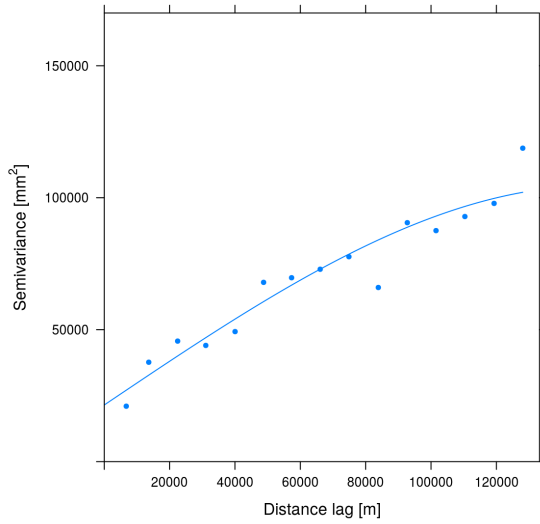


The direction of minimum variability is 55° clockwise from the North. The direction of maximum variability is 145° .

3. Compute the variograms in the direction of minimum/maximum variability.

Isotropic variogram along direction of minimum variability

Isotropic variogram along direction of maximum variability



4. Fit a spherical model on each of the directional sample variograms computed above and provide the values of the nugget, the sill and the range of the fitted models. What is the value of the anisotropy ratio?

	nugget [mm ²]	sill [mm ²]	range [km]
min. variability (55°)	21'473	104'480	149.5
max. variability (145°)	8'615	150'038	57.3

The anisotropy ratio is 0.383.

5. *Use the results obtained in 2. and 3. to define a new variogram model with geometric anisotropy.*

The new variogram model is obtained by writing the following R code:

```
vgm.anis <- vgm(psill=ps,model="Sph",range=r,nugget=n,anis=c(55,0.383))
```

where $ps=83'007$, $n=21'473$ and $r=149.5$ km represent respectively the partial sill, the nugget and the range of the variogram in the direction of minimum variability.

6. *Interpolate the precipitation values at the locations of the validation dataset.*
- (a) *using inverse distance weighting (IDW).*
 - (b) *using simple kriging and the isotropic variogram model computed in 1.*
 - (c) *using ordinary kriging and the isotropic variogram model computed in 1.*
 - (d) *using ordinary kriging and the anisotropic variogram model computed in 5.*

The interpolations are obtained by writing the following R code:

```
IDW <- krige(formula=precip~1,locations=data,newdata=newdata)
SK.iso <- krige(formula=precip~1,locations=data,newdata=newdata,model=vgm.iso,beta=mean(precip))
OK.iso <- krige(formula=precip~1,locations=data,newdata=newdata,model=vgm.iso)
OK.anis <- krige(formula=precip~1,locations=data,newdata=newdata,model=vgm.anis)
```

7. *Compare the interpolated values with the precipitation measurements of the validation dataset. Compute the bias and the root mean squared error (rmse) of the predicted values for each method. Which method performs best? And which performs worst?*

	bias [mm]	rmse [mm]
IDW	11.95	194.95
SK.iso	2.72	173.37
OK.iso	2.78	173.35
OK.anis	-1.25	169.91

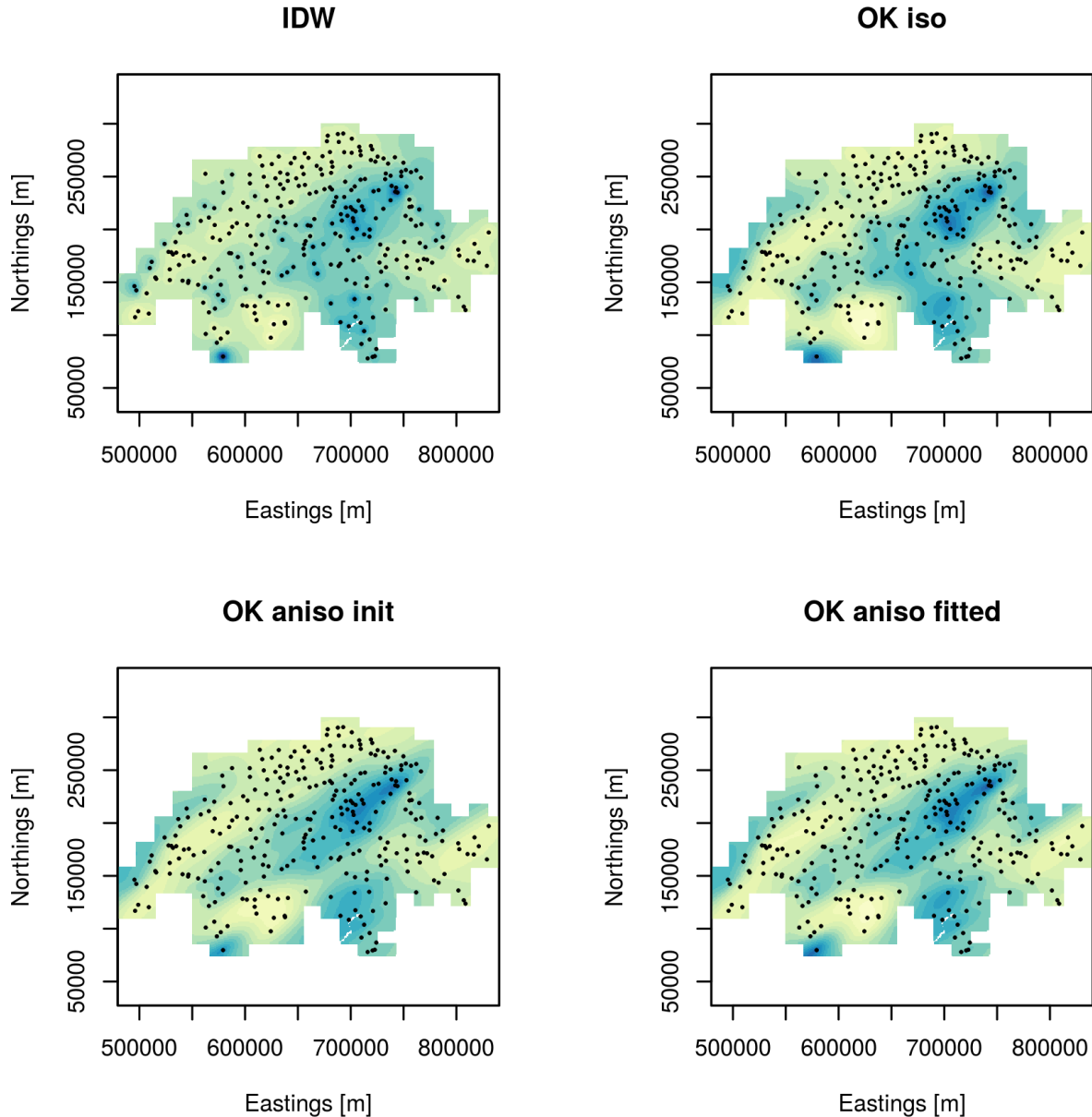
The method that performs best is the ordinary kriging with geometric anisotropy. The inverse distance weighting method performs worst. Note that because there are many data, simple kriging and ordinary kriging produce very similar results.

8. *Read the provided 1-km digital elevation model (DEM) for Switzerland and interpolate the precipitation values at the locations given by the DEM.*
- (a) *using inverse distance weighting (IDW).*
 - (b) *using ordinary kriging and the isotropic variogram model.*
 - (c) *using ordinary kriging and the anisotropic variogram model.*

The interpolations are obtained by writing the following R code:

```
IDW <- krige(formula=precip~1,locations=data,newdata=DEM)
OK.iso <- krige(formula=precip~1,locations=data,newdata=DEM,model=vgm.iso)
OK.anis <- krige(formula=precip~1,locations=data,newdata=DEM,model=vgm.anis)
```

9. Plot the maps with the interpolated precipitation values over Switzerland. What are the main differences between the different interpolation methods?



IDW does not consider the spatial structure of the data when interpolating. The influence of each measurement point on its neighborhood is therefore clearly visible. The interpolated precipitation field is very noisy and does not look realistic. Ordinary kriging, on the other hand, looks much smoother and realistic. The differences between the isotropic and the anisotropic interpolation are clearly visible over the Alps and in the Southwest of Switzerland (the Valais). It is difficult to say which method performs best (there are no reference data) but for sure, both methods are unable to adequately reproduce the different local precipitation patterns induced by the presence of mountains and valleys. Other interpolation methods (e.g., using local variograms or universal kriging or external drifts) should be considered.