Formula sheet – Part A. Nenes

R* = 8.3144 J/°K mol, 1 bar = 1 atm = 10^5 Pa, 1 atm = 1.013×10^5 Pa, 1 Pa= 1 N m^{-2}, 1 N = 1 J/m, Molar mass of Air = 29 x 10^{-3} kg mol^{-1}, $L_{lv} = 2.47\times10^6$ J kg^{-1}

$pv = RT$, R is the specific gas constant = R^*/M, where M is the molar mass of the gas.

For an ideal gas: $du = c_V\, dT,\ dh = c_P\, dT,\ c_p - c_v = R$

Combined $1^{st}+2^{nd}$ law: $du = dq - pdv = Tds - pdv$

Gibbs Free Energy $dg = -sdT + vdP$

Clausius-Clapeyron Equation: $\dfrac{de_s}{dT} = \dfrac{L_{lv} e_s}{RT^2}$

Wet bulb temperature $Tw = T - \dfrac{L_{lv}}{c_p}\left(w_v^s(T_w) - w_v\right)$, where $w_v^s(T_w)$ is the saturation water vapor mixing ratio (at temperature T_w), and w_v is the water vapor mixing ratio.

Dew point: temperature for which $w_v^*(T_d) = w_v$

Humidity variable relationships: $w_v = 0.62\ e/P$ $\quad w_v^* = 0.62\ e_s/P$ $\quad H = e/e_s$

Saturation vapor pressure of water at room temperature: 30mbar.

Enthalpy for parcel: $dH = \left(m_d c_{pd} + m_v c_{pv} + m_l c_l\right) dT + L_{lv}\, dm_v$, where the subscripts d, v, and l refer to dry air, water vapor and liquid water, respectively

Hydrostatic equation: $g = -\frac{1}{\rho}\frac{\partial p}{\partial z}$, where ρ is density.

Kelvin Equation: $\dfrac{e_s^{curved}}{e_s^{flat}} = \exp\left(\dfrac{4M_w\sigma}{RT\rho_w D_p}\right)$, where σ is the surface tension of water (=0.072 J m^{-2}), ρ_w is the density of water (=1000 kg m^{-3}), M_w is the molar mass of water and D_p is the droplet diameter.

Idealized Raoult's Law: $P = P^{sat}(T)\ \chi_w$

Lognormal distribution: $n(Dp) = \dfrac{dN}{d\log Dp} = \dfrac{N}{\sqrt{2\pi}\ln\sigma_g}\exp\left\{-\dfrac{\ln^2\left(D_p/D_{pg}\right)}{2\ln^2\sigma_g}\right\}$

Formula sheet – Part J. Gehring

Equation of motion: $\frac{d\vec{U}}{dt} + 2\vec{\Omega} \times \vec{U} - \Omega^2 \vec{r} = -\frac{1}{\rho}\vec{\nabla} p + \overrightarrow{g^*} + \vec{F}$

Total derivative: $\frac{d\vec{U}}{dt} = \frac{\partial \vec{U}}{\partial t} + (\vec{U} \cdot \nabla)\vec{U}$

Geostrophic balance: $\overrightarrow{V_g} = \frac{1}{\rho f}\vec{k} \times \nabla p$, where $f = 2\Omega \sin \varphi$ and φ is the latitude (positive in the Northern Hemisphere and negative in the Southern Hemisphere).

Hypsometric equation: $\frac{R_d \overline{T_v}}{g} ln\left(\frac{p_1}{p_2}\right) = \Delta z$, where R_d id the gas constant for dry air, $\overline{T_v}$ is the column-averaged virtual temperature.

Geopotential: $d\Phi = gdz$, where Φ is the geopotential. The geopotential height is defined as: $Z = \frac{\Phi}{g_0}$

The altimeter equation : $p_1 = p_2 e^{\frac{gz_2}{R_d \overline{T_v}}}$

The thermal wind : $\frac{\partial \vec{V}_g}{\partial p} = -\frac{R}{fp}\hat{k} \times \nabla T$. The thermal wind vector is defined as: $\vec{V}_T = -\frac{\partial \vec{V}_g}{\partial p}$

The ageostrophic wind: $\vec{V}_{ag} = \vec{V} - \vec{V}_g$.

Simplified form of the equation of the motion (retaining the three highest order terms):
$\frac{d\vec{V}}{dt} = -f\hat{k} \times \vec{V}_{ag}$, isolating the ageostrophic wind yields: $\frac{\hat{k}}{f} \times \frac{d\vec{V}}{dt} = \vec{V}_{ag}$

Continuity equation: $\nabla_p \cdot \overrightarrow{V_h} = -\frac{\partial \omega}{\partial p}$

The pressure tendency equation: $\frac{\partial p_s}{\partial t} \approx -\int_0^{p_s} (\nabla \cdot \vec{V})\partial p$

Formula sheet – Part A. Berne

Brunt-Vaisala frequency

$$N_{BV} = \sqrt{\frac{|g|}{T_v}\left(\frac{\Delta T_v}{\Delta z} + \Gamma_{d/s}\right)} \sim \sqrt{\frac{|g|}{\theta}\frac{\Delta\theta}{\Delta z}}$$

Dimensionless mountain height

$$H = \frac{1}{Fr} = \frac{N_{BV} h}{U}$$