Problem 1a.

The energy fluxes that play a role in the surface energy balance are the radiative flux, the sensible heat flux, the latent heat flux and the conductive heat flux. If the land surface is an infinitesimally thin surface without heat capacity then the net radiative flux must balance the remaining fluxes.

$$R_s \downarrow - R_s \uparrow + R_l \downarrow - R_l \uparrow = F_h + F_e + F_g$$

$R_s \downarrow$ is the incoming shortwave radiation (always positive and directed downward)
$-R_s \uparrow$ is the outgoing shortwave radiation (always negative and directed upward)
$R_l \downarrow$ is the incoming longwave radiation (always positive and directed downward)
$-R_l \uparrow$ is the outgoing longwave radiation (always negative and directed upward)
F_h is the sensible heat flux (positive if directed upward)
F_e is the latent heat flux (positive if directed upward)
F_g is the conductive heat flux (positive if directed downward)

From Figs. 9.10 and 9.9 in the Wallace and Hobbs book we can estimate the order of magnitude of such fluxes under clear sky. E.g. at 12:00 we have:
$R_s \downarrow = 900$ W/m^2 directed downward
$-R_s \uparrow = -200$ W/m^2 directed upward
$R_l \downarrow = 300$ W/m^2 directed downward
$-R_l \uparrow = -600$ W/m^2 directed upward
$F_h = 100$ W/m^2 directed upward
$F_e = 300$ W/m^2 directed upward
$F_g = 50$ W/m^2 directed downward

Problem 1b.

The growth of the BL is driven by the sensible heat flux. The growth of the BL in convective conditions develops through entrainment of air from the stable free atmosphere. This entrainment increases if convective turbulence increases in the mixed layer and decreases if the capping inversion $\Delta\theta$ increases (reducing the inertial overshoot of thermals).

$$dz_i/dt = -F_h(z_i)/\Delta\theta = A\,F_h(0)/\Delta\theta$$

where z_i is the BL height, $F_h(z_i)$ is the sensible heat flux across the capping inversion (negative as directed downward in convective conditions), $F_h(0)$ is the sensible heat flux at the surface (positive as directed upward in convective conditions) and A is the ball parameter (equal to 0.2 in free convection).

Problem 1c.

<T> = 293.15 °K; <w> = 0 m/s

T' [°K]: 0, 5, 8, 5, 3, -2, -3, -3, -7, -6
w' [m/s]: 0.05, 0.1, 0.1, 0.05, 0.05, -0.05, -0.1, -0.1, -0.05, -0.05
<w'T'> = 0.305 °K m/s
q_s = 1 x 1000 x 0.305 = 305 W/m^2

Problem 1d.
$dz_i/dt = -A\,F_h(0)/\Delta\theta = 0.2 \cdot 305/(3+273.15) \approx 22$ cm/s

Problem 2a.
Katabatic winds are driven by radiative cooling, producing a stable thermal stratification at the surface. The presence of clouds would provide longwave radiation at the surface, which would promote a more neutral condition. Synoptic forcing would generate more mixing and weaken the temperature stratification. Also the nighttime thermal flows are typically weak and a strong synoptic event would dominate the flow.

Problem 2b.
The air close to the surface is becomes colder as a result of the radiative cooling. It becomes also heavier and starts to sink vertically due to gravity. Since gravity has a non-null component in the direction tangent to the slope, we observe the typical jet close to the surface. Since the velocity profile has to be inevitably 0 at the surface, there must be a peak in the downslope velocity profile.

Problem 2c.
The lapse rate for potential temperature is:

$$\Gamma = -d\theta/dz$$

The stable thermal stratification implies a positive gradient of the potential temperature, and therefore a negative lapse rate (-5 °C/km).

Problem 2d.
The maximum magnitude occurs in the lower part of the jet, close to the surface (positive shear). The minimum magnitude (actually zero) occurs at the peak.

Problem 2e.
The momentum flux is positive in the upper part of the jet, where the velocity gradient is negative, and negative in the lower part of the jet, where the velocity gradient is positive.

Problem 2f.
The heat flux is everywhere negative, as the temperature gradient is always positive.

Problem 2g.
Considering stationary and homogeneous conditions (parallel to the surface), negligible subsidence, no synoptic forcings and negligible Coriolis effects, the momentum equation in the streamwise direction reads:

$$g\,\Delta\theta/\theta_0 \sin\alpha = \partial\overline{u'w'}/\partial z$$

Problem 2h.
Integrating the previous equation along z we obtain

$$\overline{u'w'} = g\sin\alpha\int_0^z \left(1 - z'/h_j\right)dz' + \overline{u'w'}\Big|_0$$

Considering that at the height of the peak z = z_p the turbulent momentum flux is zero (velocity gradient equal to zero) and that, close enough to the surface there must be a layer where the law of the wall holds, we can write

$$\overline{u'w'}\Big|_{z_p} = g\sin\alpha\int_0^{z_p} \left(1 - z'/h_j\right)dz' - u_*^2 = 0$$

where u_*^2 is the friction velocity. Integrating the previous equation, we obtain the flowing quadratic equation in z_p

$$\frac{g\sin\alpha}{2h_j} z_p^2 - g\sin\alpha z_p + u_*^2 = 0$$

whose solutions are:

$$z_p = h_j\left(1 \pm \sqrt{1 - \frac{2u_*^2}{h_j g\sin\alpha}}\right)$$

Because the height of the jet h_j must be larger than the height of the peak, we keep only the solution with the - sign

$$z_p = h_j\left(1 - \sqrt{1 - \frac{2u_*^2}{h_j g\sin\alpha}}\right)$$

Problem 3.
On average much more energy is received by the earth's surface in low latitudes and part of this energy is redistributed towards the poles. The main transport mechanisms are advective transport of warmer air masses via the most dominant circulation systems such as the Hadley cell close to the equator. The Hadley cell is a circulation, which brings hot and moist air to higher altitudes and then to higher latitudes and brings cooler and dryer air from higher latitudes towards the equator (trade winds).
In mid latitudes, the situation is somewhat more complicated as the coriolis force prevents a direct flow to fill the tropical surface low pressure. As the meridional temperature gradient reaches a critical value, instabilities (baroclinic waves, Rossby waves) start to develop and lead to our daily weather cycles in the

northern mid latitudes of low and high pressure systems embedded in a generally westerly flow. These instabilities with storms, fronts and the jet stream at high altitudes also bring on average warmer air towards the poles and this average yet in detail chaotic and very unstationary weather pattern is called the Ferrel cell. Sometimes, especially in the respective summers, these weather patterns bring warm air all the way to the pole, while in the respective winters, the polar vortex prevents an efficient mixing with air from lower latitudes and leads to very cold temperatures at the poles, in particular over Antarctica.

These general and idealized mechanisms are altered by the land-sea distribution which leads to secondary redistribution mechanisms such as monsoons or the Walker circulation.

Key words for points: Redistribution of radiation retrieved at low latitudes; Hadley cell; Ferrel cell; Baroclinic instability; Rossby Waves; Westerlies; Fronts; Low- and High Pressure Systems; Jet Stream; Polar Vortex; Walker circulation; Monsoon;