Physics and Chemistry of the Atmosphere
Examination
Section: Physics

I) **[3 points]**. A projectile is fired towards the North Pole at an angle of 45^o degrees with respect to the surface with velocity magnitude of 200 $m\,s^{-1}$ from a point with latitude-longitude of $0^o\,N, 0^o\,W$. Calculate the distance that the projectile travels before it crashes onto earth again.
[Bonus Question !] : Can you compute the latitude-longitude of the point where it crashes onto earth ?
Note: Assume that there is no friction, Angular velocity of the earth $\Omega = 7.29 \times 10^{-5} s^{-1}$, $g = 9.8 m s^{-2}$ *and the radius of the earth is 6370 km*

Solution I: The Coriolis force $\vec{C}$ is defined as

$$\vec{C} = -f\vec{k} \times \vec{v} \quad with \;\; f = 2\Omega \sin\phi \tag{1}$$

The Coriolis paramter f is a function of time due to the variation of ϕ (latitude) during the motion of the projectile. The velocity of the projectile ($\vec{v}$) can be decomposed along two orthogonal directions, one component normal to the surface of the earth (v_y) and the other is tangential to the surface (v_x). The initial velocities in these direction $v_{0,x}$ and $v_{0,y}$ is equal to $200\cos(45^o)\; m/s$ The equation of motion in the normal direction is

$$\frac{d\,v_y}{d\,t} = -g \tag{2}$$

Integrating the above equation in time, we get displacement in the normal direction as

$$y = v_{0,y}t - \frac{1}{2}gt^2 \tag{3}$$

where $v_{0,y}$ is the initial velocity in the y direction. Setting $y = 0$ yields two solution, one being the trivial $t = 0$. The other solution is the total time of motion of the projectile $t_a = 2v_y/g = 28.83\,seconds$. As the projectile is fired from the equator towards the north pole, the upward motion causes a Coriolis force towards the west and it can be stated as,

$$\frac{d\,v_x}{d\,t} = -2\Omega v_y \cos(\phi t) \tag{4}$$

Recalling that $v_y = v_{0,y} - gt$, and integrating the above equation with respect to time, we get,

$$v_x = -2\Omega v_{0,y}t + \Omega g t^2 \tag{5}$$

Integrating the above equation once again with respect to time,

$$b = -\Omega v_{0,y}t_a^2 + \frac{1}{3}\Omega g t_a^3 \tag{6}$$

For $t_a = 28.83\,seconds$, $b = 2.86m$ west. Note that b is the westward displacement of the projectile only due to vertical motion of the projectile. There is additionally a meridional motion of the projectile and this also causes a displacement of the projectile due to Coriolis forces. The meridional motion also causes a westward Coriolis force,

$$a_w = 2\Omega v_{0,x} \sin(\phi t) \tag{7}$$

$\phi(t)$ varies with time such as $\phi_t = v_{0,x} t / R$, with R being the radius of the earth. Substituting the expression of $\phi(t)$ and integrating the expression for a_w twice, the displacement westward due to the meridional motion can be written as

$$c = \frac{-2\Omega R^2}{v_{0,x}} \sin\left(\frac{v_{0,x} t_a}{R}\right) \tag{8}$$

The resultant displacement for $t_a = 28.83\,seconds$ is c = 467.36 m. Thus the net displacement in the westward direction is $t = b + c = 470.22$ m. The meridional displacement can be found as $d = v_{0,x}\, t_a = 4077.47m$ North. The net displacement is therefore $\sqrt{d^2 + t^2} = 4104.49\,m$.

The latitude and longitude can be easily computed as $d/R = 6.4 \times 10^{-4}$ N and $t/R = 7.38 \times 10^{-5}$ W .

II) **[2 points]**. It is getting too full on Planet Earth and half of the population has to move out. So humanity decides to install a big black ball in solar orbit, just at the right distance for a comfortable living temperature of 25 degrees Celsius. The surface albedo of this ball α_p is 0.3 and it will not have an atmosphere. Knowing that on earth (at a distance of $1.5 \times 10^{11} m$ from the sun) the flux density of solar radiation is 1368 W/m^2, what will be the distance from this new human planet to the sun.

Solution II: See Attachment 1

III) **[1 point]** . The aliens are sending us signals of electromagnetic radiation at different wavelengths to get our attention from space. They start with green and violet light. They choose a particularly clear day to send these signals

a) What kind of scattering would you expect to occur as the light travels through the atmosphere? Which light would be scattered more and by how much ?

b) [**Bonus Question !**] What if it was a rainy day when they sent the signals ? What kind of scattering would occur ?

Solution III: Rayleigh scattering, because the scattering particles (air molecules) are small compared to the wavelength, $x << 1$.

Violet is scattered more strongly. Scattering efficiency K scales inversely with the wavelength: $K_\lambda \propto \lambda^{-4}$

$$\frac{K(green)}{K(violet)} = (400nm/510nm)^4 = 0.38$$

(b) Bonus question: Geometric optics regime of scattering for raindrops

Attachment 1

It is getting too full on Planet Earth and half of the population has to move out. So humanity decides to install a big black ball in solar orbit, just at the right distance for a comfortable living temperature of 25 degrees Celsius.
The surface albedo of this ball α_p is 0.3 and it will not have an atmosphere.
Knowing that on earth (at a distance of $1.5\text{x}10^{11}$m from the sun) the flux density of solar radiation is $1368 W/m^2$, what will be the distance from this new human planet to the sun.

Solution:

We need to compute the flux density F_p on the surface of the new planet:
From radiative equilibrium with $T_p = 298.15K$:

$$(1-\alpha_p)\ F_p * A_{received} = \sigma T_p^4 * A_{radiated}$$

$$(1-\alpha_p)\ F_p = 4\sigma T_p^4$$

$$F_p = 4\sigma T_p^4/(1-\alpha_p) = 2.56\text{x}10^3$$

Using the inverse square law we can now estimate the distance R_p that the new planet will have to have from the sun (i.e. the radius of the planets solar orbit):

$$R_p = \text{sqrt}(F_{earth}/F_p)*R_{earth} = 1.0965\text{x}10^{11}\text{m}$$