Chapter 5: Atmospheric Turbulence and Boundary Layers

Introduction:

In this section, we give a short summary of the basis of surface layer transport in the atmospheric boundary layer (ABL). Since the time and length scales are long compared scale to the microscopic the fluid may be treated as a continuum. The surface layer is the first turbulent layer above the surface in which fluxes of momentum and heat are constant with height. The reasoning behind this assumption is that any flux divergence (non-constant flux) would lead to local non-stationarity until the constant flux is re-established. As a result, we can "measure" the surface flux by measuring the flux at some altitude say a few meters above the surface. This simplifies the problem because the turbulent surface layer is more accessible to measurements than the ill-defined surface itself.

a) Prandtl mixing length theory

To grasp the principle of turbulent transfer in the surface layer, we first look at the transfer of momentum. Momentum transport in an incompressible fluid with negligible density variations is the same as "velocity" transport. Assume some sort of mean velocity profile close to the surface (Figure WindProfile). With mean velocity profile, we denote the wind speed averaged over some time period, for which the assumption of stationarity holds, i.e. typically between 10 and 60 minutes in the surface layer. We therefore de-compose the wind speed (and other quantities) in the mean and the deviation from the mean (Reynold's decomposition):

$$u = \bar{u} + u', \quad \bar{u} = \int_{\Delta t} u(t)\, dt$$

Here, u is the instantaneous streamwise wind velocity and Δt is typically 20 minutes. In practice, when we have a time series of wind measurements at certain time intervals, we can get the mean by a simple average:

$$\bar{u} = \frac{1}{N}\sum_{i=1}^{N} u_i$$

Based on this notation convention, we can define a quantity called Turbulent Kinetic Energy (TKE), which is basically the amount of energy contained in the deviations from the mean wind speed. For convenience, we do not include the mass (air density) in the definition of the turbulent kinetic energy, $[e] = \frac{m^2}{s^2}$ and write:

$$\bar{e} = \overline{u'^2} + \overline{v'^2} + \overline{w'^2} = \sigma_u^2 + \sigma_v^2 + \sigma_w^2$$

By analogy to momentum transport in a laminar fluid, we want to relate the shear stress $[\tau_{xz}] = \frac{N}{m^2}$ to the gradient of the mean velocity:

$$\tau_{xz} = -\rho \nu_t \frac{\partial \overline{u}}{\partial z},$$

where $[\rho] = \frac{kg}{m^3}$ is the mass density of the air and $[\nu_t] = \frac{m^2}{s}$ is a turbulent viscosity, which will be called turbulent exchange coefficient K in the following. An important step is now to realize that the vertical momentum or velocity transport in a turbulent flow is the covariance between streamwise and vertical velocity:

$$\frac{\tau_{xz}}{\rho} = Cov(u,w) = \frac{1}{N}\sum_{i=1}^{N}\left(u_i - \overline{u}\right)\left(w_i - \overline{w}\right) = \overline{u'w'}$$

The concept of momentum transport by turbulent eddies is illustrated in Fig. WindProfile. The momentum transport or shear stress is also used to define a velocity scale, called the friction velocity $[u_*] = \frac{m}{s}$, which is the square root of the constant shear stress close to the surface:

$$\frac{\tau_{xz}}{\rho} = \overline{u'w'} =: u_*^2 = -\nu_t \frac{\partial \overline{u}}{\partial z} = -K\frac{\partial \overline{u}}{\partial z}$$

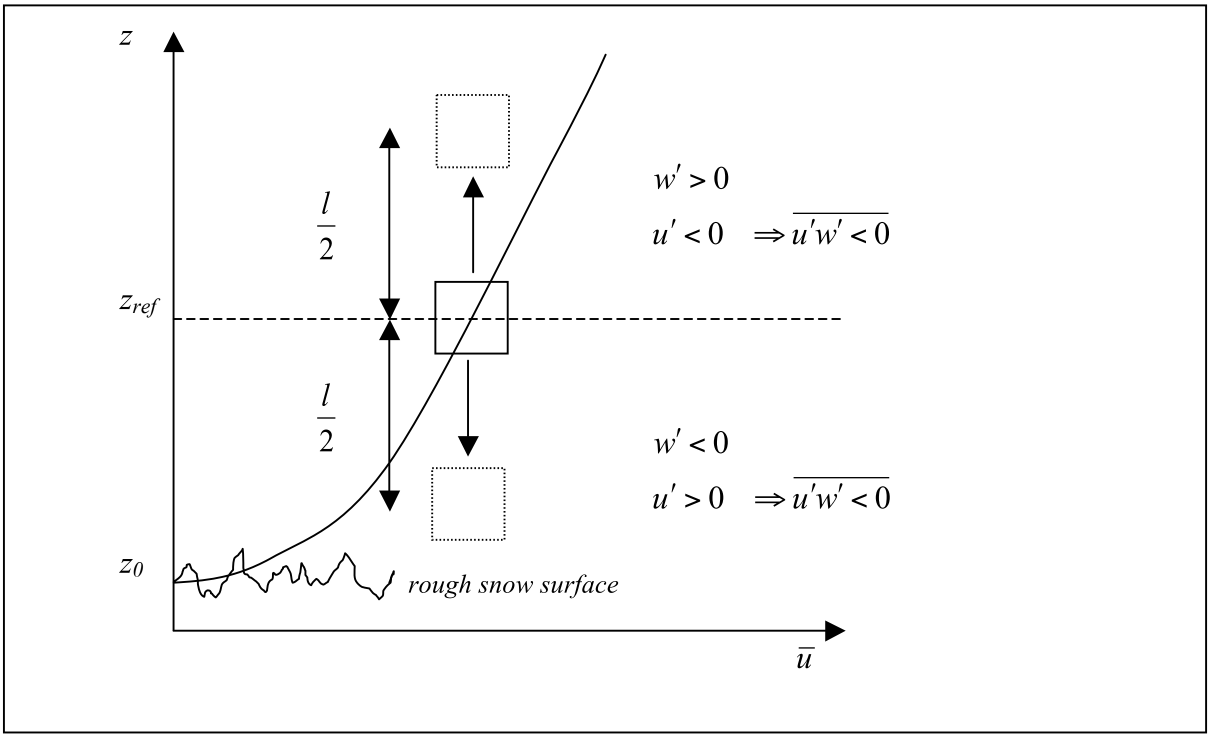

Figure WindProfile: Conceptual picture of turbulent eddy movement in the surface layer as a basis to derive Prandtl mixing layer theory for surface fluxes

This fundamental equality of velocity transport and covariance in a turbulent flow leads directly to the mixing length argument. First, we look at the mean velocity difference when moving an air parcel or eddy up or down by $l/2$ relative to some reference height z_{ref}:

$$\Delta\overline{u} = \overline{u}(z_{ref} + \frac{l}{2}) - \overline{u}(z_{ref} - \frac{l}{2})$$

This velocity difference can be locally approximated by (Taylor series):

$$\Delta\overline{u} = l\frac{\partial\overline{u}}{\partial z}$$

If we further assume isotropic turbulence, i.e. that the velocity variance is independent of direction and velocity fluctuations are equally strong in the vertical and in the streamwise dircction:

$$|u'| \approx |w'|,$$

we can write:

$$-\overline{u'w'} = (\Delta\overline{u})^2 = l^2\left|\frac{\partial\overline{u}}{\partial z}\right|\frac{\partial\overline{u}}{\partial z}.$$

Let us further assume that the eddy length scale $l = kz$ is proportional to the distance from the ground with some constant k, which is called the von Karman constant to arrive at an estimation of the turbulent exchange coefficient based on the Prandtl mixing length concept:

$$K = l^2\left|\frac{\partial\overline{u}}{\partial z}\right| = k^2 z^2\left|\frac{\partial\overline{u}}{\partial z}\right|$$

The mixing length is a distance that a fluid parcel will keep its original characteristics before dispersing them into the surrounding fluid.

b) Description of Profiles:

Directly from Eq. (MixingLength) above together with the definition of the friction velocity, u_*, but also from similarity considerations we can find:

$$\frac{\partial\overline{u}}{\partial z}\frac{z}{u_*} = \frac{1}{k}\left(= \psi_M = f(\frac{z}{L})\right)$$

This equation states that a unique form of a non-dimensional wind profile exists. The expression in round brackets expresses that this form may be a function of a stability parameter $\left[\frac{z}{L}\right] = 1$, which describes effects of atmospheric stratification close to the ground. This is further discussed below. Now we can integrate the non-dimensional wind profile between some height at the surface called the roughness length z_0 and some arbitrary height above the surface often taken as the measurement height.

$$k \int_{u(z_0)=0}^{u(z)} d\bar{u}' = u_* \int_{z_0}^{z} \frac{dz'}{z'} \quad ; \quad k(\bar{u}(z) - \bar{u}(z_0)) = u_*(\ln(z) - \ln(z_0))$$

and arrive at the famous logarithmic wind profile:

$$\bar{u}(z) = \frac{u_*}{k} \ln(\frac{z}{z_0})$$

c) Application to Sensible and Latent Heat:

For turbulent transport of sensible heat $\left[q_s\right] = \frac{W}{m^2}$ the same principle applies as for momentum: the eddies do not only carry velocity but also heat, moisture and other scalar quantities. Therefore, we can also write following the same principle as for momentum (described above):

$$\frac{q_s}{\rho c_p} = \overline{T'w'} = -K_H \frac{\partial \overline{T}}{\partial z} =: T_* u_*$$

Here $\left[c_p\right] = \frac{W}{kg\,K}$ is the heat capacity, $\left[K_H\right] = \frac{W}{m^2}$ the turbulent exchange coefficient for heat and we have defined the temperature scale $\left[T_*\right] = K$. Note that we again assume that the flux is constant over some distance from the ground and that therefore the temperature and velocity scales are also constant with height and time (stationarity). By using the same similarity formalism as for the wind profile we can write the non-dimensional wind profile:

$$\frac{\partial \overline{T}\, z}{\partial z T_*} = \frac{1}{k} (= \psi_H = f(\frac{z}{L}))$$

$$k \int_{T(z_H)}^{T(z)} d\overline{T}' = T_* \int_{z_H}^{z} \frac{dz'}{z'} \quad ; \quad k(\overline{T}(z) - \overline{T}(z_H)) = T_*(\ln(z) - \ln(z_H))$$

Here we have used a new roughness length, z_H, which formally takes into account that surface features may have a different effect on scalar (heat) transport then on momentum transport. Experiments confirm that an identical von Karman constant $k = 0.4$ applies.

We can now formulate temperature profile and heat flux:

$$\overline{T}(z) - \overline{T}(z_H) = \frac{T_*}{k} \ln(\frac{z}{z_H}) \, ; \quad \frac{q_s}{\rho c_p} = -\overline{T'w'} =: -T_* u_* = \frac{k u_*}{\ln(\frac{z}{z_H})} (\overline{T}(z) - \overline{T}(z_H))$$

For convenience, we often directly use the measured mean wind speed $\overline{u}$, the air temperature $\overline{T}_a$ and (if available) a measured surface temperature $\overline{T}_S$, such that the sensible heat flux can be written as:

$q_s = C_H \rho c_p \overline{u} (\overline{T}_a - \overline{T}_s)$ The heat exchange coefficient in Eq. „HeatFlux“ is then given by comparison with Eq. SimpleHeatFlux and using Eq. WindProfile:

$$C_H = \frac{k^2}{\ln(\frac{z}{z_H})\ln(\frac{z}{z_0})}$$

The development of the corresponding equation for latent heat is analogous. The only complication for latent heat is that phase change at the snow surface happens as atmospheric moisture gets deposited or as moisture gets evaporated (if the surface is wet) or sublimated (if the surface is dry ice) into the atmosphere. The moisture gradient is the driving force for latent heat transfer. Note that in case of a snow or ice surface, the water vapour pressure at the surface can be approximated by the saturation vapour pressure at the surface temperature. Therefore, the latent heat exchange, Q_l (W m^{-2}) can be expressed as the difference in vapour pressure between the surface and the air with the identical exchange coefficient as for sensible heat and considering the latent heat for the phase change of water or ice to vapour, $L^{w/i}$ (2256/2838 kJ kg^{-1}):

$$q_l = 0.622 L^{w/i} C_H \frac{\rho}{p_a} \bar{u}(p_v(\bar{T}_a) - p_v^s(\bar{T}_s))$$

Here the numerical constant 0.622 is the ratio of the gas constant for dry air over that of water vapor and p_a (N m^{-2}) is the atmospheric pressure. The partial vapour pressure in the air, p_v (N m^{-2}) is usually obtained from a relative humidity measurement, rH (%) and the saturation vapour pressure for the air temperature:

$$p_v(\bar{T}_a) = rH\ p_v^s(\bar{T}_a)).$$

d) Stability Effects:

One complication is that the atmospheric surface layer may show effects of stratification. If warm air moves over a cold snow surface, or if the high snow emissivity leads to very low snow surface temperatures, the air immediately above the snow will cool. Either of these cases can lead to a strong temperature gradient in the air. Cold dense air tends to remain at the surface. While this is obvious we must also take note that a rising air parcel results in a pressure drop, which also cools the air, causing stratification to persist. Conversely, if the surface temperature is higher, it may sufficiently raise the air temperature, in which case buoyant warm air parcels may move upward.

The stability correction functions $\psi_{M/H}$ introduced formally above describe the effect. With the Ansatz:

$\psi = 1 + g(\frac{z}{L})$, where L is the Obukhov length: $L =: \frac{u_*^{\,2}}{k\frac{g}{\overline{T}}T_*}$, one can simply express the stability correction as an additive correction term in the integrative form of the wind profile or the temperature profile, e.g.:

$$\overline{u}(z) = \frac{u_*}{k}\ln(\frac{z}{z_0} - \Psi_M(\frac{z}{L}))$$

The exchange coefficient for sensible heat takes the form:

$$C_H = \frac{k}{\ln(\frac{z}{z_0}) - \Psi_M(\frac{z}{L})}\frac{k}{\ln(\frac{z}{z_H}) - \Psi_H(\frac{z}{L})}$$

The correction functions ψ or Ψ are empirically determined and formulations can be found in Paulson, Businger and Dyer, or Stearns and Weidner. The problem is discussed again in Chapter 10, where particular characteristics of exchange over snow covered surfaces are discussed.