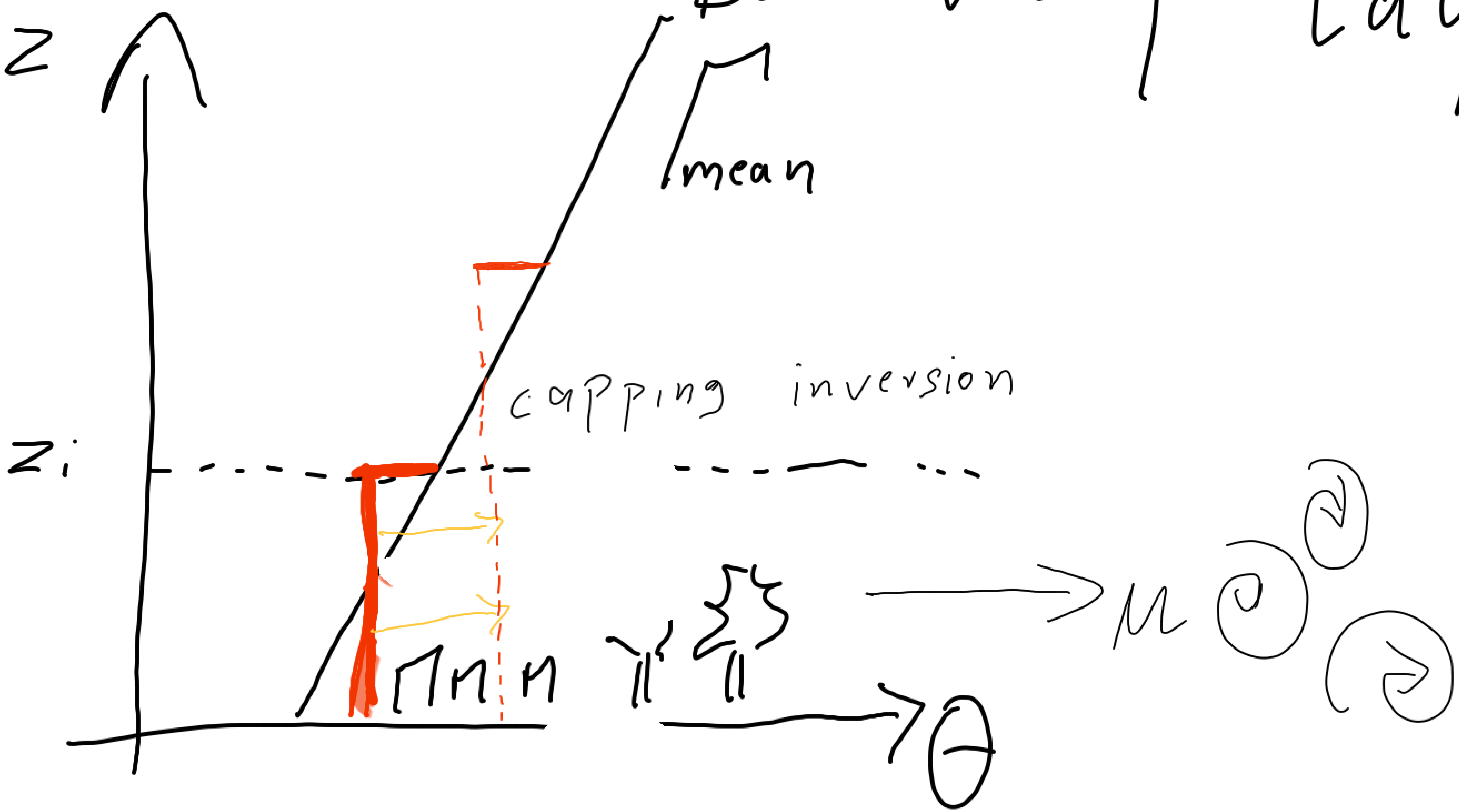
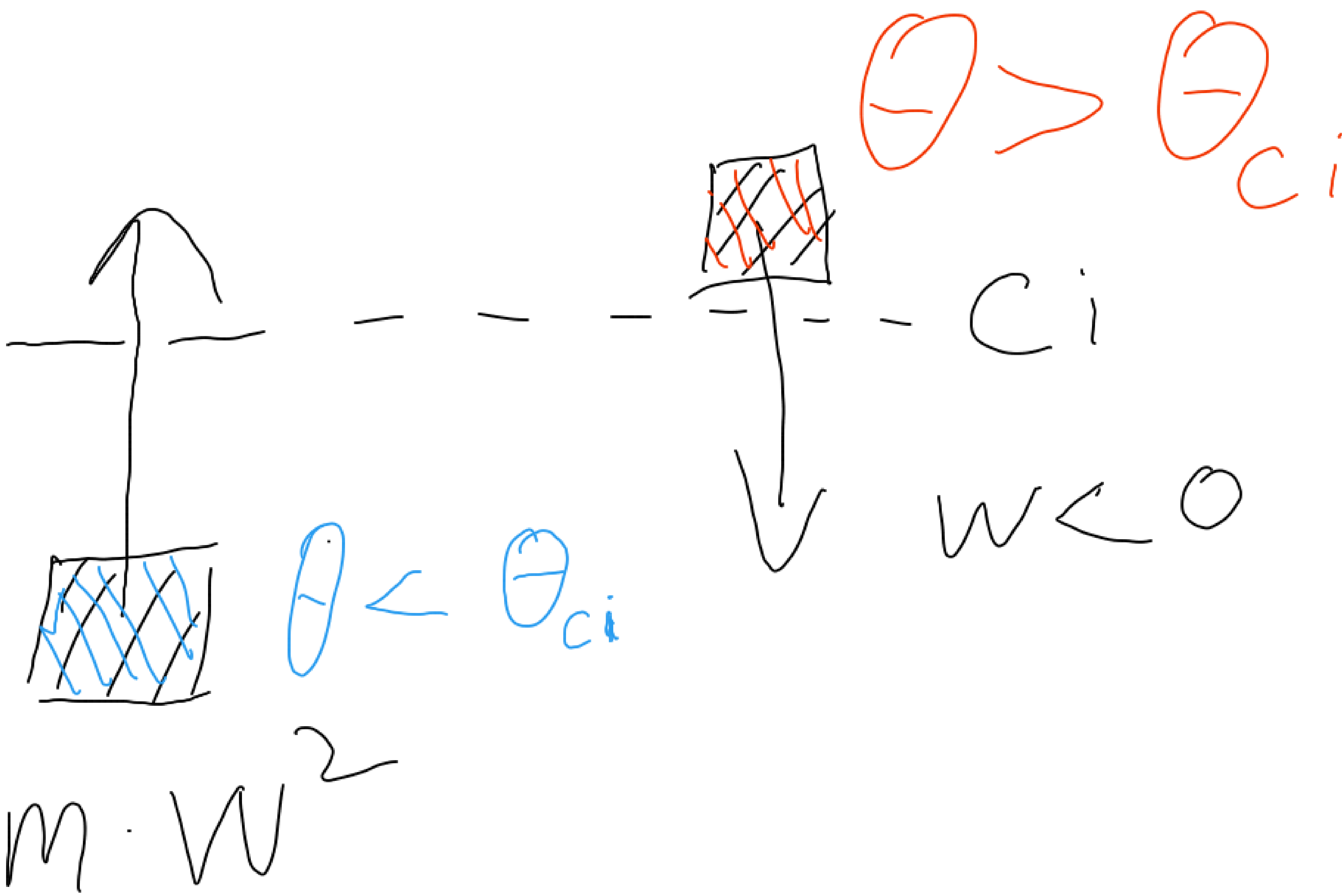
Session: 521197
118137

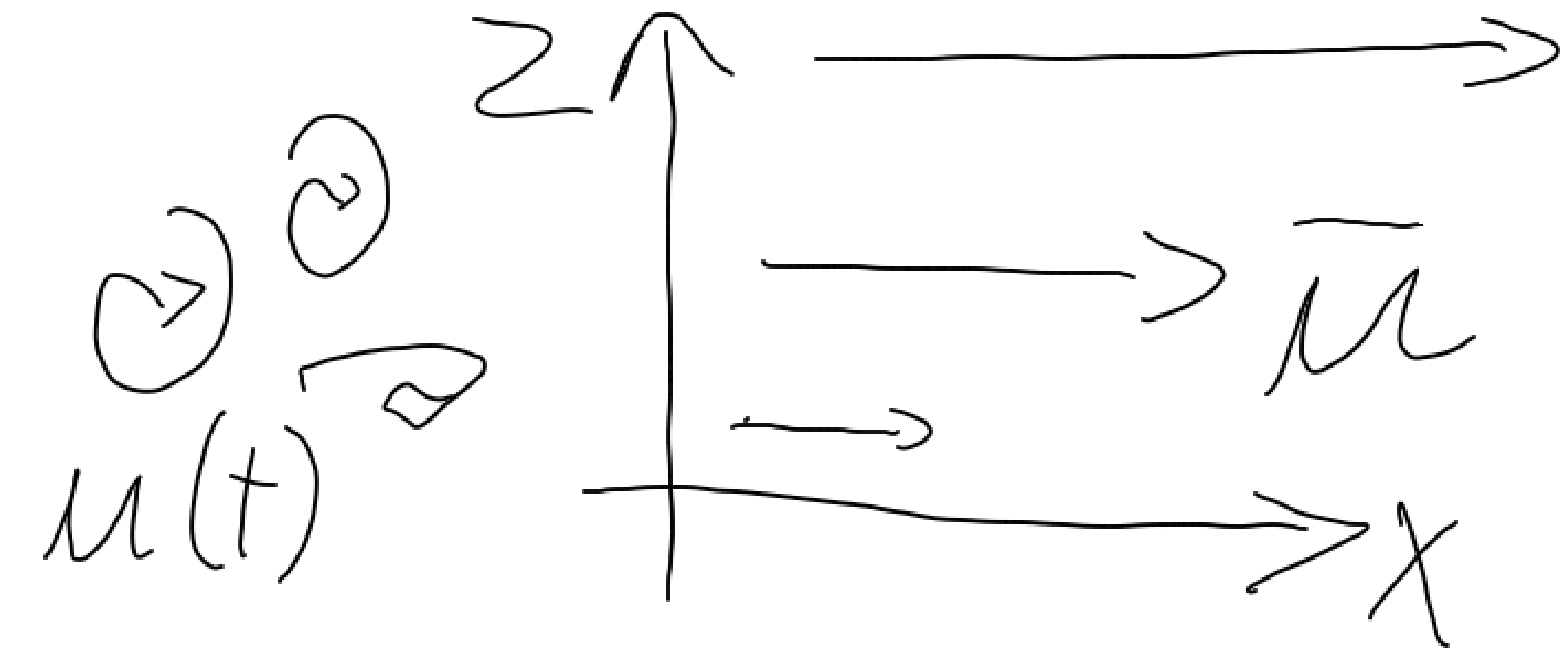
Mixing in the Atmospheric Boundary Layer




u


$\theta > \theta_{ci}$
c_i
$w < 0$
$\theta < \theta_{ci}$
$m \cdot w^2$

Reynolds Averaging



X: direction of mean wind

$$u = \bar{u} + u'$$

$$\bar{u} = \frac{1}{\Delta t}\int_{\Delta t} u(t)\, dT$$

$$\bar{u} = \frac{1}{N}\sum_{i=1}^{N} u_i$$

Discrete Measurements

$$\overline{w} = 0$$

Turbulent kinetic Energy

$$TKE = \frac{m}{2}\left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2}\right)$$

$$\boxed{\overline{u'^2} = \frac{1}{N}\sum_{i=1}^{N} u_i'^2}$$

$$e = \frac{TKE}{m}$$

$$\boxed{u_i' = u_i - \overline{u_i}}$$

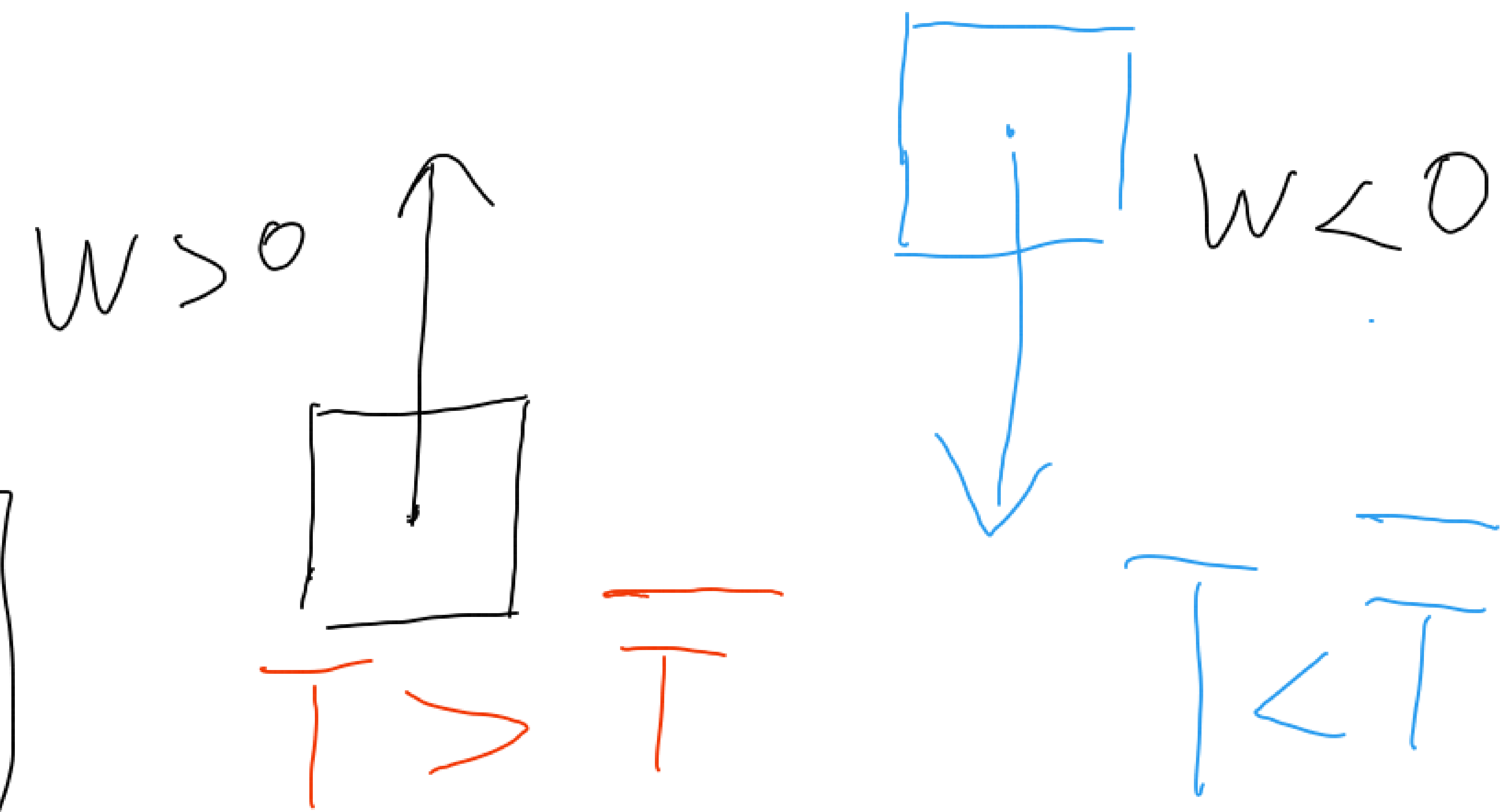
$$e = \frac{1}{2}\left(\sigma_u^2 + \sigma_v^2 + \sigma_w^2\right)$$

Covariance: $\mathrm{Cov}(w, T)$

$$= \frac{1}{N} \sum_{i=1}^{N} \overbrace{(w - \overline{w})}^{w'} \overbrace{(T - \overline{T})}^{T'}$$

$$= \overline{w'T'}$$

$$\boxed{\Rightarrow \text{Fluxes}}$$



TKE budget equation:

$$\frac{de}{dt} = \underbrace{\frac{g}{\overline{\theta}}\,\overline{w'\theta'}}_{\text{I}} - \underbrace{\overline{u'w'}\,\frac{\partial \overline{u}}{\partial z}}_{\text{II}}$$

$$\underbrace{\frac{\partial\, \overline{w'e}}{\partial z}}_{\text{III}} - \underbrace{\frac{1}{\overline{\rho}}\,\frac{\partial\, \overline{w'p'}}{\partial z}}_{\text{IV}} - \underbrace{\varepsilon}_{\text{V}}$$

I : Buoyancy Production

II : Shear Production

III : Turbulent Transport

IV : Pressure Correlation

V : Dissipation

Re-distribute

$$\left[\frac{de}{dt}\right] = \frac{m^2}{s^3}$$

Description of Stability:

I vs. II

Richards number:

$$Ri = \frac{\mathrm{I}}{\mathrm{II}}$$

Approximate turbulent fluxes by gradients : $\overline{w'\theta'} = K_1 \dfrac{\partial \bar{\theta}}{\partial z}$

$$\Rightarrow$$

$$R_i = \frac{\dfrac{g}{\bar{\theta}} \dfrac{\partial \bar{\theta}}{\partial z}}{\left(\dfrac{\partial \bar{u}}{\partial z} \right)^2}$$

Fluxes:

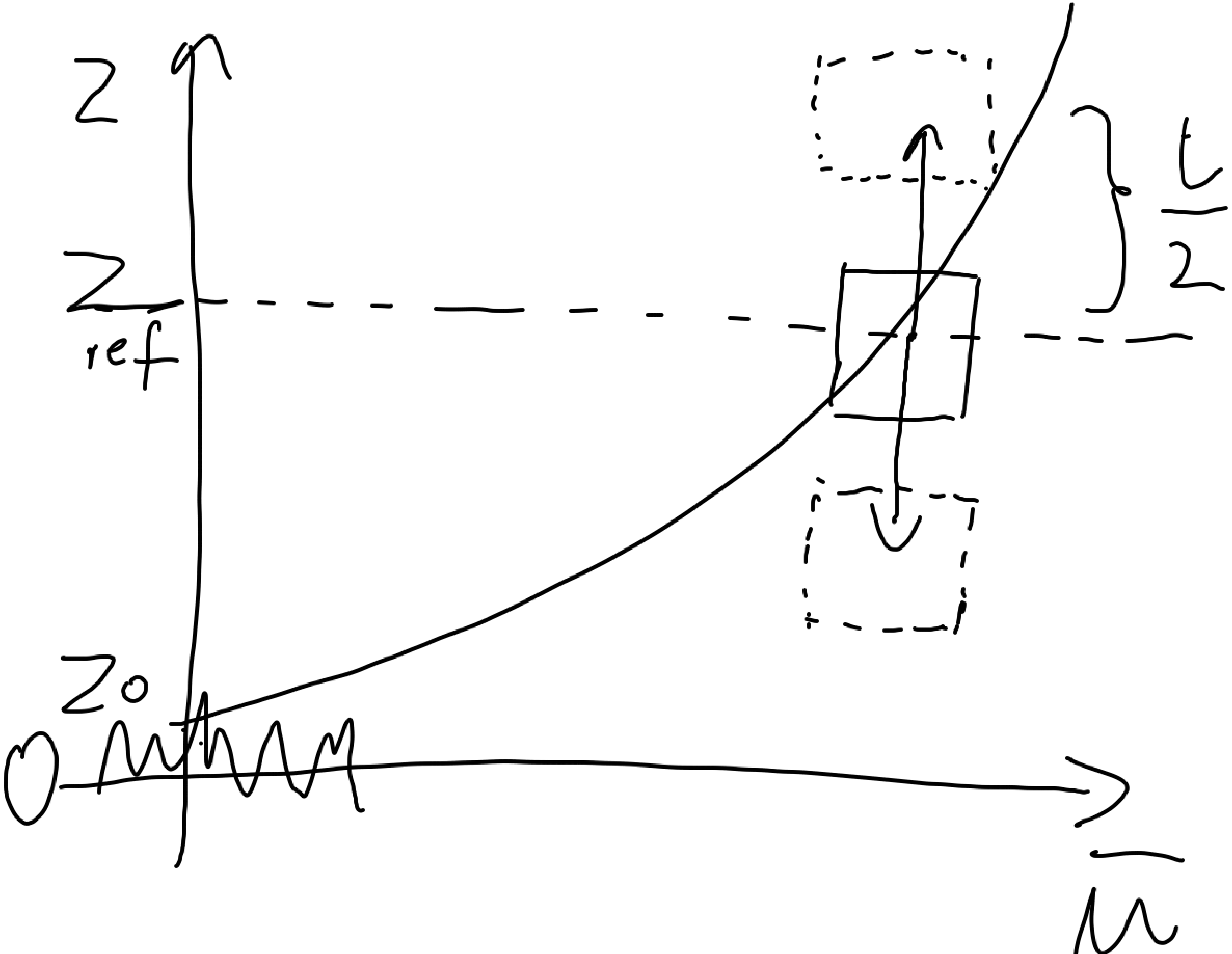
Laminar: $\tilde{\tau}_{xz} = -\rho \nu \frac{\partial \bar{u}}{\partial z}$

ν : viscosity

turbulent:

$$\tilde{\tau}_{xz} = -\rho \bar{K} \frac{\partial \bar{u}}{\partial z}$$

$\tilde{\tau}_{xz}$: shear stress or momentum flux



$$\left.\begin{array}{l} w' > 0 \\ u' < 0 \end{array}\right\}$$

$$\left.\begin{array}{l} w' < 0 \\ u' > 0 \end{array}\right\}$$

From the Figure:

Moving a slower fluid parcel up:

$w' > 0$
$u' < 0$ $\Rightarrow u'w' < 0$ ...... $\overline{u'w'} < 0$

Moving a fluid parcel down:

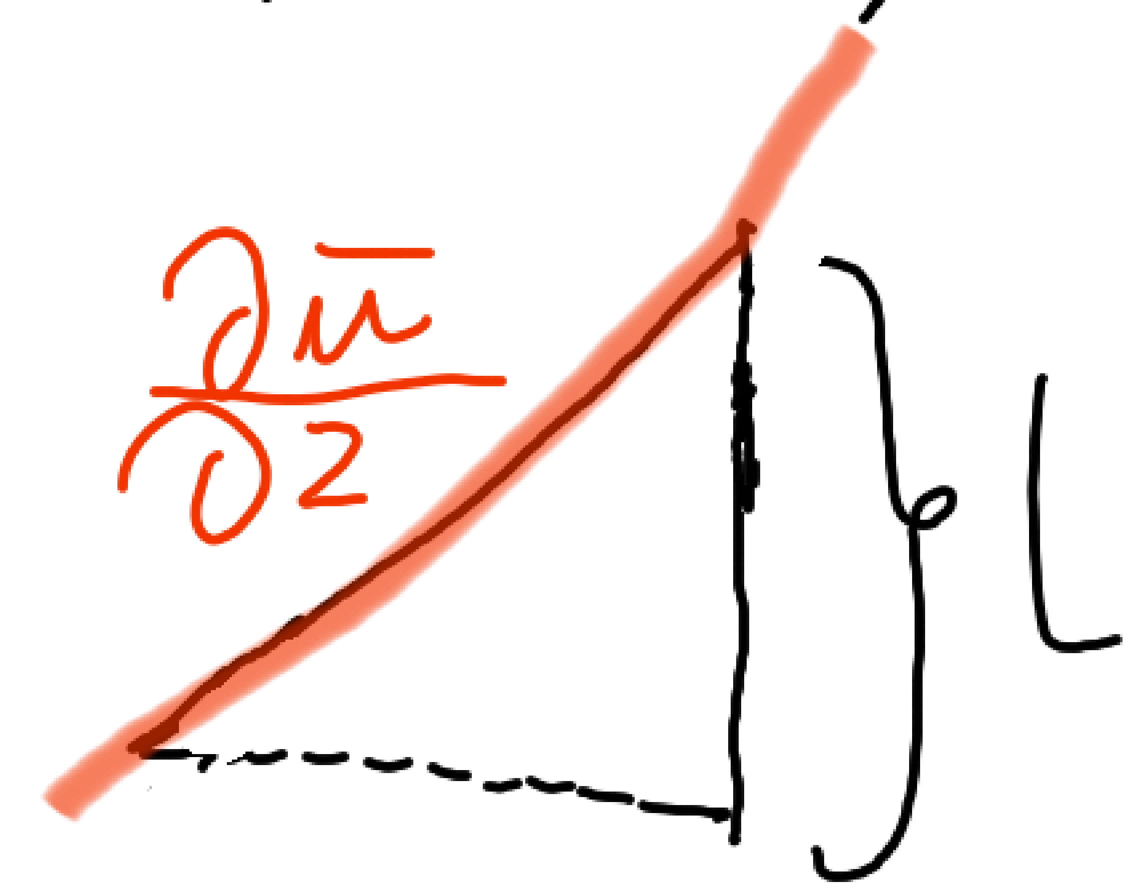
$w' < 0$
$u' > 0$ $\Rightarrow u'w' < 0$ ...... $\overline{u'w'} < 0$

Derivation of the Prandtl (1922) wind profile:

Looking at the velocity difference over vertical distance l

$$\Delta \bar{u} = \bar{u}\left(z_{ref} + l/2\right) - \bar{u}\left(z_{ref} - l/2\right)$$

Taylor: $\Delta \bar{u} = l \dfrac{\partial \bar{u}}{\partial z}$



Assume: $|u'| \approx |w'|$

$$\Rightarrow \overline{-u'w'} = \Delta \bar{u}^2 = l^2 \left| \frac{\partial \bar{u}}{\partial z} \right| \frac{\partial \bar{u}}{\partial z}$$

Assume: $l = kz$ "eddy size scales with distance from surface"

k : von Karman constant

Momentum Flux:

$$\frac{\tau_{xz}}{\rho} = \overline{-u'w'} = \bar{K} \frac{\partial \bar{u}}{\partial z} =: u_*^2$$

u_* : friction velocity

$$K = l^2 \left| \frac{\partial \bar{u}}{\partial z} \right| = k^2 z^2 \left| \frac{\partial \bar{u}}{\partial z} \right| = \frac{u_*^2}{\frac{\partial \bar{u}}{\partial z}}$$

$$\Rightarrow \frac{\partial \bar{u}}{\partial z} = \frac{u_*}{k z} ; \quad k \int_{\bar{u}(z_0)}^{\bar{u}(z)} d\bar{u}' = u_* \int_{z_0}^{z} \frac{dz'}{z} ;$$

$$\boxed{\bar{u}(z) = \frac{u_*}{k} \ln\left(\frac{z}{z_0}\right)}$$

Only if: $u_* \neq f(z)$;

Typical Flux Formalation:

$$\frac{\tau_{xz}}{\rho} = u_*^2 =: C_D \, \bar{u}(z)^2$$

$$\Rightarrow \quad C_D = \frac{k^2}{\ln^2\left(\frac{z}{z_0}\right)}$$

Heat Flux:

$$q_s = \rho c_p C_H \bar{u} \Delta T$$

$$\Rightarrow C_H \; (= C_D) = \frac{k^2}{\ln^2\left(\frac{z}{z_0}\right)}$$

Questions

1. Stationarity
2. Constant Flux (with height)