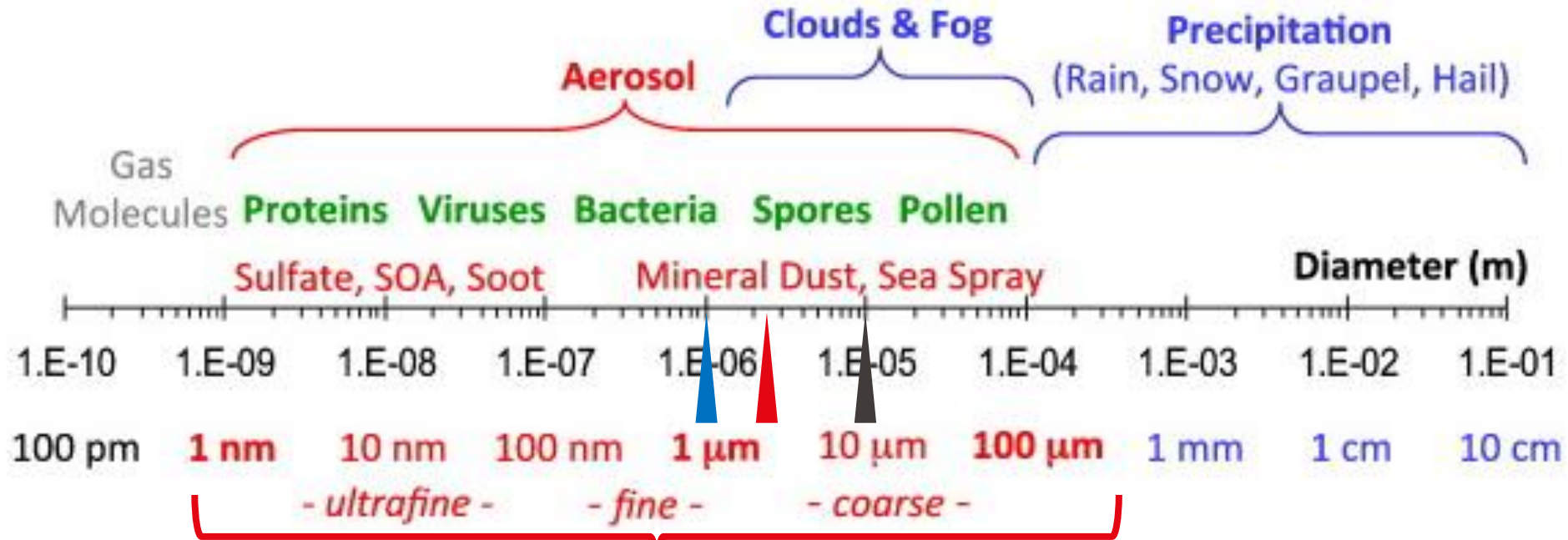
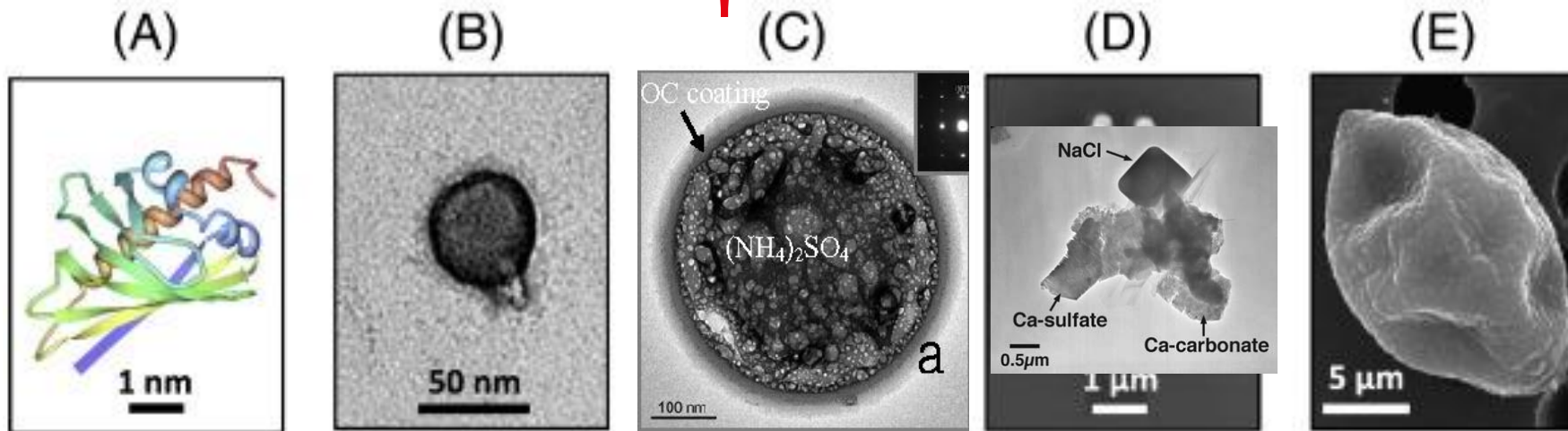


- Know sources of aerosols and factors determining lifetime
- Distinguish between primary and secondary particles
- Understand the principles of various particle diameters and particle motion
- Be able to calculate aerosol size distributions
  - Number, surface and volume
  - Lognormal size distribution
- Understand the principles of gas / particle partitioning
  - Saturation vapor pressure
  - Gibbs energy
  - Henry's and Raoult's laws



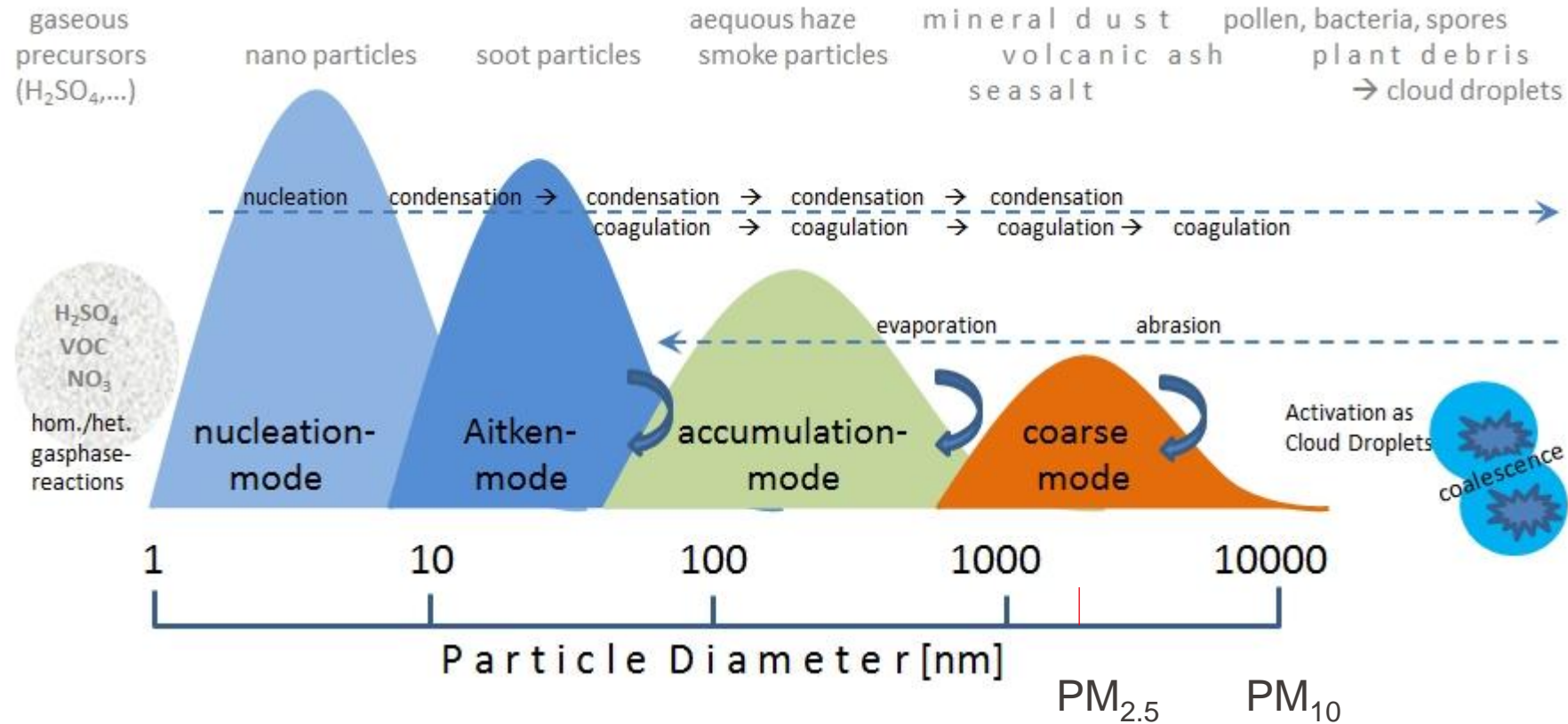
**PM<sub>1</sub>**  
**PM<sub>2.5</sub>**  
**PM<sub>10</sub>**

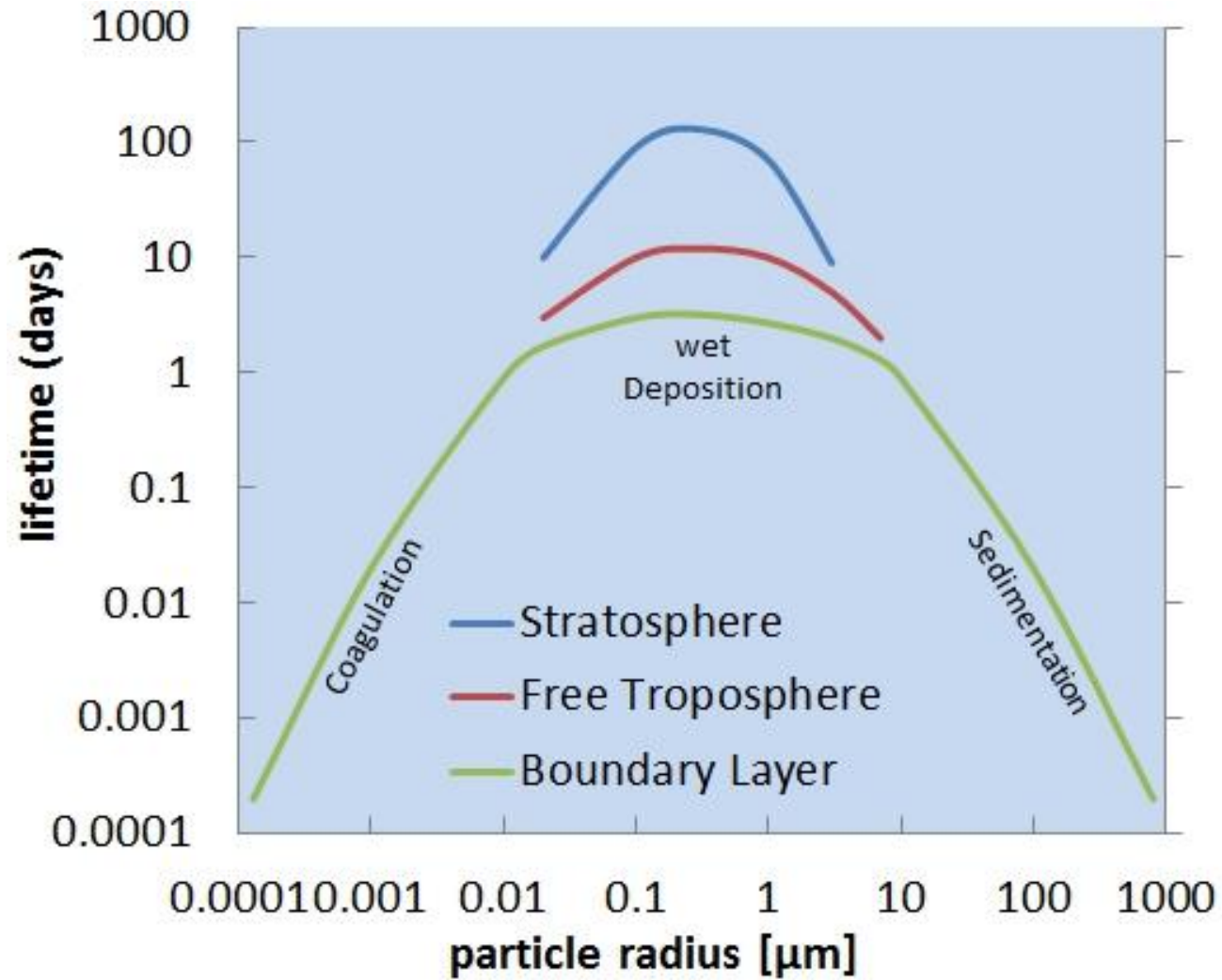
Particulate matter with a diameter equal to or smaller than  $x \mu$ m (air quality)



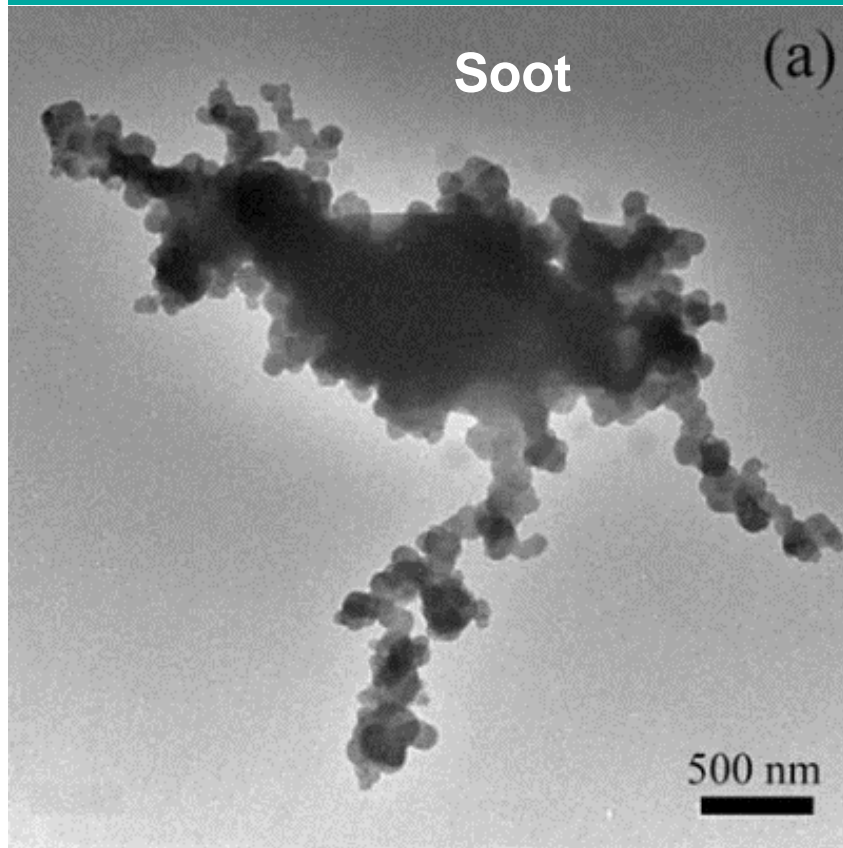
- (A) protein,
- (B) virus
- (C) Urban particle
- (D) Mineral dust
- (E) pollen grain





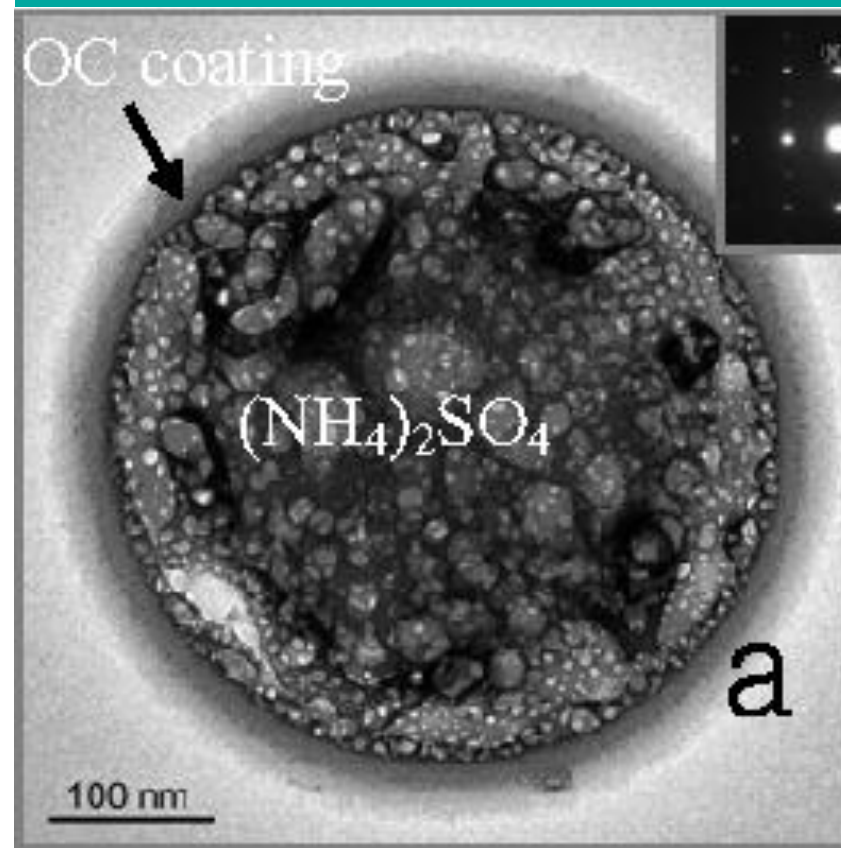


## Primary



Primary particles are emitted from their source as particles. Examples: soot for combustion, mineral dust from soil.

## Secondary



Secondary particles are NOT emitted as particles, but they are formed from gases in the atmosphere through condensation or chemical reactions (in clouds). Examples: Ammoniumsulfate, ammoniumnitrate, secondary organic aerosol

Images are from a Transmission Electron Microscope (TEM).

Links to papers: [right](#), [left](#)



# Sources of aerosols

## Mixed



Forest fires



Sea spray



Dust



Volcanic eruptions



Traffic / Transport



Domestic activities



Industry



Agriculture

## Anthropogenic

**TABLE 2.20 Global Emission Estimates for Major Aerosol C**

Source	Estimated Flux, Tg yr <sup>-1</sup>
Natural	
Primary	
→ Mineral dust	
0.1–1.0 μm	48
1.0–2.5 μm	260
2.5–5.0 μm	609
5.0–10.0 μm	573
→ 0.1–10.0 μm	1490
Seasalt	10,100
Volcanic dust	30
Biological debris	50
Secondary	
Sulfates from DMS	12.4
Sulfates from volcanic SO <sub>2</sub>	20
Organic aerosol from biogenic VOC	11.2
Anthropogenic	
Primary	
Industrial dust (except black carbon)	100
Black carbon	12 <sup>a</sup>
Organic aerosol	81 <sup>a</sup>
Secondary	
Sulfates from SO <sub>2</sub>	48.6 <sup>b</sup>
Nitrates from NO <sub>x</sub>	21.3 <sup>c</sup>

<sup>a</sup>Tg C.

<sup>b</sup>Tg S.

<sup>c</sup>Tg NO<sub>3</sub><sup>-</sup>.

**Table 5.3** Estimates (in Tg per year) for the year 2000 of (a) direct particle emissions into the atmosphere and (b) *in situ* production

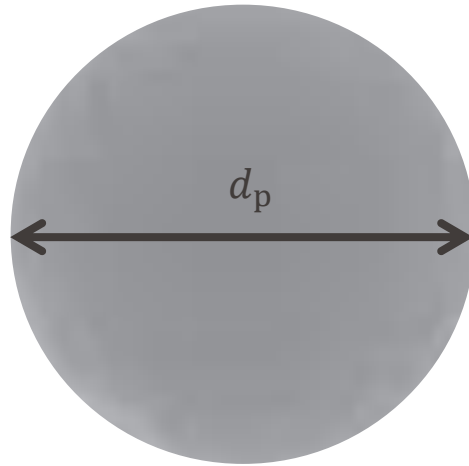
(a) Direct emissions			(b) <i>In situ</i>		
	Northern hemisphere	Southern hemisphere		Northern hemisphere	Southern hemisphere
Carbonaceous aerosols			Sulfates (as NH <sub>4</sub> HSO <sub>4</sub> )	145	55
Organic matter (0–2 μm) <sup>a</sup>			Anthropogenic	106	15
Biomass burning	28	26	Biogenic	25	32
Fossil fuel	28	0.4	Volcanic	14	7
Biogenic (>1 μm)	—	—	Nitrate (as NO <sub>3</sub> <sup>-</sup> )		
Black carbon (0–2 μm)			Anthropogenic	12.4	1.8
Biomass burning	2.9	2.7	Natural	2.2	1.7
Fossil fuel	6.5	0.1	Organic compounds		
Aircraft	0.005	0.0004	Anthropogenic	0.15	0.45
Industrial dust, etc. (>1 μm)			Biogenic	8.2	7.4
→ Sea salt			<sup>a</sup> Sizes refer to diameters. [Adapted from Intergovernmental Panel on Climate Change, <i>Climate Change 2001</i> , Cambridge University Press, pp. 297 and 301, 2001.]		
<1 μm	23	31			
1–16 μm	1,420	1,870			
Total	1,440	1,900			
→ Mineral (soil) dust					
<1 μm	90	17			
1–2 μm	240	50			
2–20 μm	1,470	282			
Total	1,800	349			

Note: It is not straight forward to estimate the total emission fluxes, hence there can be deviations between publications.



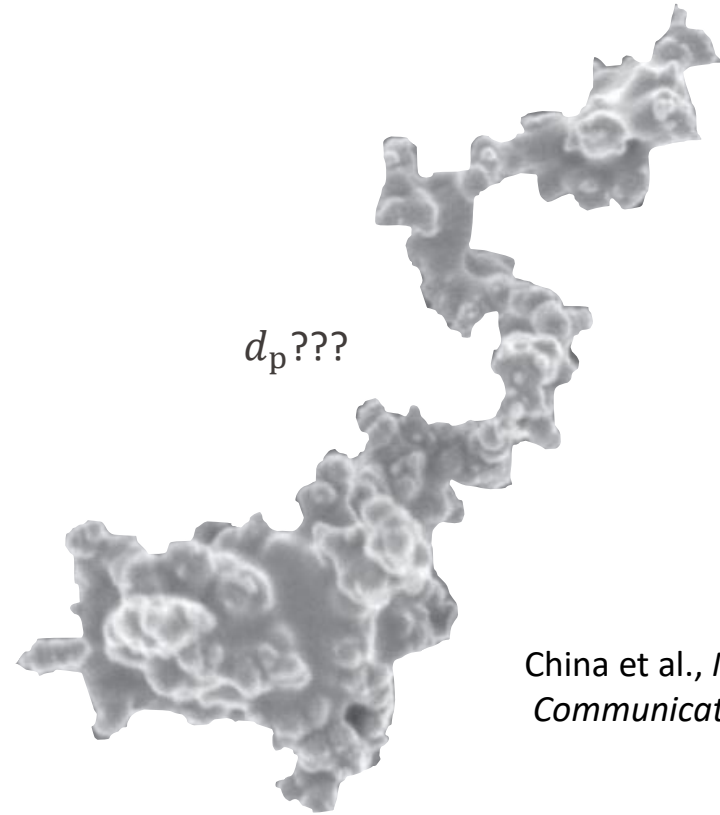
## Spherical particle

Geometric diameter  $d_p$



## Fractal aggregate

$d_p ???$



China et al., *Nature Communications*, 2013

To define particle sizes precisely we use the concept of an **equivalent diameter**:  
«Diameter of a sphere, having the same value of a particular physical property (that is measurable) as the particle under consideration».

Definition of the **volume equivalent diameter** (through «reference particle»):  
«Diameter of a sphere, having the same volume as the particle under consideration».

This means:

$$V_{\text{ref}}(d_{\text{geom}}^{\text{ref}}) = V_{\text{particle}} \xrightarrow{\text{use: } d_{\text{ve}} := d_{\text{geom}}^{\text{ref}}} \frac{\pi}{6} d_{\text{ve}}^3 = V_{\text{particle}}$$
$$\Rightarrow d_{\text{ve}} = \left( \frac{6}{\pi} V_{\text{particle}} \right)^{1/3}$$

There are other diameter definitions that mainly depend on the measurement principle (not treated in this lecture), e.g.:

- Optical diameter
- Electrical mobility diameter
- Aerodynamic diameter
- Vacuum aerodynamic diameter

Consider that aerosol particles move in the atmosphere, i.e. a gas. A useful dimensionless parameter to describe the interaction between the particles and gas is the Reynolds number  $Re$ , the ratio of ***inertial*** to ***viscous*** forces. For an object of characteristic length scale  $L$ ,

$$Re = \frac{\rho_g v L}{\eta} \quad \rightarrow \quad \frac{\text{inertial forces}}{\text{viscous forces}}$$

where

$\rho_g$ : the density of the gas, for air  $\rho_{g,air} = 1.2 \text{ kg/m}^3$  ( $T = 293 \text{ K}$ ,  $p = 1 \text{ atm}$ )

$v$ : the velocity of the gas relative to the object

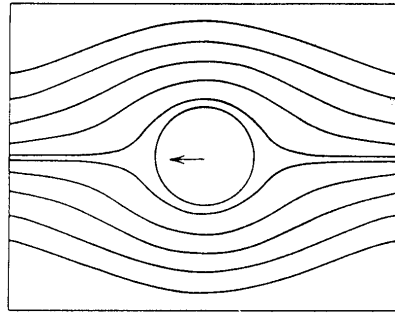
$\eta$ : the viscosity of the gas, for air  $\eta_{air} = 1.8 \times 10^{-5} \text{ Pa.s}$  ( $T = 293 \text{ K}$ ,  $p = 1 \text{ atm}$ )

For a particle ‘submerged’ in air  $L$  can be considered to be the  $d_{ve}$  of the particle (other choices also possible)

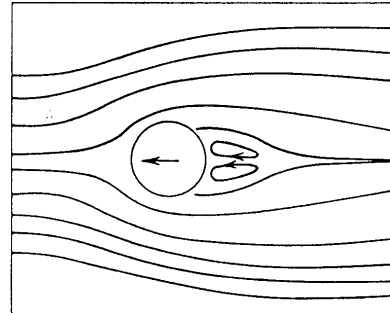
Note:  $Re$  is also useful when describing aerosol flow in a tube. In our work we use it to characterize particle losses, so we can deliver quantitative results. In this case  $L := d_{\text{tube}}$



**Laminar** (smooth) flow for **low  $Re$**  ( $< 1$ )

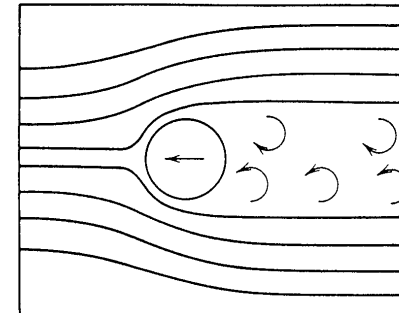


$Re = 0.1$



$Re = 2$

**Turbulent** flow for **high  $Re$**



$Re = 250$

Hinds, 1999

And which  $Re$  applies in the atmosphere?

**TABLE 9.2 Reynolds Number for Particles in Air Falling at Their Terminal Velocities at 298 K**

Diameter, $\mu\text{m}$	$Re$
0.1	$7 \times 10^{-9}$
1	$2.8 \times 10^{-6}$
10	$2.5 \times 10^{-3}$
20	0.02
60	0.4
100	2
300	20

In most situations the flow is laminar  
 $Re (< 1)$  around atmospheric particles

Stokes Law describes the drag force  $F_{\text{drag}}^p$  acting on an individual particle moving in a gas.

$$F_{\text{drag}}^p(v, d_{\text{ve}}) = 3\pi\eta d_{\text{ve}}v$$

$\eta$  gas viscosity,

$v$  particle velocity relative to the gas

$d_{\text{ve}}$  volume equivalent diameter

The aerodynamic drag force in the continuum regime is proportional to:

- particle diameter
- velocity of the particle (relative to the surrounding gas)

The simplifying assumptions required to derive Stokes law from the Navier-Stokes equation are:

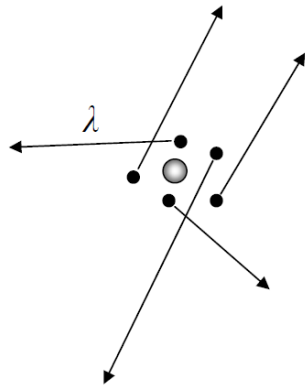
- Inertial forces negligibly small relative to viscous forces  $\rightarrow Re < 1$
- Gas velocity at the particle surface is 0 ('no slip condition')  $\rightarrow$  not true at small  $d_{\text{ve}}$
- Spherical, rigid particle with constant velocity relative to the gas  $\rightarrow$  not always true either
- Incompressible gas and no other particles or boundaries nearby

The behavior of a particle in a gas is characterised by the ratio of the mean free path of the gas molecules to the diameter of the particle. This ratio is called *Knudsen number* ( $Kn$ ):

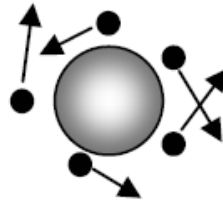
$$Kn = \frac{2\lambda_g}{d_{ve}}$$

$\lambda_g$  represents the mean distance travelled by a gas (thermal velocity) molecule between collisions with other gas molecules.

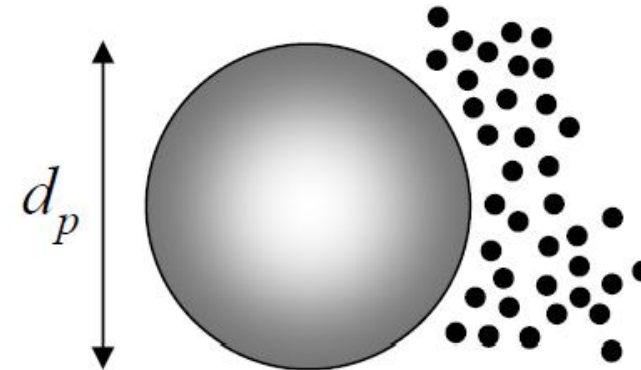
## Flow regimes



Free molecular (kinetic):  
 $Kn \gg 1$



Transition:  $Kn \sim 1$



Continuum:  $Kn \ll 1$   
(Stokes Law valid, see next slides)

At  $T = 298$  K and  $p = 1$  atm,  $\lambda_g = 66$  nm  $\rightarrow$   $Kn$  is 1 for a particle with  $d_{ve} = 132$  nm.

We typically consider the free molecular regime for particles  $< 10$  nm (nucleation mode), and the continuum regime for particles  $> 10$   $\mu$ m.



# Cunningham slip correction (Stokes for all regimes)

Stokes' law, which includes the **boundary condition that the gas velocity at the particle surface is zero** (i.e. drag force caused by shear forces), is **only valid** in the **continuum regime** (i.e. if  $K_n \ll 1$ , large particles).

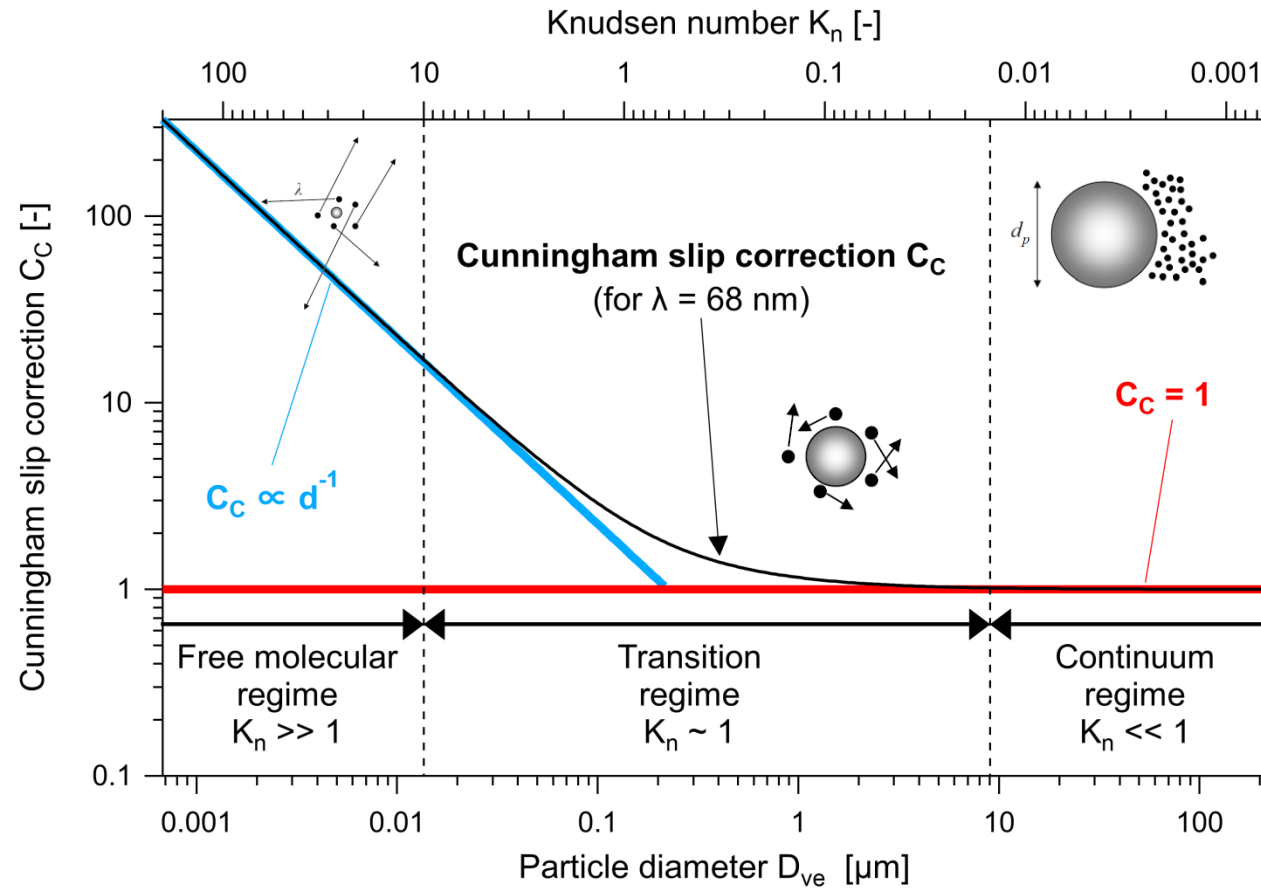
Stokes' law becomes **invalid in the free molecular regime**, where the mean free path of the molecules is much greater than the particle diameter (i.e. if  $K_n \gg 1$ , small particles), the **drag force is fully caused by collisions of individual molecules with the particle** (viscous forces dominate).

This problem is solved by modifying Stokes' law with the **Cunningham slip correction ( $C_c$ )**, which makes it more generally **applicable** for spherical particles **across all size regimes** (and laminar flow):

$$F_{\text{drag}}^p(v, d_{\text{ve}}) = \frac{3\pi\eta d_{\text{ve}} v}{C_c(d_{\text{ve}})}$$

$C_c$  depends on the ratio of mean free path to particle diameter and thereby also on the Knudsen number.

# The Cunningham slip correction



$C_c$  is needed to e.g. calculate the terminal settling velocity of a particle ( $\rightarrow$  see dry deposition from earlier lecture)

**Free molecular regime:**  
 $C_c \propto d^{-1} \Leftrightarrow F_{\text{drag}}^p \propto d^2$

**Transition regime**  
 $F_{\text{drag}}^p \propto \frac{d}{C_c(d)}$

**Continuum regime:**  
 $C_c = 1 \Leftrightarrow F_{\text{drag}}^p \propto d$

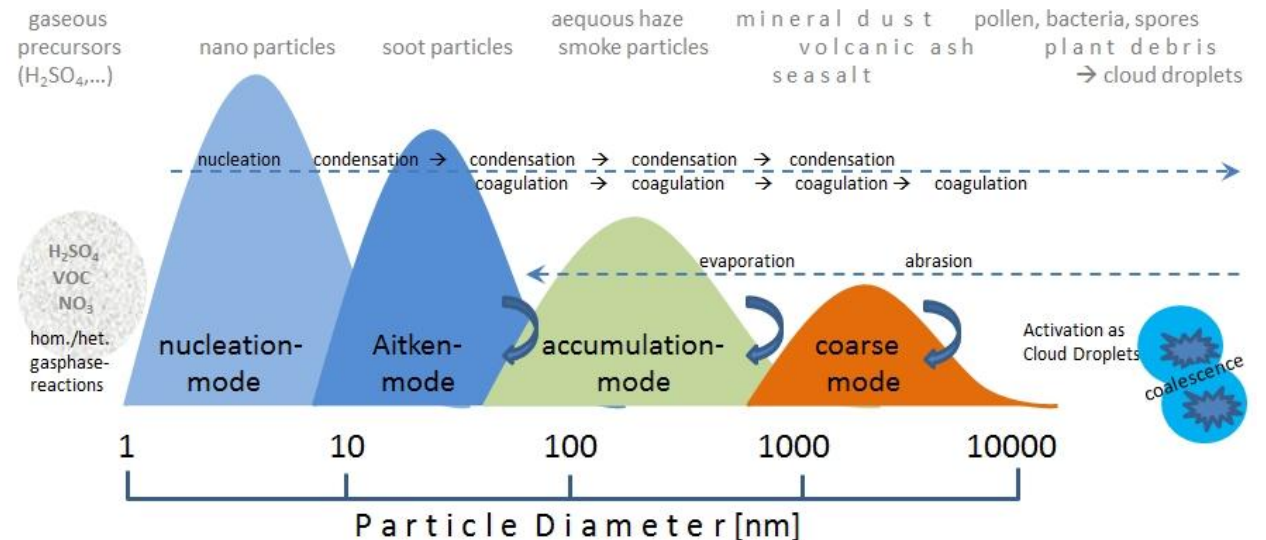
A monodisperse aerosol (i.e., all particles have the same size) does not exist in the ambient atmosphere; in the laboratory only as an approximation. Hence we have to deal with **polydisperse** aerosol.

Thus, it is necessary to describe the sizes of the aerosol particles with a size distribution: a distribution that gives the concentration of particles having a certain size (i.e. diameter).

In practice, a number concentration can be determined only in a certain size range

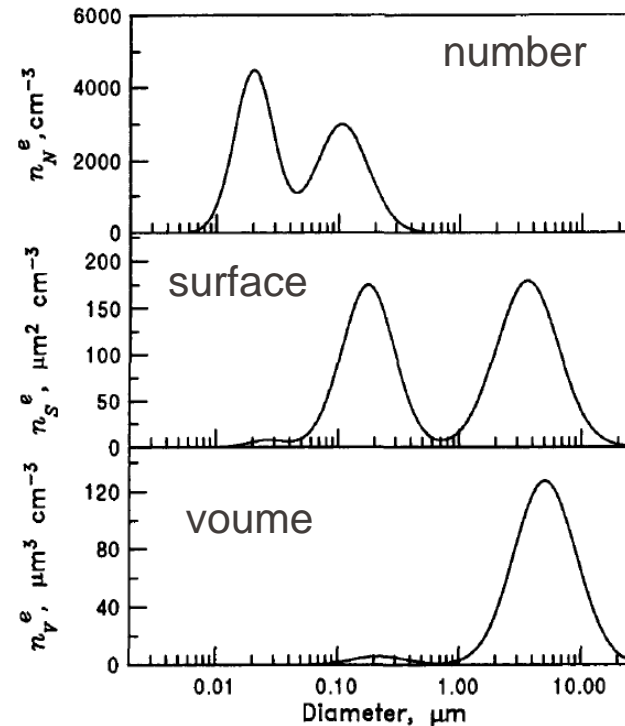
$$d_{p(1)} - d_{p(2)} = \Delta d_p \text{ or } dd_p$$

(i.e. number of particles with diameters between  $d_{p(1)}$  and  $d_{p(2)}$ ).





# Total particle number, surface area, and volume



$$\frac{dN}{d\log D_p}$$

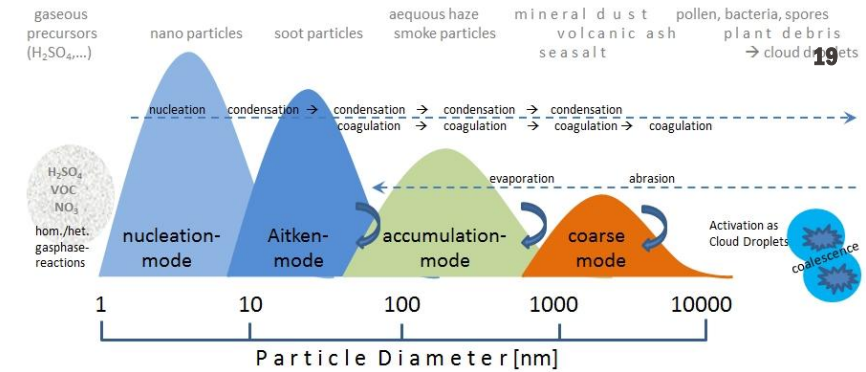
$$\frac{dS}{d\log D_p}$$

$$\frac{dV}{d\log D_p}$$

Detailed explanation  
in exercises.

**FIGURE 8.6** The same aerosol distribution as in Figures 8.4 and 8.5 expressed as a function of  $\log D_p$  and plotted against  $\log D_p$ . Also shown are the surface and volume distributions. The areas below the three curves correspond to the total aerosol number, surface, and volume, respectively.

# From gas to particle to cloud droplet



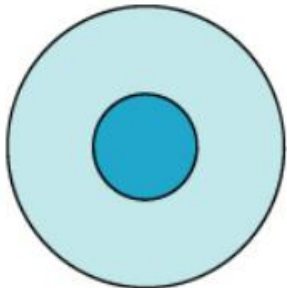
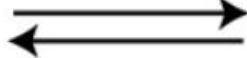
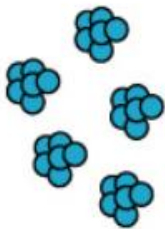
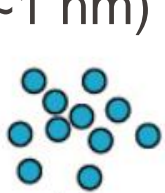
Gas to particle conversion  
(formation of critical clusters  
~1 nm)

nucleation mode aerosol  
(~few nm)

accumulation mode aerosol  
(<80 nm)

cloud condensation nuclei  
(>80 nm)

cloud droplets  
(~20 μm diameter)



nucleation

coagulation  
condensation

coagulation  
condensation

activation

- Here we consider **equilibrium conditions** (see next slide) of a two phase system (gas and condensed - solid, liquid – phase).
- We consider a **closed system** (the system under consideration and the environment are normally considered as one system, where the environment dictates temperature and pressure).
- We consider **isothermal-isobaric conditions**, i.e. fixed temperature and pressure, which refer to the conditions at the boundary of the system, i.e. temperature and pressure of the system are fixed by its surroundings. This is representative of atmospheric conditions.
  
- Vapor is defined as gas that can condense.
- In the atmosphere, air is a gas carrier that determines temperature. In the atmospheric boundary layer there are:
  - $10^{19} \text{ cm}^{-3}$  air molecules,
  - $10^{17} \text{ cm}^{-3}$  water molecules,
  - $10^{10} \text{ cm}^{-3}$  sulfuric acid molecules
- When colliding with particles, air molecules do not stick to particles, but substances like sulfuric acid will.

## Thermodynamic state variables (state functions, state parameters or thermodynamic variables):

$T$ :	Absolute temperature	}	<b>intensive</b> properties (independent of system size)
$p$ :	Pressure		
$\mu$ :	Chemical potential		
$S$ :	Entropy	}	<b>extensive</b> properties (scale with system size)
$V$ :	Volume		
$n_i$ :	Number of moles of species $i$ .		

## Thermodynamic potentials:

$U$ :	Internal energy
$H$ :	Enthalpy
$G$ :	<b>Gibbs energy</b> (Gibbs function, Gibbs free energy, or free enthalpy)

$\mu$  see following slides

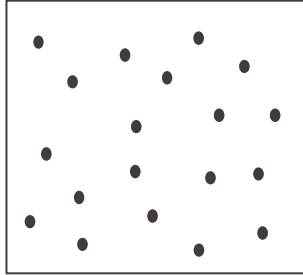
$S$  is associated with a state of disorder, randomness, or uncertainty of the system.

$H$  is the sum of the system's internal energy.

$G$  see next slides



Any thermodynamic **system “a”**:



state variables:

$$\begin{array}{ll} T_a & S_a \\ p_a & V_a \\ \mu_{i,a} & n_{i,a} \end{array}$$

potentials:

$$\begin{array}{l} U_a \\ H_a \\ G_a \end{array}$$

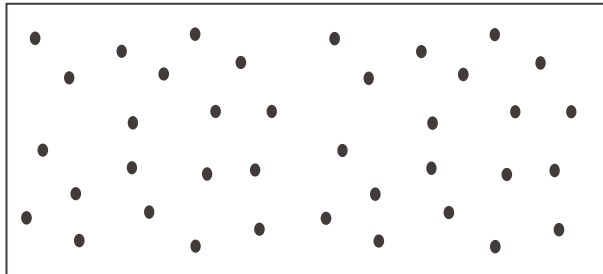
State variables

$T$ :	Absolute temperature
$p$ :	Pressure
$\mu$ :	Chemical potential
$S$ :	Entropy
$V$ :	Volume
$n_i$ :	Number of moles of species $i$ .

potentials

$U$ :	Internal energy
$H$ :	Enthalpy
$G$ :	<b>Gibbs energy</b> (Gibbs function, Gibbs free energy, or free enthalpy)

**System “b”**: exactly equal to system “a” except for scaling the system with factor of 2 (doubled system size and mass):



state variables:

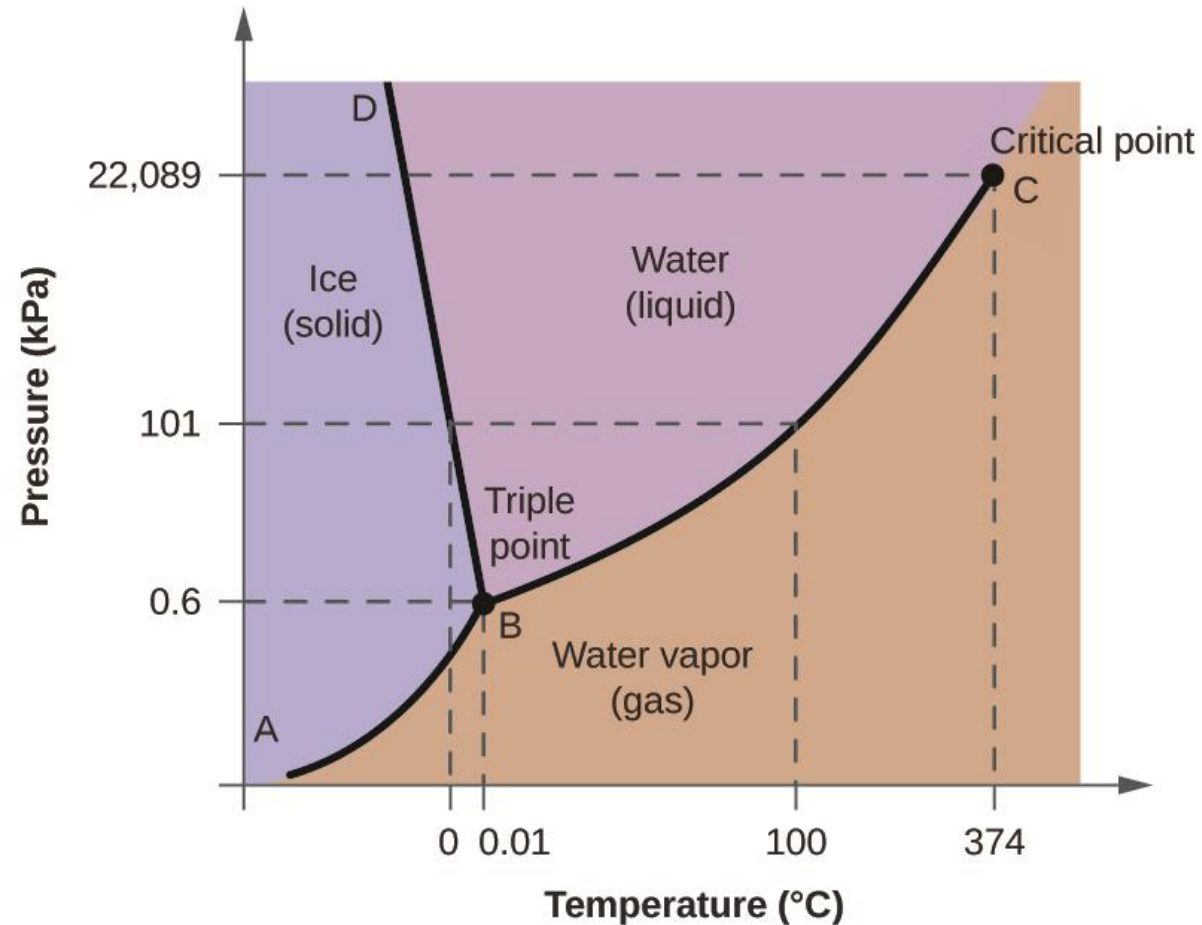
$$\begin{array}{l} T_b = T_a \\ p_b = p_a \\ \mu_{i,b} = \mu_{i,a} \end{array}$$

potentials:

$$\begin{array}{ll} S_b = 2 \cdot S_a & U_b = 2 \cdot U_a \\ V_b = 2 \cdot V_a & G_b = 2 \cdot G_a \\ n_{i,b} = 2 \cdot n_{i,a} & H_b = 2 \cdot H_a \end{array}$$

independent of system scaling  
→ **intensive properties**

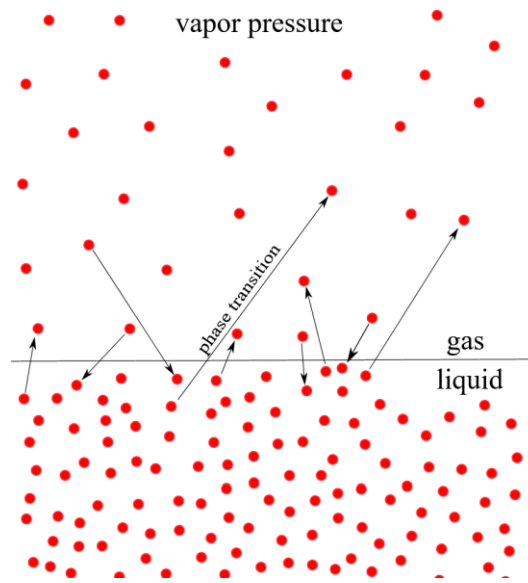
proportional to system scaling factor  
→ **extensive properties**



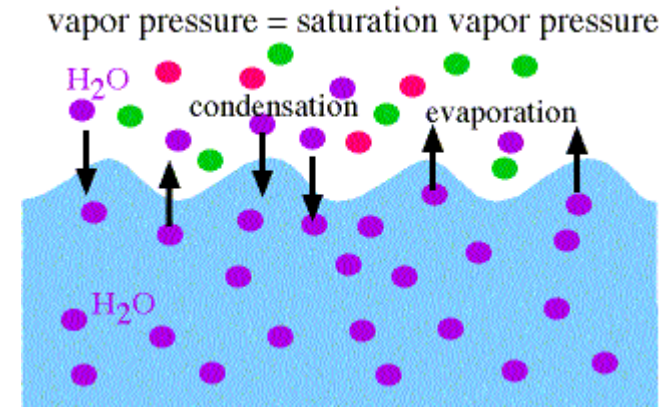
- Solid lines are the phase boundaries
- Critical point: liquid and gas are indistinguishable
- Triple point (273.16 K and a partial vapor pressure of 611.657 Pa): all three phases co-exist

→ Here we are mostly interested in the gas-liquid relationship

# Vapor pressure and Saturation vapor pressure



When the number of molecules that condense and evaporate is the same, the system is in equilibrium and we speak of the saturation vapor pressure.



The vapor pressure of a liquid or solid is the pressure exerted by the vapor in equilibrium with the condensed (solid/liquid) phase.

The **equilibrium vapor pressure**  $p_{\text{sat},i}$  for substance  $i$  is also referred to as its **saturation vapor pressure**, and determines the amount of substance that can be “held” in the gas phase at equilibrium.

The **saturation ratio** ( $S_A$ ) of a vapour of species  $i$  is defined as:

$$S_A := \frac{p_i}{p_{\text{sat},i}(T)}, \text{ where } p_i \text{ is the partial pressure and } p_{\text{sat},i} \text{ the saturation vapor pressure of species } i.$$

# Reminder: 1<sup>st</sup> law of thermodynamics

The first law of thermodynamics (conservation of energy) can be written as:

$$dU = TdS - pdV + \sum_{i=1}^k \mu_i dn_i$$

The internal energy  $U$  is the corresponding thermodynamic potential with its natural extensive state variables  $S, V$  and  $n_1, \dots, n_k$ :

$$U(S, V, n_1, \dots, n_k) = TS - pV + \sum_{i=1}^k \mu_i n_i$$

**Total kinetic energy**

**Potential energy tied to  
volume expansion work**

**Potential energy  
associated with latent  
heat and chemical bonds**



# Thermodynamic Equilibrium / 2<sup>nd</sup> law of thermodynamics <sup>26</sup>

2<sup>nd</sup> law of thermodynamics: «in a closed system, entropy increases in all spontaneous reactions, arriving at a state of thermodynamic equilibrium, which means minimizing the free energy of the system under consideration» → Gibbs energy:  $dG = 0$

The Gibbs energy is defined as:

$$G := U + pV - TS \quad (1)$$

Inserting “ $U = TS - pV + \sum_{i=1}^k \mu_i n_i$ ” (from previous slide) directly provides:

$$G = \sum_{i=1}^k \mu_i n_i \quad (2)$$

The total differential of Equation 1 is:

$$dG = dU + Vdp + pdV - TdS - SdT \quad (3)$$

Inserting the 1<sup>st</sup> law of thermodynamics ( $dU = TdS - pdV + \sum_{i=1}^k \mu_i dn_i$ ) provides the exact differential of the Gibbs energy:

$$dG = Vdp - SdT + \sum_{i=1}^k \mu_i dn_i \quad (4)$$

Note: the natural variables of the Gibbs energy are  $p, T$  and  $n_1, \dots, n_k$ .

$$dG = Vdp - SdT + \sum_{i=1}^k \mu_i dn_i$$



simplifies under **isothermal-isobaric** conditions  
(i.e.  $dT = 0$ ,  $dp = 0$ ):

$$dG = \sum_{i=1}^k \mu_i dn_i$$




And the condition for **thermodynamic equilibrium under isothermal-isobaric conditions** eventually becomes:

$$dG = 0 \xRightarrow{(2^{nd} \text{ law})} \sum_{i=1}^k \mu_i dn_i = 0$$

The chemical potential ( $\mu_i$ ) of substance  $i$  is the partial molar Gibbs energy, i.e. the change of the Gibbs free energy if one mole of substance  $i$  is added to a system under isothermal-isobaric conditions (and otherwise constant composition). **Lowering pressure of a pure gas lowers its chemical potential.**

$$\mu(T, p) = \mu^0(T, p^0) + RT \ln \frac{p}{p^0}$$


$$pV = nRT$$

(ideal gas law)

Symbols:

$T$ : temperature

$n$ : Number of moles

$V$ : Volume

$p$ : Pressure

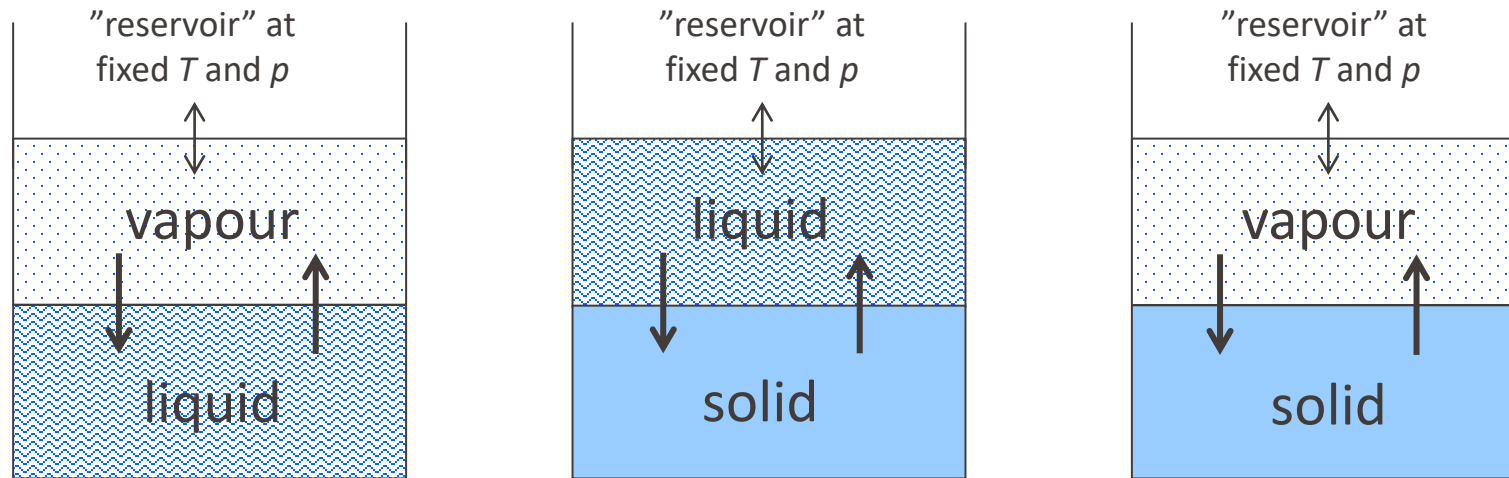
$p^0$ : Pressure of the standard state (1 atm)

$\mu$ : Chemical potential

$\mu^0$ : Chemical potential of the standard state

Note: we don't derive above equation here fully, this might be treated in another class in the future.

# Equilibrium Between Two Phases of a Substance



The condition for equilibrium under isothermal-isobaric conditions is:

$$dG = \sum_i \mu_i dn_i = 0$$

For transitions of species  $i$  between two phases applies:

$$dn_{i,\text{phase1}} = -dn_{i,\text{phase2}} \quad \text{and} \quad \text{total number of molecules doesn't change}$$

and we directly obtain:

$$\mu_{i,\text{phase1}} = \mu_{i,\text{phase2}} \quad (\text{for phase transitions of a species})$$

$\Rightarrow$  **The chemical potentials of a species in two different phases are equal in equilibrium.**

$G$ : Gibbs free energy

$\mu$ : Chemical potential

$n$ : Number of moles

# Chemical Potential of a Liquid/Solid

The chemical potentials of a species in two different phases are equal in equilibrium:

$$\mu_{i,\text{phase1}} = \mu_{i,\text{phase2}}$$

For a pure condensed phase (liquid or solid) in equilibrium with its saturated vapor follows:

$$\mu_{\text{cond}}^*(T) = \mu_{\text{vap}}(T, p_{\text{sat}}(T))$$

➔ **The chemical potential of the pure condensed phase is constrained by its saturation vapour pressure.**

Symbols:

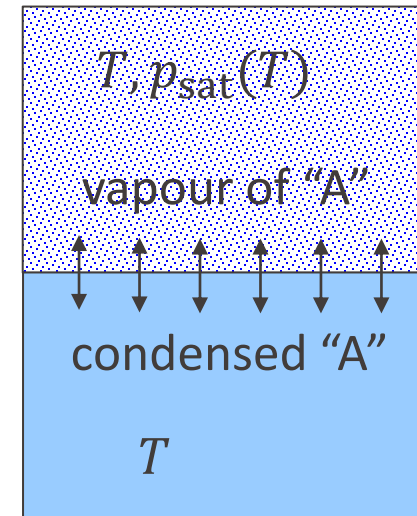
$\mu_{\text{vap}}$ : Chemical potential of the vapour phase

$\mu^0$ : Chemical potential of the standard state of a gas

$\mu_{\text{cond}}^*$ : Chemical potential of the pure condensed phase (solid or liquid)

$p^0$ : Pressure of the standard state of a gas (1 atm)

$p_{\text{sat}}$ : Saturation vapour pressure of the pure liquid/solid

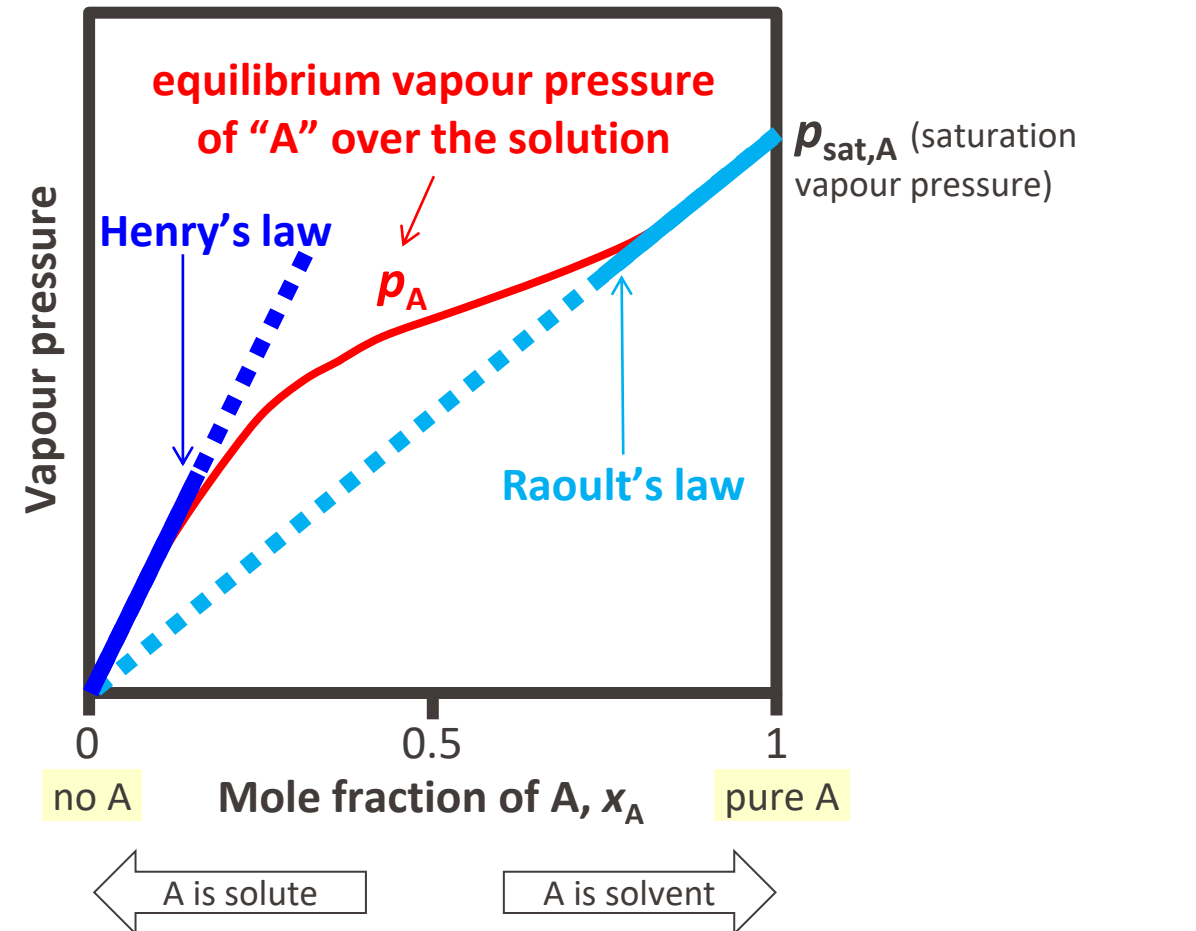


The two phases can be **mixtures**, as long as species  $i$  can change between the two phases independently of all other species



# Two types of mixtures: Raoult's and Henry's law

- **Raoult's law** applies when  $x_A \rightarrow 1$  (ideal solution)
  - $p_A(T) = p_{\text{sat},A}(T)x_A$
  - Raoult's law states that the equilibrium partial pressure of species *A* over an **ideal solution** is equal to the saturation vapour pressure of this species in pure form times its mole fraction in the solution.
- **Henry's law** applies when  $x_A \rightarrow 0$  (ideal-dilute solution)
  - $p_A = H'_A x_A$
  - Henry's law denotes the non-reactive absorption of gases in aqueous solution.
  - The equilibrium vapour pressure of *A* over an aqueous solution containing small amounts of *A*.

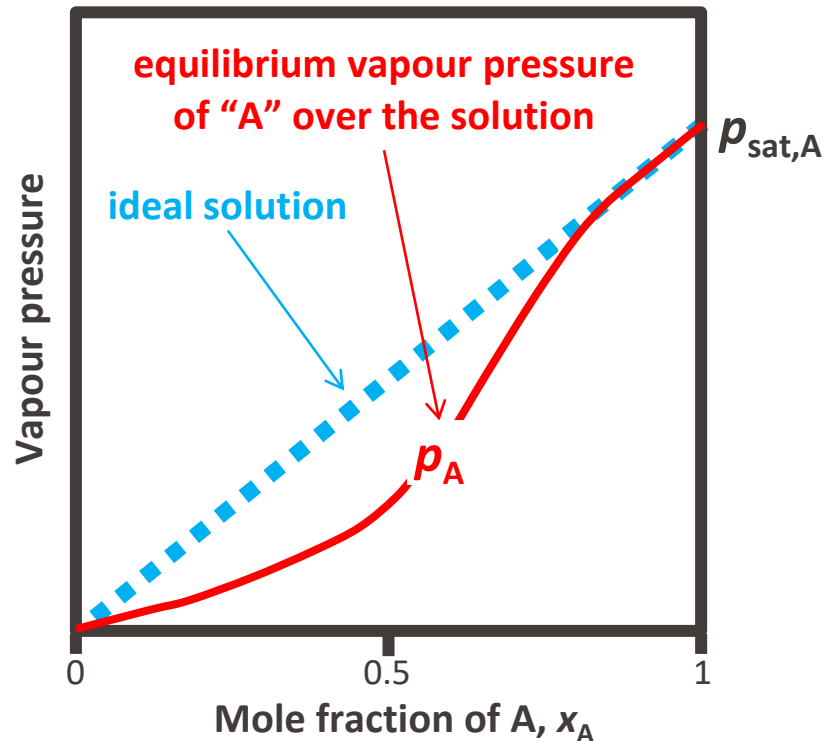


$p_A$  vapor pressure  
 $p_{\text{sat},A}$  saturation vapor pressure  
 $x_A$  mole fraction of substance A  
 $H'_A$  Henry constant for A

# Equilibrium Vapour Pressure over Non-Ideal Solutions

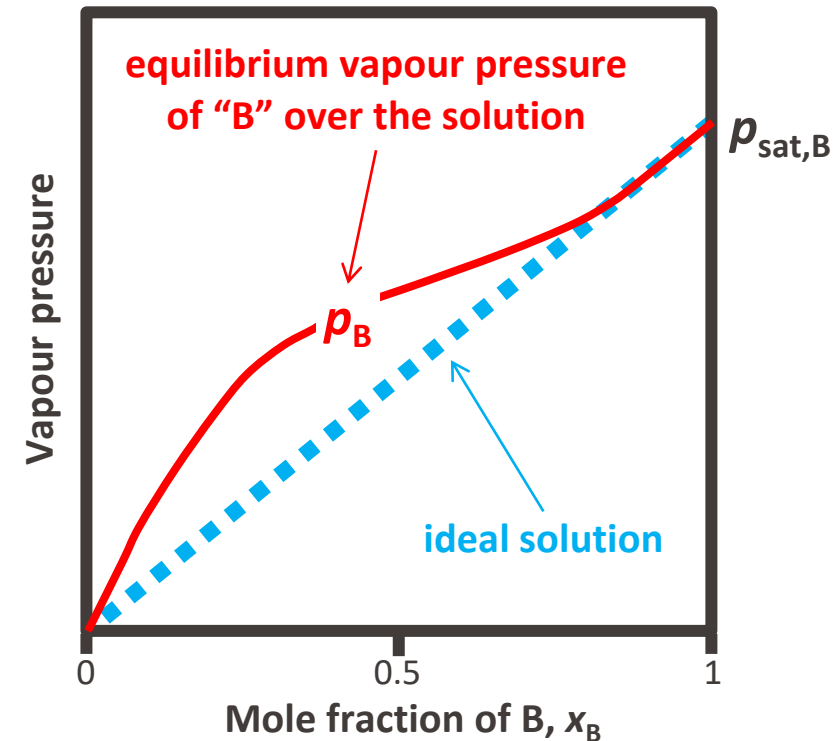
Example: Species A is experiencing  
**positive/attractive interactions**  
in the solution

→ decreased vapour pressure



Example: Species B is experiencing  
**negative/repelling interactions**  
in the solution

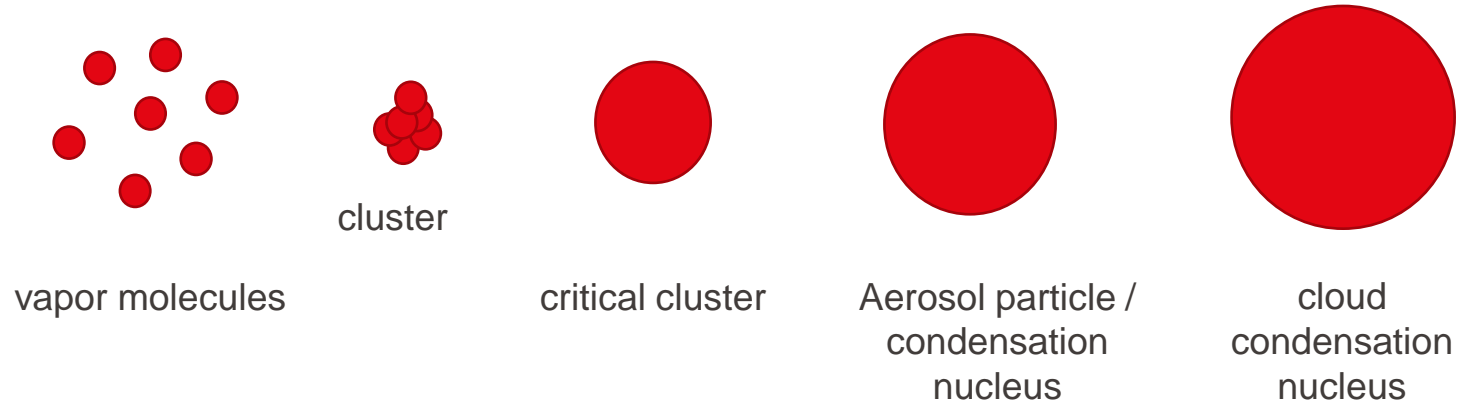
→ increased vapour pressure



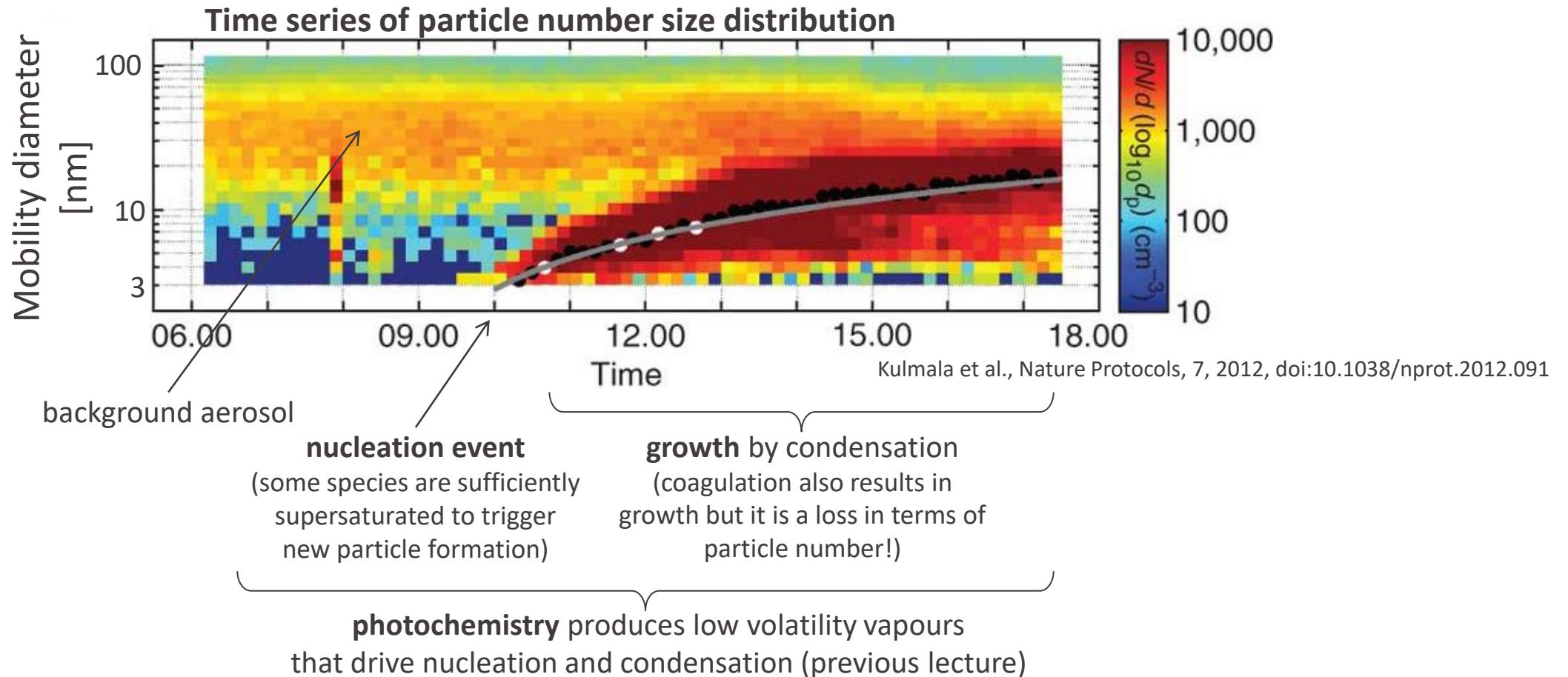
- We now understand the saturation ratio ( $S$ ) of a gas over the condensed phase.
  - When  $S < 1$  there is vapor undersaturation and the liquid evaporates until we reach equilibrium.
  - When  $S > 1$ , the vapor is supersaturated and condenses, provided there is already a planar liquid surface.
  - If such a liquid does not exist, the vapor faces a dilemma, because the state that minimizes the Gibbs energy (2<sup>nd</sup> law!) is the bulk liquid rather than supersaturated gas.
  - However, to form a liquid, the interface between gas and liquid must be created first, which costs free energy ( $dG \neq 0$ ).
- The stochastic, kinetic process through which supersaturated vapors form clusters that overcome this energy barrier is called **nucleation**.

# Mind map of a liquid drop model for nucleation

- We describe the cluster formation (nucleation) process as if they were liquid droplets with bulk liquid density and surface tension.
- The gas phase is treated as an ideal mixture of ideal gases.
- This results in the **classical nucleation theory** (CNT).  
(Extensions exist, which we will not treat in this lecture.)
- In CNT we look at the change in the Gibbs free energy ( $dG/dt$ ).



# «Banana-Type» Nucleation Event in the Atmosphere



- Nucleation events are frequently observed in the atmosphere.
- Secondary particles from nucleation give a substantial contribution to the total number of cloud condensation nuclei (see lectures by Thanos Nenes).



- There are natural and anthropogenic sources of aerosols. Global quantification is a challenge, particularly for natural sources.
- Primary particles are emitted as particles, secondary particles are formed in the atmosphere from vapors.
- Aerosol lifetime is a function of particle size and removal by precipitation.
- Particle diameters are a matter of definition.
- With Stoke's law we can describe the movement of particles through air.
- Particles occur in polydisperse manner and form a size distribution. Size distributions can be characterized by modes
  - Nucleation
  - Aitken
  - Accumulation
  - Coarse
- The equilibrium vapor pressure, Gibbs energy and the 2nd law of thermodynamics help us understand the partitioning between gas and particle.
- Henry's and Raoult's laws define the equilibrium vapor pressure over a solution.