Introduction to Atmospheric Physics

Block II: Dynamics and Horizontal Motions

What has happened thus far and next

- Thermodynamics

Describing the processes of moving (horizontally, vertically) air parcels. As they change pressure with height, they also change temperature and density. If the air parcel is dry, the change is described (to first order) by energy conservation (1. law) and is called adiabatic. If condensation / evaporation processes occur, the corresponding heat has to be considered and the air parcels cool less as they rise. Whether or not air parcels want to rise or stay is described by static stability.

- Atmospheric Dynamics

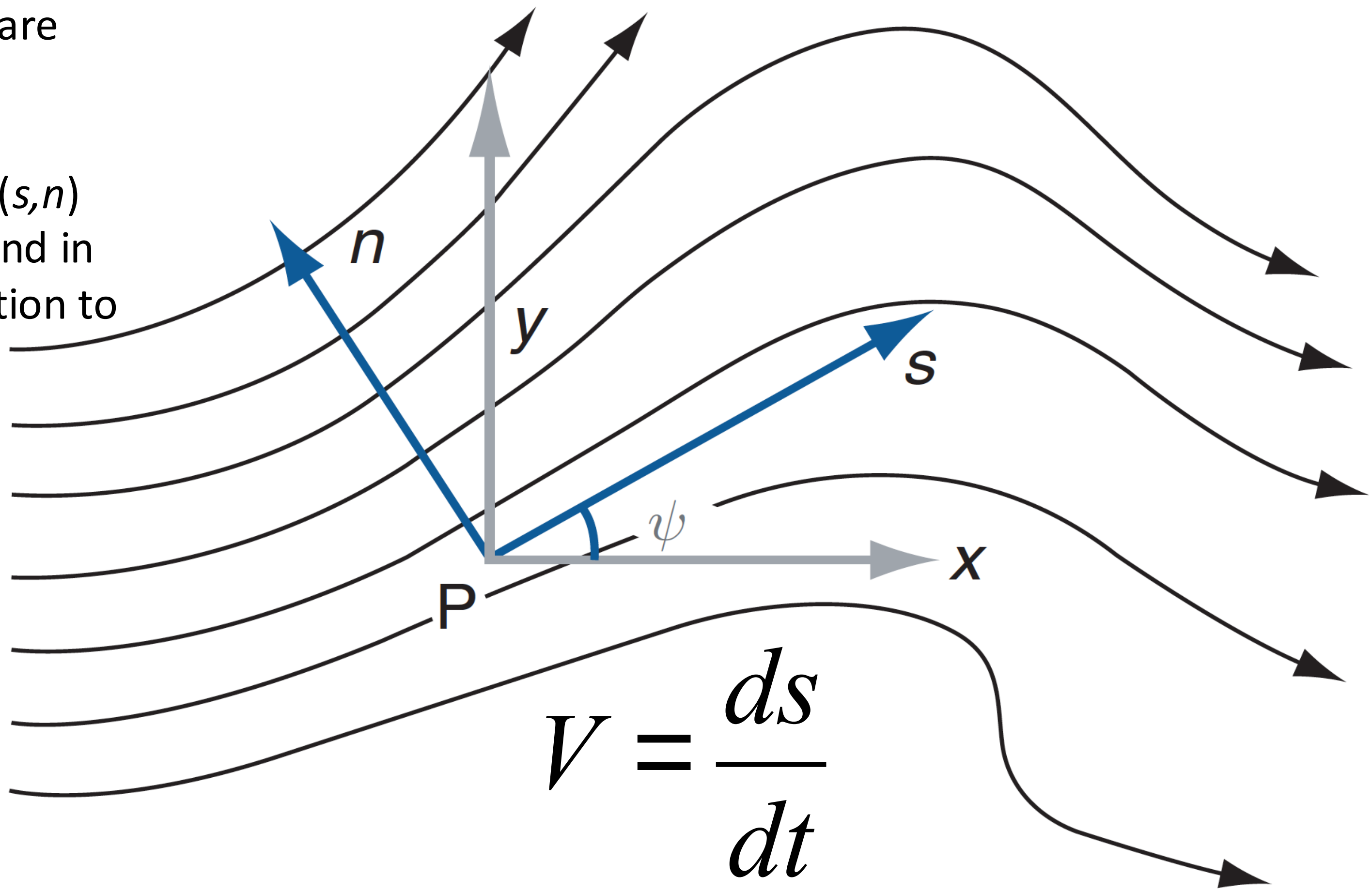
Thermodynamics is the basis for large scale atmospheric motion. But today, we will not look at it with the view of isolated air parcels but regarding the atmosphere as a whole and try to understand how motion at one place is causing (compensating) motion somewhere else. At the end, it all is driven by trying to transport heat from hot to cold places.

Outline:

- Horizontal Motions
- Apparent and Real Forces
- Geostrophic, Ekman and Gradient Winds
- Primitive Equations as a Basis for Weather Forecasting

Horizontal Motions: Natural Coordinates

Shown are streamlines, which depict a snapshot of the flow at a certain time. A streamline is obtained by connecting the origins of fluid-particle velocity vectors. Under stationary conditions, streamlines are identical to fluid parcel trajectories. The natural coordinate system axes (*s*,*n*) point in flow direction and in the perpendicular direction to the left.



$$V = \frac{ds}{dt}$$

Some Notation and Concepts (Optional)

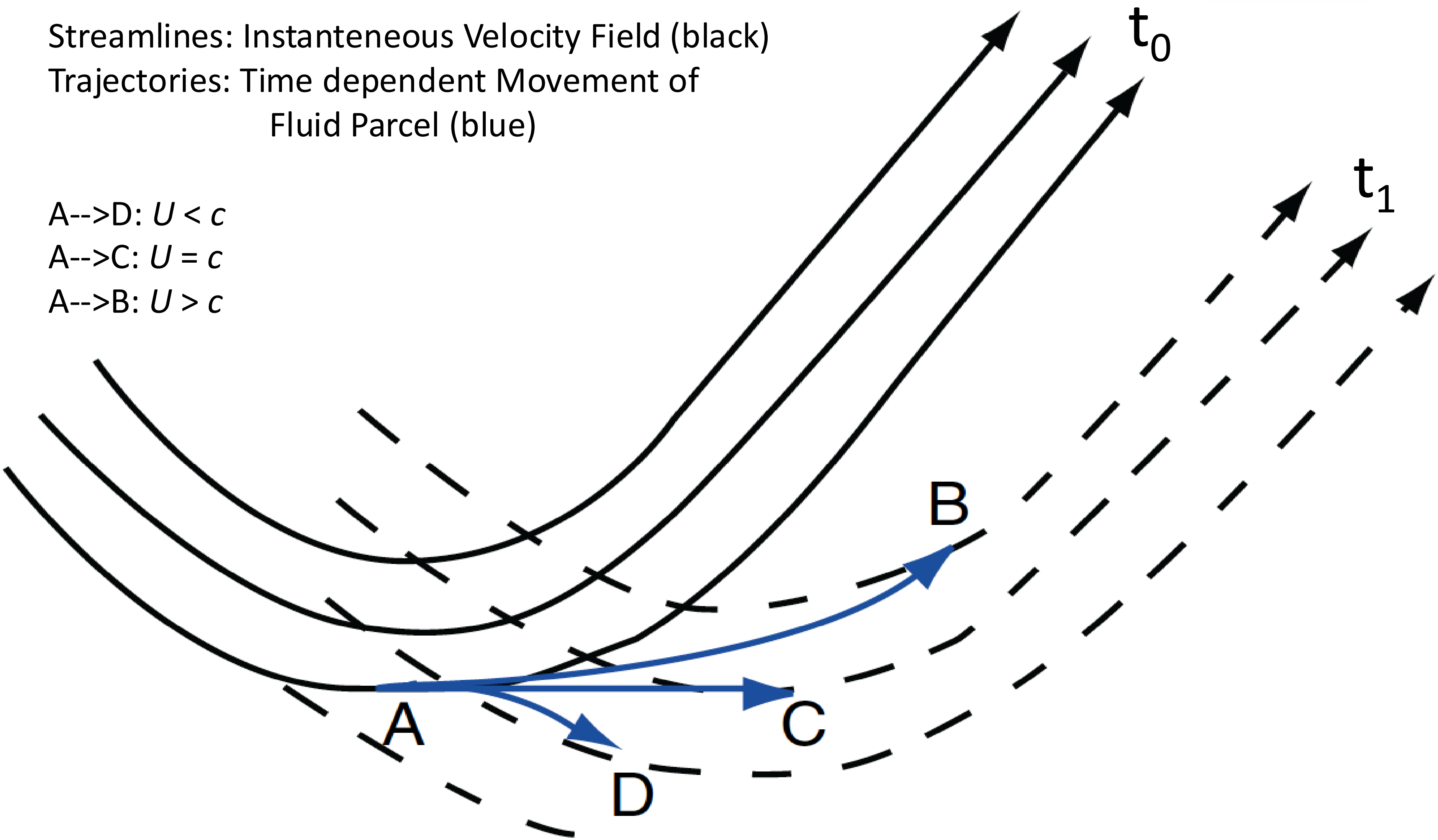
Table 7.1 Definitions of properties of the horizontal flow

	Vectorial	Natural coords.	Cartesian coords.
Shear		$-\frac{\partial V}{\partial n}$	
Curvature		$V\frac{\partial \psi}{\partial s}$	
Diffluence		$V\frac{\partial \psi}{\partial n}$	
Stretching		$\frac{\partial V}{\partial s}$	
Vorticity ζ	$\mathbf{k} \cdot \nabla \times \mathbf{V}$	$V\frac{\partial \psi}{\partial s} - \frac{\partial V}{\partial n}$	$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$
Divergence $\mathrm{Div}_H \mathbf{V}$	$\nabla \cdot \mathbf{V}$	$V\frac{\partial \psi}{\partial n} + \frac{\partial V}{\partial s}$	$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$
Deformation			$\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}; \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$

Streamlines versus Trajectories (Optional)

Streamlines: Instanteneous Velocity Field (black)
Trajectories: Time dependent Movement of Fluid Parcel (blue)

A-->D: $U < c$
A-->C: $U = c$
A-->B: $U > c$

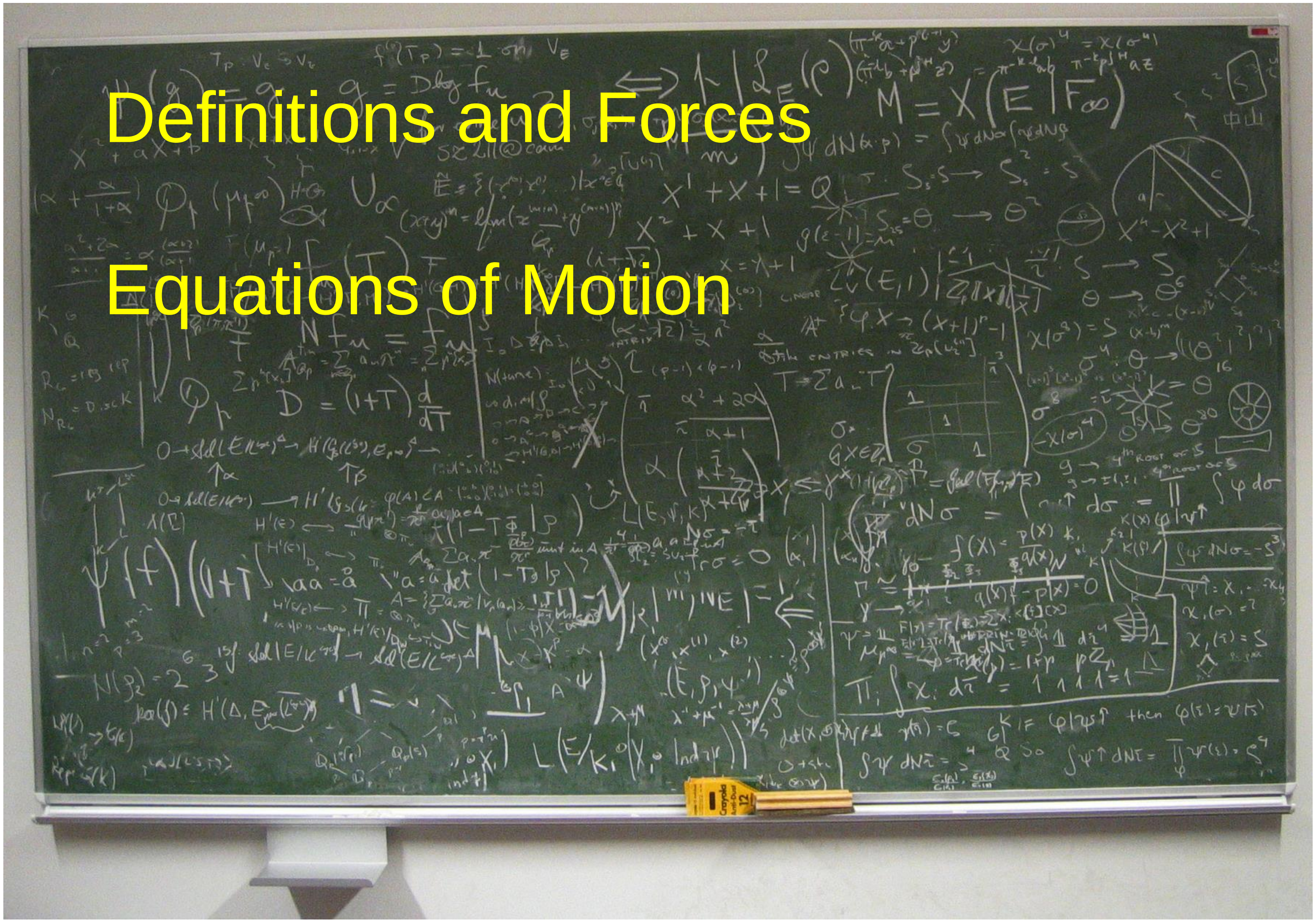


„Eastward“ moving wave (phase speed *c)* on uniformly westerly flow (speed *U*)

How can you measure an approximation of atmospheric streamlines

A. Use a Doppler Radar to get the velocities of tiny rain droplets in the air
B. Use a Lidar to measure the concentration of aerosol particles in the air
C. Release a large number of small balloons and track their path

Black board…



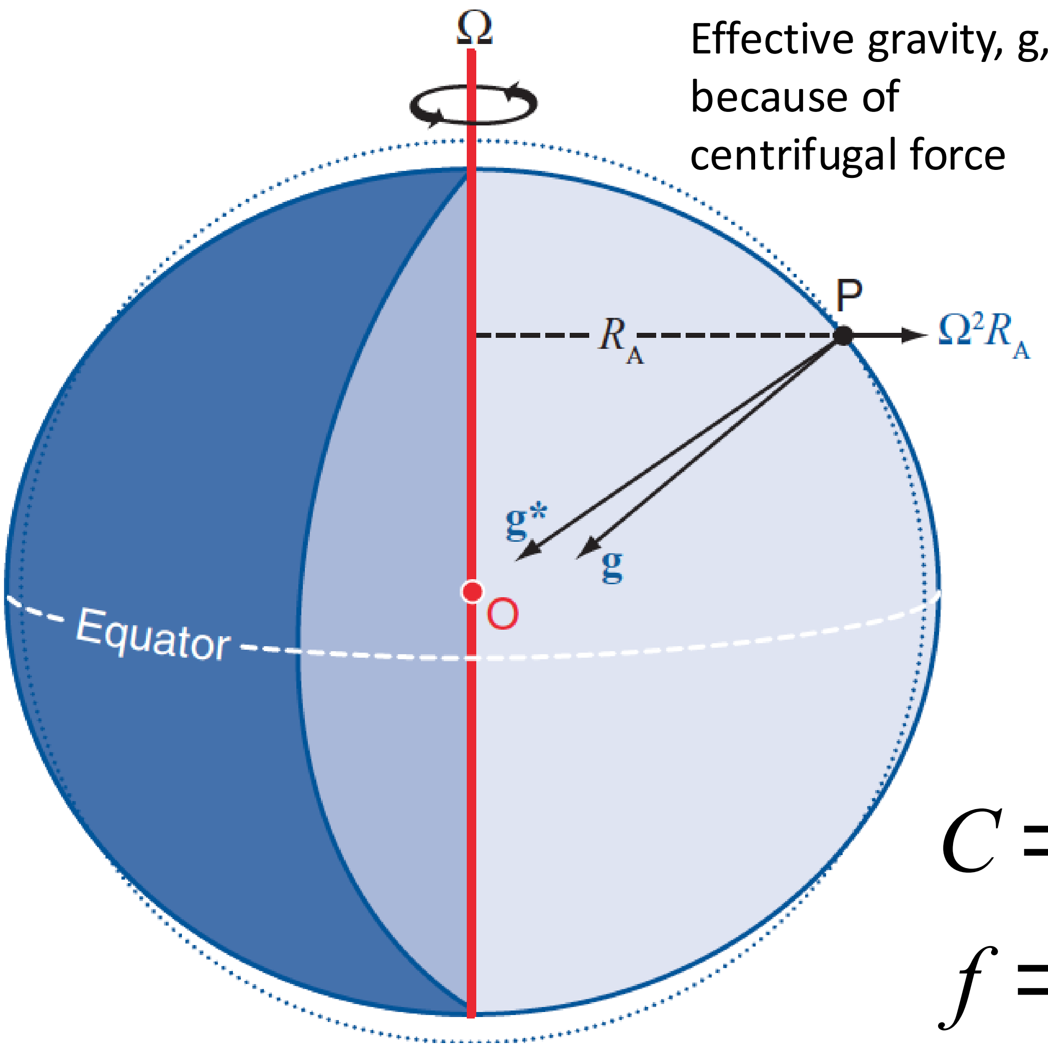
Definitions and Forces

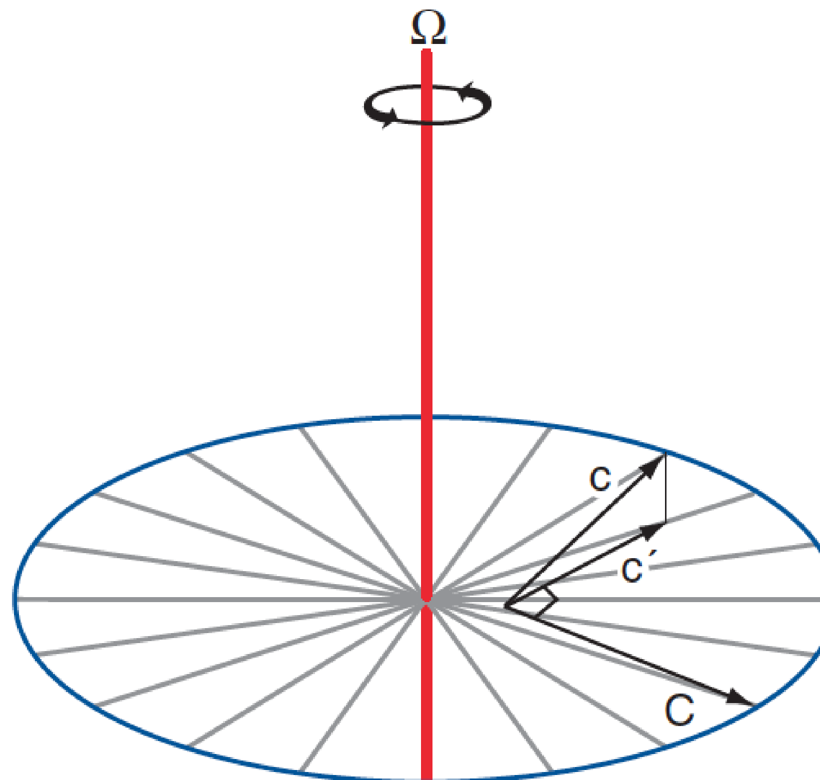
Equations of Motion

When is the pressure force encircled on the blackboard not causing vertical motion?

A. The air parcel is adiabatic
B. The pressure force is balanced by friction at the earth’s surface
C. The pressure force is balanced by gravity

Apparent Forces – gravity, g and Coriolis, C





$$C = -\, f\mathbf{k}´\ \mathbf{V};$$

$$f = 2\mathrm{W}\sin f$$

For an intuitive explanation video see: https://www.youtube.com/watch?v=49JwbrXcPjc

Derivation of Coriolis Force I: Additional Centrifugal Force

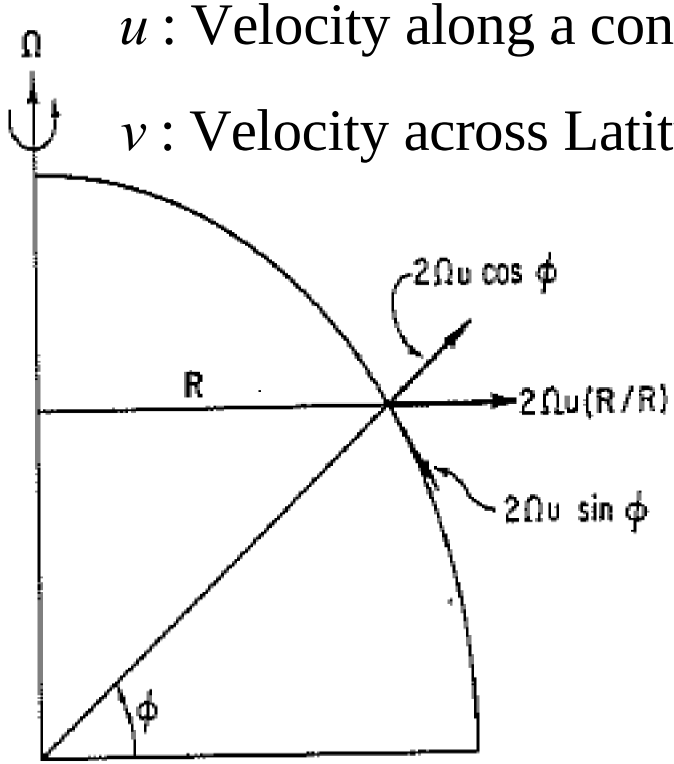
$$\left(W+\frac{u}{R}\right)^2 \boldsymbol{R} = W^2\boldsymbol{R} + \frac{2Wu\boldsymbol{R}}{R} + \frac{u^2\boldsymbol{R}}{R^2}$$

$\boldsymbol{R}$: Position Vector, on a plane perpendicular to axis of rotation

W: Magnitude of Angular Velocity

u : Velocity along a constant Latitude

v : Velocity across Latitudes (perpendicular to u)



Adapted from Holton's text: An introduction to Dynamic Meteorology

Components of the Coriolis force owing to relative motion along a latitude circle.

Derivation of Coriolis Force I

Total centrifugal force is a combination of the one due to rotation of the earth plus the one from the fluid particle movement (*u*), which is assumed to be eastward along a constant latitude. For synoptic scale motions, the third term after expansion can be neglected, the first term has been included already in g, and the remaining force can be split in meridional and vertical components. The vertical component is indistinguishable from g (has the same direction) and is therefore typically neglected.

The Coriolis parameter, f (s^{-1}), is defined as $f = 2\Omega \sin \phi$ and is constant for a given latitude.

$$|u| << \Omega R$$

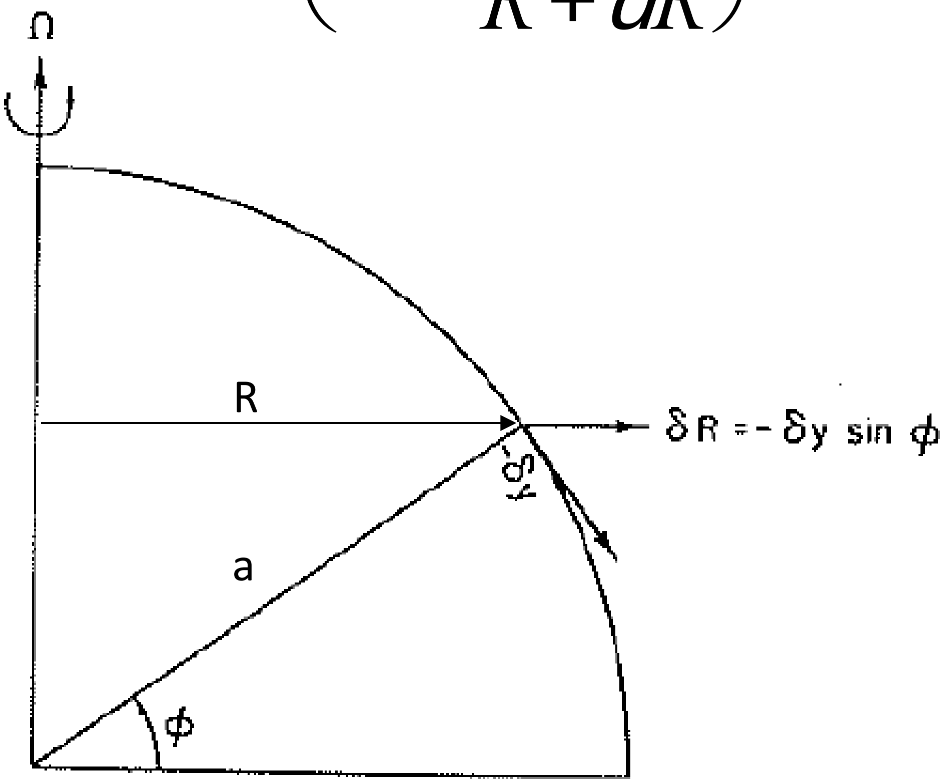
$$\text{Meridional Coriolis: } \left(\frac{dv}{dt}\right) = -2\Omega u \sin \phi = -f\,u$$

$$\text{Vertical Coriolis: } \left(\frac{dw}{dt}\right) = 2\Omega u \cos \phi$$

Derivation of Coriolis Force II

Fluid particle moving across latitudes (meridional), e.g. equatorward. Due to conservation of angular momentum, as R increases, the particle gets deflected westward.

$$\Omega R^2 = \left(\Omega + \frac{\delta u}{R+\delta R}\right)(R+\delta R)^2; \quad \delta u = u(\phi_0 + \delta\phi)$$



Relationship of δR and $\delta y = a\,\delta\phi$ for an equatorward displacement.

Derivation of Coriolis Force II

Expanding the conservation equation above, neglecting second order differentials and solving for the zonal velocity:

$$\text{Using } dR = -a\,df \sin f_0 :$$

$$du = -2W dR = +2W a\,df \sin f_0$$

$$D\text{ividing by } dt \text{ and taking the limit as } dt \to 0 :$$

$$\text{Zonal Coriolis: } \left(\frac{du}{dt}\right) = 2W \frac{a\,df}{dt} \sin f_0 = 2W v \sin f = f\, v$$

$$\text{Including Vertical Motion: } \left(\frac{du}{dt}\right) = 2W v \sin f - 2W w \cos f$$

For a rigorous derivation see:
https://physics.stackexchange.com/questions/383/derivation-of-the-centrifugal-and-coriolis-force

Real Forces (per unit mass: N kg^{-1}):

Pressure Gradient Force: $$\boldsymbol{P} = -\frac{1}{\rho}\nabla p$$

Friction Force: $$\boldsymbol{F} = -\frac{1}{\rho}\frac{\partial \boldsymbol{\tau}}{\partial z}$$

(τ: shear)

Horizontal Balance of Motion (Navier-Stokes):

$$\frac{d\mathbf{V}}{dt} = -\frac{1}{\rho}\nabla p - f\mathbf{k} \times \mathbf{V} - \frac{1}{\rho}\frac{\partial \boldsymbol{\tau}}{\partial z}$$

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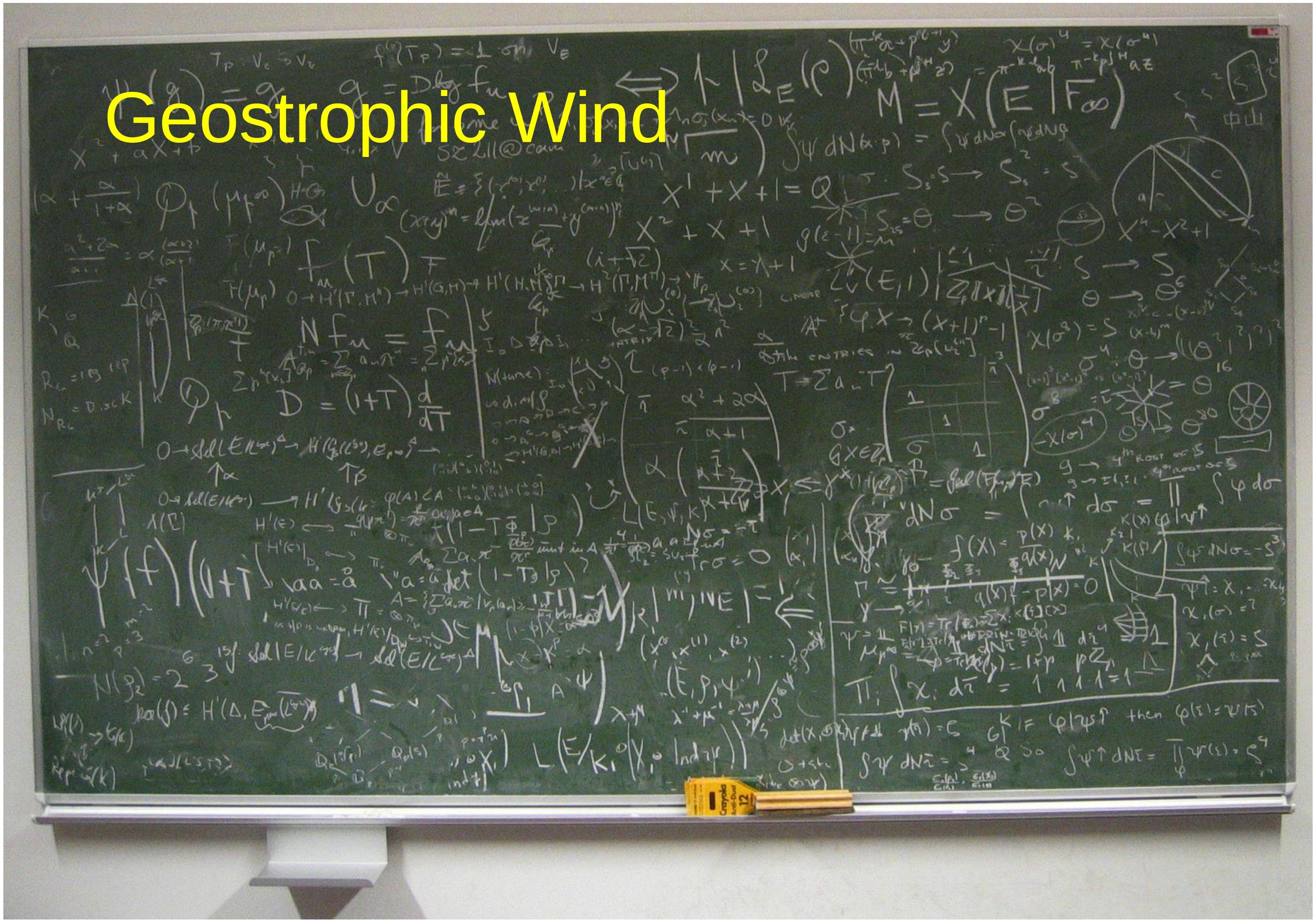
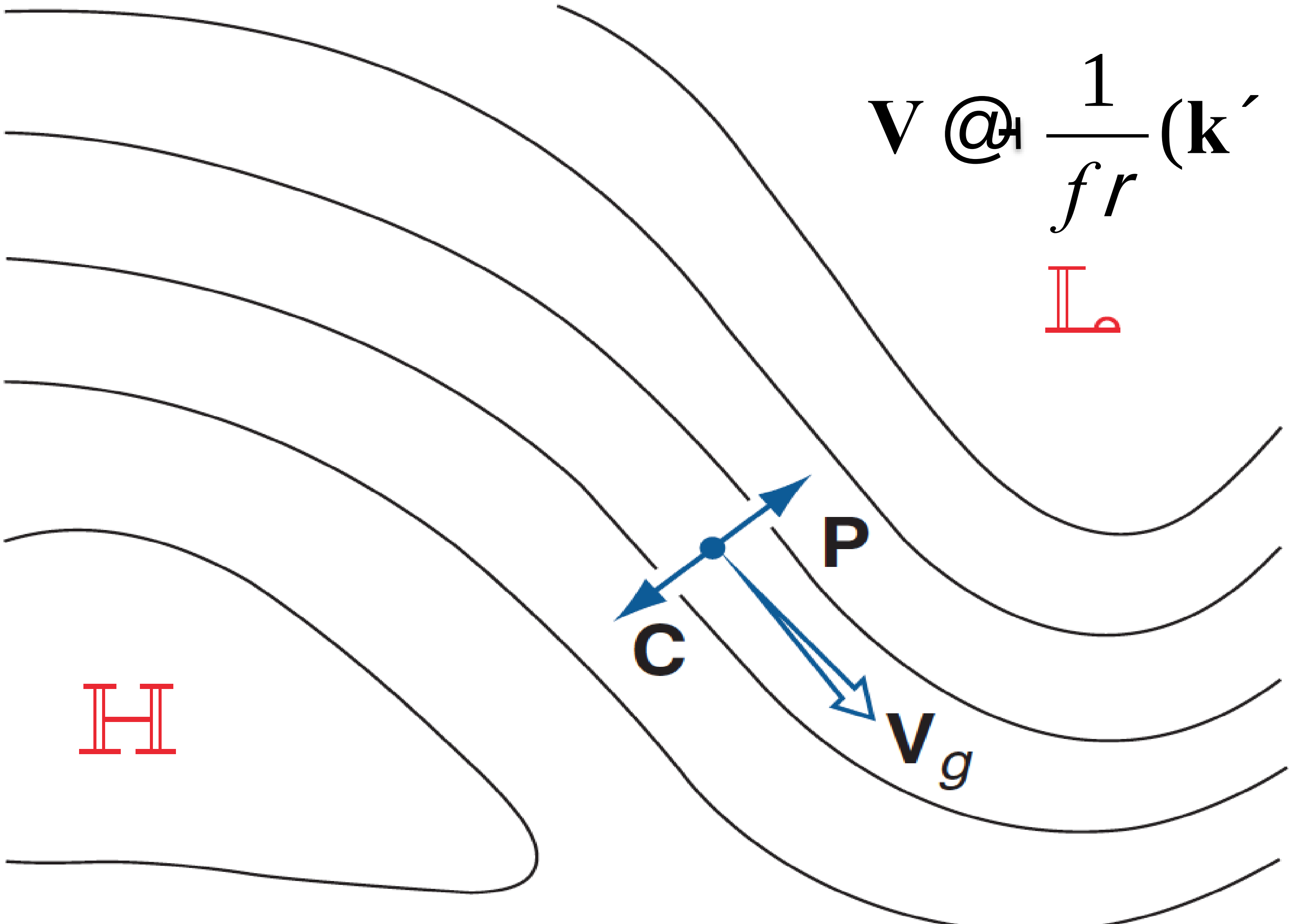


Geostrophic Wind

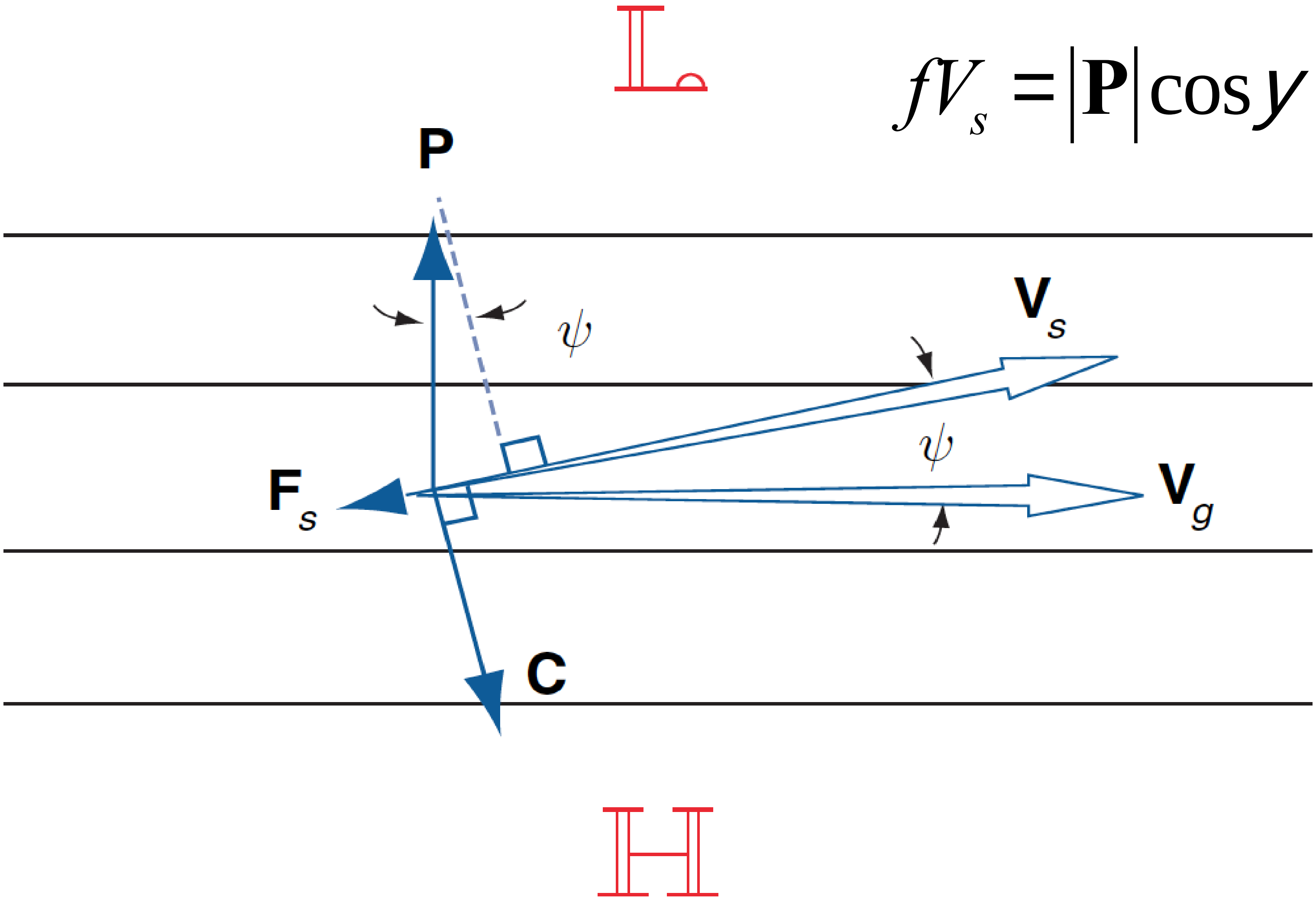
$$\mathbf{V} \approx -\frac{1}{f\rho}(\mathbf{k} \times \nabla p)$$



The geostrophic wind describes the balance between pressure gradient and Coriolis forces. It is often a good approximation of real winds observed high up in the troposphere. A prominent example is the jet stream, which is often close to geostrophic balance.

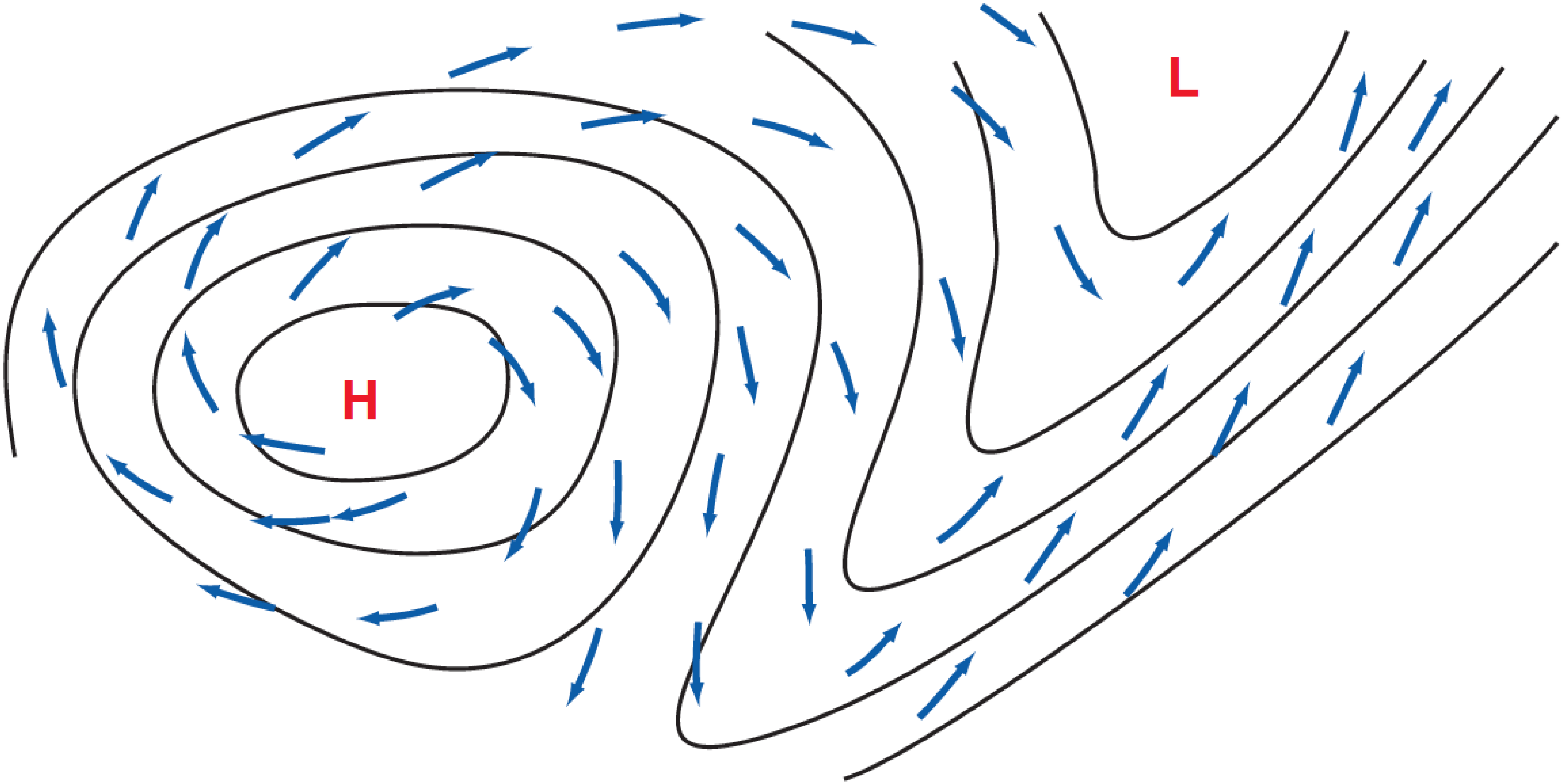
Ekman Drift when approaching the surface

$$fV_s = |\mathbf{P}| \cos y$$



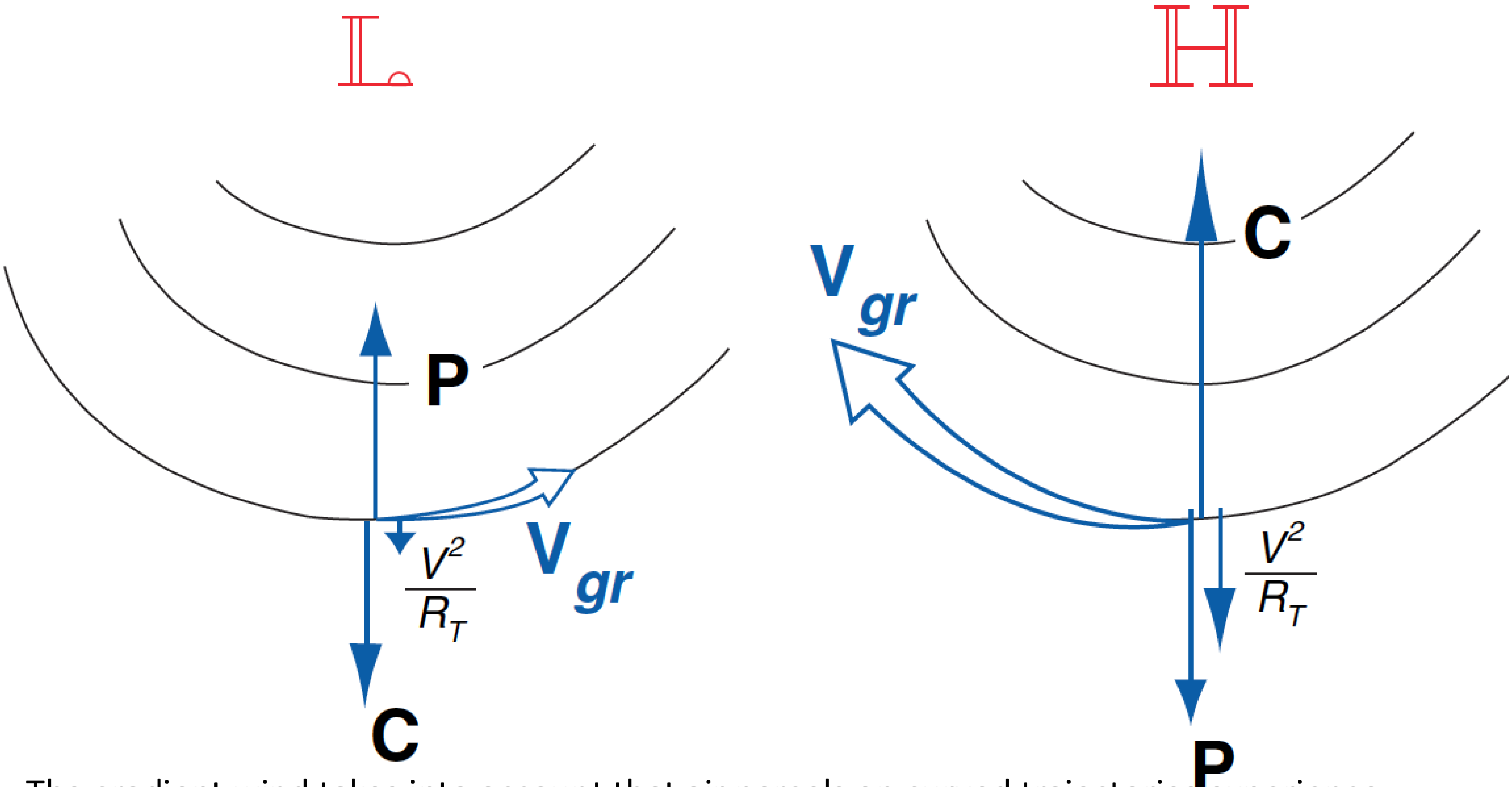
As you get closer to the surface, friction "deflects" the geostrophic wind in direction of the pressure gradient. Since the effect gradually increases, an "Ekman" spiral may result.

Schematic Surface Flow



The Ekman wind helps to lower pressure differences as it leads to cross-isobar flows induced by frictional drag close to the earth's surface.

Gradient Wind



The gradient wind takes into account that air parcels on curved trajectories experience themselves a centrifugal acceleration.

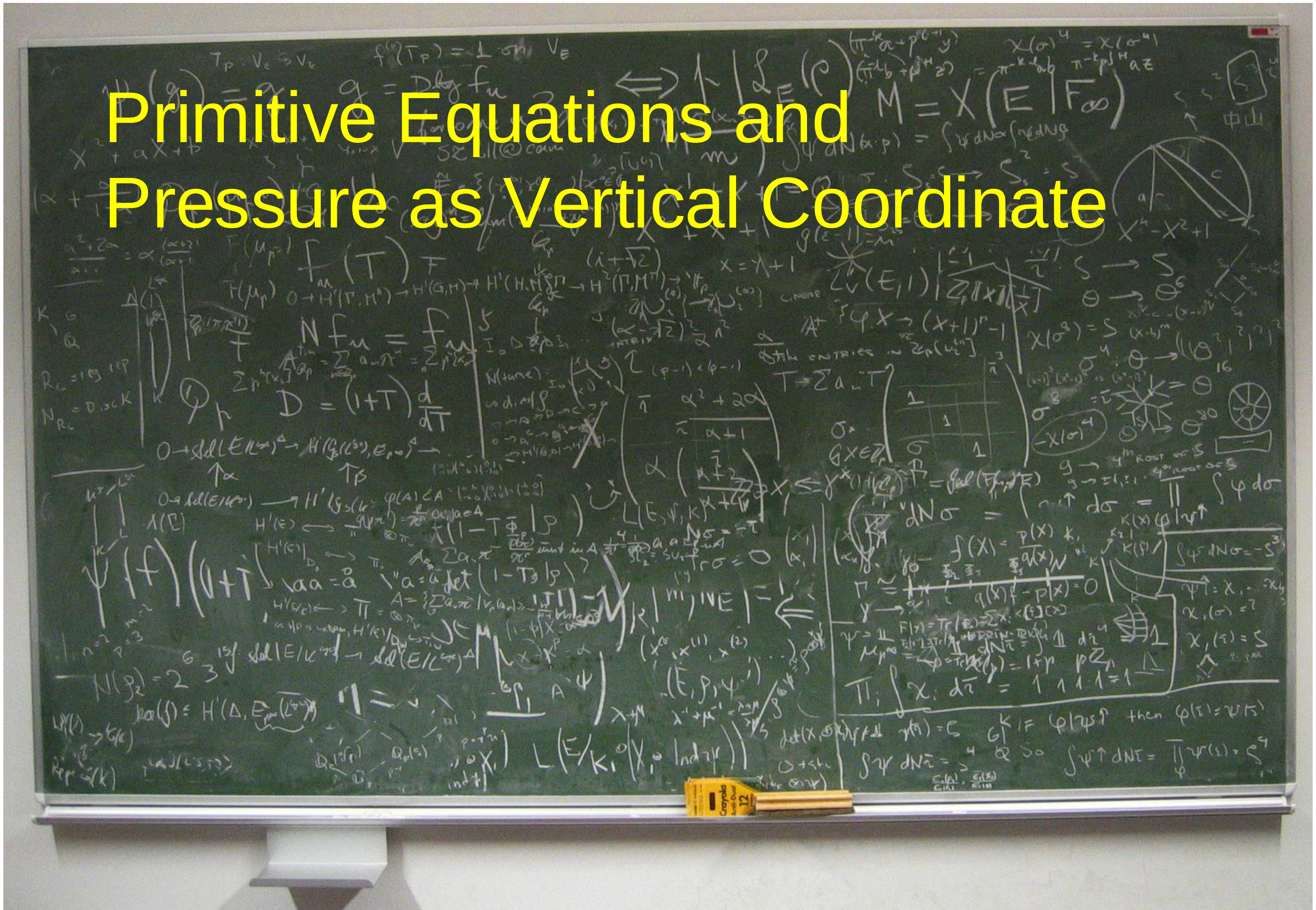
Why is there no Ekman spiral to be observed close to the ground in Switzerland?

A. Switzerland is not in the EU
B. There is not enough wind (e.g. for wind energy)
C. The terrain generates local pressure differences
D. Switzerland has no coastline

The gradient wind is

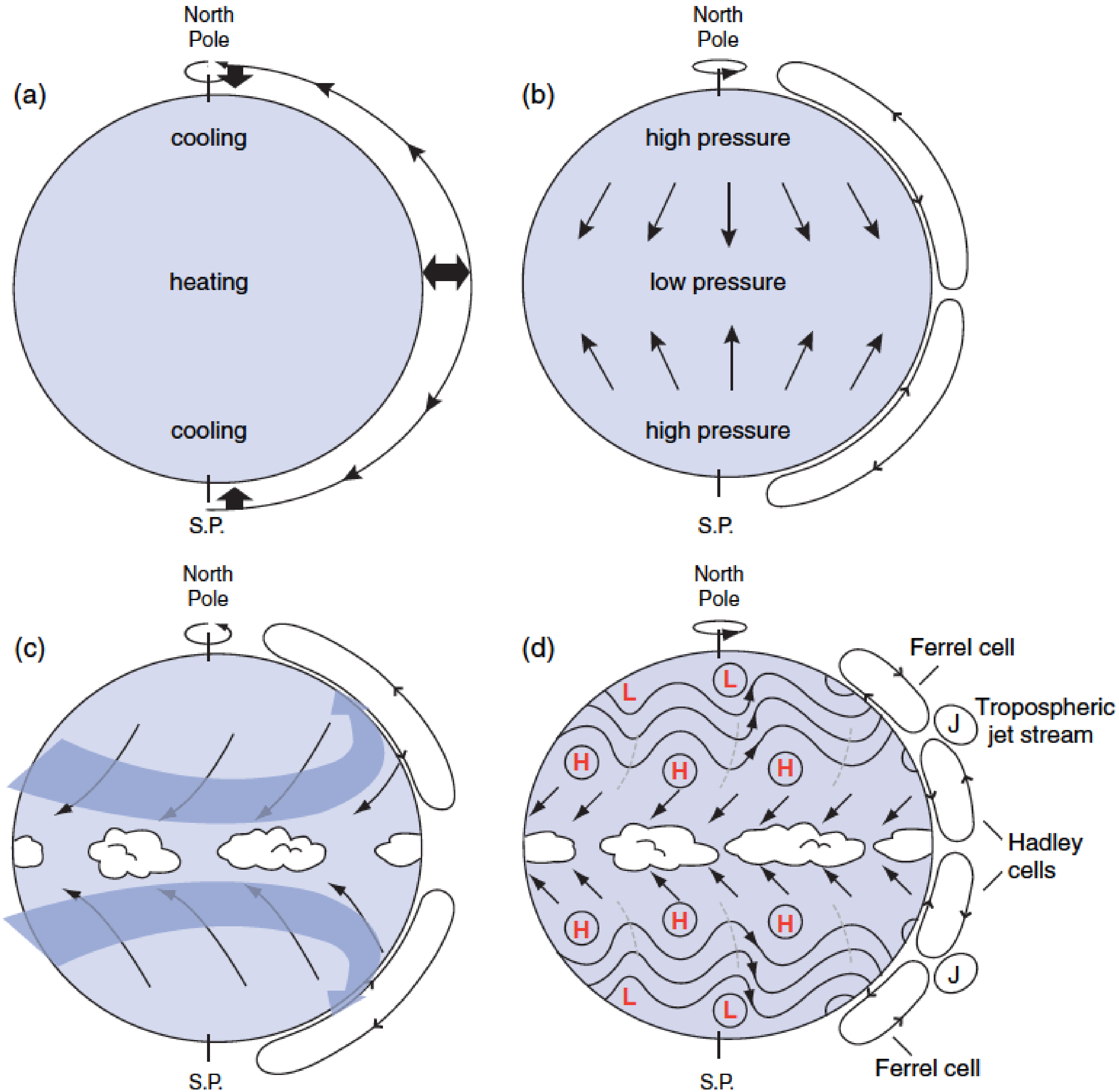
A. Only important if there is a strong pressure gradient
B. A better approximation of true wind speeds in the atmosphere than the geostrophic wind
C. A better approximation of true wind speeds in the atmosphere than the Ekman wind

Black board…



Primitive Equations and Pressure as Vertical Coordinate

Solution of Primitive Equations



Integrating the primitive equations in time, a global “Hadley” cell develops first (a and b), then a first version of the “trade winds” start at lower elevations and westerly flows at higher elevations (c) and finally (d) the most significant elements with Rossby waves, jet streams and Ferrel cells develop due to instabilities in the geostrophic wind balance.

Take Home Messages

- Large-scale horizontal motions in the atmosphere extend over scales of 10^3 km.

- Kinematic descriptions of flows can be made on the basis of flow streamlines in a natural coordinate system. Quantities such as shear, curvature, diffluence and stretching can be defined as simple derivatives of streamlines. Vorticity, divergence and deformation are “secondary” quantities, which can also be defined in Cartesian coordinates.

- Know the difference between streamlines and trajectories.

- Force balances determine motions, there are real and apparent forces. The most important real forces are pressure gradients, shear gradients (and centrifugal forces), the most important apparent forces are due to centrifugal forces and lead to Coriolis and effective acceleration of gravity.

Take Home Messages

- Geostrophic wind is determined by the balance between pressure gradient and Coriolis and makes the wind turn around low pressure systems.

- Close to the ground, friction starts to play a role and leads to deviations from the geostrophic wind with some component of the wind going towards low pressure and helping to diminish the pressure gradient.

- (The thermal wind equation relates vertical changes in geostrophic wind to temperature gradients. It helps to diagnose cold or warm advection of air.)

- The primitive equations are a complete set of equations that can be integrated on a (super-)computer to fully describe atmospheric motions. They form the basis for weather forecasting. They consist of the equations of motion for the three velocity components, the equation describing energy conservation and the equation of diagnosing the vertical velocity from the divergence of the horizontal flow, which is called the continuity equation. Note that the ideal gas law is assumed to hold and that the hydrostatic (or hypsometric in pressure coordinates) equation is still used as the vertical force balance despite the fact that vertical velocities are now present.