Exercise set 7: Properties of turbulence

Atmospheric turbulence data is measured with a sonic anemometer and a hygrometer during a field campaign at the Seedorf lake (FR) to investigate turbulent fluxes over water bodies. You are given three sample data files collected at the same time and place:

u.out – streamwise velocity (u) measured at 20 Hz (units: m/s)

w.out – vertical velocity (w) measured at 20 Hz (units: m/s)

q.out – specific humidity (q) measured at 20 Hz (units: g/m^3)

Figure 1 – The Seedorf lake.

Note: All measurements were taken at a height of 2 m above the ground.

(a.) Plot u, w and q as a function of time. On your plots, indicate the mean values of u, w and q ($\bar{u}$, $\bar{w}$ and $\bar{q}$) as well as the total sampling time.

(b.) Verify that $\overline{u'} = 0$, $\overline{w'} = 0$ and $\overline{q'} = 0$. The overbar denotes a temporal average and u' means the fluctuating component of the streamwise velocity.

(c.) Plot u', w' and q' as a function of time. Calculate the standard deviation of those variables (σ_u, σ_w, σ_q).

(d.) Determine the turbulent fluxes of momentum ($\overline{u'w'}$) and water vapour ($\overline{w'q'}$).

(e.) Based on what you found in (d), what can you tell about the time of the day when these measurements were taken?

(f.) Knowing that the vertical gradient of the mean water vapour concentration at the measurement height is $\frac{d\bar{q}}{dz} = -0.5\frac{g/m^3}{m}$, obtain the eddy-diffusion coefficient for water vapor. Is it larger or smaller than the molecular diffusion coefficient ($D_m = 2.42 \times 10^{-5}\ m^2/s$ at an ambient temperature of 20C)?

(g.) Compute the auto-correlation function of u' (normalized so the maximum correlation is 1.0) and plot the function. (Hint: you can use the auto-correlation function " autocorr" in the Matlab software).

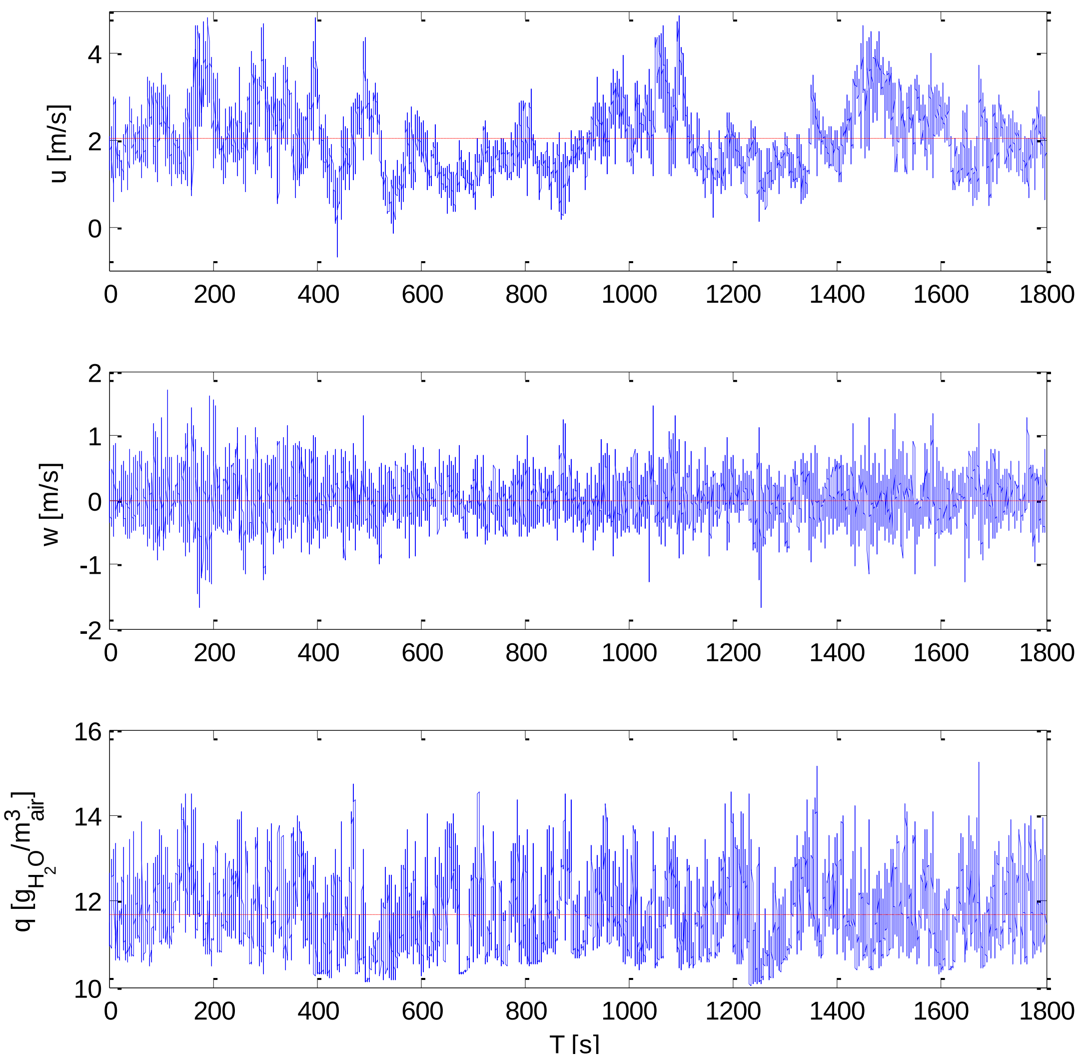
(h.) Calculate the integral time scale (integrate the auto-correlation function between 0 and the time when the correlation function becomes zero).

(i.) Estimate the "typical" size of the largest eddies (find the integral length scale). Hint: You can use the mean velocity to calculate the integral length scale from the integral time scale.

(j.) Estimate the Reynolds number based on the mean velocity and integral length scale. Based on that, is the flow laminar or turbulent?

(a)

$$\bar{u} = \frac{1}{N}\sum_{i=1}^{N} u_i = 2.03 \ \left[{}^{m}\!/_{s}\right], \quad \bar{w} = \frac{1}{N}\sum_{i=1}^{N} w_i = -1.14\times10^{-5} \ \left[{}^{m}\!/_{s}\right], \quad \bar{q} = \frac{1}{N}\sum_{i=1}^{N} q_i = 11.67 \ \left[{}^{g}\!/_{m^3}\right]$$



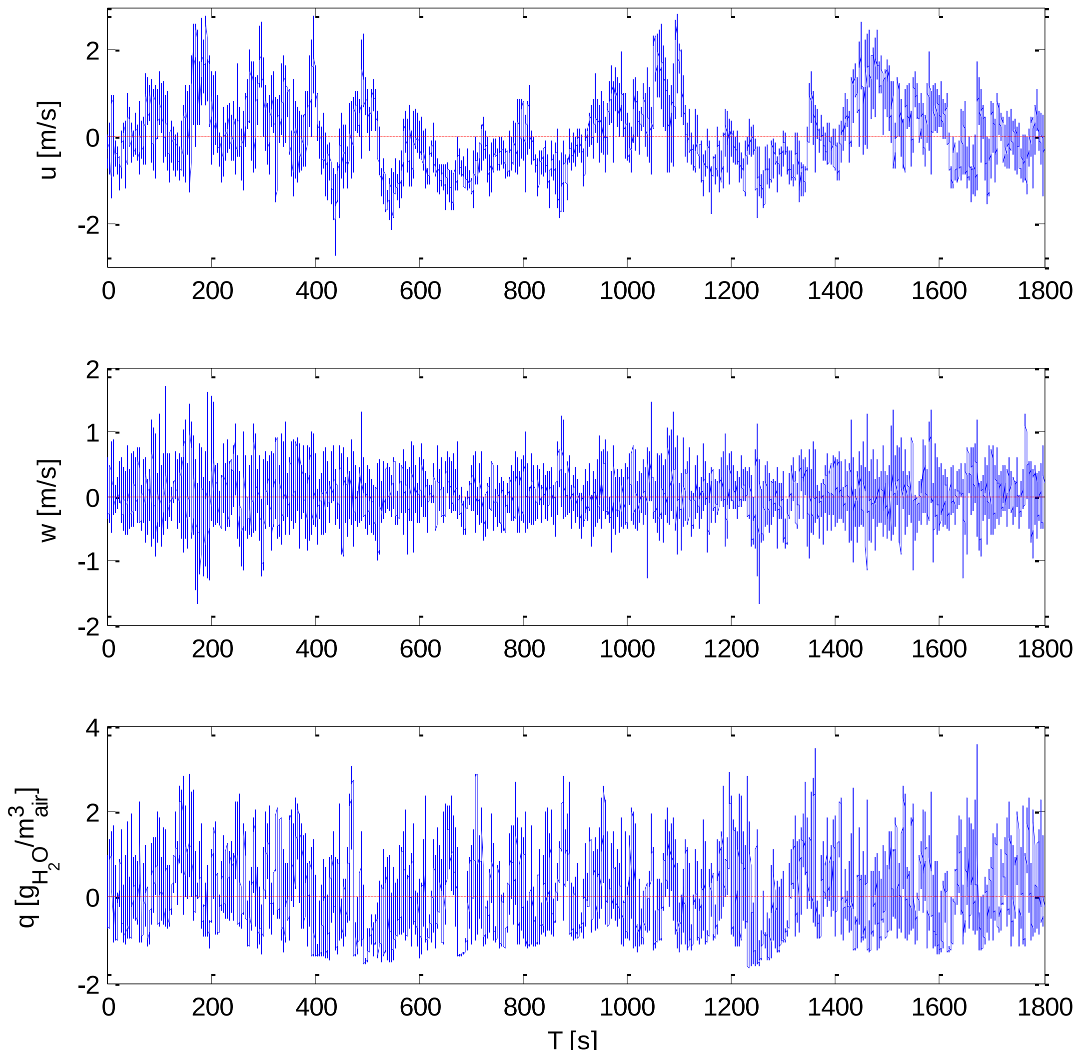
The solid lines denote *u*, *w* and *q*. The dashed lines denote $\bar{u}$, $\bar{w}$ and $\bar{q}$.

The total sampling time is 30 minutes.

(b~c)

Firstly, we prove mathematically that $\overline{a'} = 0$. ➔ $a = \bar{a} + a' \rightarrow \overline{a = \bar{a} + a'} \rightarrow \bar{a} = \bar{\bar{a}} + \overline{a'} \rightarrow \overline{a'} = 0$

$\overline{u'} = 1.96\times10^{-14} \left[{}^{m}\!/\!_{s}\right]$ $\overline{w'} = -1.67\times10^{-17} \left[{}^{m}\!/\!_{s}\right]$ $\overline{q'} = -2.74\times10^{-14} \left[{}^{m}\!/\!_{s}\right]$



The solid lines denote *u'*, *w'* and *q'*. The dashed lines denote $\overline{u'}$, $\overline{w'}$ and $\overline{q'}$.

$\sigma_u = 0.77$ [m/s] $\sigma_w = 0.31$ [m/s] $\sigma_q = 0.87$ [g/m^3]

(d~e)

$$\overline{u'w'} = -0.0511 \ \left[m^2 / s^2 \right] \qquad \overline{w'q'} = 0.1324 \ \left[g / m^2 s \right]$$

The mean vertical flux of water vapor is positive (upward flux) due to evaporation from the relatively moist surface. Evaporation typically occurs during day time conditions.

(f)

From the eddy-diffusion model: $\overline{w'q'} = -D_t \dfrac{\partial \overline{q}}{\partial z}$,

we can obtain $D_t = -\dfrac{\overline{w'q'}}{\partial \overline{q} / \partial z} = -\dfrac{0.1324 \ gm^{-2}s^{-1}}{-0.5 \ gm^{-4}} \approx 0.265 \ m^2 s^{-1}$

As expected, due to the high mixing and transport efficiency of turbulence, $D_t >> D_m$.

(g ~ j)

To calculate the auto-correlation function, we use its definition:

$$\rho(\Delta t) = \frac{\overline{u'(t)u'(t+\Delta t)}}{\overline{\left(u'(t)\right)^2}}$$

The integral scale can be calculated from the autocorrelation function using

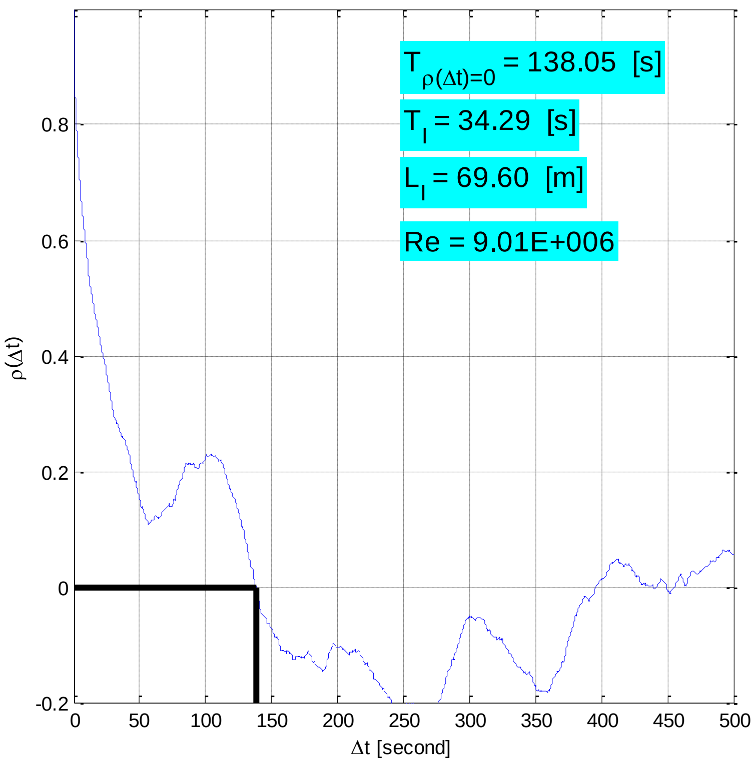
$$t_I = \int_0^{\infty} \rho\left(\Delta t\right) d\left(\Delta t\right)$$

Here, we take the upper bound at the time when the autocorrelation becomes zero.

$$T_{\rho(\Delta t)=0} = 138.05 \ [s] \ \Rightarrow \ t_I = \int_0^{T_{\rho(\Delta t)=0}} \rho\left(\Delta t\right) d\left(\Delta t\right) = 34.29 \ [s]$$

$$L_I \approx \overline{u} \times t_I = 2.03 \times 34.29 = 69.6 \ [m]$$

$$\mathrm{Re} = \frac{\overline{u} \times L_I}{\nu} = \frac{2.03 \times 69.6}{15.68 \times 10^{-6}} \approx 9.01 \times 10^6 \ \Rightarrow \ \text{The flow is highly turbulent.}$$



T ρ(Δt)=0 = 138.05 [s]
T I = 34.29 [s]
L I = 69.60 [m]
Re = 9.01E+006
ρ(Δt)
Δt [second]
0.8
0.6
0.4
0.2
0
-0.2
0
50
100
150
200
250
300
350
400
450
500