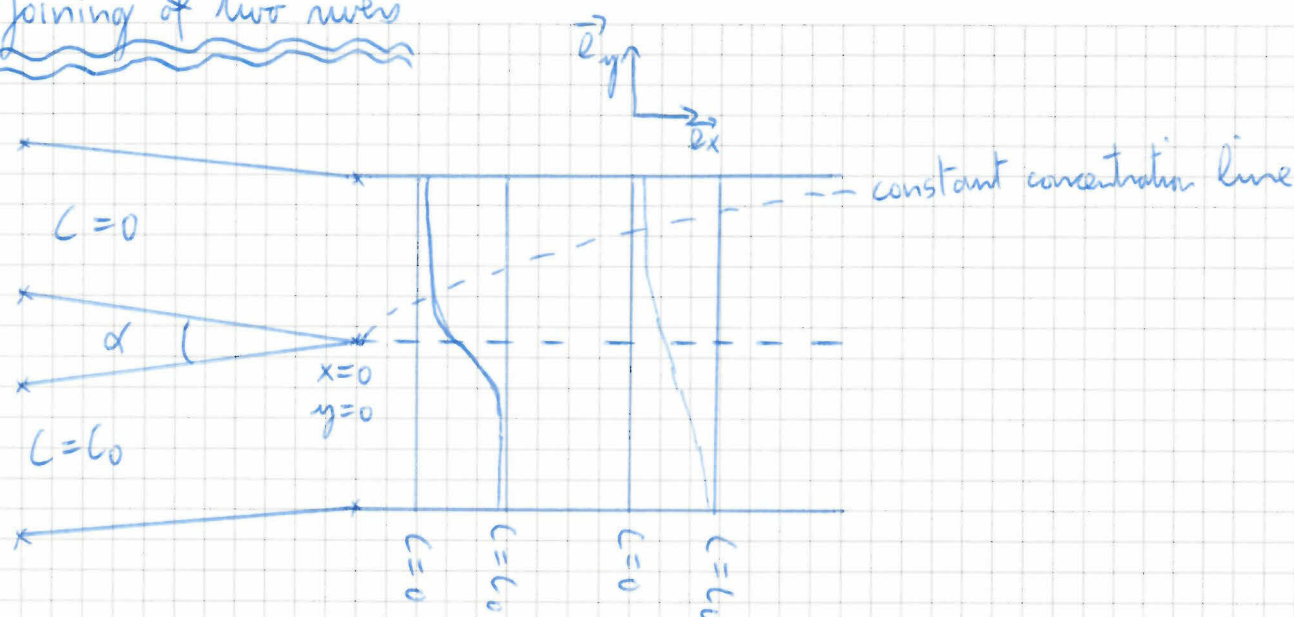




(1)

Joining of two rivers



$$C(x=0, y>0) = 0$$

$$C(x=0, y<0) = C_0$$

$$C_0 \theta(-y)$$

We look for a steady state $\Rightarrow$ no I.C.

$$C(x, y \rightarrow \infty) = 0$$

$$C(x, y \rightarrow -\infty) = C_0$$

wide river

$\alpha \ll 1$ small angle $\Rightarrow$ negligible advection in the transverse direction

ADE (2D)

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_x \frac{\partial^2 C}{\partial x^2} + D_y \frac{\partial^2 C}{\partial y^2}$$

Steady state

no transverse advection

$Pe_x = \frac{D_x}{u^2 t}$

$$\ll 1$$

Formal proof later in the lecture

The 2D approach is valid for a shallow river



(2)

The 2D ADE reduces to

$$u \frac{\partial C}{\partial x} = D_y \frac{\partial^2 C}{\partial y^2}$$

We set $h = \frac{x}{u}$ and $\zeta = y$

↳ Flow time from the junction

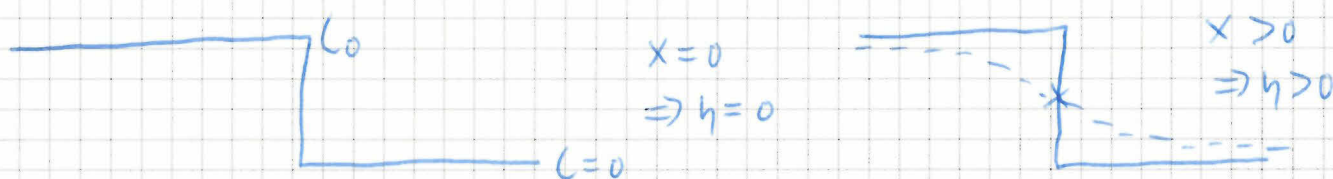
$$\frac{\partial}{\partial x} \cdot = \frac{\partial h}{\partial x} \frac{\partial}{\partial h} \cdot + \frac{\partial \zeta}{\partial x} \frac{\partial}{\partial \zeta} \cdot = \frac{1}{u} \frac{\partial}{\partial h} \cdot$$

$$\frac{\partial}{\partial y} \cdot = \frac{\partial}{\partial \zeta} \cdot$$

The 2D ADE in the new coordinates becomes

$$\frac{\partial C}{\partial h} = D_y \frac{\partial^2 C}{\partial \zeta^2} \quad (*) \quad \text{D.E. (no advection)}$$

B.C. on x gives the I.C for the D.E. (*)



This initial condition corresponds to the gate opening problem with the solution

$$C(h, \zeta) = \frac{C_0}{2} \left[1 - \operatorname{erf} \left(\frac{\zeta}{\sqrt{4D_y h}} \right) \right]$$

$$\Rightarrow C(x, y) = \frac{C_0}{2} \left[1 - \operatorname{erf} \left(\frac{y}{\sqrt{4D_y \frac{x}{u}}} \right) \right] \quad \text{No time (steady state)}$$