Environmental transport phenomena: Lecture I

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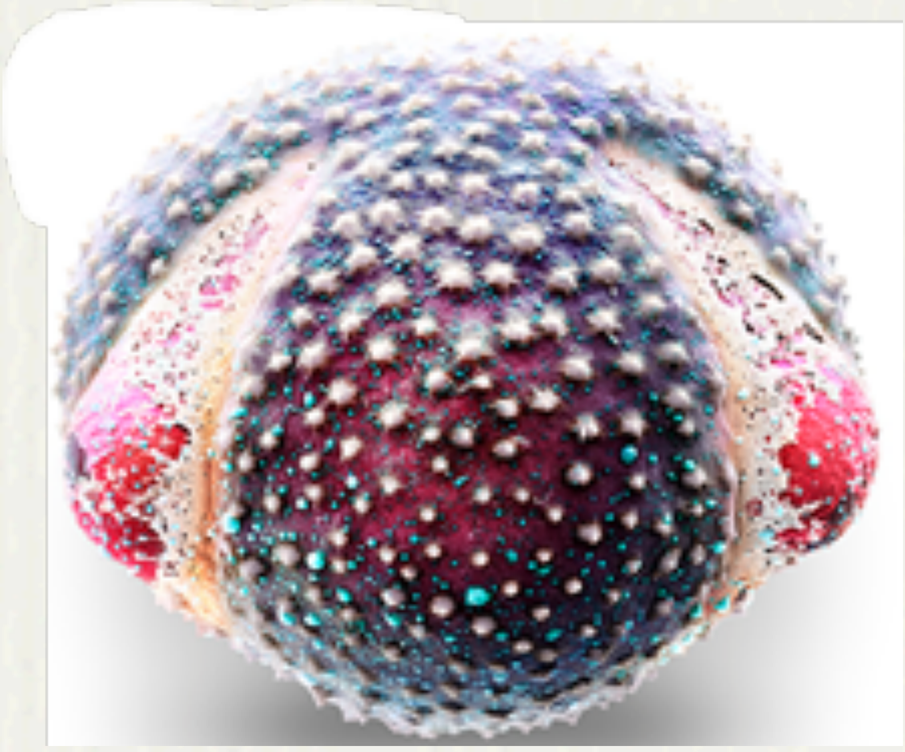
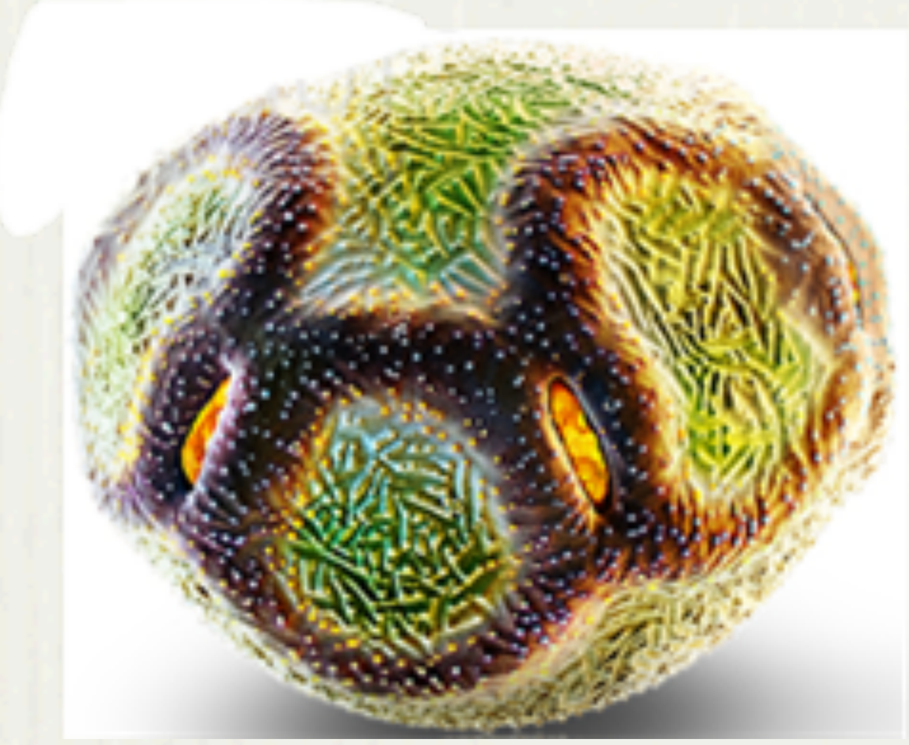




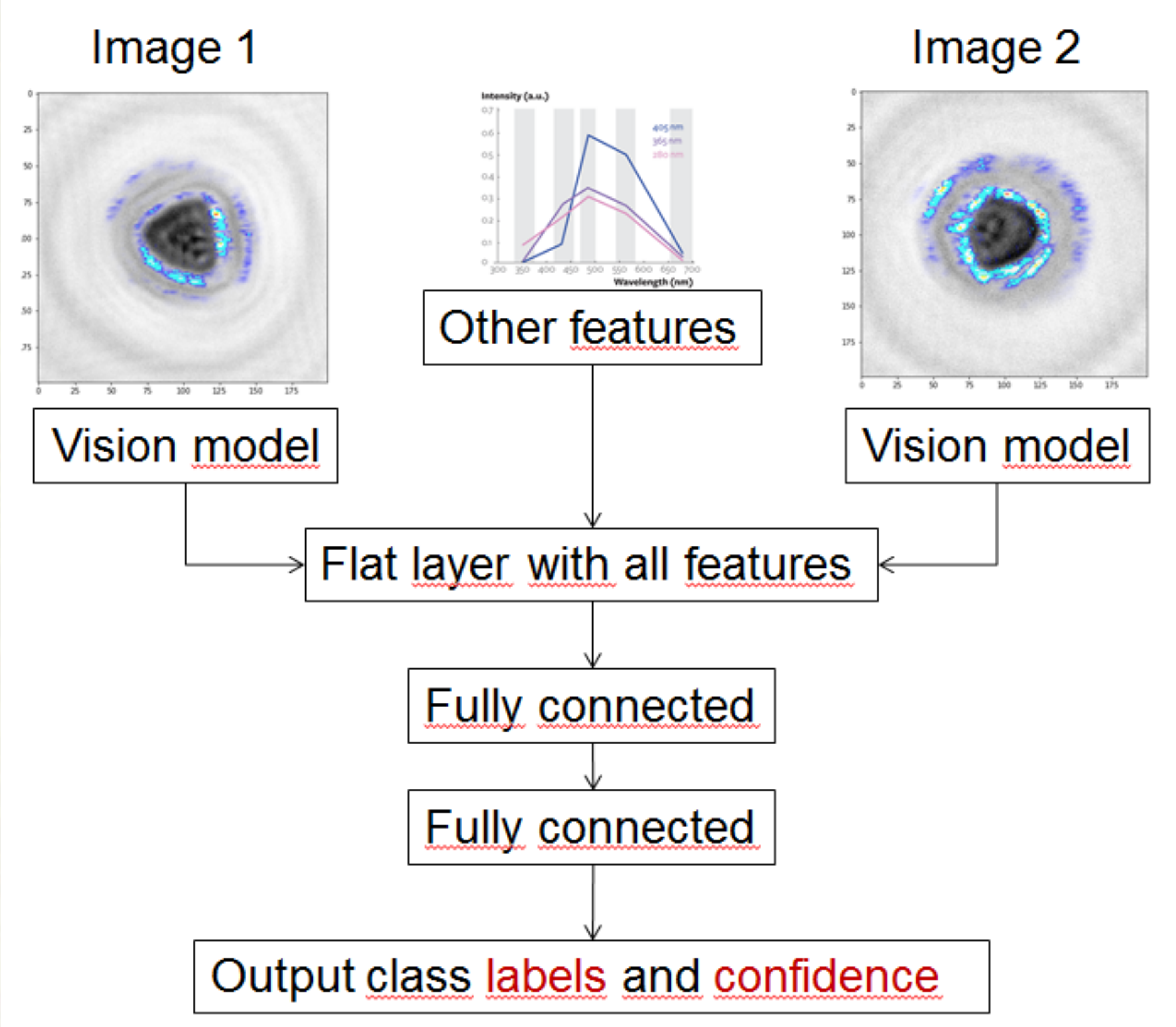
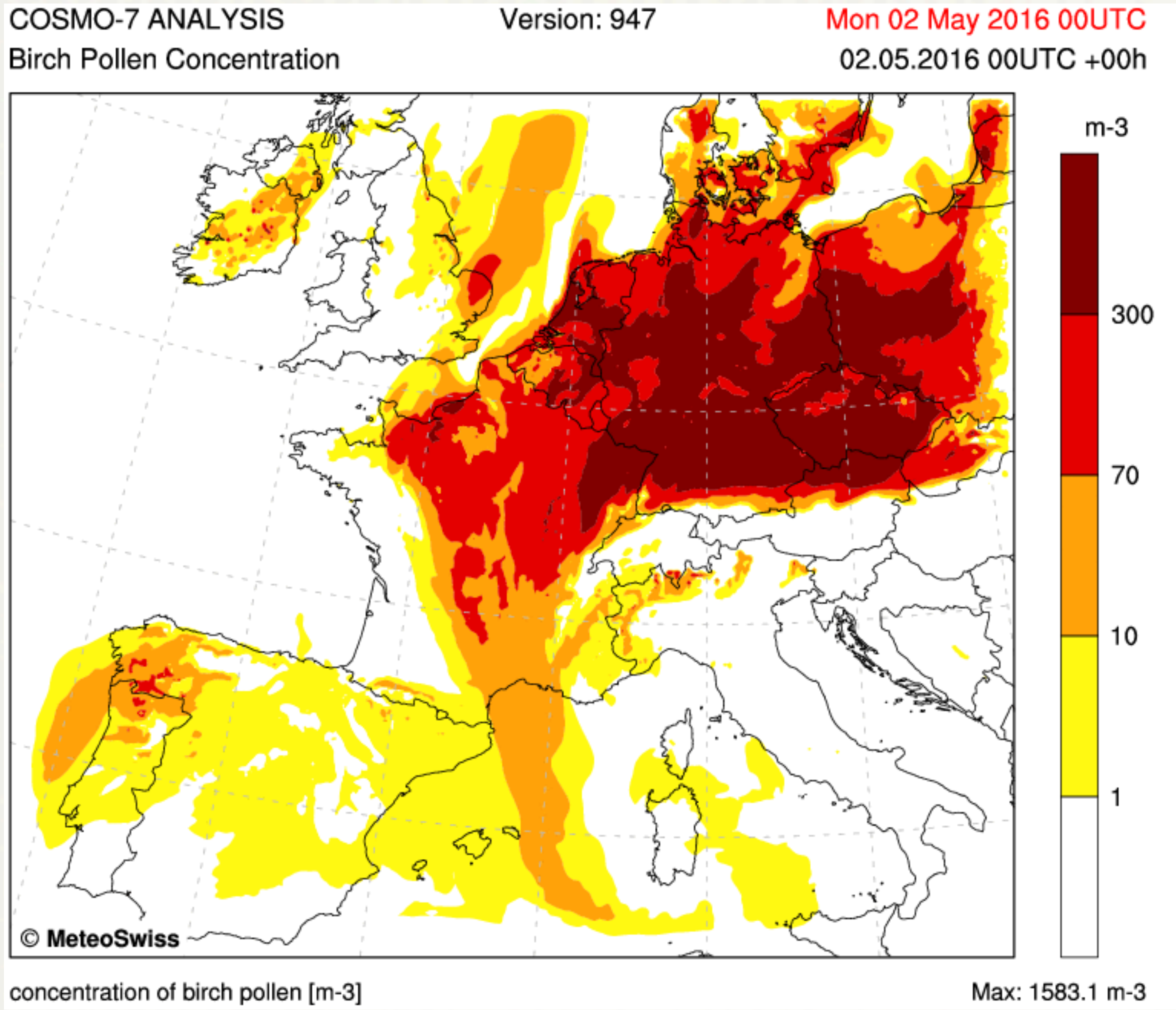
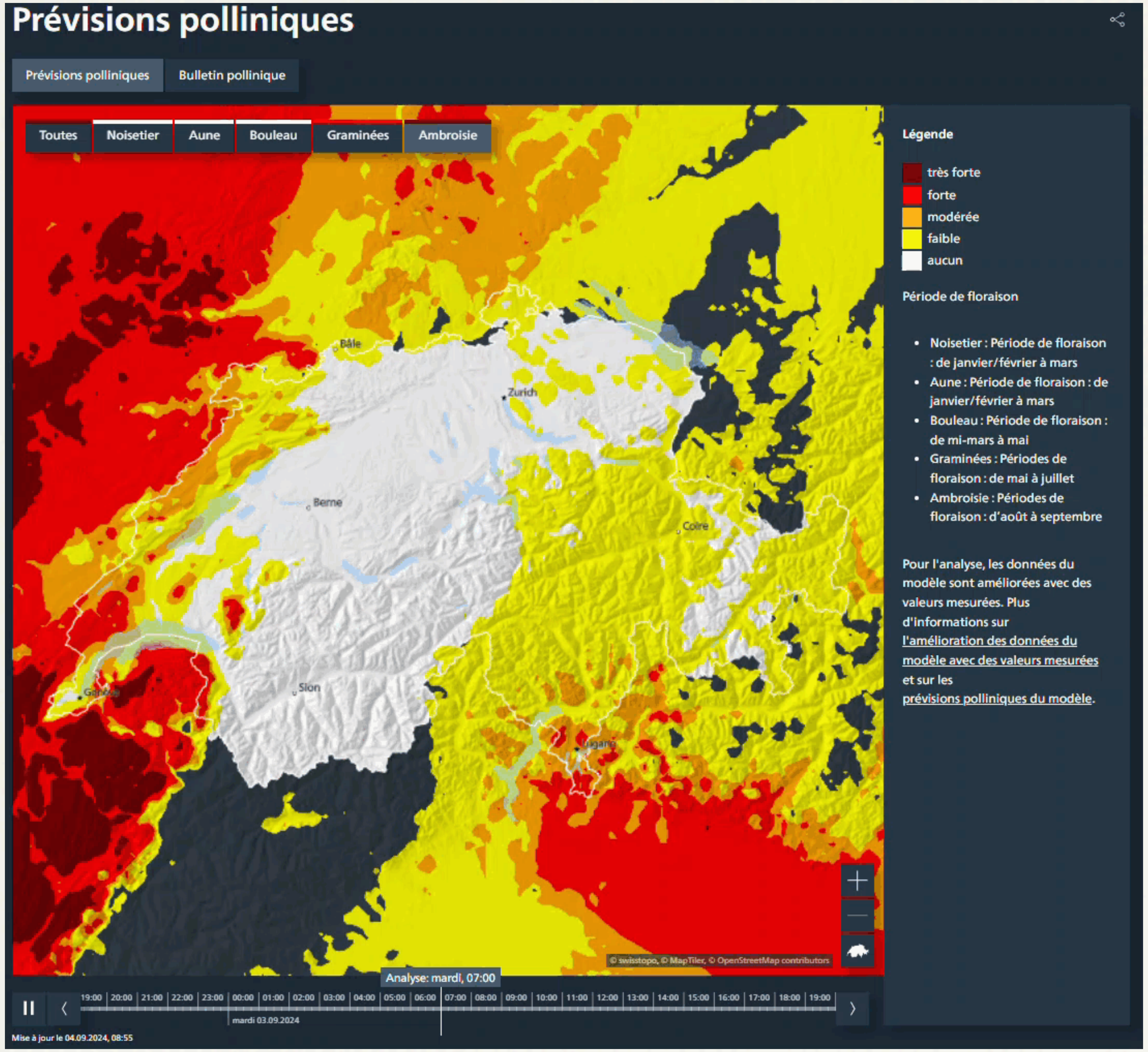


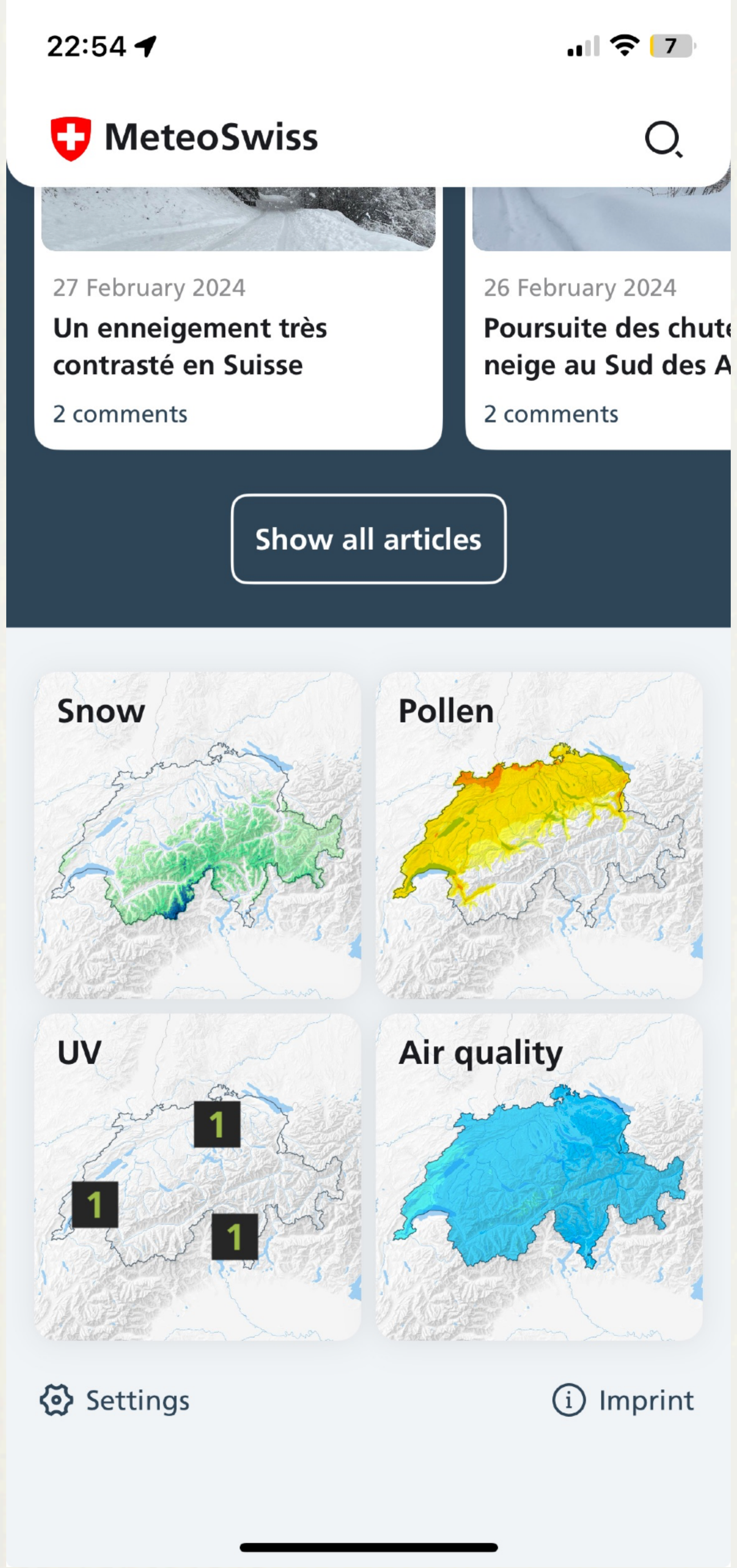
Image 1
Image 2
Other features
Vision model
Vision model
Flat layer with all features
Fully connected
Fully connected
Output class labels and confidence



COSMO-7 ANALYSIS
Version: 947
Mon 02 May 2016 00UTC
Birch Pollen Concentration
02.05.2016 00UTC +00h
m-3
300
70
10
1
© MeteoSwiss
concentration of birch pollen [m-3]
Max: 1583.1 m-3



Prévisions polliniques
Prévisions polliniques
Bulletin pollinique
Toutes
Noisetier
Aune
Bouleau
Graminées
Ambroisie
Bâle
Zurich
Berne
Coire
Sion
Lugano
© swisstopo, © MapTiler, © OpenStreetMap contributors
Analyse: mardi, 07:00
19:00 20:00 21:00 22:00 23:00 00:00 01:00 02:00 03:00 04:00 05:00 06:00 07:00 08:00 09:00 10:00 11:00 12:00 13:00 14:00 15:00 16:00 17:00 18:00 19:00
mardi 03.09.2024
Mise à jour le 04.09.2024, 08:55
Légende
très forte
forte
modérée
faible
aucun
Période de floraison
Noisetier : Période de floraison : de janvier/février à mars
Aune : Période de floraison : de janvier/février à mars
Bouleau : Période de floraison : de mi-mars à mai
Graminées : Périodes de floraison : de mai à juillet
Ambroisie : Périodes de floraison : d'août à septembre
Pour l'analyse, les données du modèle sont améliorées avec des valeurs mesurées. Plus d'informations sur l'amélioration des données du modèle avec des valeurs mesurées et sur les prévisions polliniques du modèle.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MeteoSwiss
27 February 2024
Un enneigement très contrasté en Suisse
2 comments
26 February 2024
Poursuite des chute
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Snow
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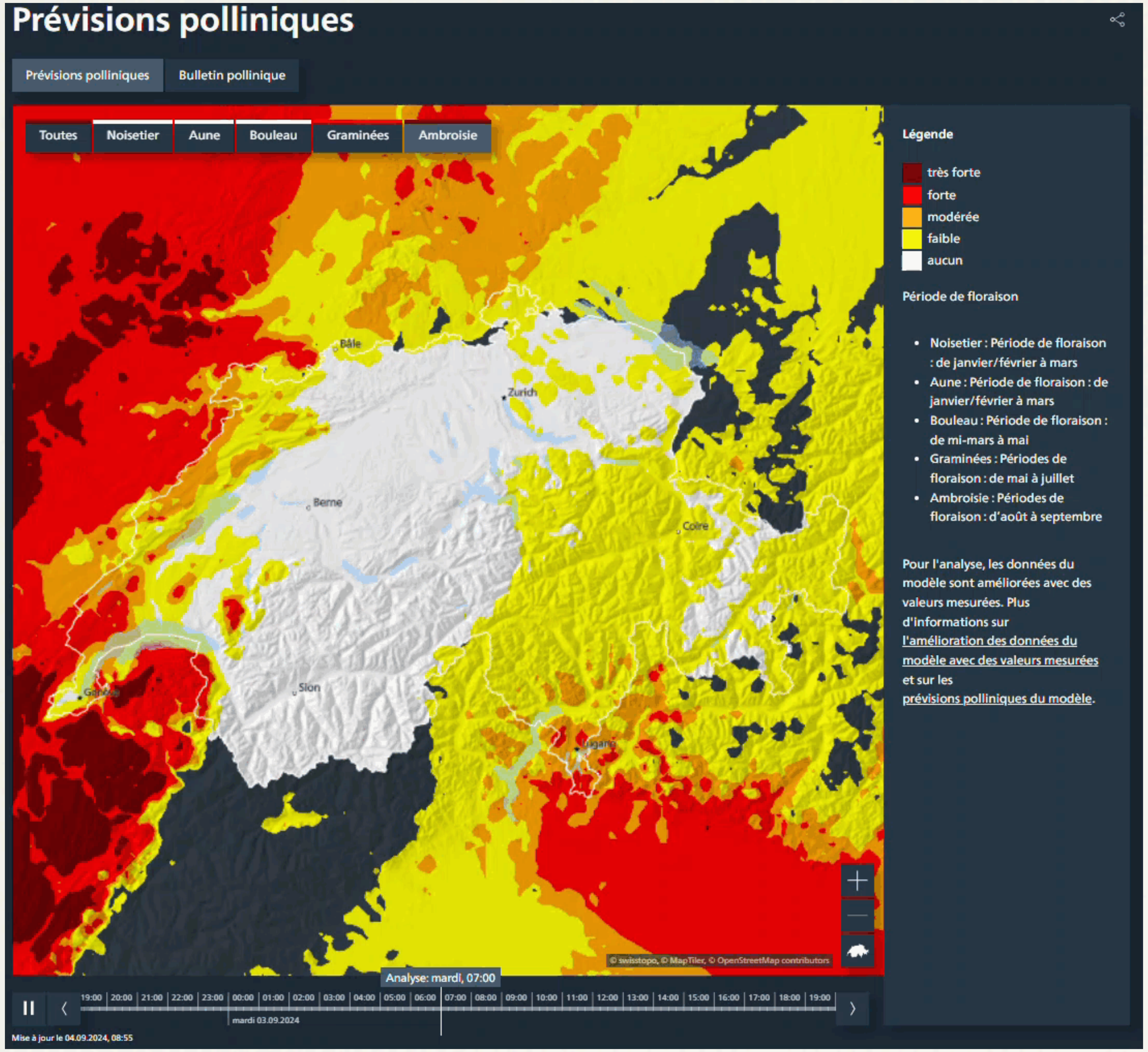
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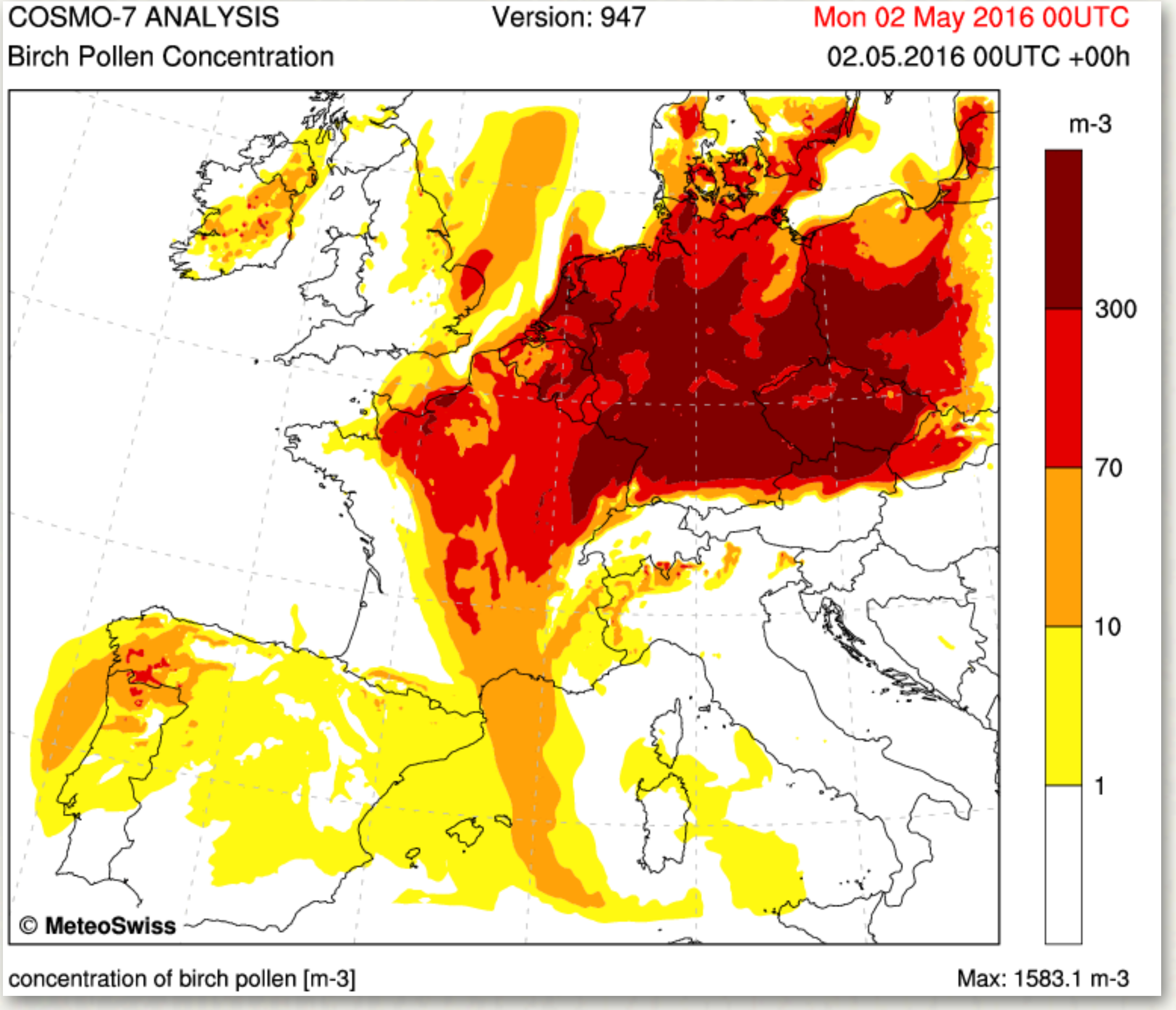
Analyse: mardi, 07:00
mardi 03.09.2024
Mise à jour le 04.09.2024, 08:55

Summary

"The course aims at introducing basic **physical** aspects of **molecular and turbulent diffusion**, as well as of **dispersion** processes, their **mathematical modeling**, solutions and related **environmental applications**"



Lecture goals

Describe and interpret the physical processes

Solve and elaborate simple physical **models**

Apply computational fluid dynamics (CFD) **models**

Develop numerical transport **models with FLUENT**: problem formulation, **modeling**, and **interpretation** of the results

Course organisation

I) Part I: 6 weeks teacher Benoît Crouzy (benoit.crouzy@meteoswiss.ch)

II) Part II: 4 weeks teacher Fernando Porte-Agel (fernando.porte-agel@epfl.ch)

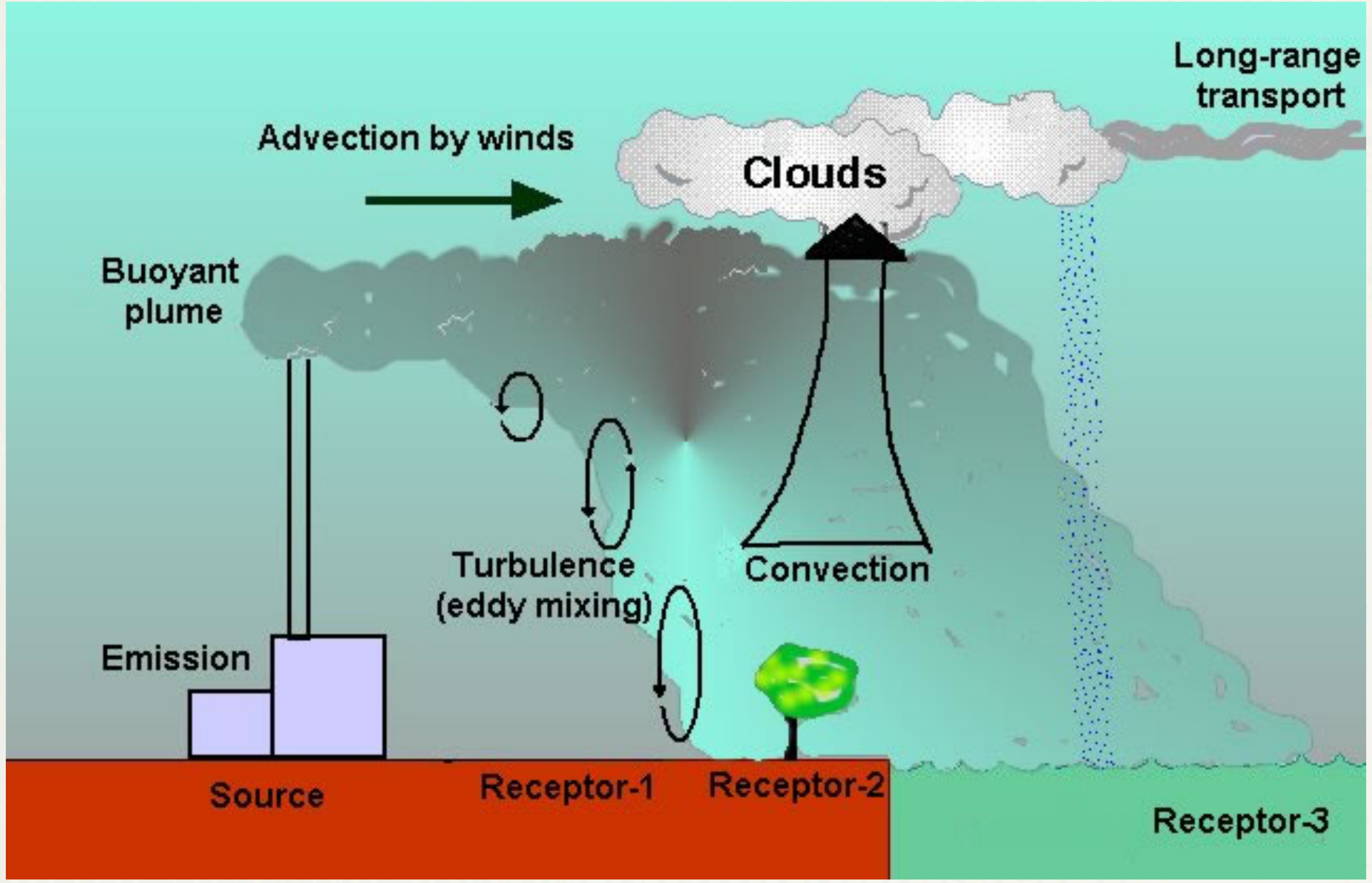
III) Fluent project: 4 weeks

Slides + Lecture notes + Reference (book)

Course grade: **written** exam 70% + **project** grade 30%

Contents

- **Advection and diffusion**
- **Point source pollution**
- **Introduction to turbulence**
- **Turbulent dispersion**
- Mixing in rivers, lakes and in reservoirs
- Atmospheric boundary layer
- Computational fluid dynamics



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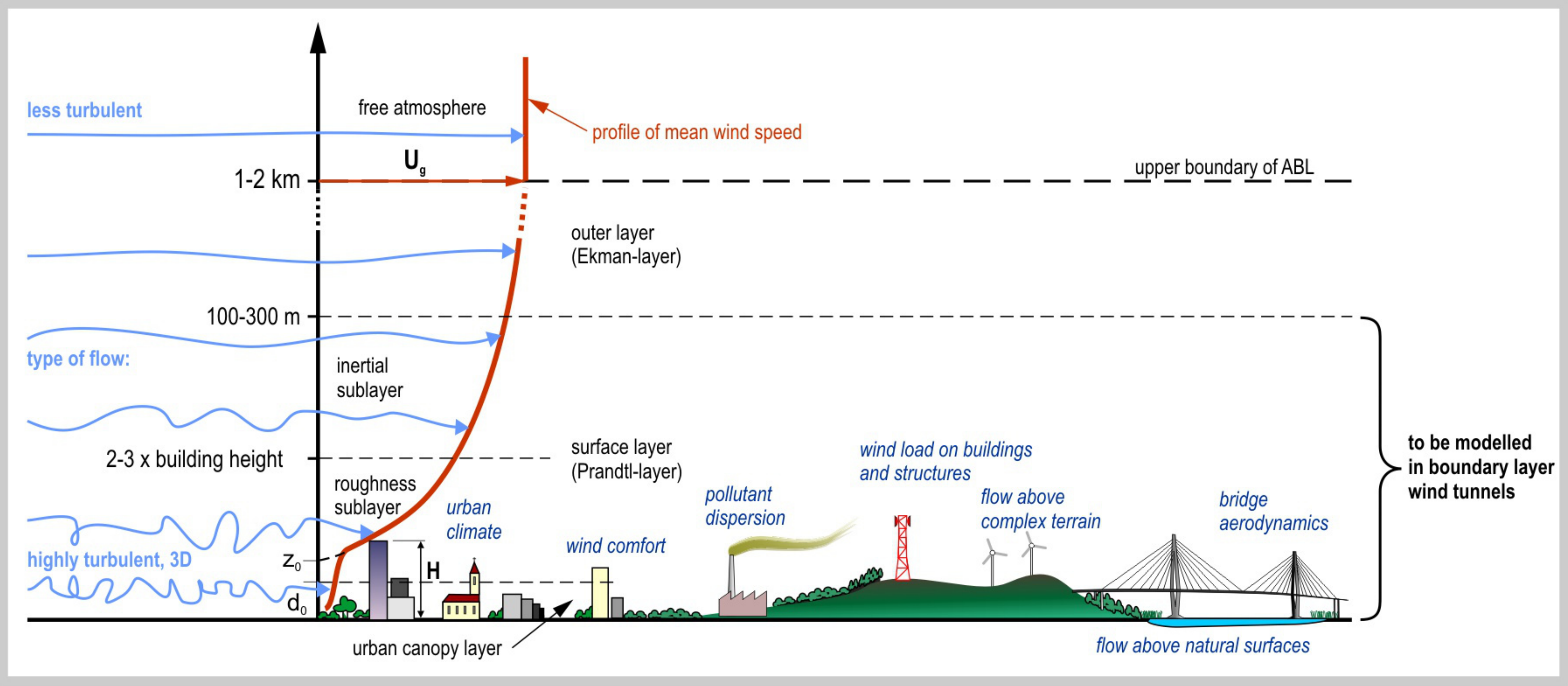

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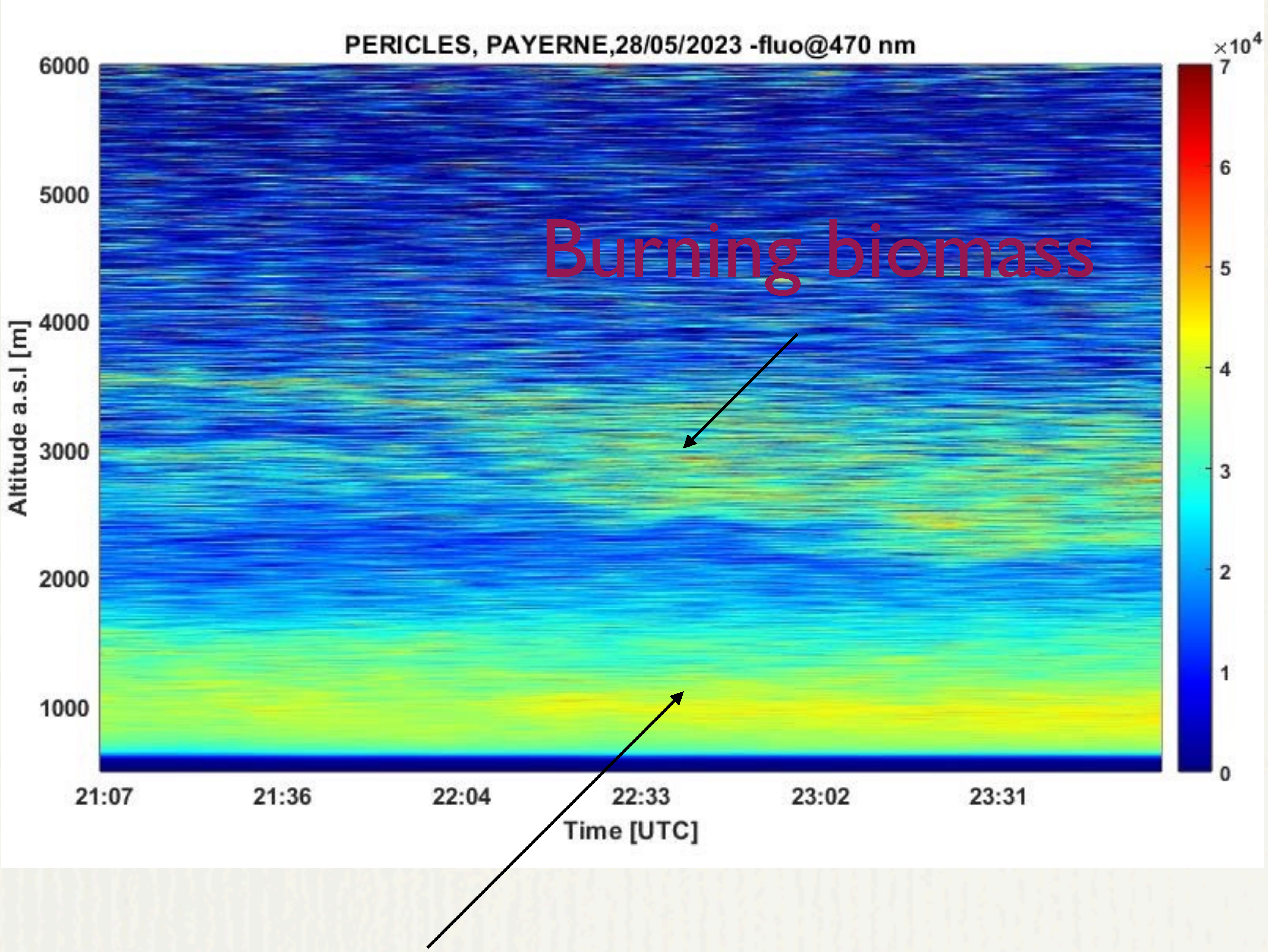
Fluorescent particles



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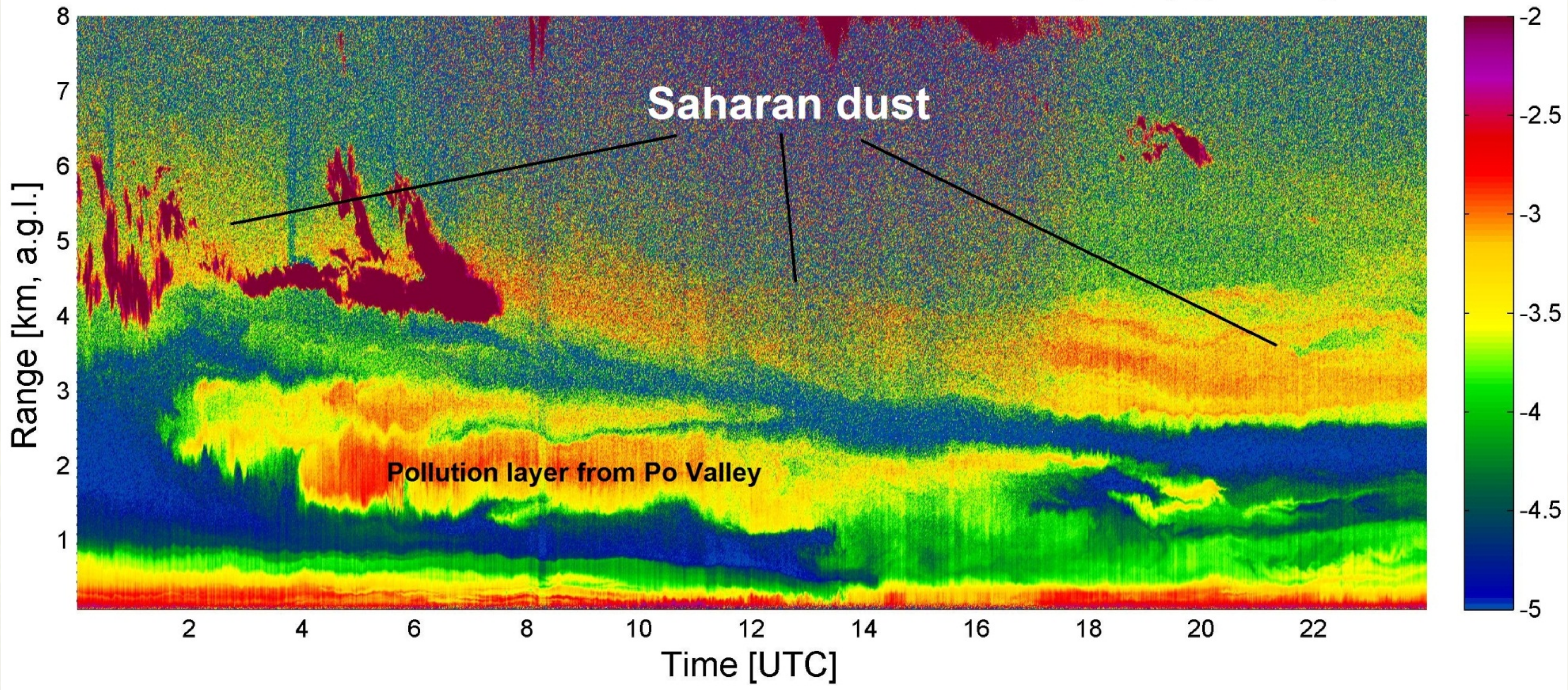

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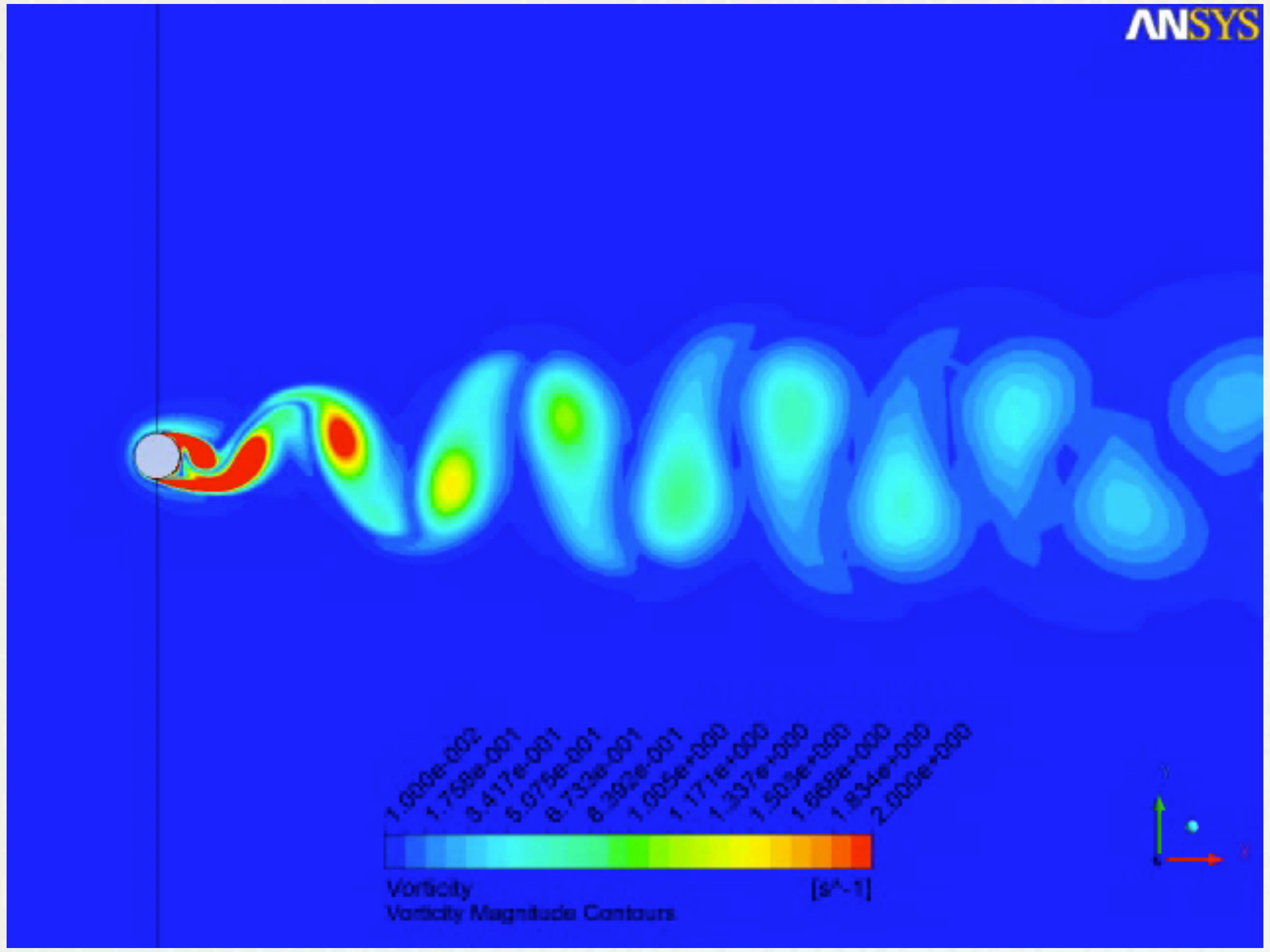


Fluent project introduction

- Introductory session **today** (over exercise session)
- **Tutorial** at your own pace (Q&A after exercise week 3-4)
- **Mini-project** (Q&A after exercise week 7-8)

Outline (Part I of the lecture)

I) **Dimensional** analysis

II) **Fluid dynamics** (summary of concepts needed)

III) Molecular **diffusion**

- Diffusion equation, Fick's laws
- Solutions for various **initial** and **boundary conditions**
- Introduction to **partial differential equations** (methods and classification)
- **Advection-Diffusion-Reaction** equation

I) Dimensional analysis

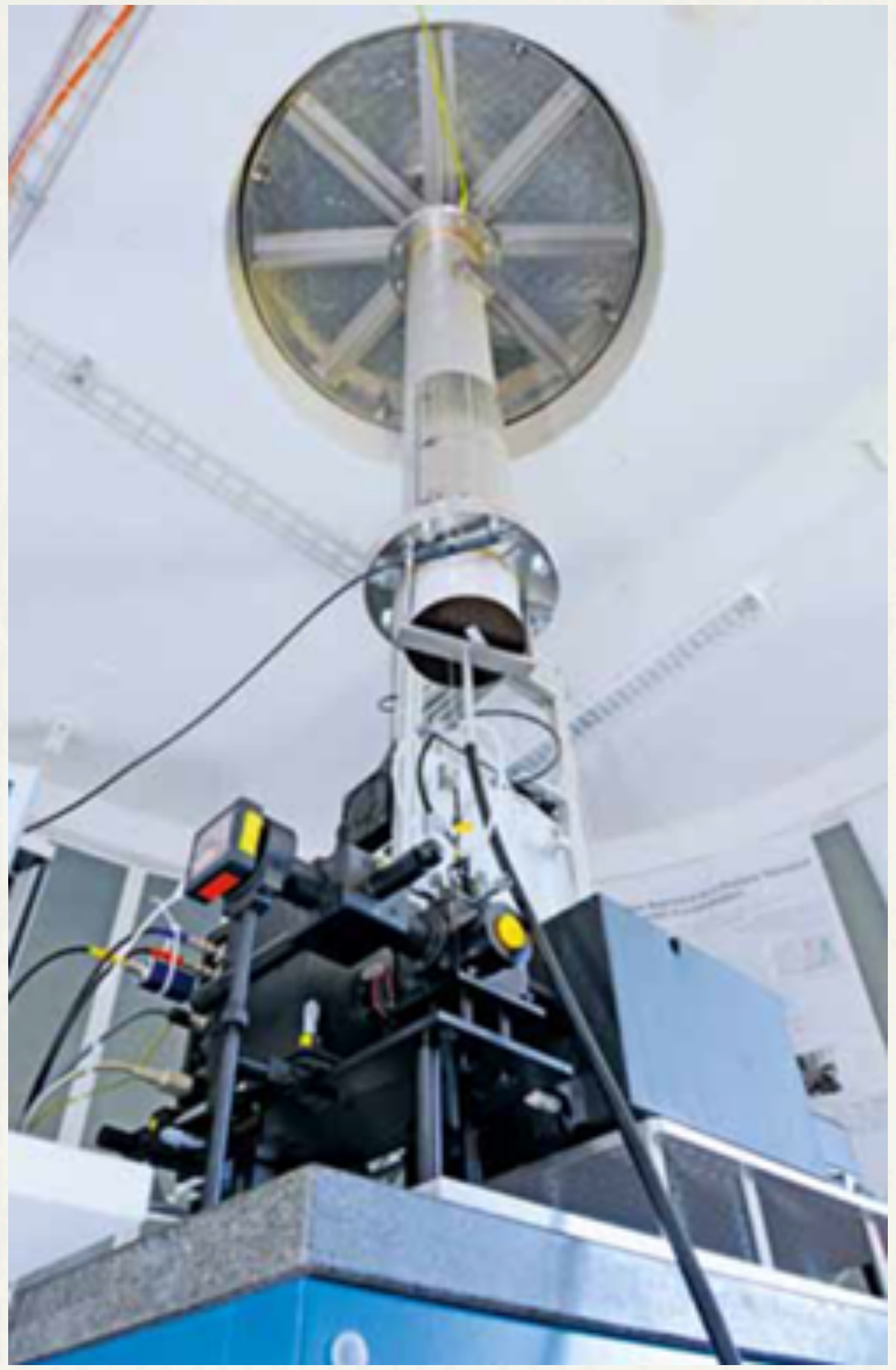
Dimensional analysis

- Review on **units** and **dimensions**
- Simple approach to **tackle complex problems**
- Physical modeling (**scaled** models)
- Interpretation of experimental data (**relative** importance of physical phenomena)

Metrology and environmental sciences

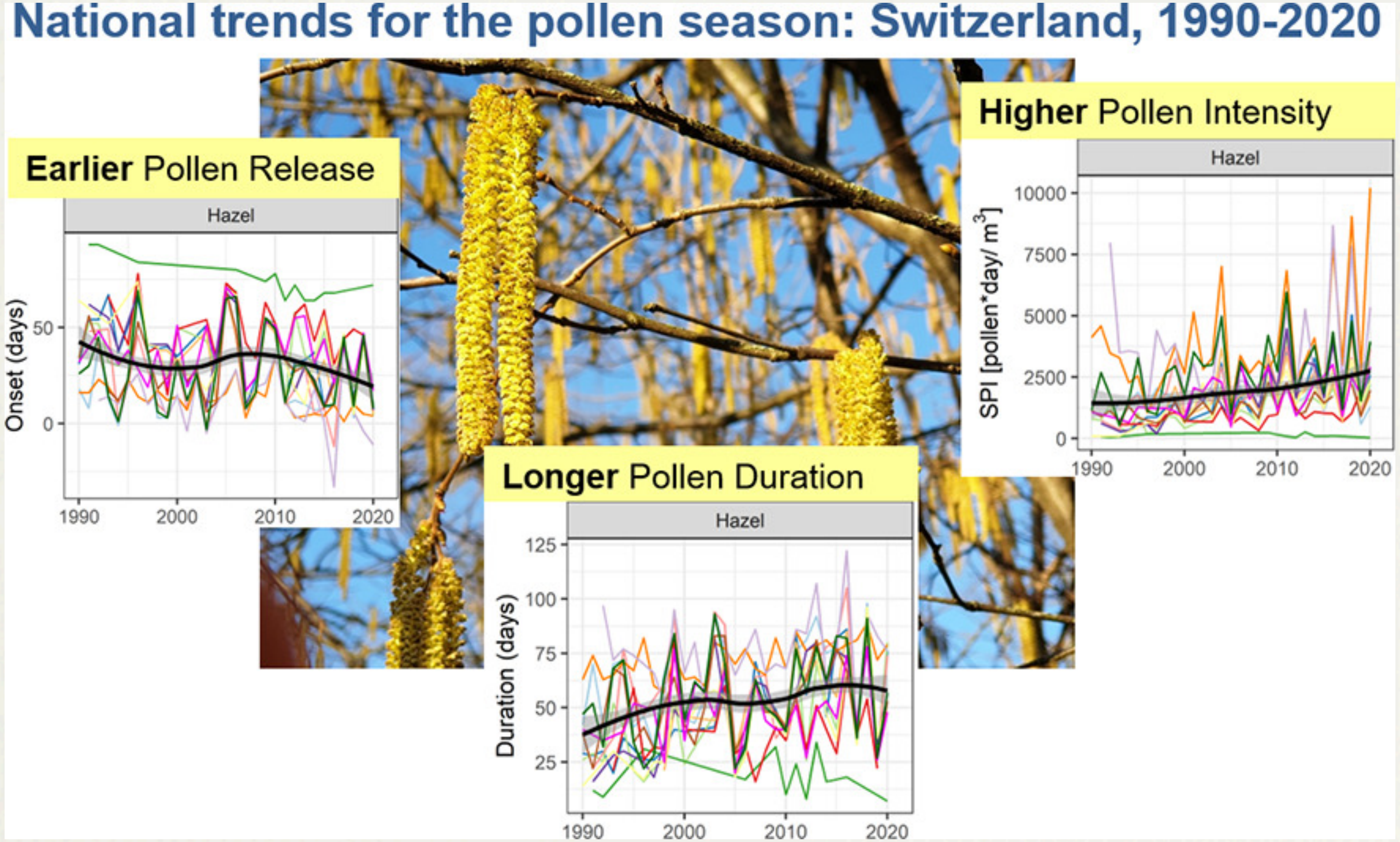
Standardisation of measurement / units assured by metrology institutes traditionally domain of physical sciences and engineering. However,

- need for **traceability** / reproducibility in environmental sciences (e.g. **climate change** studies)
- need to combine **various sources of measurement** for global environmental studies



Calibration of optical particle counters at the Swiss institute for Metrology

Metrology and environmental sciences



climate analysis affected by technical changes ?

Dimension and units

Dimension: **type** of physical quantity, **numerical value** assigned by unit (SI units)

- **fundamental** dimensions (eg. mass, length, time, electric current, temperature, amount of substance and luminous intensity) and units (kg, m, s, A, K, mol, cd)

- **derived** dimensions

$$[Q_i] = L^{\alpha_i} M^{\beta_i} T^{\gamma_i}$$

$$[\mu_i] = L^{-1} M^1 T^{-1}$$

Always monomial power law !

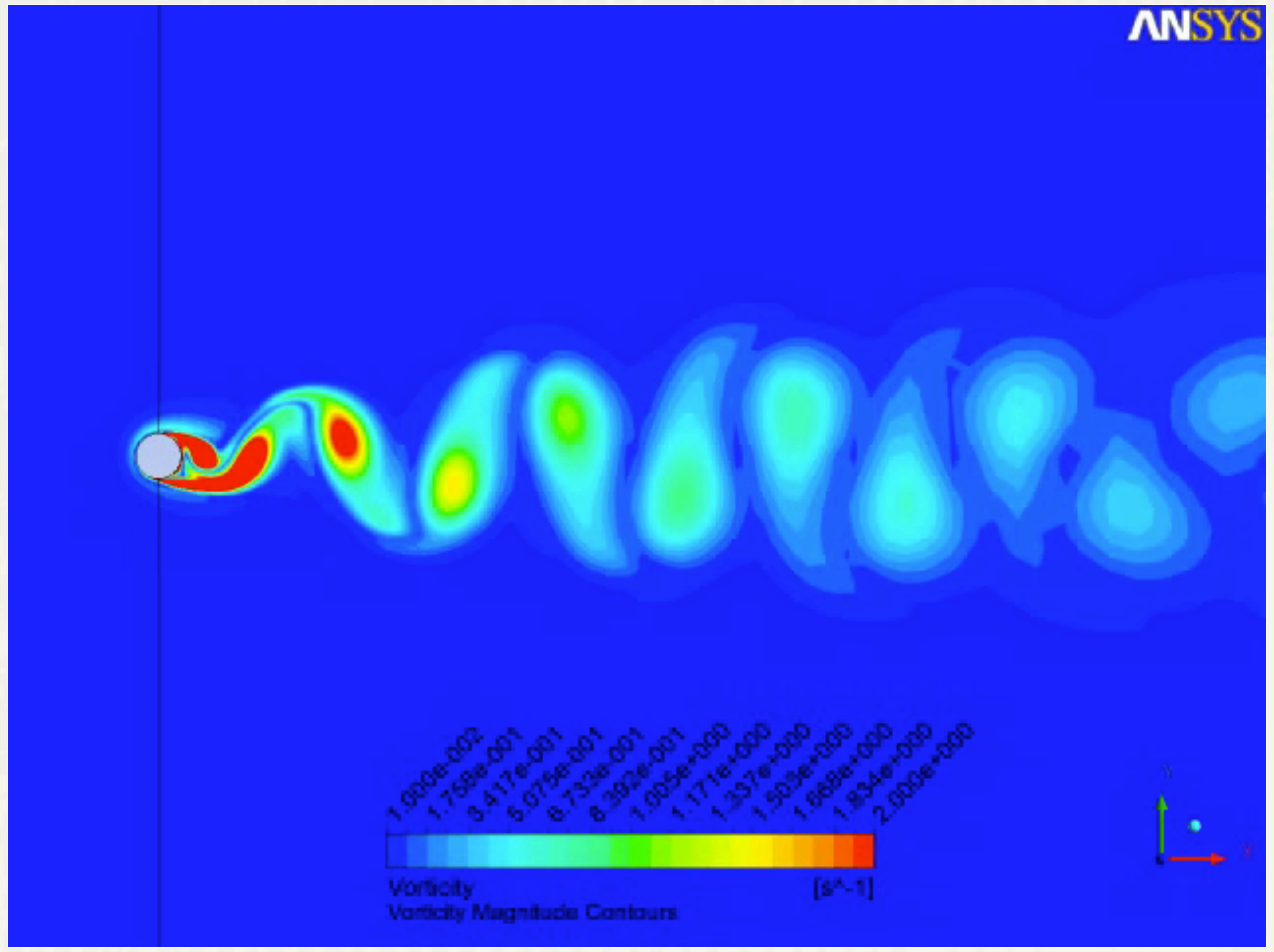
Dimensional **homogeneity**: all additive terms in physical equations must have equal dimensions (check your equations !)

Dimensional independence

Condition for independence of Q_1, Q_2 and Q_3

$$[Q_i] = L^{\alpha_i} M^{\beta_i} T^{\gamma_i}$$

$$det \begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{pmatrix} \neq 0$$



Counter-example: object in a fluid, obstacle dimension **D**, flow velocity **V,** dynamic viscosity **μ** and fluid density **ρ** can be combined into a **dimensionless** number.

$$Re = \frac{DV\rho}{\mu}$$

Buckingham π-theorem

Systematic method for computing **dimensionless parameters** from physical variables

- Any physical equation can be written in the form (n physical variables, written in terms of k independent units)

$$f(q_1, q_2, \ldots, q_{n-1}) = q_n$$

- It can be rewritten in the form (**p=n-k**)

$$F(\pi_1, \pi_2, \ldots, \pi_{p-1}) = \pi_p$$

with $\pi_i = q_1^{a_1^i} q_2^{a_2^i} \ldots q_n^{a_n^i}$

form of F a-priori unknown (use experimental data), **exception** for p=1 then F=cste

Buckingham π-theorem

How to proceed to build dimensionless variables:

- List the relevant **physical quantities**

$$[Q_i] = L^{\alpha_i} M^{\beta_i} T^{\gamma_i}$$

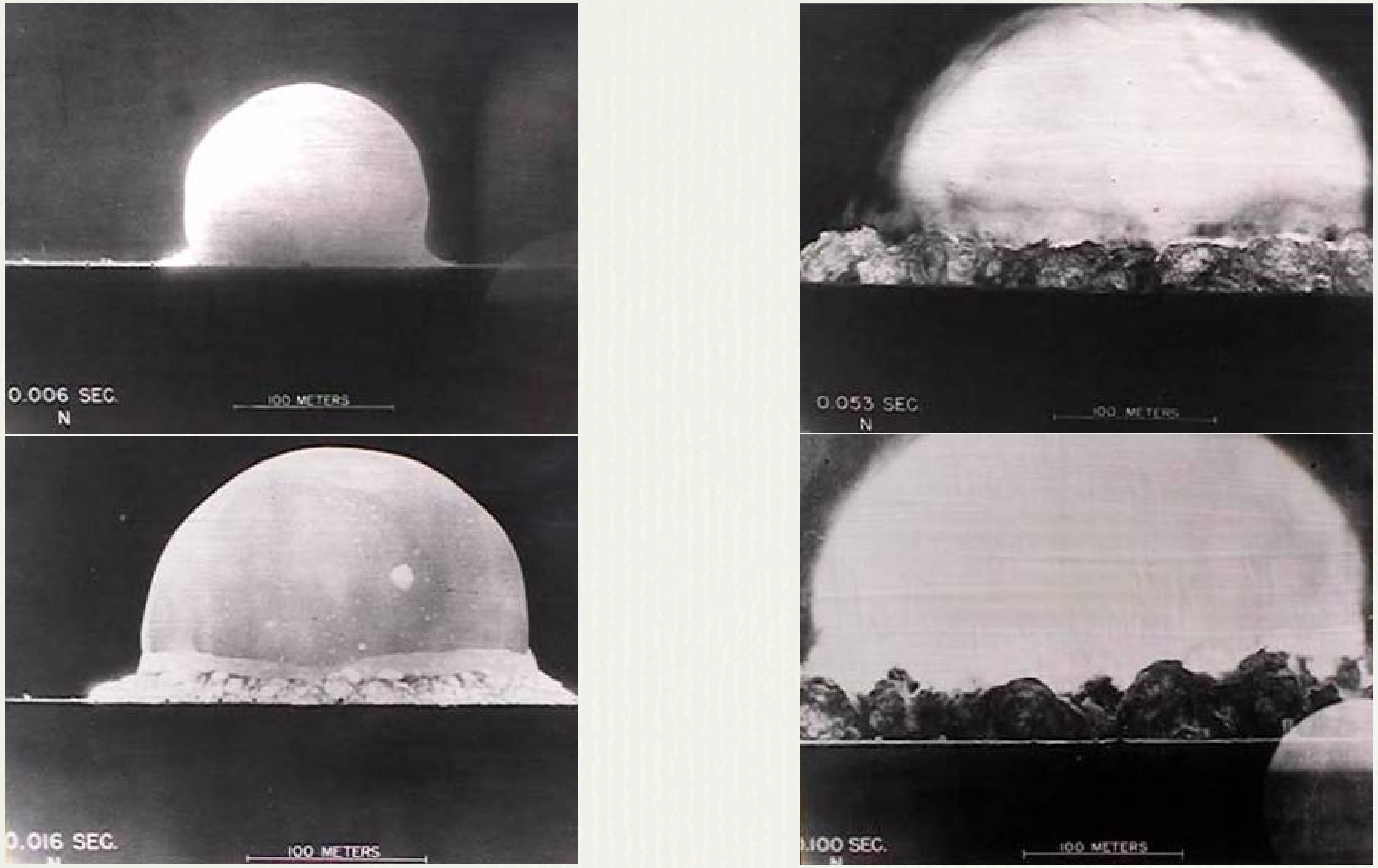
- List the involved **fundamental dimensions**

- Write down the **dimensional matrix**

$$\begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{pmatrix}$$

- Find a basis of the **kernel** of this matrix (see basic linear algebra, Gauss-Jordan elimination)

Buckingham π-theorem (Example 1)



Estimation of Trinity atomic test released energy from pictures (blackboard derivation)

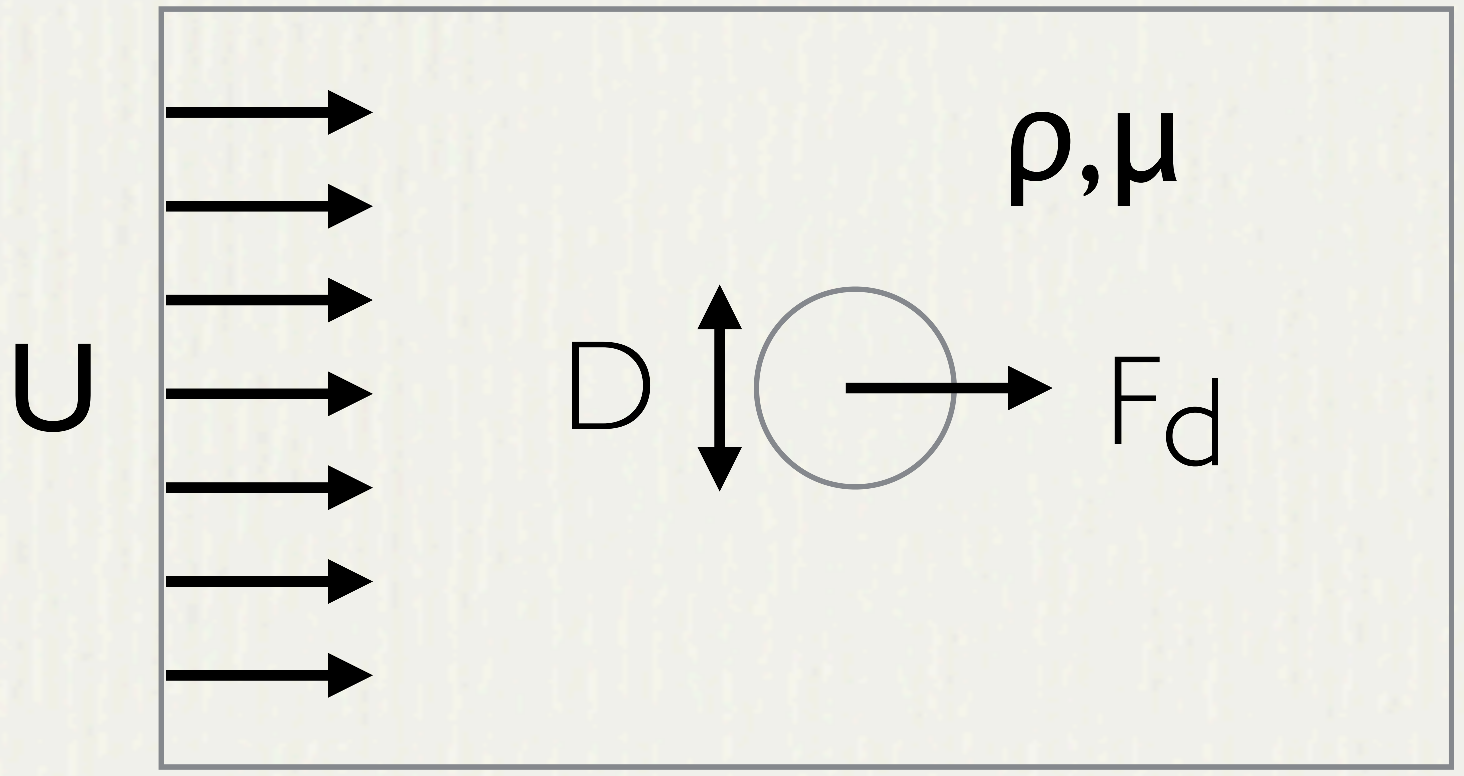
Buckingham π-theorem

Remaks on the technique

- Dimensional analysis simply states that there is a relationship between quantities. It doesn't (**except** in the case of a single **π**, which must, therefore, be constant) states what the relationship is. For the specific relationship one must appeal to theory or, more commonly, to experimental data.

- The choice of the kernel basis is **not unique**, intuition or trial/error is needed to obtain meaningful results (see examples).

Buckingham π-theorem (Example 2)



$$F_d = f(\rho, \mu, U, D)$$

$$\pi_1 = \frac{F_d}{\rho U^2 D^2}$$

p=n-k=5-3=2

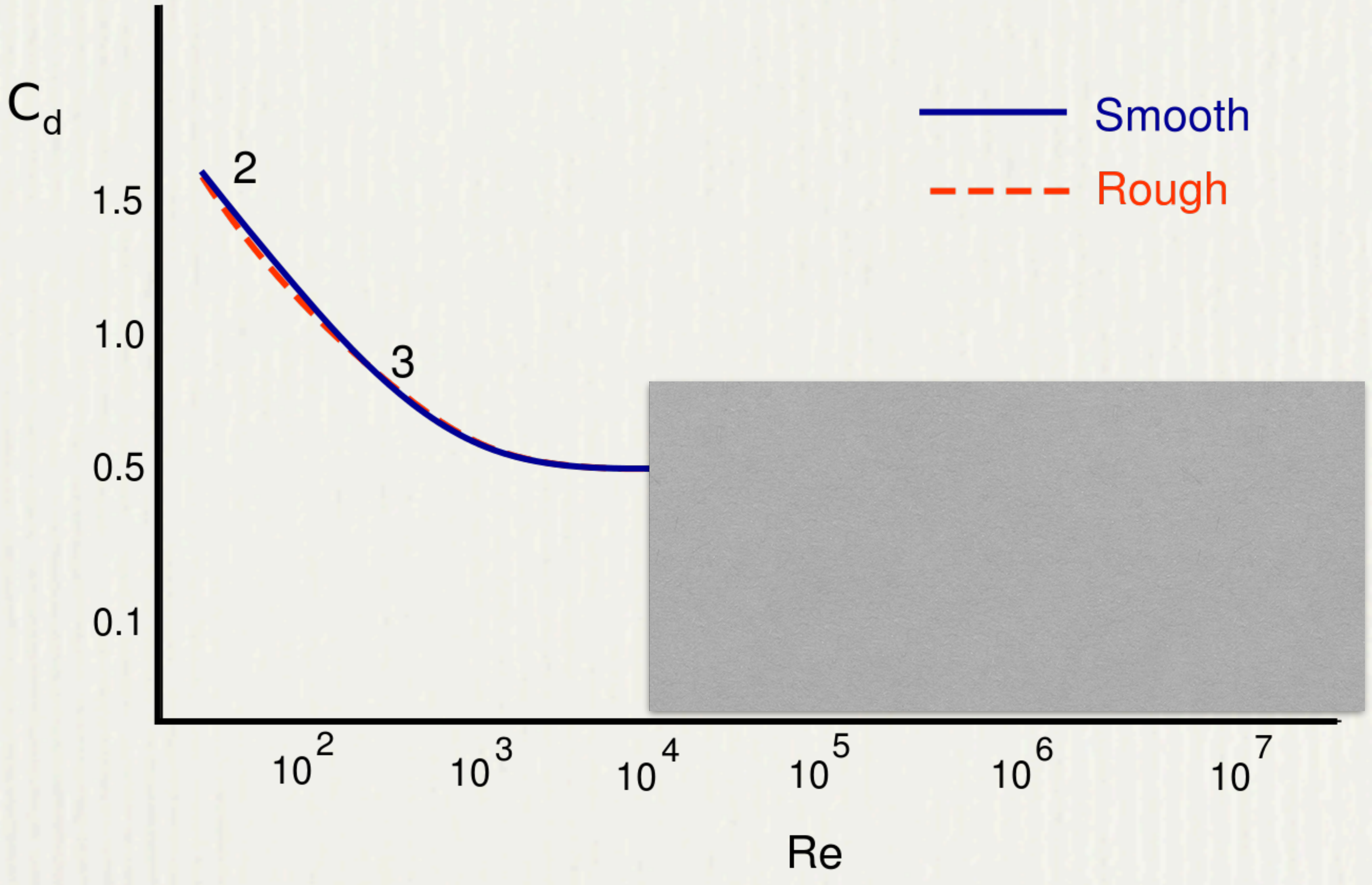
$$\pi_2 = \frac{\rho U D}{\mu} = Re$$

$$\frac{F_d}{\rho U^2 D^2} = F(Re) \rightarrow Cst$$

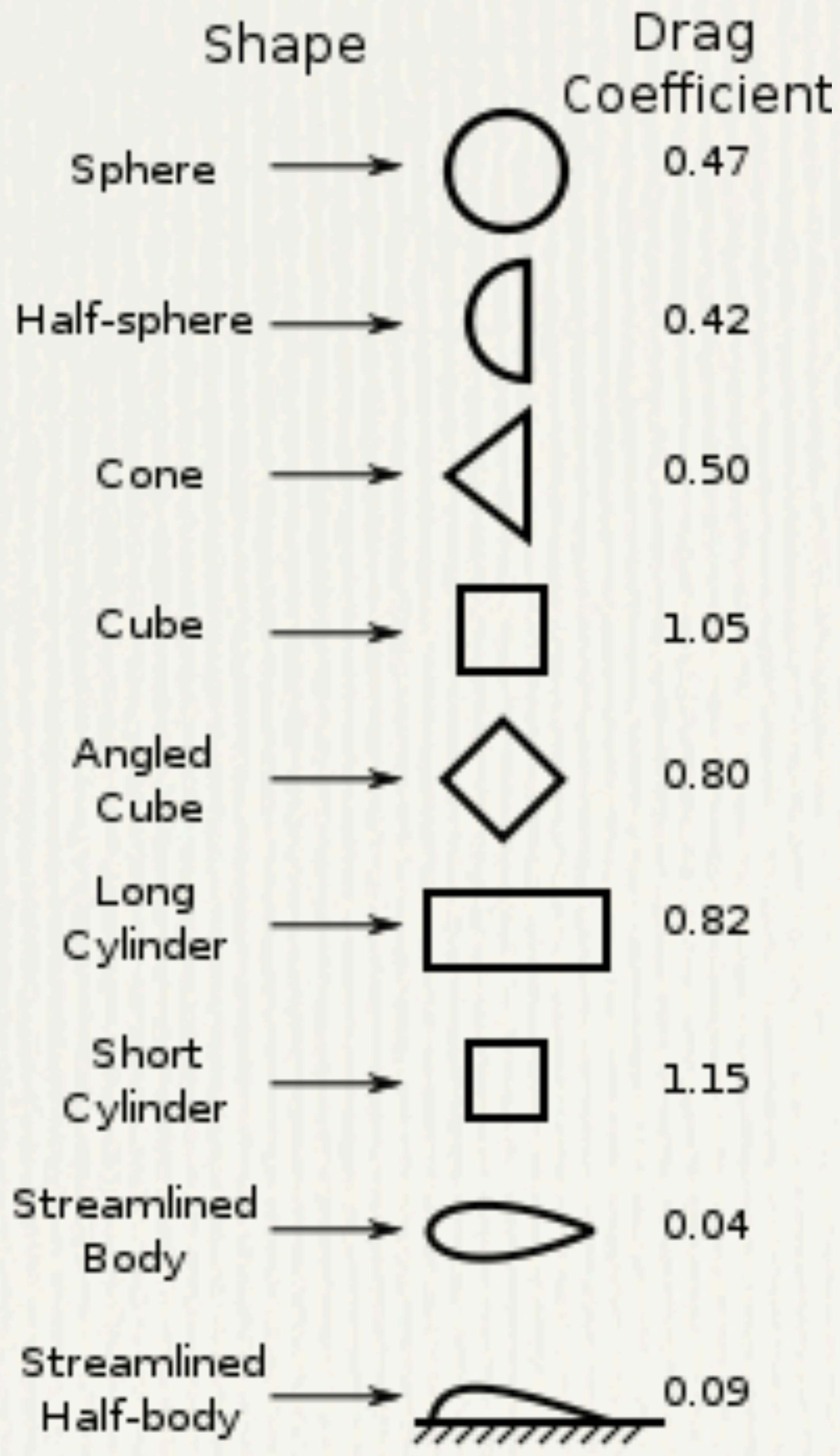
for Re>1000 (viscous forces negligible compared to inertial forces)

Buckingham π-theorem (Example 2)

$$\frac{2F_d}{\rho U^2 A} = C_d$$



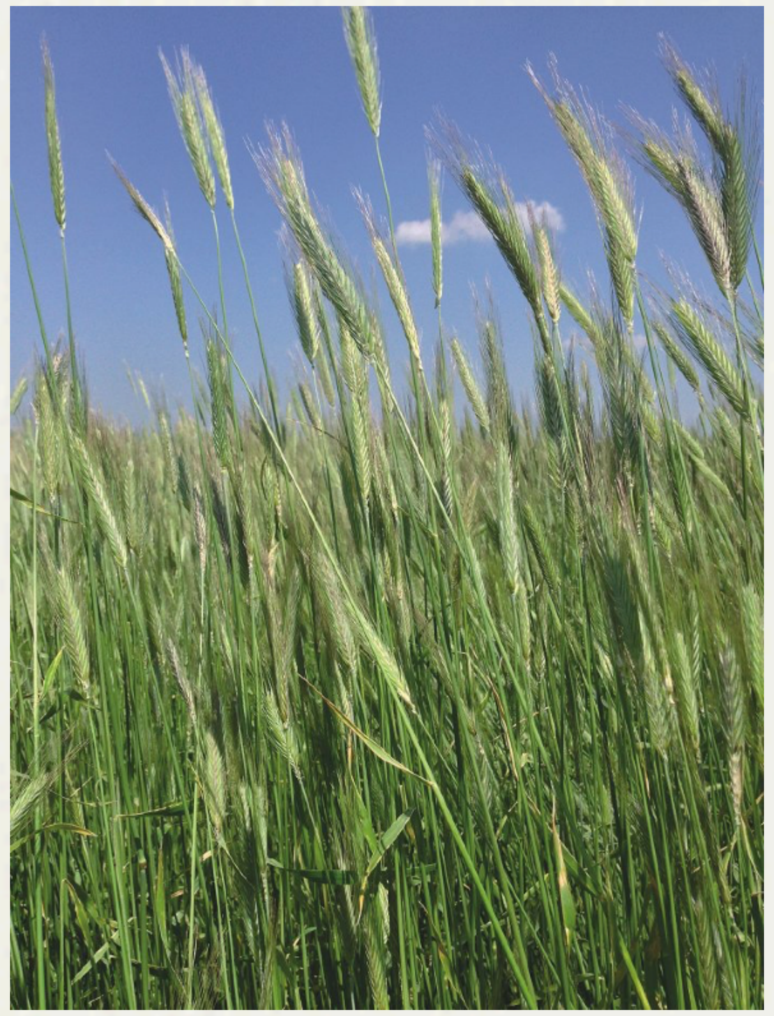
From lab experiments (sphere)

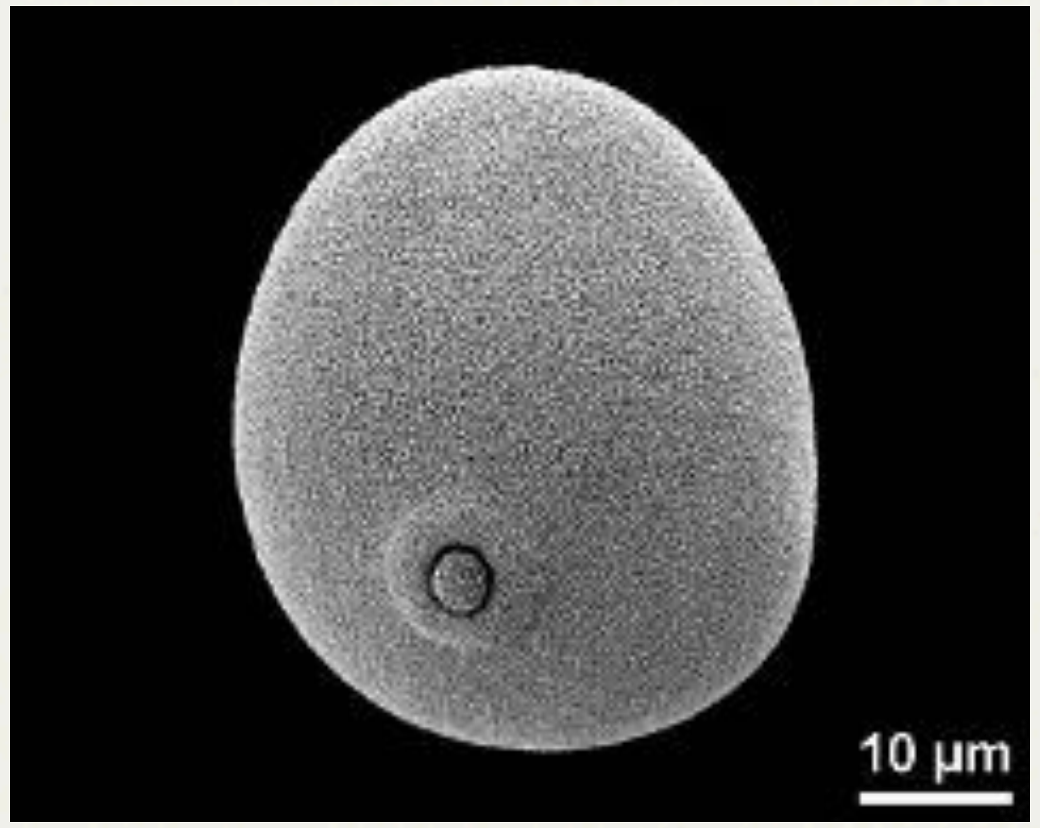


Measured Drag Coefficients

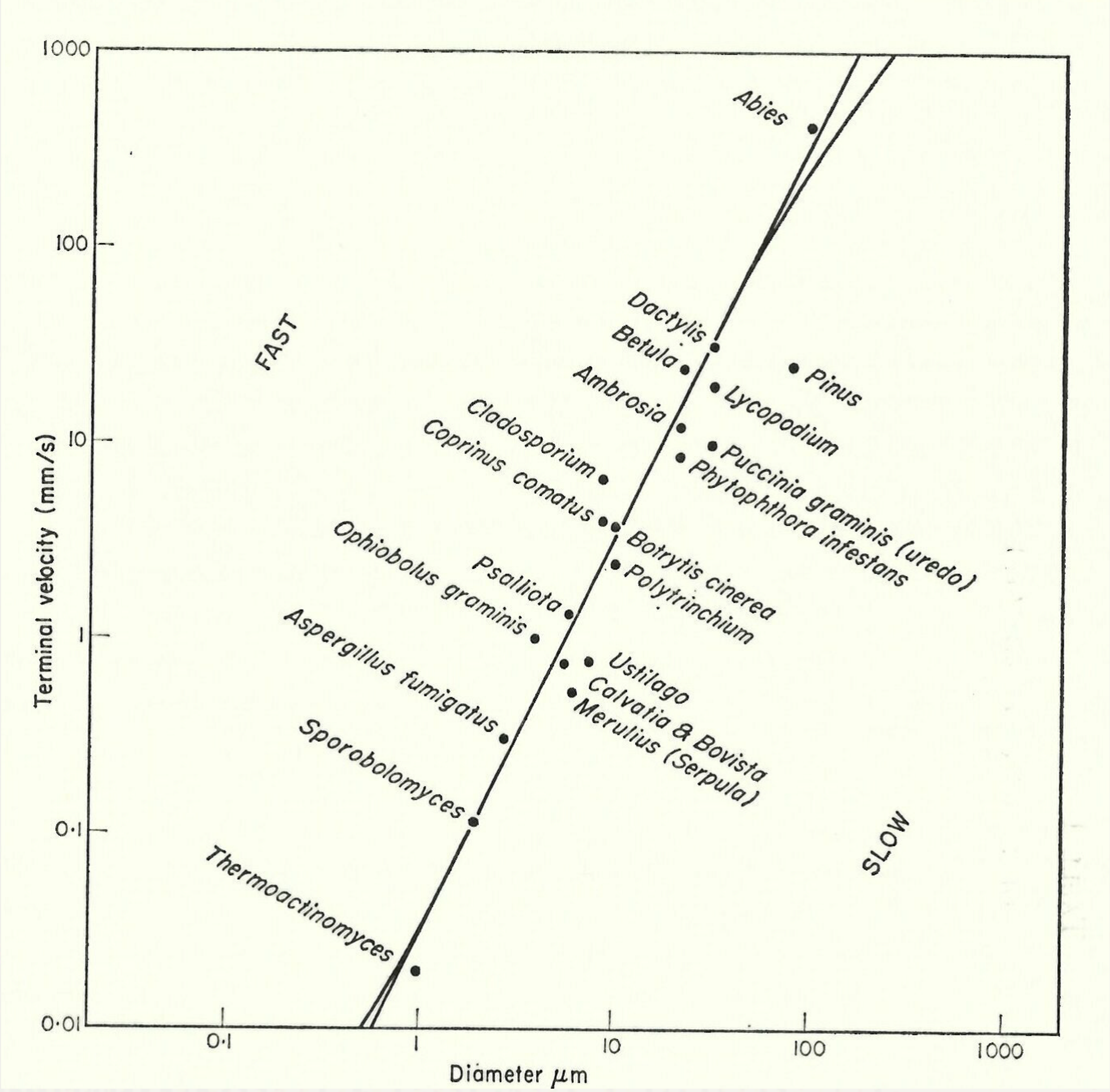
Buckingham π-theorem (Example 3)

$$V_{set} \propto \frac{gD^2\left(\rho_{pollen} - \rho_{air}\right)}{\mu}$$



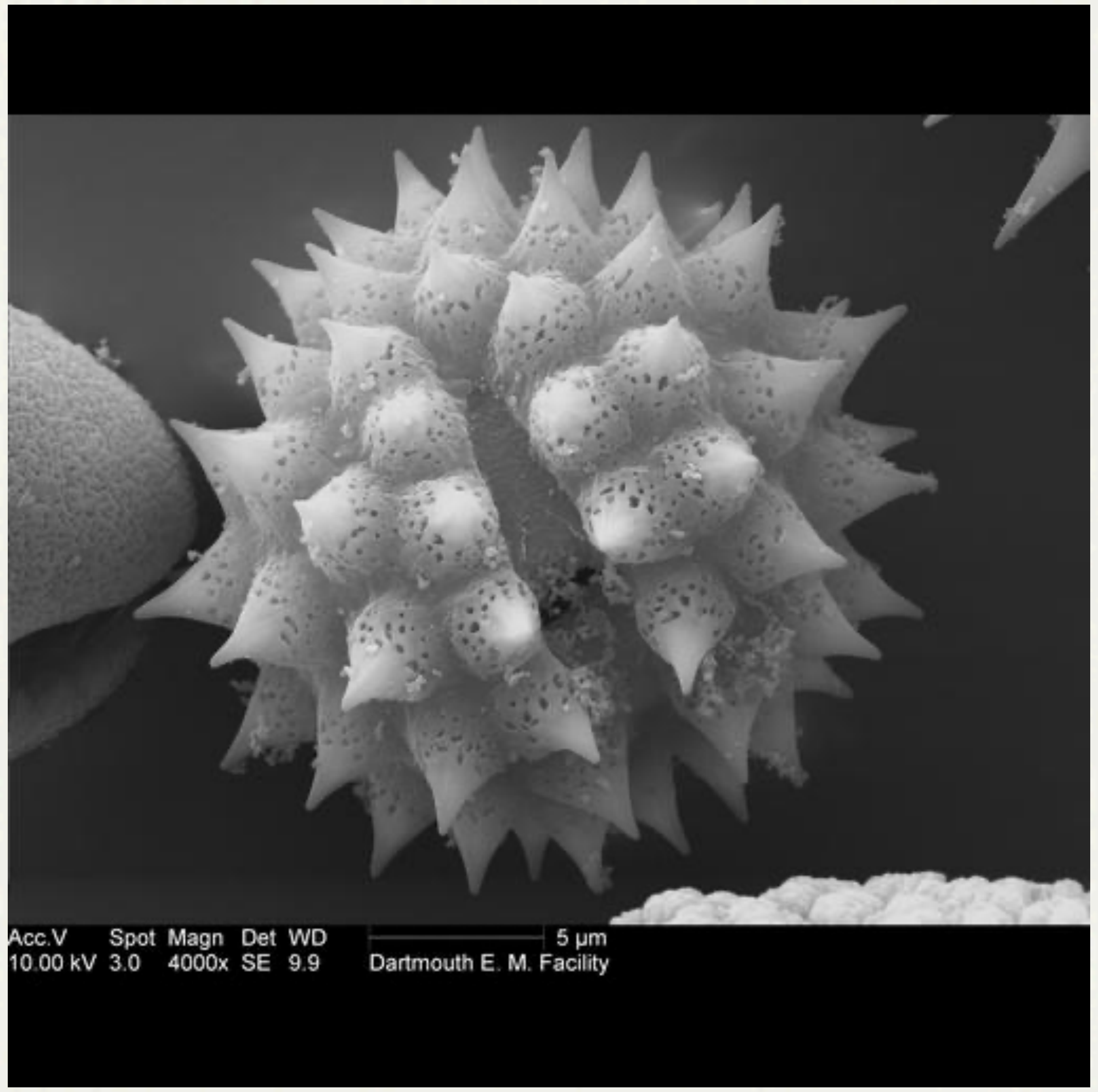


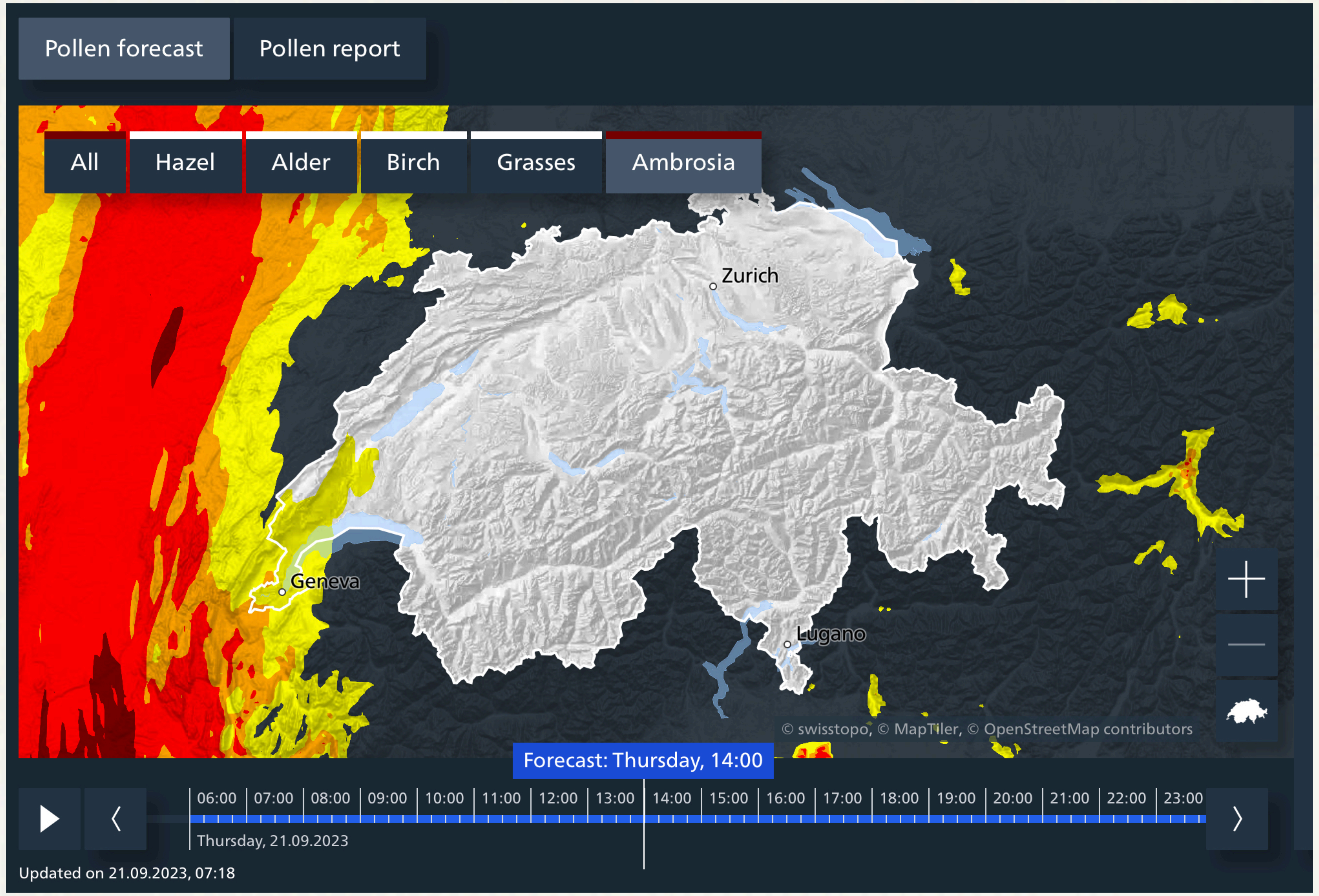
Rye pollen very allergenic but poor flyer due to large size



Estimation of **pollen settling velocity** (blackboard derivation)

Buckingham π-theorem (Example 3)





Use of geometric size for non-spherical pollen grains:
underestimation of settling velocity

Complete self similarity

Complete similarity occurs if the following condition is satisfied.

$$\lim_{\pi_{n-1}\to 0,\infty} F(\pi_1, \pi_2, \ldots, \pi_{n-1}) = Cst$$

Then $\boldsymbol{\pi}_{n-1}$ can be removed from the functional link the problem can be simplified.

In example 2 this is the case in the limit of large Reynolds number.

$$\pi_2 = \frac{\rho U D}{\mu} = Re$$

Incomplete self-similarity

In limits where

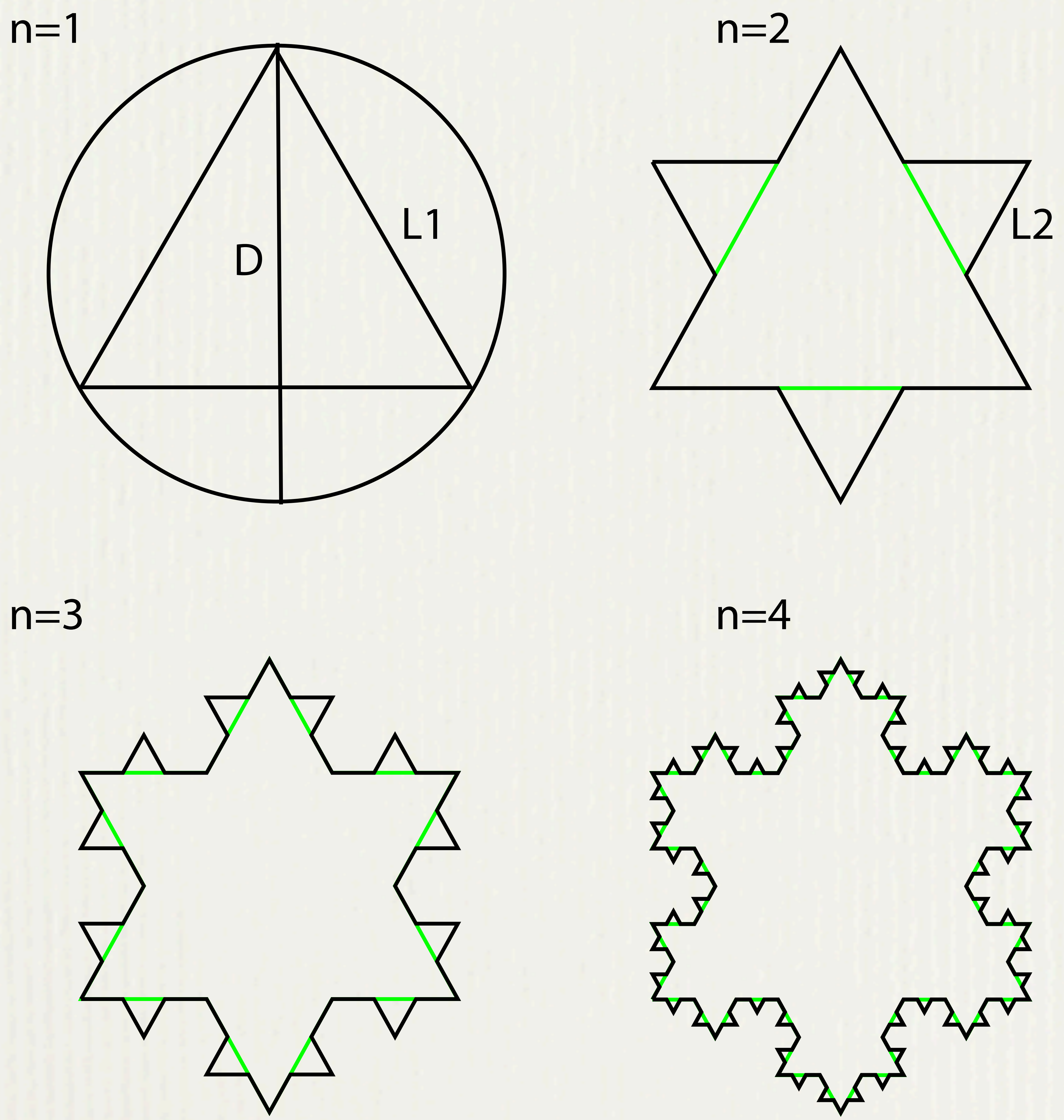
$$\lim_{\pi_{n-1} \to 0,\infty} F(\pi_1, \pi_2, \ldots, \pi_{n-1}) = 0, \infty$$

π_{n-1} cannot removed from the functional link but the problem **may still be simplified** for $\pi_{n-1} >> 1$ or for $\pi_{n-1} \sim 0$ where the following approximations can be made

$$F(\pi_1, \pi_2, \ldots, \pi_{n-1}) \approx \pi_{n-1}^{\alpha} \tilde{F}(\pi_1, \pi_2, \ldots, \pi_{n-2})$$

Additional insight (theory, experiment) is needed to obtain the coefficient α. See following example Koch snowflake.

Incomplete self-similarity (example: Koch flake)



When n goes to infinity the resulting object has an infinite perimeter (dimension between 1 and 2, **fractal**).

$$\frac{P_n}{D} = \left(\frac{L_n}{D}\right)^{-\alpha} \cdot Cst \qquad \alpha > 0$$

Fractals: real-world relevance

Massstab 200 km
-> 2400 km

Massstab 100 km
-> 2800 km

Massstab 50 km
-> 3450 km

What is the length of the coastline of Britain ?

Alaska-Canada boundary dispute:

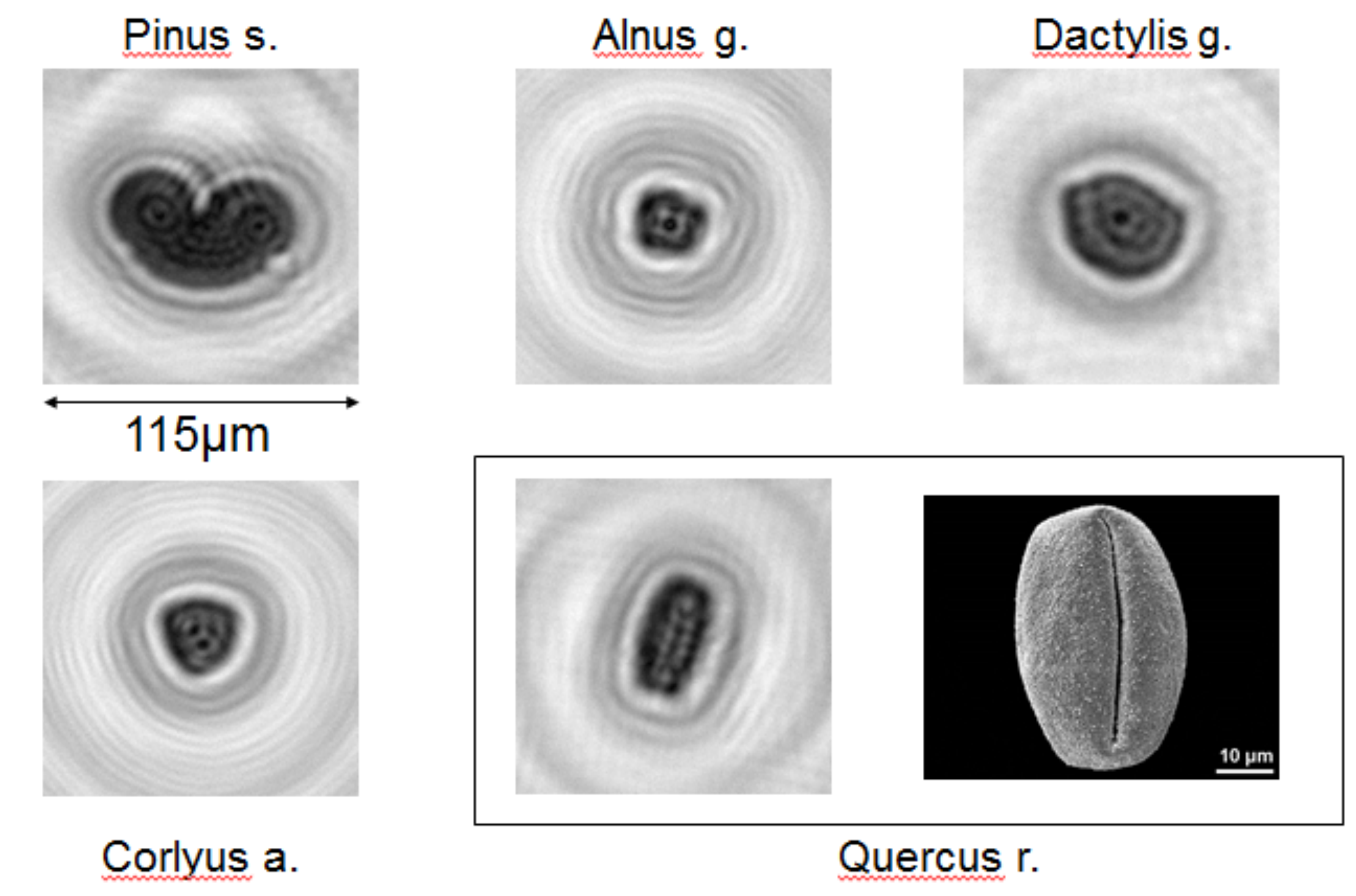
"the limit... shall be formed by a line parallel to the winding of the coast, and which shall never exceed the distance of ten marine leagues therefrom"

https://www.google.ch/maps/@57.2685656,-135.2953876,482068m/data=!3m1!1e3

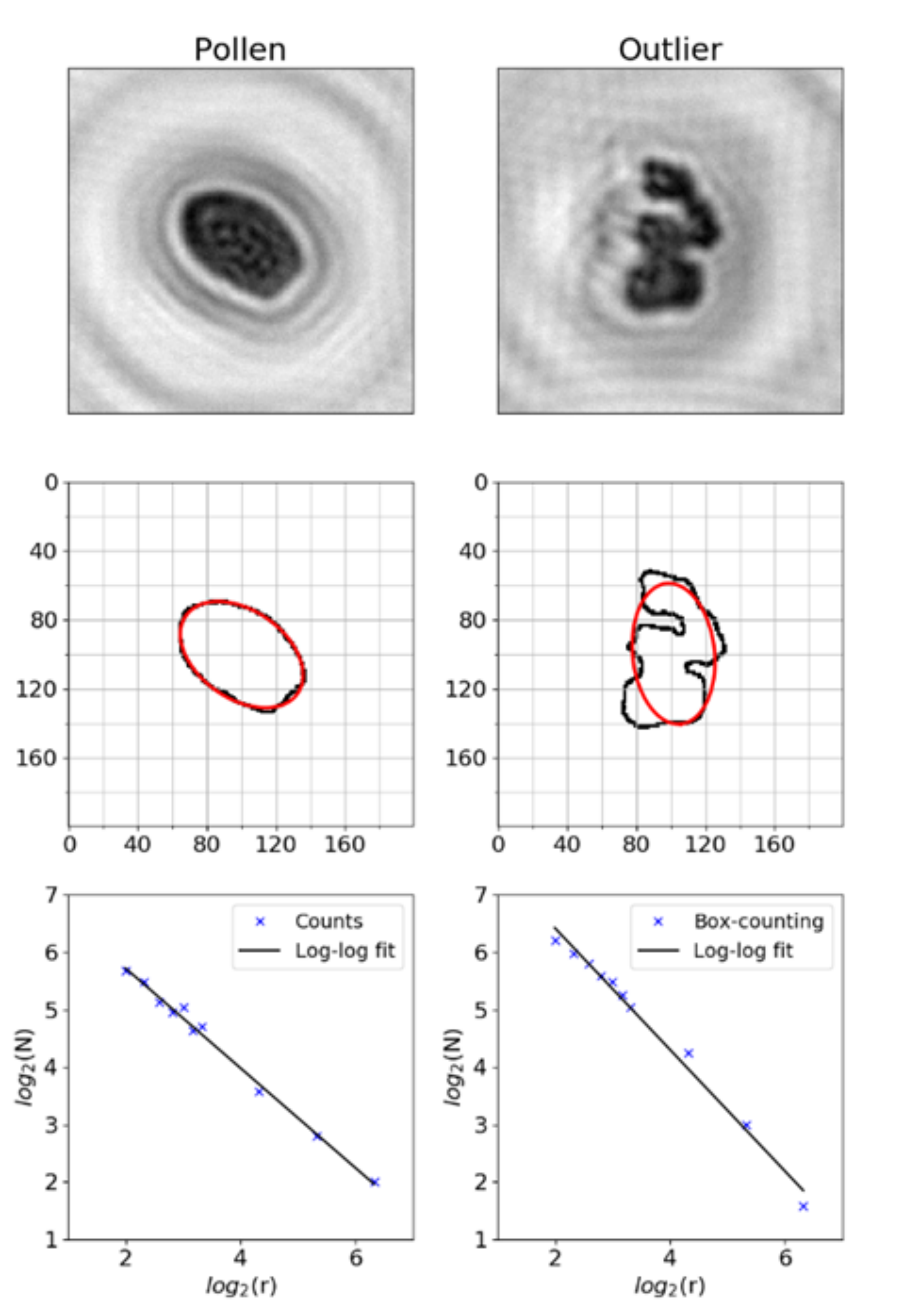
Fractals: identification of bioaerosols

Digital holography used for on-line pollen monitoring

Fractal dimension allows to distinguish coarse particulate matter from bioaerosols

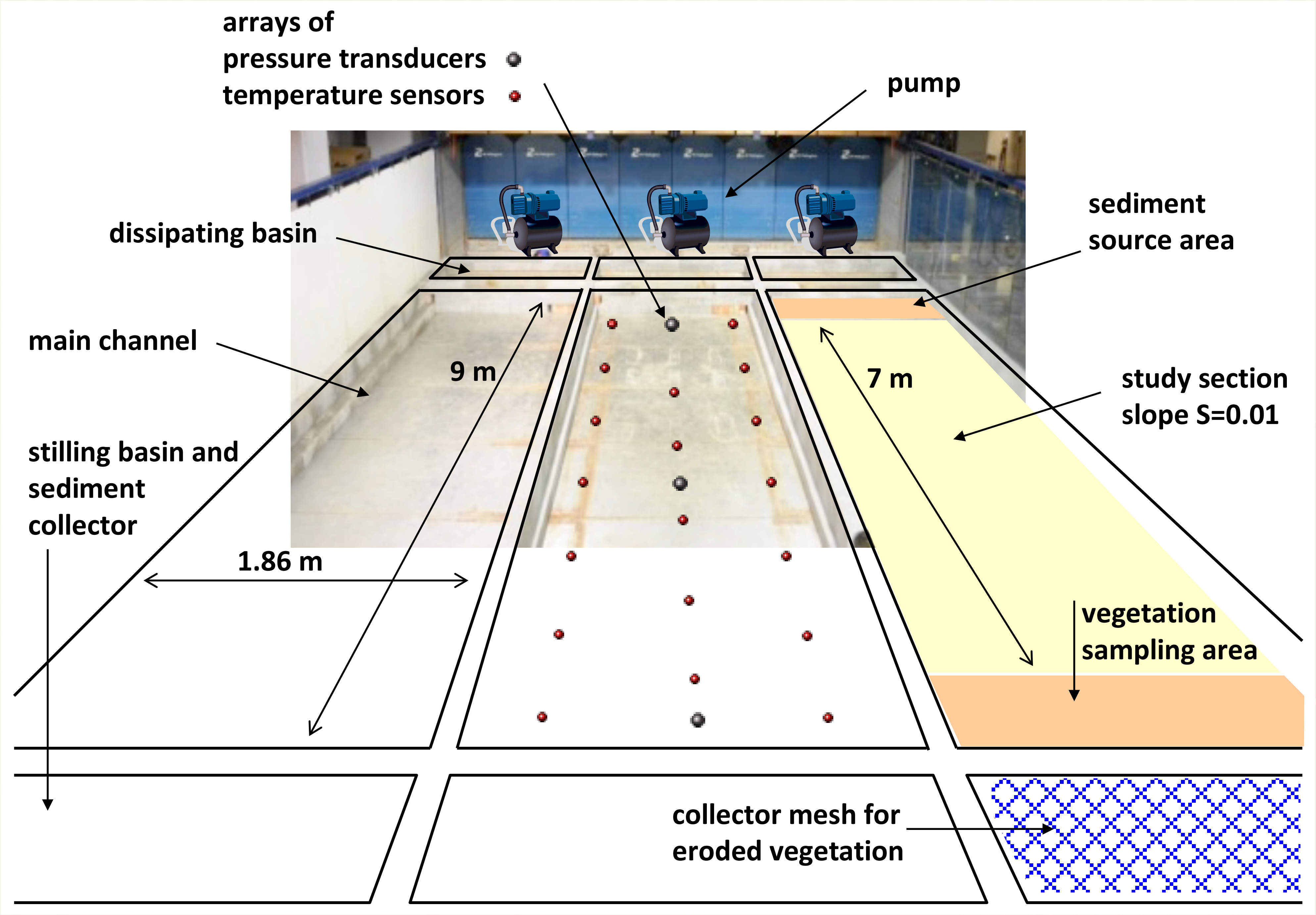


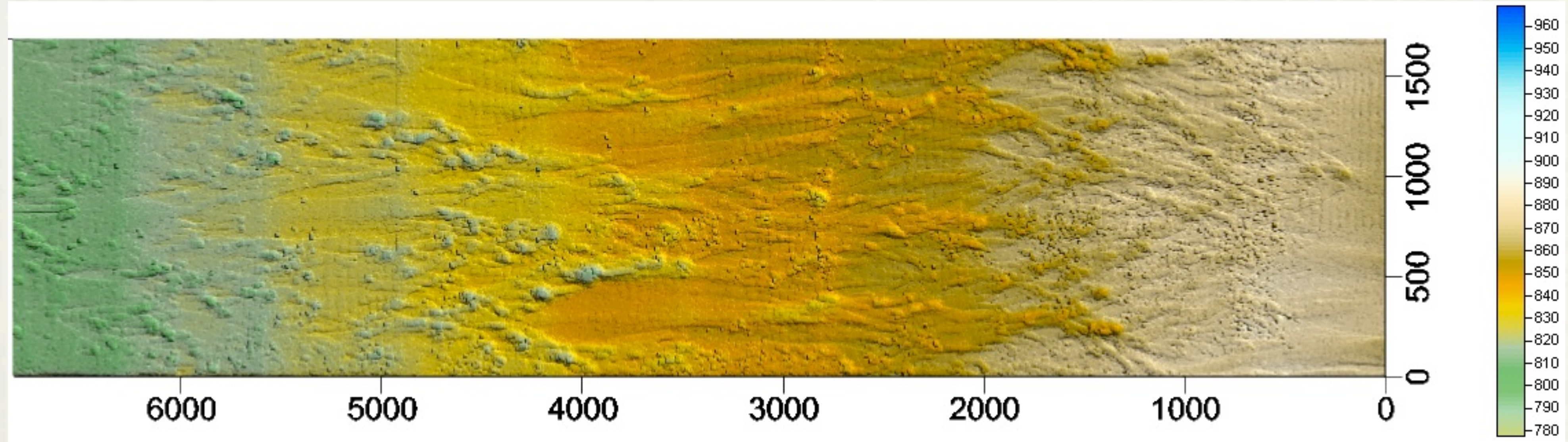
https://amt.copernicus.org/articles/13/1539/2020/



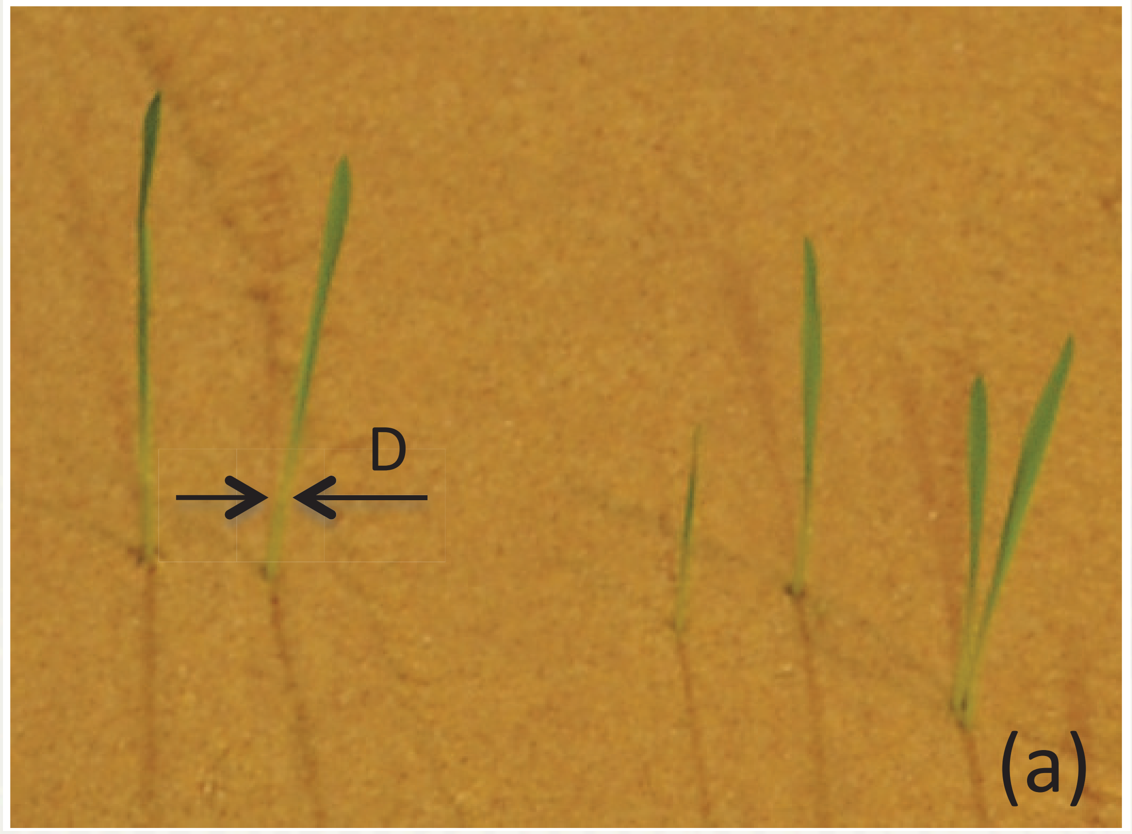
Model Theory (environmental fluid mechanics)

- How to make a model (e.g. laboratory) of a real-world system ?





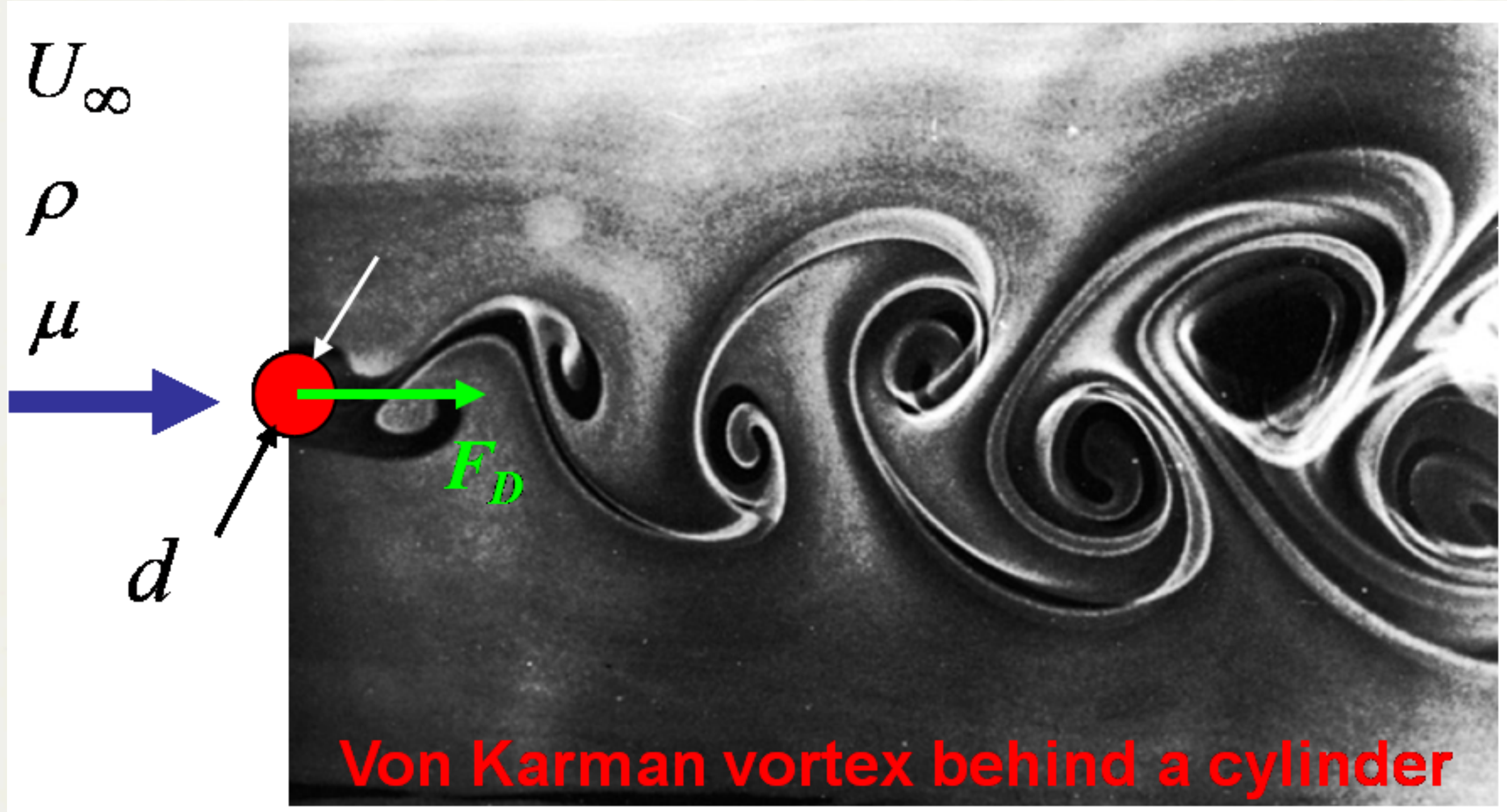
laboratory

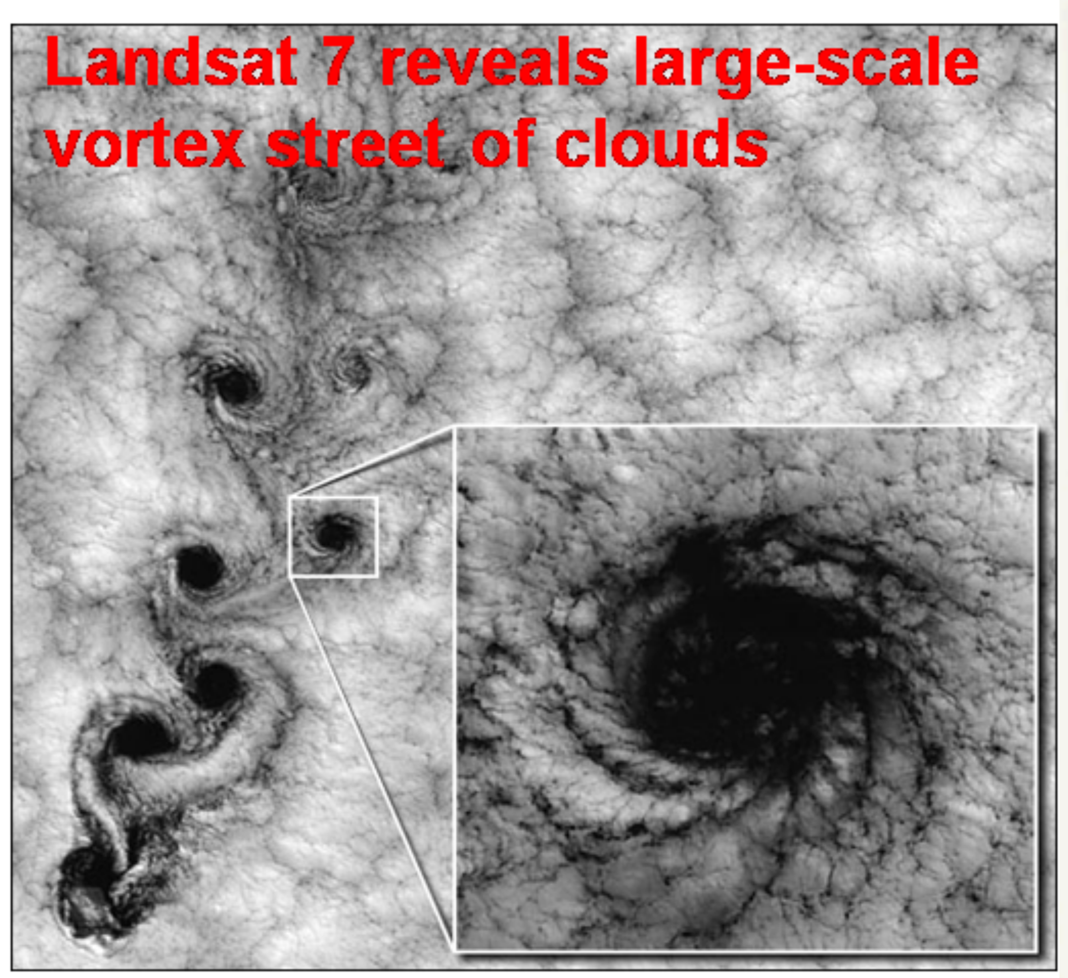


real-world

Model Theory (environmental fluid mechanics)

- Good model has comparable behavior to original
- Physical behavior governed by physical equations
- **π** theorem states that physical equations can be rewritten in dimensionless form.
- Model and original are **similar** if they have the same dimensionless numbers.





Model Theory (environmental fluid mechanics)

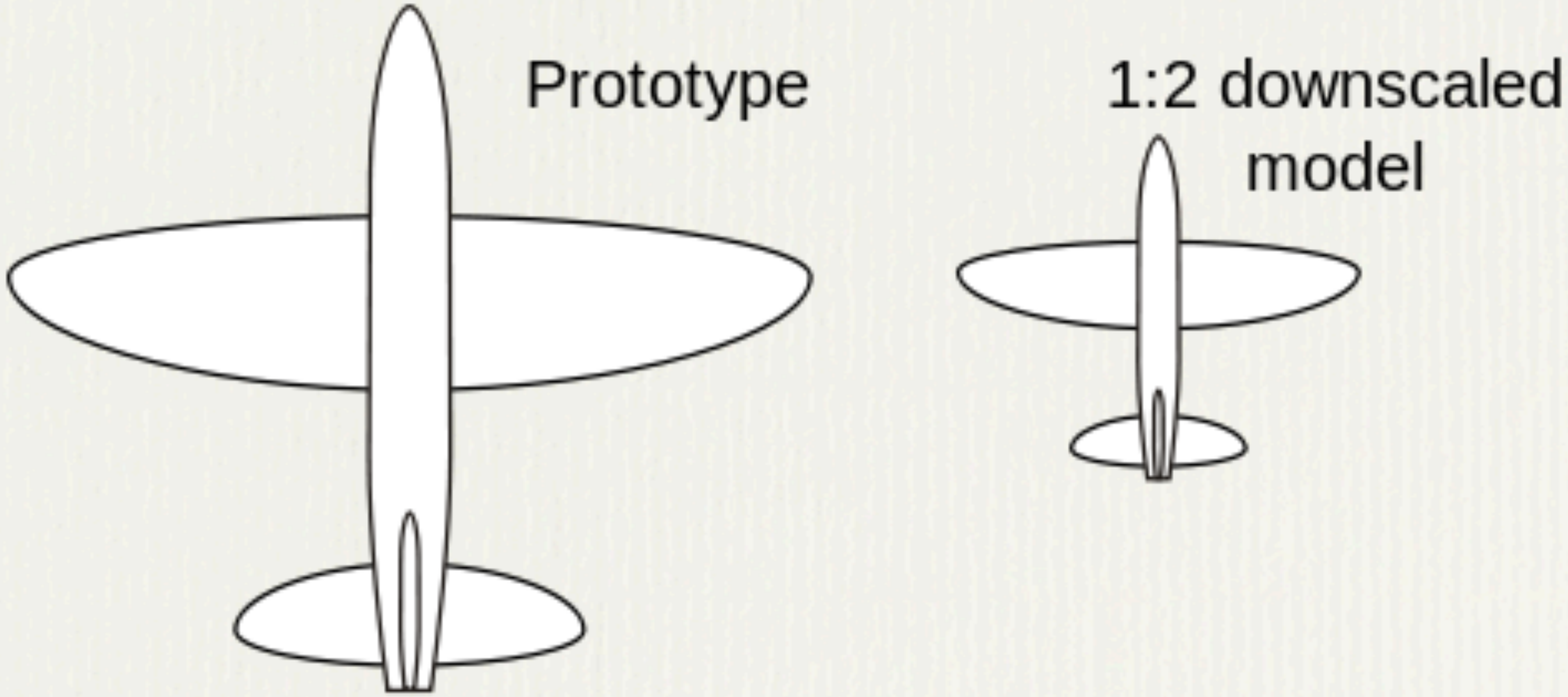
In environmental fluid mechanics we distinguish between

- **complete** similarity
- **incomplete** similarity: geometric (identical shapes), kinematic (ratio between lengths and times are identical), dynamic (ratio of all forces identical, e.g. Reynolds -> inertial vs. viscous or Froude -> inertial vs. gravitational, numbers are the same)

Reynolds similarity often **difficult to achieve**: if the velocity is fixed by the Froude similarity then viscosity has to be adjusted.

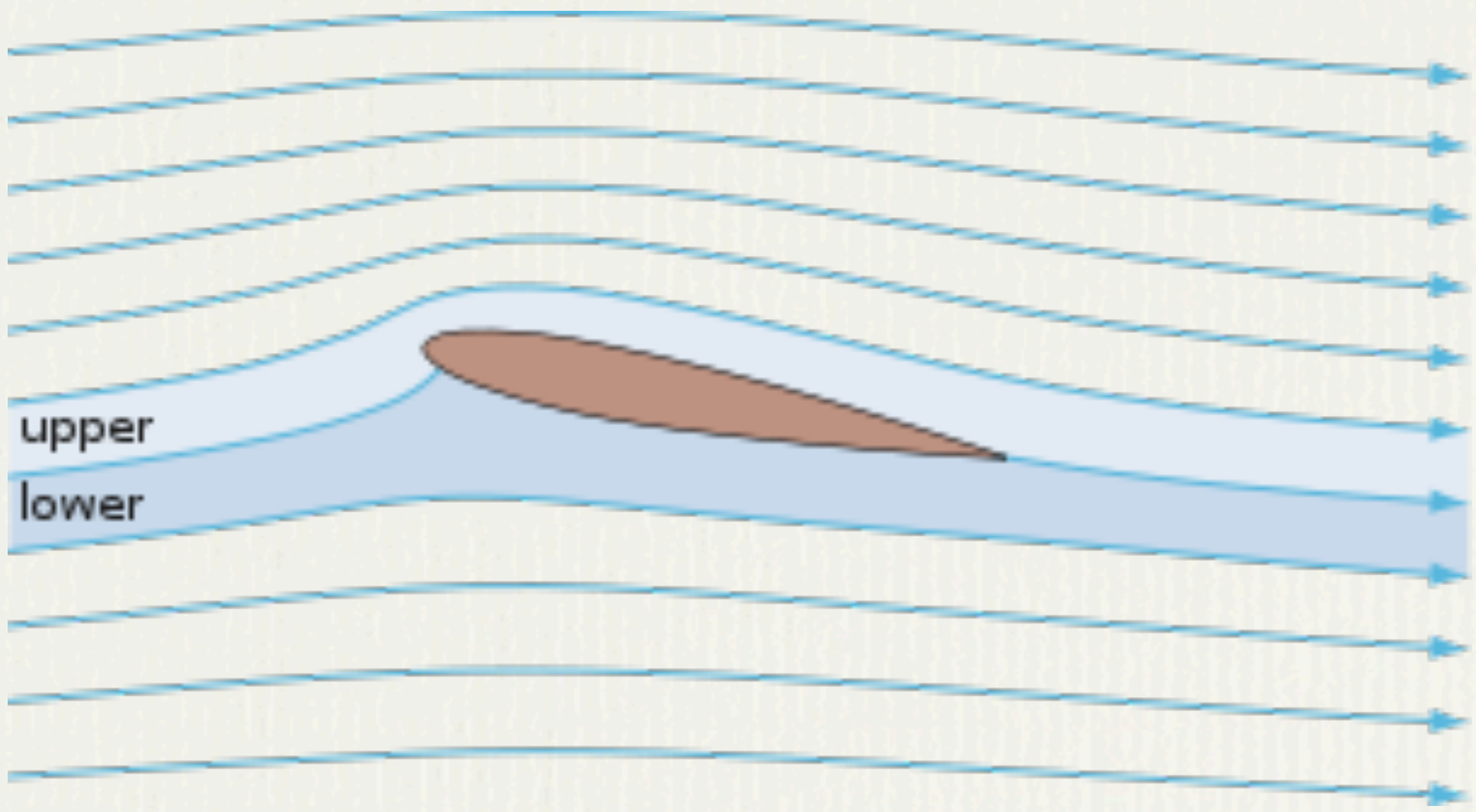
- as long as the flow is fully turbulent (viscosity forces negligible) this similarity is often not necessary

Model Theory: geometric similarity



Ratios between corresponding **lengths** in model and application (or prototype) are the same.

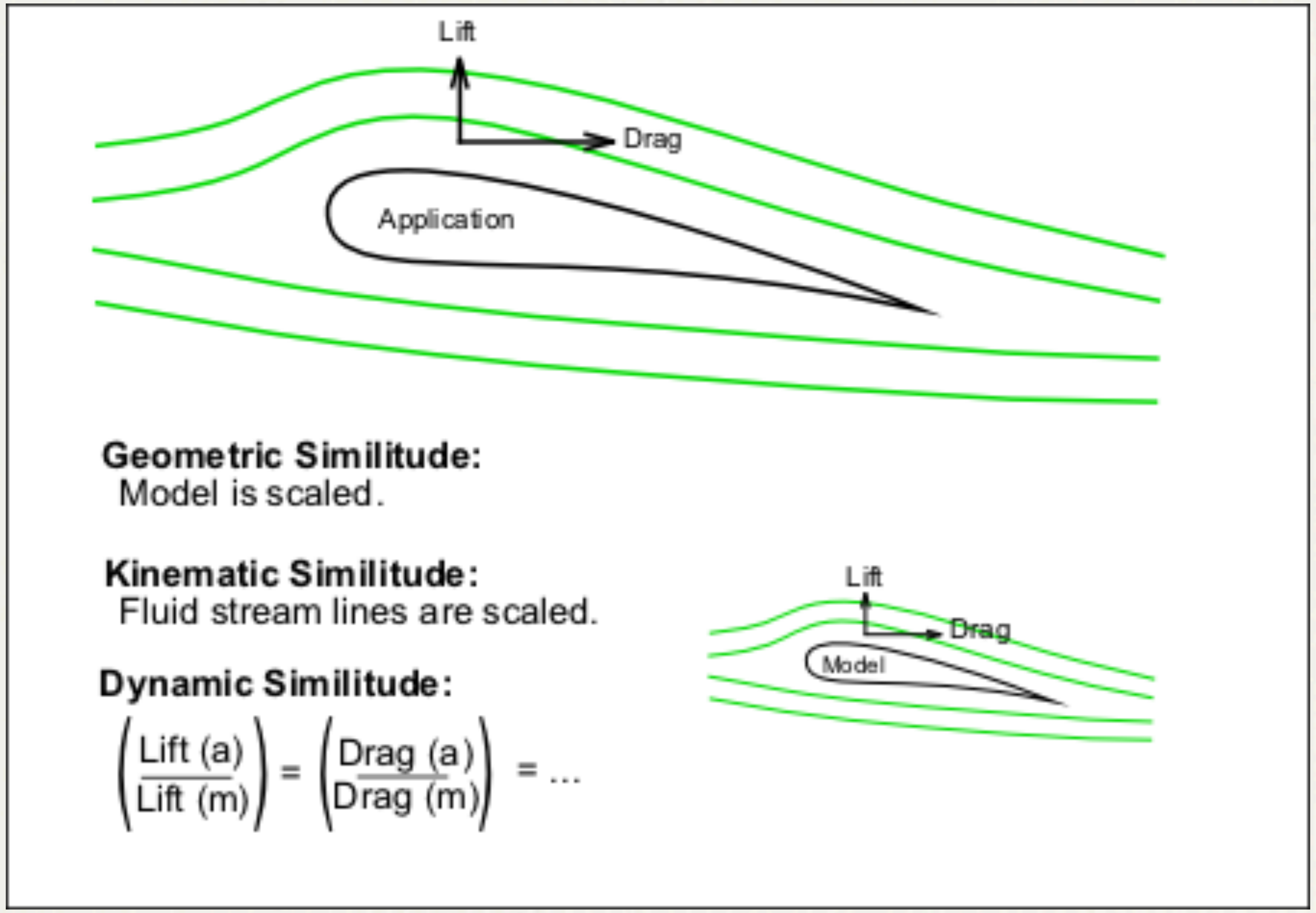
Model Theory: kinematic similarity



Flow field in prototype and model have the same shape and the ratios of corresponding velocities and accelerations are the same.

Geometrically similar streamlines are kinematically similar.

Model Theory: dynamic similarity



Ratio between forces in application and model must be constant.

Geometric and kinematic similarity necessary but insufficient conditions

Summary of Lecture I: Uses of dimensional analysis

- Recognise (self)-**similarity** -> model theory

- **Reduce** the number of variables

- Variables known, physics unknown (equations, boundary conditions): **simple approach** to tackle complex problems

- Interpretation of experimental data: from the dimensionless variables identify the **relative importance** of physical phenomena

Summary of Lecture I: common pitfalls

- Review on **units and dimensions**: check your equations for consistency
- Beware of **incomplete** set of independent quantities
- **Superfluous** independent quantities complicate the result (be pragmatic !)
- Functional dependence can be more or less difficult to recognise depending on the choice of dimensionless variables

II) Fluid Mechanics

Fluid mechanics: review

- Basis for transport phenomena

- Velocity field specified by

$$\mathbf{V}(x, y, z, t) = u(x, y, z, t)\mathbf{e}_x + v(x, y, z, t)\mathbf{e}_y + w(x, y, z, t)\mathbf{e}_z$$

- **State of the fluid** specified by the velocity components u, v, w, pressure p and density ρ.

- 5 equations (and appropriate **initial** and **boundary** conditions) are needed to describe the evolution of the fluid

- In the presence of additional substances, their **concentration** C(x,y,z,t) has to be added. Correspondingly, an additional equation is needed.

Fluid mechanics: review

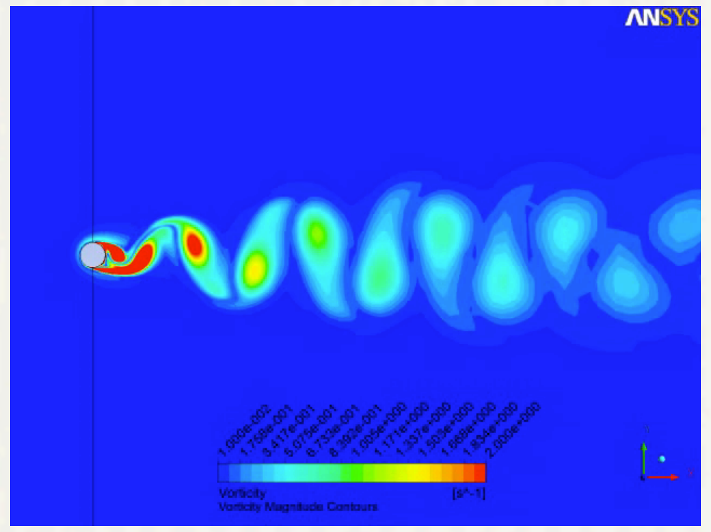
- Regardless of the form of the fluid mechanics physical equations, dimensional analysis can be applied and relevant dimensionless quantities can be found.

- Velocity U, viscosity **μ**, density **ρ**, characteristic length L, gravitational acceleration g, time t, pressure p and diffusion coefficient D (incompressible fluid).

$$Re = \frac{\rho L U}{\mu} \qquad Fr = \frac{U}{\sqrt{gL}} \qquad Pe = \frac{UL}{D}$$

inverse characteristic time e.g. vortex shedding frequency

$$Ne = \frac{p}{\rho U^2} \qquad St = \frac{fL}{U}$$



$\dots$

Fluid mechanics: review

- **Newtonian** fluid: shear stress $\tau = \mu \frac{du}{dy}$, with u velocity component parallel to the direction of shear and y displacement in perpendicular direction.

- **Navier-Stokes equation** for incompressible flow of Newtonian fluid reads (momentum conservation)

inertia

divergence of stress

$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} \right) = -\nabla p + \mu \nabla^2 \mathbf{v} + \mathbf{f}$$

variation

convection

source

Left hand side material derivative $\rho \frac{D\mathbf{v}}{Dt}$

With $\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla$ (local+convective term)

Fluid mechanics: review

- Continuity equation (mass conservation) $$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{v} = 0$$

incompressible flow $$\boxed{\nabla \cdot \mathbf{v} = 0}$$ Solenoidal flow (e.g. magnetic field)

- Compressible flow p and **ρ** are linked (state equation energy conservation)

$$\frac{D\rho}{Dt} = \frac{\rho}{\epsilon}\frac{Dp}{Dt} \qquad Ma = U\sqrt{\frac{\rho}{\epsilon}}$$ Mach number

- Bernoulli equation along a streamline (curve tangent to velocity field), valid for steady, incompressible flow of perfect fluid (no viscosity).

$$\frac{1}{2}v^2 + \frac{P}{\rho} + gh = Cste.$$

Fluid mechanics: review

- Navier-Stokes equation can be rewritten in dimensionless form

$$\mathbf{f} = \rho g \mathbf{e}_z$$

$$\left(\frac{\partial \mathbf{v}^\star}{\partial t^\star} + (\mathbf{v}^\star \cdot \nabla)\mathbf{v}^\star\right) = -\nabla p^\star + \frac{1}{Re}\nabla^2 \mathbf{v}^\star + \frac{1}{Fr^2}\mathbf{e}_z^\star$$

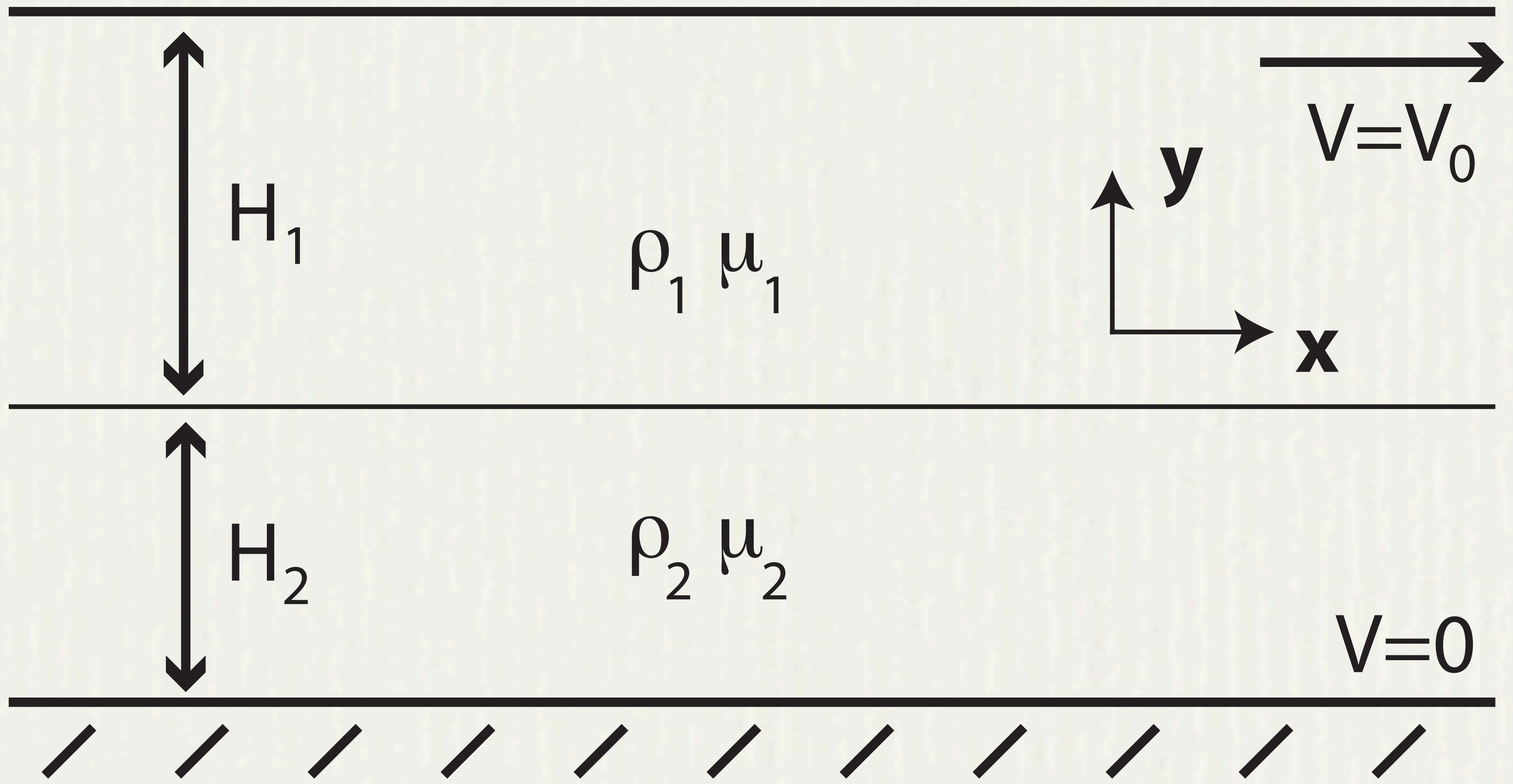
- with the **scales**

characteristic length	L	time	$\frac{L}{U}$
flow velocity	U	$p^\star = \frac{p}{\rho U^2}$	

- Stokes regime (Re<<1), Euler regime (Re>>1, inertia dominates)

Fluid mechanics: example

- Example (exercise set): laminar motion of two layers of Newtonian fluids (stationary process or steady flow)



$$\mathbf{v} = u(y)\mathbf{e}_x$$

$$\frac{\partial^2 u}{\partial y^2} = 0$$

- Boundary conditions: no-slip condition (zero velocity relative to boundary and equality of stresses + velocities at the boundary)

$$\begin{aligned} u(0) &= 0 \\ u(H_1 + H_2) &= V_0 \\ \mu_1 \frac{du}{dy}|_{y=H_2+\epsilon} &= \mu_2 \frac{du}{dy}|_{y=H_2-\epsilon} \\ u(H_2+\epsilon) &= u(H_2-\epsilon) \end{aligned}$$

- No initial condition (boundary condition in time) due to stationarity

Fluid mechanics: turbulent regime