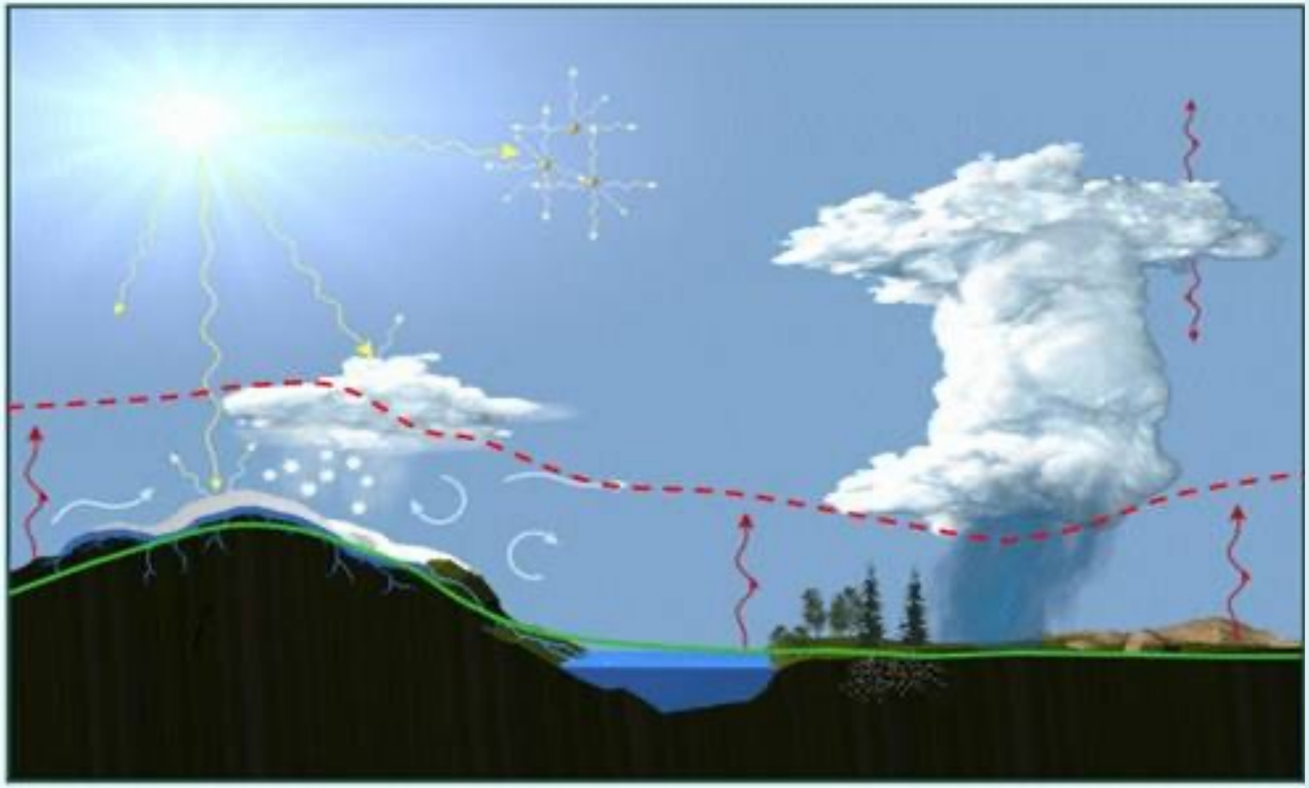
Environmental Transport Phenomena

The Atmospheric Boundary Layer



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Atmospheric Boundary Layer Turbulence

I. The atmospheric boundary layer

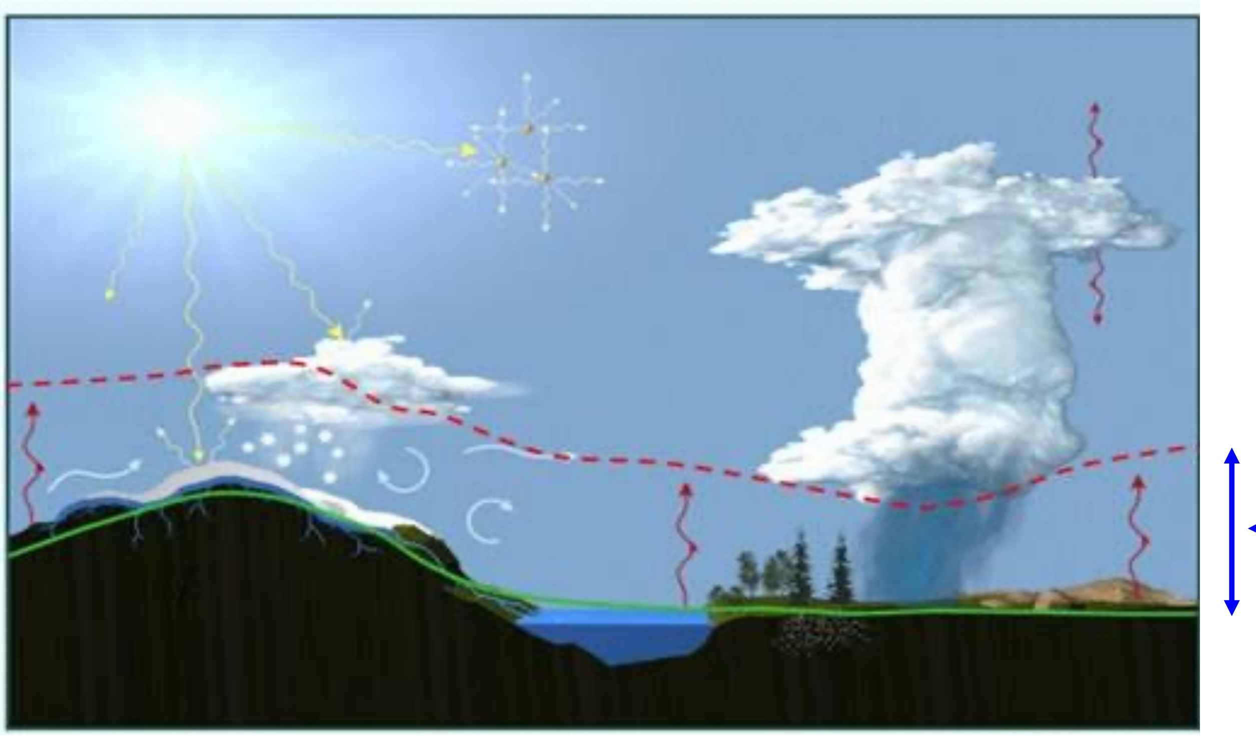
II. Structure of the atmospheric boundary layer

III. Effects of buoyancy

IV. Turbulent mixing in three dimensions

Reference: Book, Chapter 6

The Atmospheric Boundary Layer



Source: NOAA ESRL - http://www.esrl.noaa.gov

ABL ~ 1 km

(Re ~ 10^8-10^9)

- **Highly turbulent boundary layer flow**
- **Strongly affected by:**
  - Thermal effects
  - Surface heterogeneity
  - Topography

The Atmospheric Boundary Layer

Definition:

- The lowest part of the atmosphere that is in direct interaction with the Earth's surface and responds to *surface forcings* with a time scale of about an hour or less. It is ***highly turbulent***.

Scale:

- Boundary layer depth is variable, typically between 100-3000 m
- Ratio of boundary layer depth to radius of earth: 1 km/6400 km

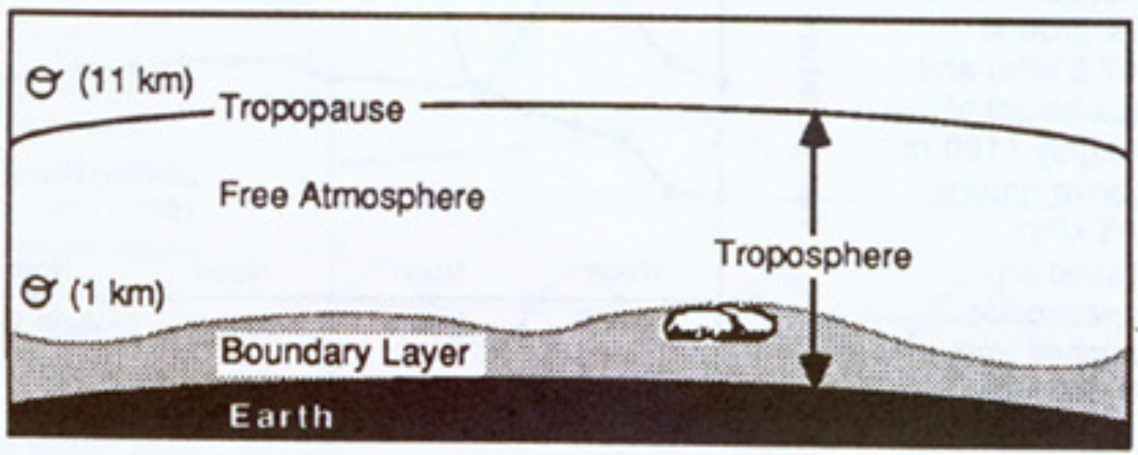


Fig. 1.1 The troposphere can be divided into two parts: a boundary layer (shaded) near the surface and the free atmosphere above it.

The turbulent eddy scales

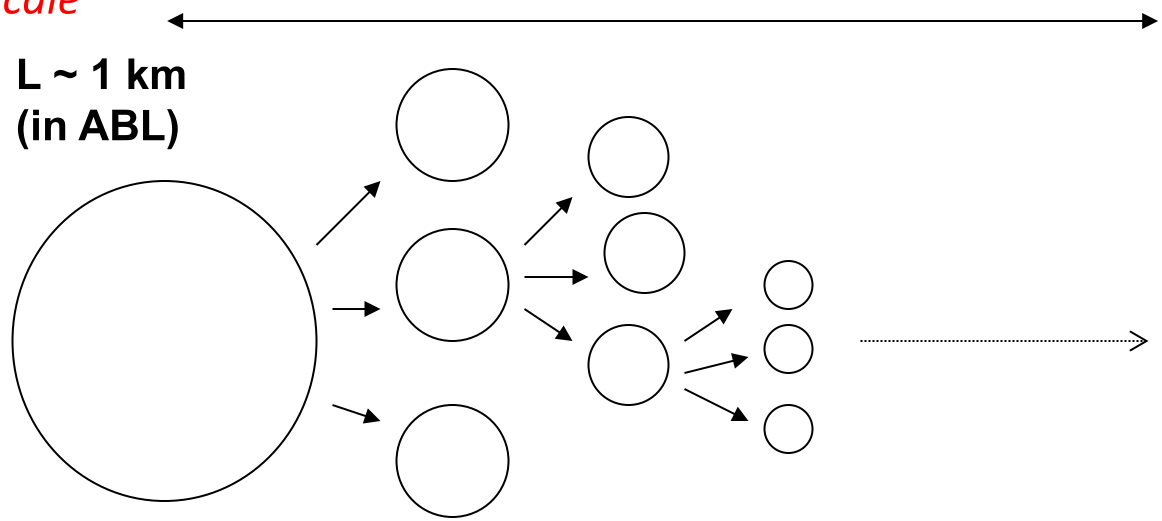
Integral scale

Range of flow scales

Kolmogorov scale

L ~ 1 km (in ABL)

L_K ~ (in ABL)



$$\frac{L_I}{L_K} \sim \mathrm{Re}^{3/4}$$

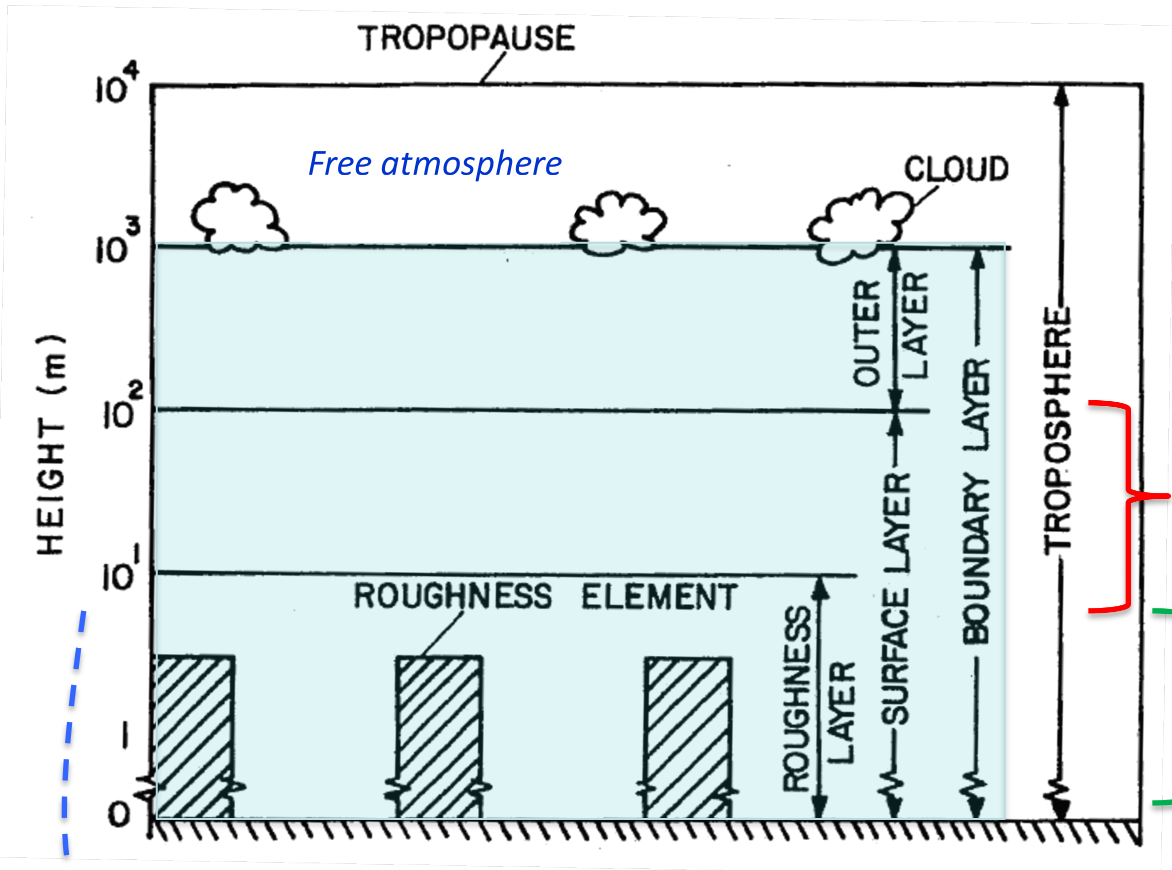
Energy production → ***(Energy cascade)*** → ***Energy dissipation***

(Inertial effects)

(Viscous effects)

- ***<u>Question</u>***: *What is approximately the size of the Kolmogorov (dissipation) scale? (=size of the smallest eddies)*

Turbulent boundary layer flow over a rough surface



(logarithmic scale)

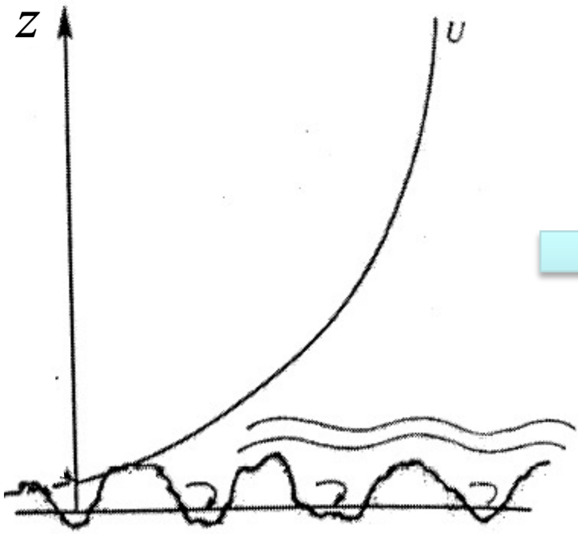
Log layer occupies the **lowest 10-20% of BL. Turbulent fluxes change by less than 10%**

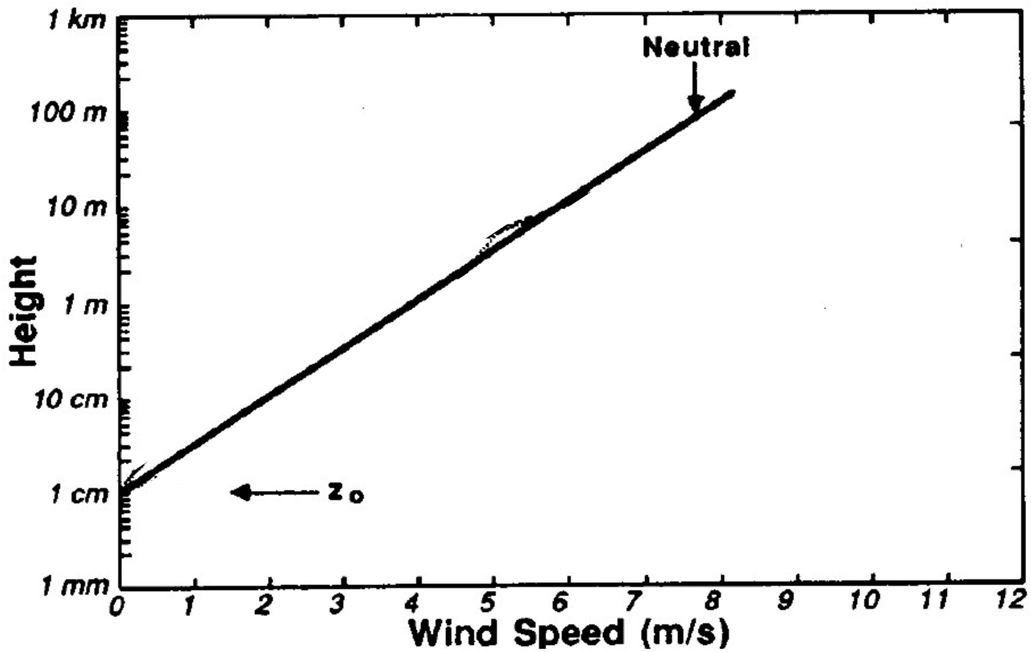
Roughness sublayer (Canopy sublayer) Lowest portion of the surface layer, in which the influence of individual roughness elements (soil roughness, plants, buildings) can readily be discerned. The flow is **horizontally heterogeneous**. In city centers, it may comprise a significant portion of the urban boundary layer.

Typical velocity profiles in NEUTRAL turbulent boundary layers over ‘rough’ surfaces

Linear scale

Semi-log scale





The Logarithmic Wind Velocity Profile:
In the **Surface Layer** (approximately Lowest 10-20% of the boundary layer)

Logarithmic velocity profile: Derivation 1

Using non-dimensional analysis (Pi Theorem):

Important variables: $u,\ z,\ u_*,\ z_o$

u_* : friction velocity
z_o : aerodynamic roughness length

Non-dimensional groups: $\dfrac{u}{u_*},\ \dfrac{z}{z_o}$ → $\dfrac{u}{u_*} = f\left(\dfrac{z}{z_o}\right)$

Experimental data used to find relation between groups

$$\frac{\bar{U}}{u_*} = \frac{1}{k}\ln\left(\frac{z}{z_o}\right)$$

k: von Karman constant (k≈0.4)

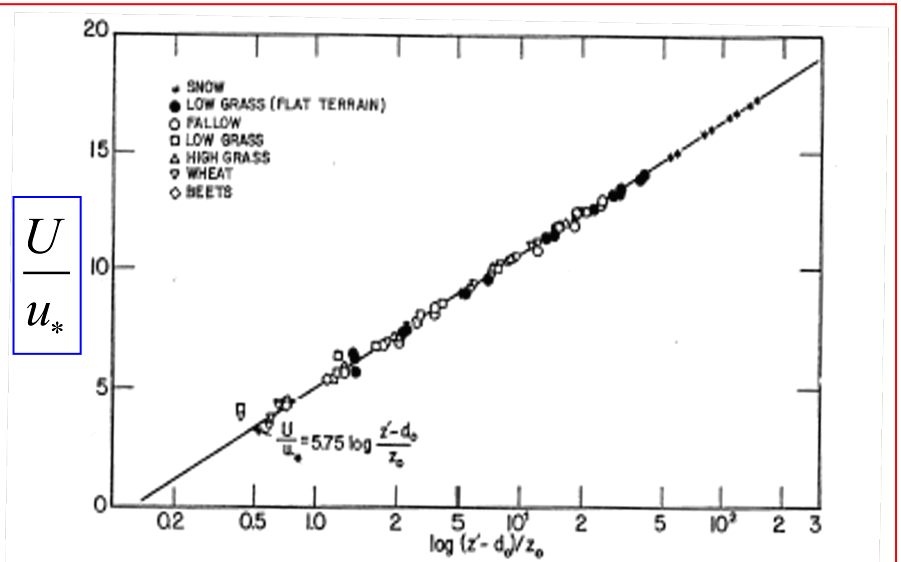


Figure 10.9 Comparison of observed velocity profiles over crops with the log law [Equation (10.14)]. [From Plate (1971).]

Logarithmic velocity profile: Derivation 2

Turbulent flux of momentum (kinematic flux):

$$\overline{u'w'}$$

Q: What is the sign in the surface layer and why?

Eddy-viscosity model:

$$\overline{u'w'} = -\nu_T \frac{\partial \bar{U}}{\partial z}$$

ν_T: Turbulent (Eddy) viscosity

Using a a length scale and a vel. scale:

$$\nu_T \propto l_{scale} \cdot u_{scale}$$

Characteristic velocity scale in surface layer: $u_{scale} \approx u_*$

Characteristic length scale in surface layer: $l_{scale} \propto z$
(size of eddies limited by the presence of the earth's surface)

$$\nu_T = k\, z\, u_*$$

k: von Karman constant (k≈0.4) (empirical)

Logarithmic velocity profile: Derivation (continued)

$$\overline{u'w'} = -k\, z\, u_* \frac{\partial \bar{U}}{\partial z}$$

$$\overline{u'w'} = -u_*^2$$

$$\frac{\partial \bar{U}}{\partial z} = \frac{u_*}{kz}$$

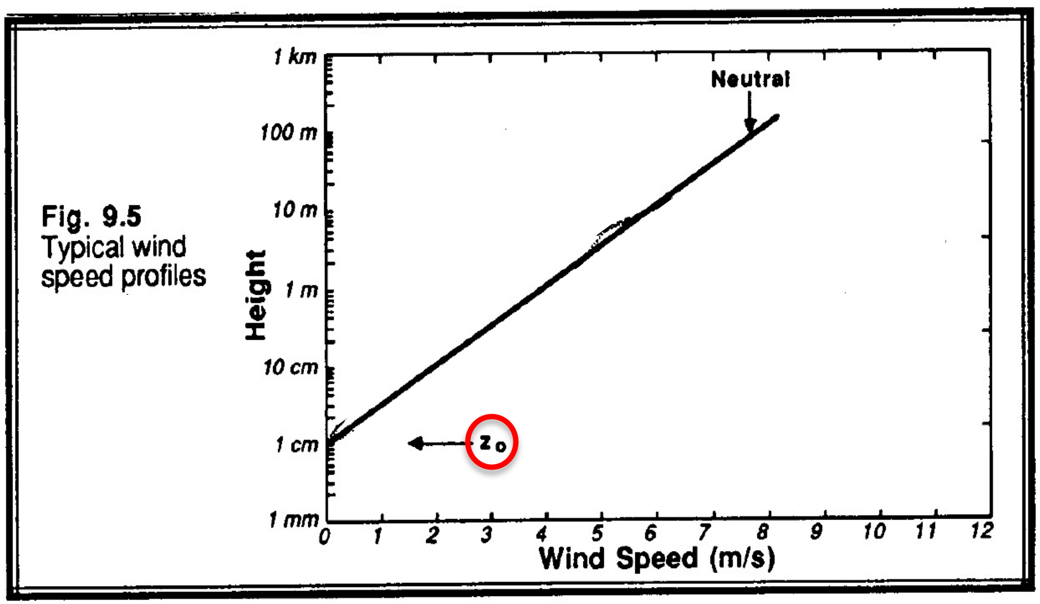
$$\phi_M = \frac{kz}{u_*}\frac{\partial \bar{U}}{\partial z} = 1$$

Turbulent stress/flux is nearly constant in the surface layer (and equal in magnitude to the surface shear stress tao=u*2)

Integrating between any two heights z_1 and z_2 :

$$\int_{\bar{U}_1}^{\bar{U}_2} dU = \frac{u_*}{k}\int_{z_1}^{z_2}\frac{dz}{z}$$

$$\bar{U}_2 - \bar{U}_1 = \frac{u_*}{k}\ln\left(\frac{z_2}{z_1}\right)$$



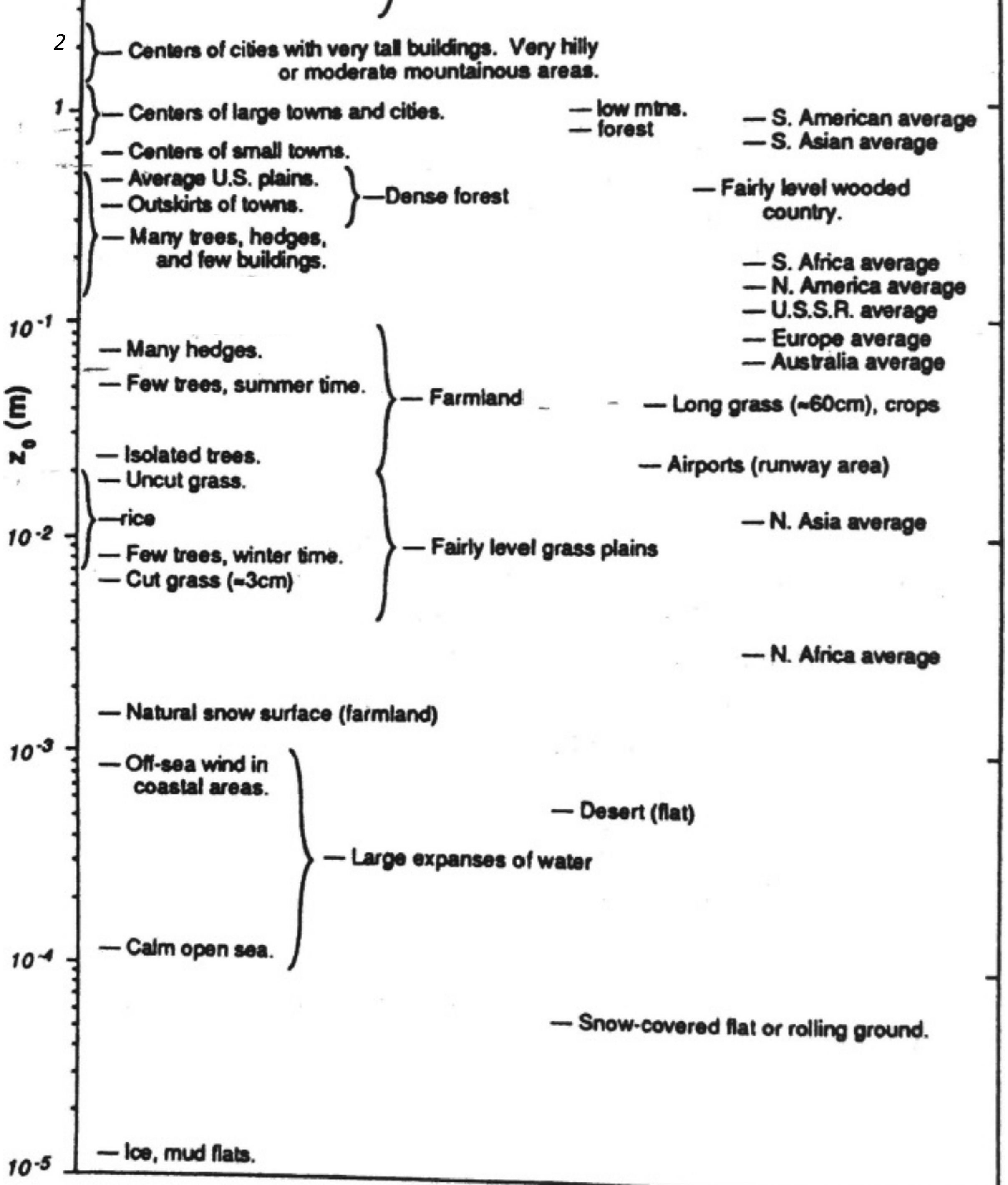
*Defining the **aerodynamic height z_o** as the height where **the extrapolation of the log law gives U=0***

$$\bar{U} = \frac{u_*}{k}\ln\left(\frac{z}{z_o}\right)$$

(over rough surfaces)

Aerodynamic roughness length/height (z_o)

- Defined as the height where wind speed becomes zero (extrapolating the log law)
- It only depends on the SURFACE (**unique for a given surface**)
- 'Aerodynamic' because it is **determined from measurements of wind speed at various heights**
- **Graphically**: Extrapolate the straight line (in the semi-log graph of wind velocity vs. height) to the height where U=0
- **It is NOT equal to the height of the individual roughness elements**
- Other factors: -density of elements;
  -shape of the elements.
- The aerodynamic roughness is **always smaller than the height of individual roughness elements**



From: 'An Introduction to Boundary Layer Meteorology' by R.B. Stull (1988)

Turbulence intensities

Definitions:

$$i_x = \frac{\left(\overline{u'^2}\right)^{1/2}}{U(z)} = \frac{\sigma_u}{U} \qquad i_y = \frac{\left(\overline{v'^2}\right)^{1/2}}{U(z)} = \frac{\sigma_v}{U} \qquad i_z = \frac{\left(\overline{w'^2}\right)^{1/2}}{U(z)} = \frac{\sigma_w}{U}$$

In a turbulent boundary layer, T.I. decreases with height (max. at surface)

Measurements of Panofsky (1967) in a **NEUTRAL SURFACE LAYER**:

$$\sigma_u = 2.2u_* \qquad \sigma_v = 2.2u_* \qquad \sigma_w = 1.25u_*$$

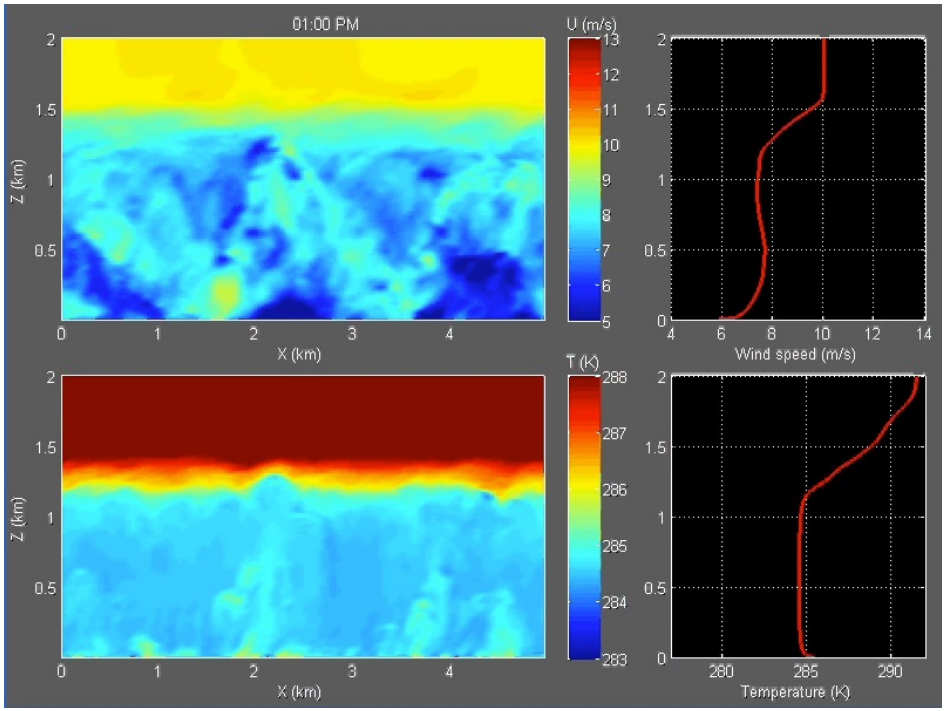
Combining these relationships with logarithmic velocity profile yields (*to do as exercise*):

$$i_x = i_y = \frac{0.88}{\ln(z/z_0)} \qquad i_z = \frac{0.50}{\ln(z/z_0)}$$

Diurnal evolution of the ABL: Buoyancy effects

Day time Unstable (convective) boundary layer

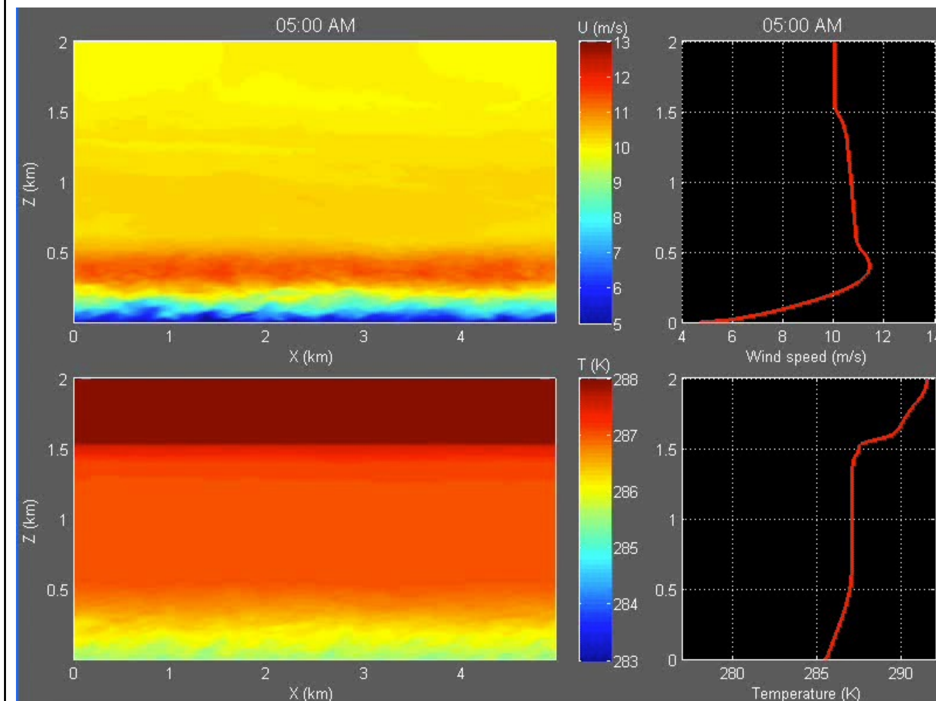
1:00 PM



Velocity (Top)
Potential Temperature (Bottom)

NocturnalStable (stratified) boundary layer

5:00 AM



Velocity (Top)
Potential Temperature (Bottom)

Buoyancy effects in the ABL

The **density of the air (and its buoyancy)** depends on the temperature and the pressure. We can define a **'potential temperature' that accounts for the 2 effects.**

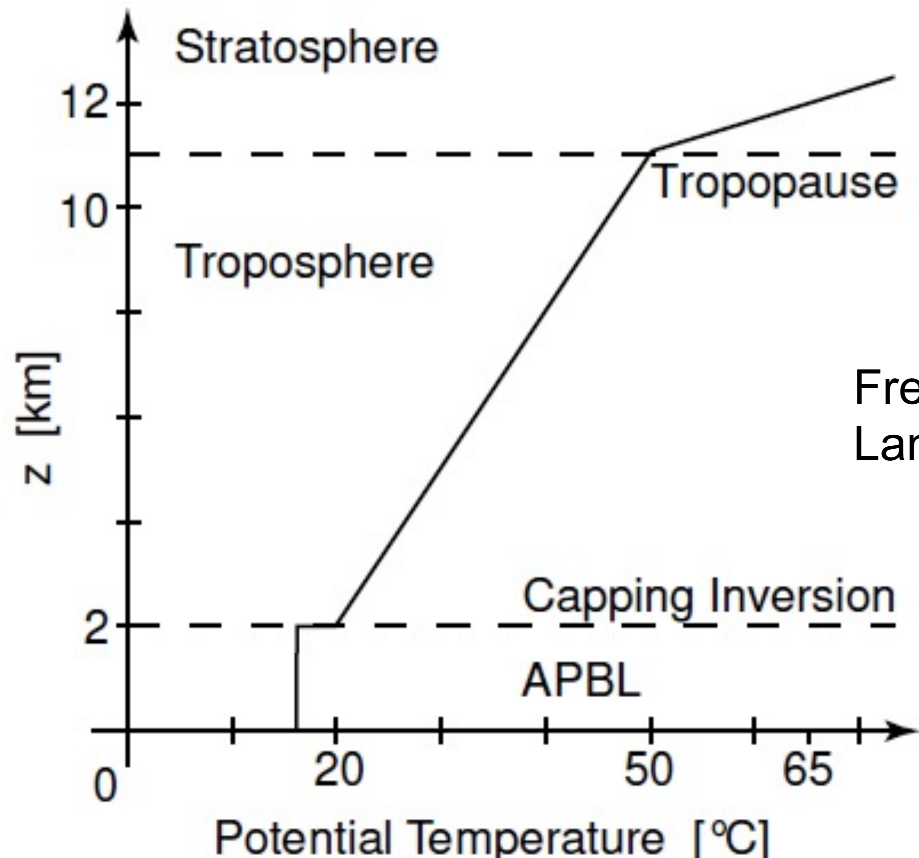
Potential Temperature:

$$\theta = T\left(\frac{P_o}{P}\right)^{\frac{R_{air}}{C_p}} = T\left(\frac{P_o}{P}\right)^{0.286}$$

Temperature that an air parcel with absolute temperature T and pressure P would have if brought adiabatically to the pressure P_o of 1000-mb (100 KPa)

Also: $$\theta = T + \frac{g}{C_p} z$$

For dry air: C_p=1005 J K^{-1}



Free Atmosphere:
Laminar Flow

ABL:
Turbulent Flow

Potential temperature (θ): Derivation

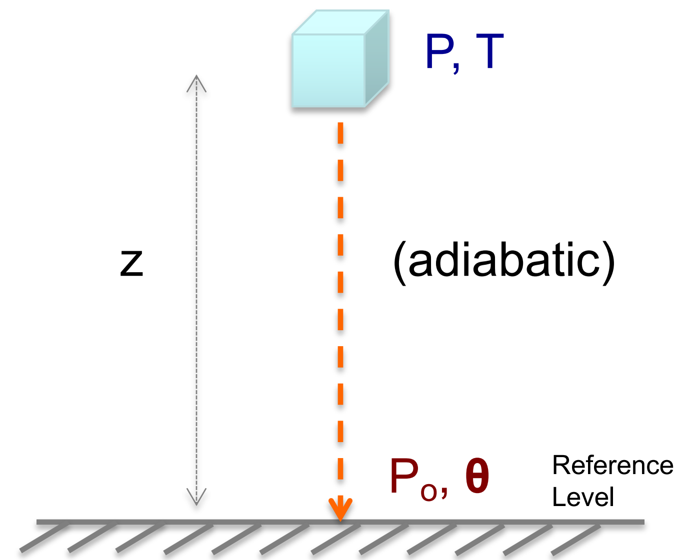
- Hydrostatic Equation: $$\frac{\partial P}{\partial z} = -\rho g$$ *[1]*

- First Law of Thermodynamics: $$dU = dH + dW$$

Change in internal energy → $$dU = \rho C_p dT$$

Heat exchange

Work on the volume → $$dW = dP$$



Assuming adiabatic conditions (i.e., no exchange of heat with surroundings, i.e., *dH=0*)

$$\rho C_p dT = dP$$ *[2]*

Combining equations [1] and [2]: $$\Gamma_{ad} = \left(\frac{\partial T}{\partial z}\right)_{ad} = -\frac{g}{C_p} = -9.8 \text{ K km}^{-1}$$ (dry adiabatic lapse rate)

Integrating from height z to the reference height (z=0)

$$\int_T^{\theta} dT = -\frac{g}{C_p}\int_z^0 dz$$

$$\theta = T + \frac{g}{C_P} z$$

Note that for **neutral (adiabatic) conditions**: $$\frac{\partial \theta}{\partial z} = 0$$

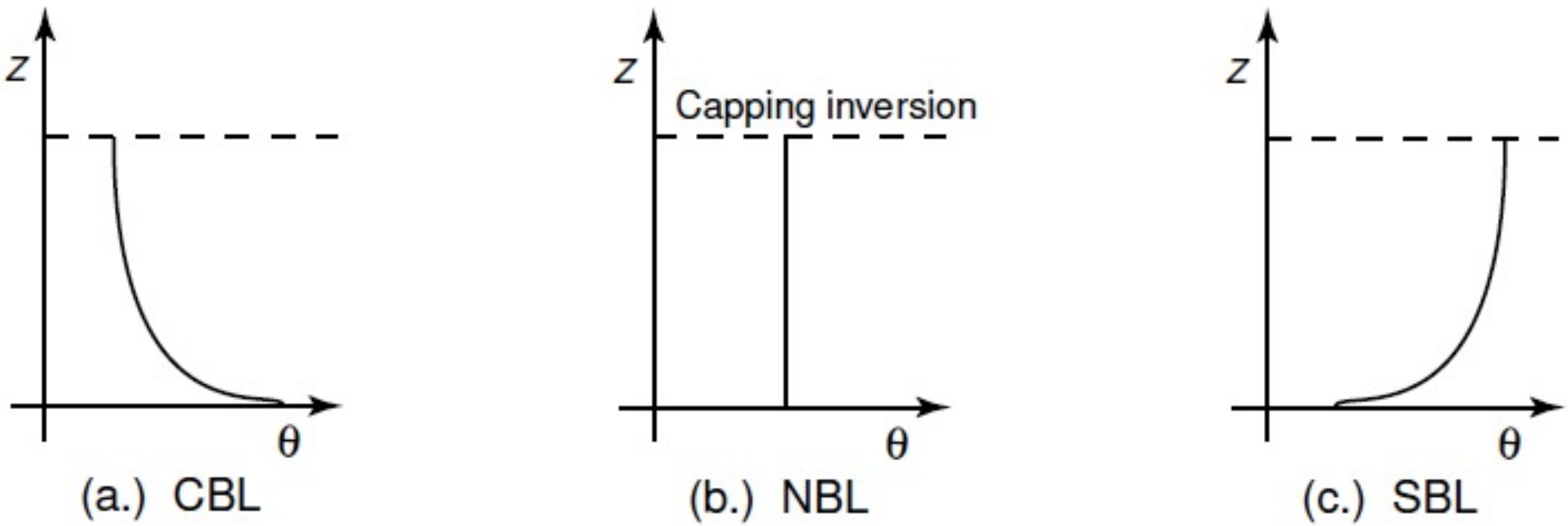
Buoyancy (thermal) effects

Day time Unstable **(convective) boundary layer (CBL)**

Ground is hotter than air

Stable boundary layer (SBL)

Ground is colder than air (typically at night)



Atmospheric stability:

- **NEUTRAL** ➔ $\left(\frac{\partial \theta}{\partial z} = 0\right)$
- **UNSTABLE** ➔ $\left(\frac{\partial \theta}{\partial z} < 0\right)$ ⇒ Turbulence enhancement
- **STABLE** ➔ $\left(\frac{\partial \theta}{\partial z} > 0\right)$ ⇒ Turbulence reduction

***FLUX**: Transfer rate of a quantity per unit area per unit time*

Quantity	Flux		Kinematic Flux	
Heat	Q^*	$\left[\frac{J}{m^2 s}\right]$	$Q = \frac{Q^*}{\rho_{fluid} C_p}$	$\left[K \frac{m}{s}\right]$
Pollutant	$q^*_{pollut.}$	$\left[\frac{kg_{pollut.}}{m^2 s}\right]$	$q_{pollut} = \frac{q^*_{pollut}}{\rho_{air}}$	$\left[\frac{kg_{pollut}}{kg_{fluid}} \frac{m}{s}\right]$
Momentum	F^*	$\left[\frac{kg \cdot m \cdot s^{-1}}{m^2 s}\right]$	$F = \frac{F^*}{\rho_{fluid}}$	$\left[\frac{m}{s} \frac{m}{s}\right]$

Mean Fluxes

$$\overline{W} \cdot \overline{\theta}$$

$$\overline{W} \cdot \overline{q}$$

$$\overline{W} \cdot \overline{U}$$

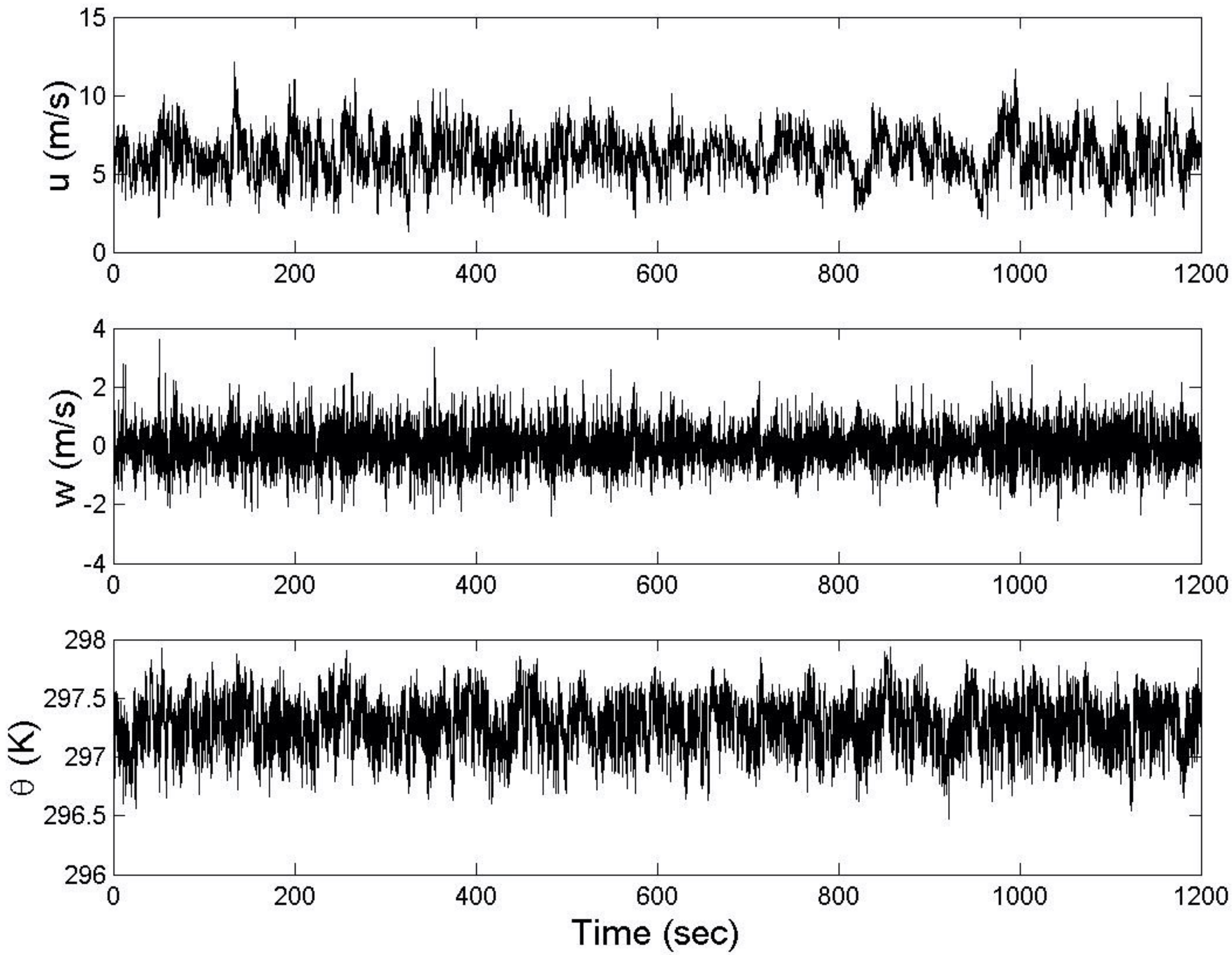
Turbulent (Reynolds) Fluxes

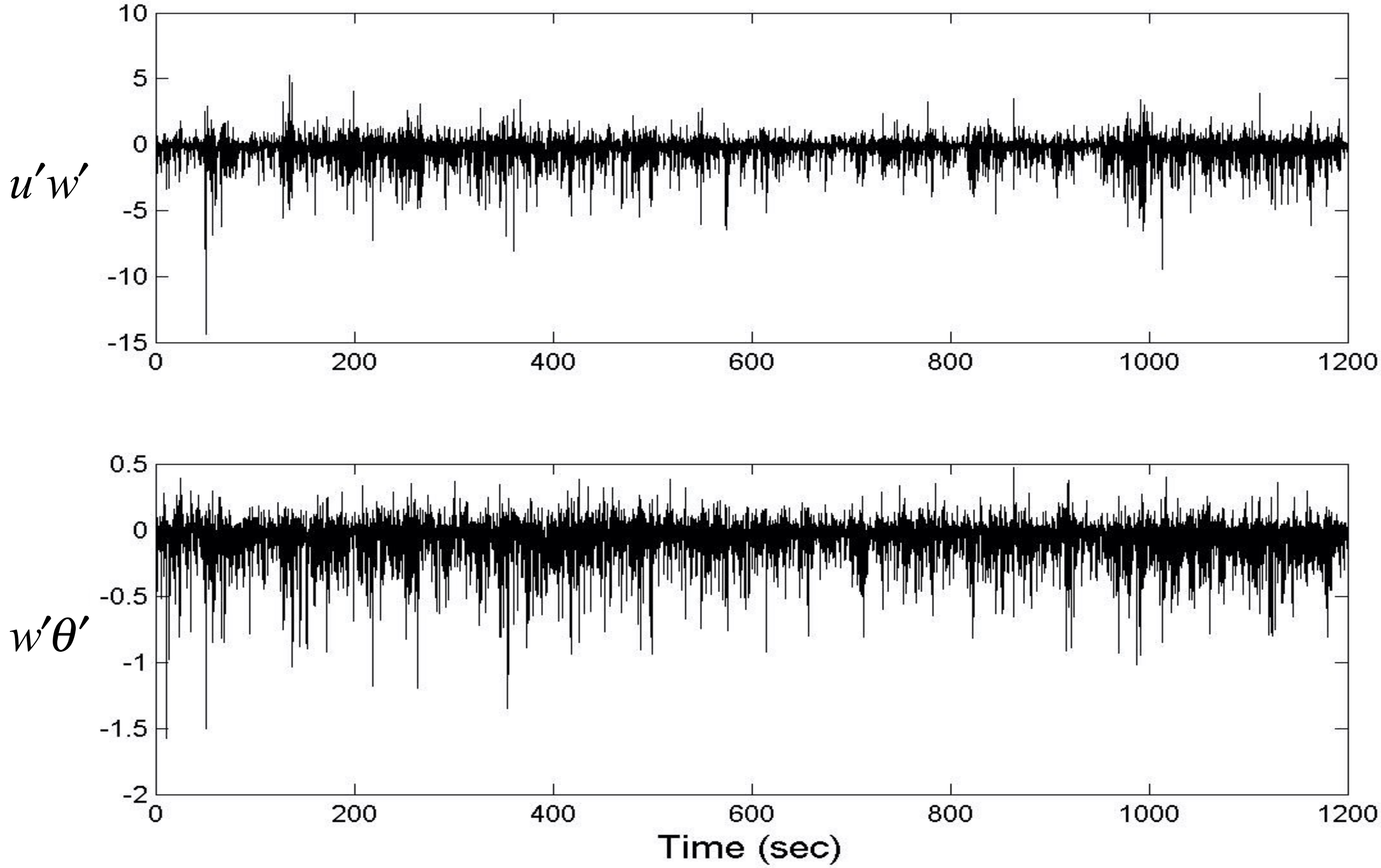
$$\overline{w'\theta'}$$

$$\overline{w'q'}$$

$$\overline{w'u'}$$

Time series of velocity and temperature collected with a sonic anemometer at 20 Hz.





Turbulent (kinematic) momentum flux:

$$\boxed{\overline{u'w'} < 0}$$

Turbulent (kinematic) heat flux:

Stable boundary layers: $\boxed{\overline{w'\theta'} < 0}$

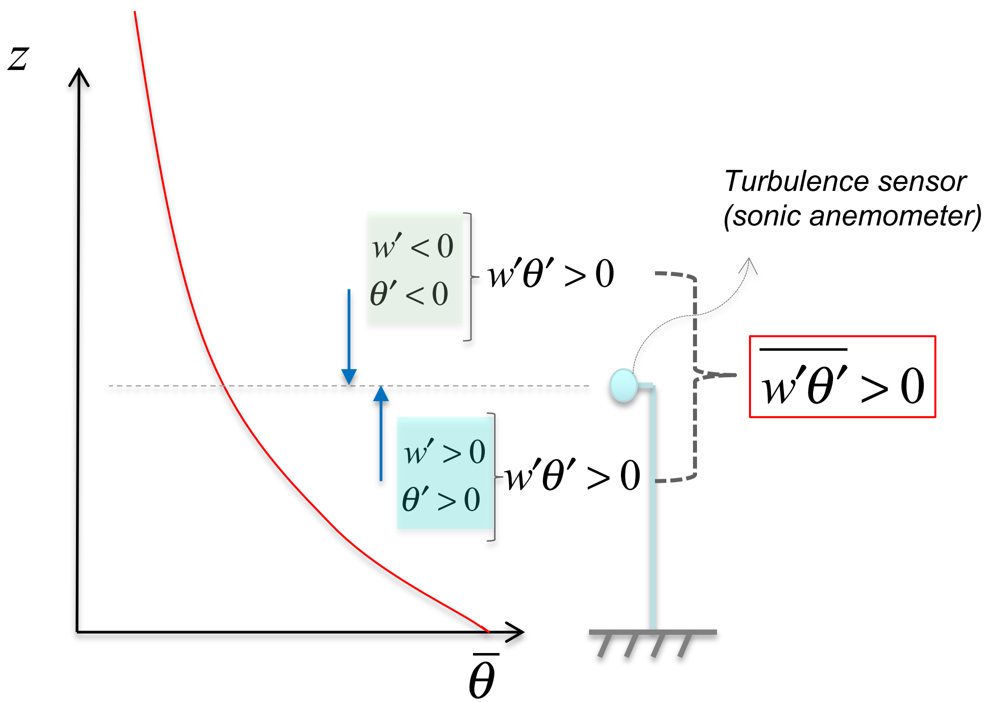
Untable boundary layers: $\boxed{\overline{w'\theta'} > 0}$

Physical explanation for turbulent flux and its sign

- Based on mean temperature profile and the sign of the *w'* and *θ'* fluctuations, we can demonstrate what is the sign of the **vertical turbulent heat flux** ($\overline{w'\theta'}$) for convective and stable conditions.

CASE: Unstable (convective) boundary layer

Under unstable conditions, turbulent eddies bring towards the sensor relatively cooler air from above (when there is a downward motion: w'<0), and they also bring relatively warmer air from below (when there is an upward motion: w'>0)



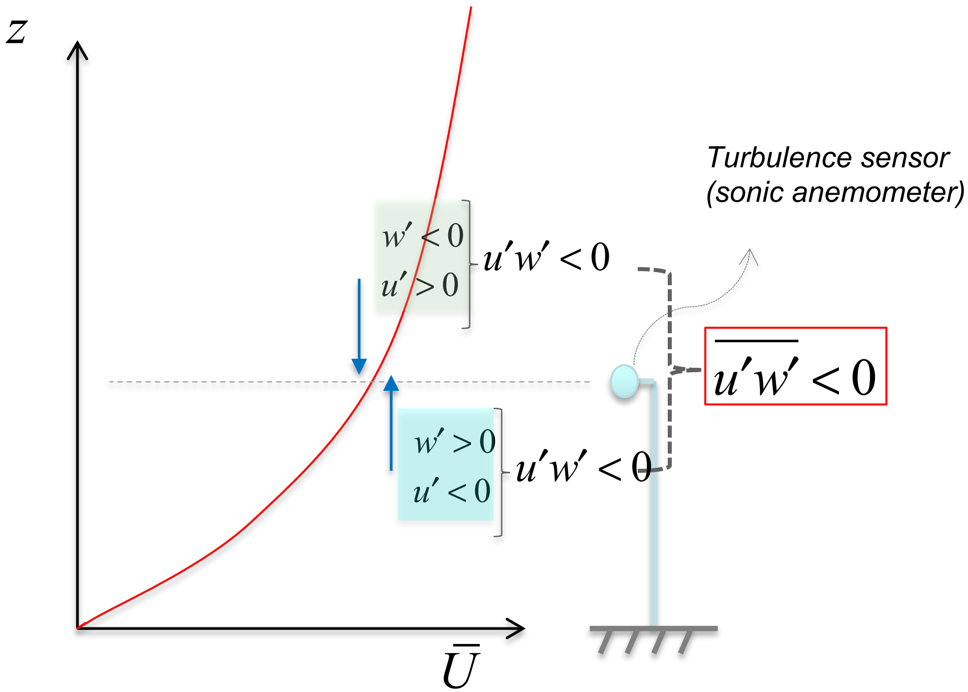
- Note: this is consistent with the eddy-diffusion model:

$$\overline{w'\theta'} = -D_\theta \frac{d\overline{\theta}}{dz}$$

(positive) (negative)

Physical explanation for turbulent flux and its sign

- Based on the mean velocity profile in the surface layer, we can also demonstrate what is the sign of the **vertical turbulent flux of streamwise momentum** ($\overline{u'w'}$).



- Note: again, this is consistent with the eddy-viscosity model:

$$\overline{u'w'} = -D_{t,mom} \frac{d\bar{U}}{dz}$$

(negative) (positive)

- Note that the sign of the momentum flux is always negative because of the shape of the velocity profile.
- Physical meaning: Mean streamwise velocity (and thus streamwise momentum) decreases as one gets closer to the surface; therefore there has to be a downward flux of streamwise momentum. It is also said that the surface is a 'sink' of momentum (since the velocity goes to zero there).

Buoyancy (thermal) effects

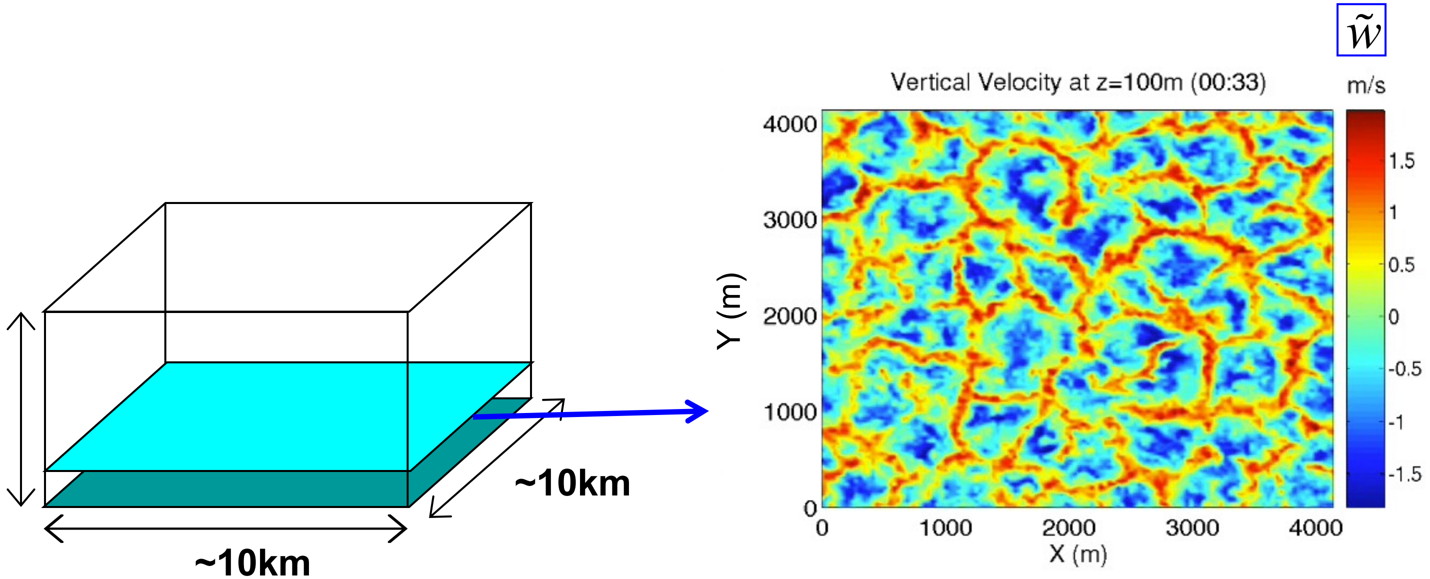
Pasquill stability categories:

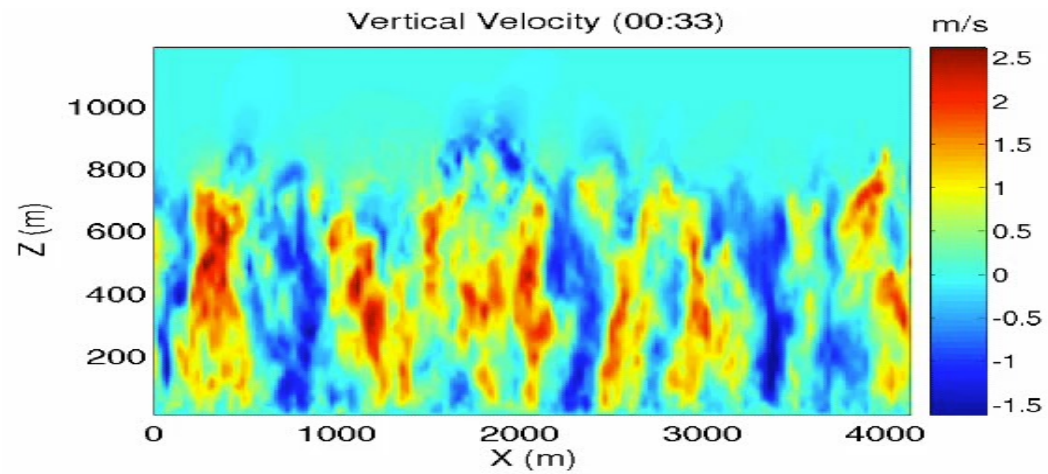
Table 6.1. Pasquill stability categories taken from Csanady (1973).

Surface wind speed in [m/s]	Solar insolation			Night conditions	
	Strong	Moderate	Slight	mainly overcast or $\geq 4/8$ low cloud	$\leq 3/8$ Low cloud
2	A	A–B	B	–	–
2-3	A–B	B	C	E	F
3-5	B	B–C	C	D	E
5-6	C	C–D	D	D	D
6	C	D	D	D	D

A - Extremely unstable, B - Moderately unstable, C - Slightly unstable, D - Neutral, E - Slightly stable, F - Moderately stable.

Free convection (Extremely unstable - very calm summer day conditions)





Buoyancy (thermal) effects

Effect of buoyancy on Turbulence Intensities:

Table 6.2. Typical turbulence intensities near the ground level

Thermal stratification	i_y	i_z
Extremely unstable	0.40–0.55	0.15–0.55
Moderately unstable	0.25–0.40	0.10–0.15
Near neutral	0.10–0.25	0.05–0.08
Moderately stable	0.08–0.25	0.03–0.07
Extremely stable	0.03–0.25	0.00–0.03

- **Under <u>unstable</u> (convective) conditions**, **turbulence is generated** by both shear (due to friction with the surface) and **buoyancy**.
- **Under <u>stable</u> conditions, turbulence** is generated by shear (friction), but it is **damped by negative buoyancy (thermal stratification)**.

Turbulent mixing in 3 dimensions: Pollutant plume

Turbulent mixing in 3 dimensions: Pollutant plume

http://www.24heures.ch/vaud-regions/usine-thevenazleduc-flammes/story/30833723

Turbulent mixing in 3 dimensions

- Recall: In Chapter 3 we derived turbulent advection-diffusion equations for turbulent flows
- Transport equation for MEAN CONCENTRATION field C:

$$\frac{\partial \overline{C}}{\partial t} + \frac{\partial \overline{u}_i \overline{C}}{\partial x_i} = D_{x,t} \frac{\partial^2 \overline{C}}{\partial x^2} + D_{y,t} \frac{\partial^2 \overline{C}}{\partial y^2} + D_{z,t} \frac{\partial^2 \overline{C}}{\partial z^2}$$

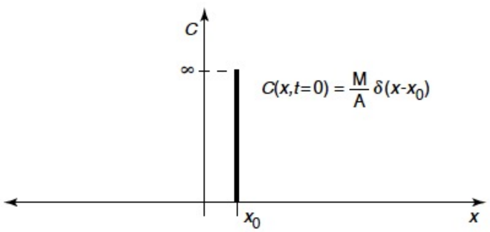
which is equivalent to:

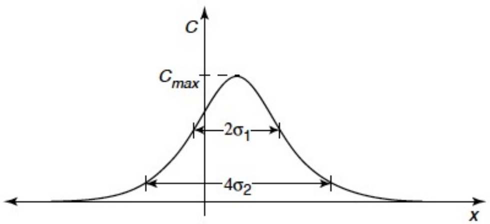
$$\frac{\partial \overline{C}}{\partial t} + \frac{\partial \overline{u}\overline{C}}{\partial x} + \frac{\partial \overline{v}\overline{C}}{\partial y} + \frac{\partial \overline{w}\overline{C}}{\partial z} = D_{x,t} \frac{\partial^2 \overline{C}}{\partial x^2} + D_{y,t} \frac{\partial^2 \overline{C}}{\partial y^2} + D_{z,t} \frac{\partial^2 \overline{C}}{\partial z^2}$$

Table 2.1: Table of solutions to the diffusion equation

Schematic and Solution

Instantaneous point source, infinite domain





$$C(x,t) = \frac{M}{A\sqrt{4\pi Dt}} \exp\left[-\frac{(x-x_0)^2}{4Dt}\right]$$

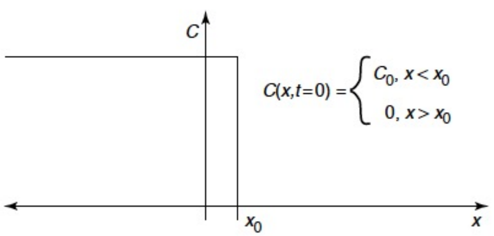
$$C_{max}(t) = \frac{M}{A\sqrt{4\pi Dt}}$$

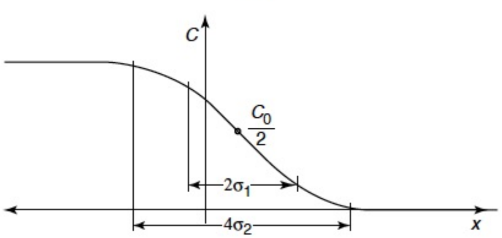
$$q_x(x,t) = \frac{M(x-x_0)}{2At\sqrt{4\pi Dt}} \exp\left[-\frac{(x-x_0)^2}{4Dt}\right]$$

Let $\sigma = \sqrt{2Dt}$ and $(2\sigma)^2 = 8Dt$.
For $x_0 = 0$:
$C(\pm\sigma, t) = 0.61 C_{max}(t)$

Let $\sigma = \sqrt{2Dt}$ and $(4\sigma)^2 = 32Dt$.
For $x_0 = 0$:
$C(\pm 2\sigma, t) = 0.14 C_{max}(t)$

Instantaneous distributed source, infinite domain





$$C(x,t) = \frac{C_0}{2}\left[1 - \text{erf}\left[\frac{(x-x_0)}{\sqrt{4Dt}}\right]\right]$$

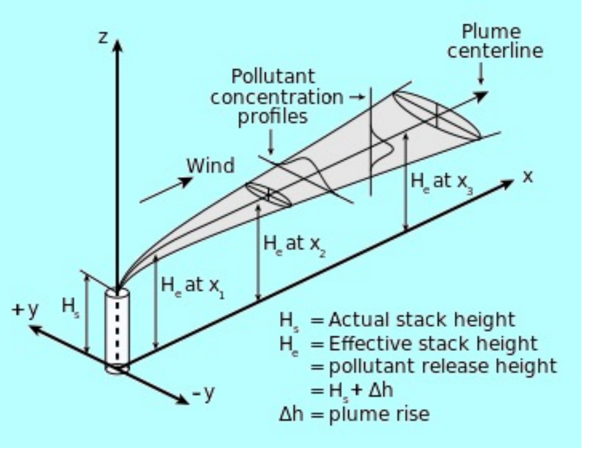
$$C_{max}(t) = C_0$$

$$q_x(x,t) = \frac{C_0\sqrt{D}}{\sqrt{4\pi t}} \exp\left[-\frac{(x-x_0)^2}{4Dt}\right]$$

Let $\sigma = \sqrt{2Dt}$ and $(2\sigma)^2 = 8Dt$.
For $x_0 = 0$:
$C(+\sigma, t) = 0.16 C_0$
$C(-\sigma, t) = 0.84 C_0$

Let $\sigma = \sqrt{2Dt}$ and $(4\sigma)^2 = 32Dt$.
For $x_0 = 0$:
$C(+2\sigma, t) = 0.02 C_0$
$C(-2\sigma, t) = 0.98 C_0$

Turbulent mixing in 3 dimensions: Pollutant plume



Limiting solutions for *D*:

Near-field: $(x \to 0)$

(Linear) $$D_{x,t} \approx \frac{\sigma_x^2}{2 \cdot t} = \frac{\sigma_x^2}{2}\frac{\bar{u}}{x}$$

(Linear) $$D_{y,t} \approx \frac{\sigma_y^2}{2 \cdot t} = \frac{\sigma_y^2}{2}\frac{\bar{u}}{x}$$

(Linear) $$D_{z,t} \approx \frac{\sigma_z^2}{2 \cdot t} = \frac{\sigma_z^2}{2}\frac{\bar{u}}{x}$$

Example: Continuous point release – **Gaussian plume model**

Source strength: $\dot{m} \quad \left[\frac{M}{T}\right]$

Source height: $h \quad [L]$
(smokestack height)

To enforce **solid boundary condition at *z=0***, we use an **image source**.

Solution for the **slender plume assumption** (*streamwise turbulent diffusion is negligible* compared with advection) is given by Csanady (1973):

$$C(x,y,z)=\frac{\dot{m}}{2\pi\bar{u}\sigma_y\sigma_z}\left[\exp\left\{-\frac{y^2}{2\sigma_y^2}-\frac{(z-h)^2}{2\sigma_z^2}\right\}+\exp\left\{-\frac{y^2}{2\sigma_y^2}-\frac{(z+h)^2}{2\sigma_z^2}\right\}\right]$$

- Question: Ho to get $\sigma_y,\ \sigma_z$?

Gaussian Plume Model: Near-field solution

Taylor's theorem (Taylor, 1921), based on the fact that lateral/vertical mixing is driven by the spanwise/vertical turbulence velocity fluctuations:

$$\sigma_y \approx \sigma_v \cdot t = i_y \cdot x$$

$$\sigma_z \approx \sigma_w \cdot t = i_z \cdot x$$

$$\sigma_y = i_y \cdot x$$

$$\sigma_z = i_z \cdot x$$

$$\sigma_w \cdot t = \sigma_w \frac{x}{\overline{u}} = i_z \cdot x$$

Note: at a distance x, the plume has been spreading for a time: $t = \frac{x}{\overline{u}}$

Where turbulence intensities ***i_y* and *i_z* can be obtained from Table 6.2** (book & previous slides).

NOTE: This solution is valid for a considerable range, often up to the distance where the plume grows so large that touches the ground (where maximum ground concentration is observed)

Caution: This section has some typos in the book.

Buoyancy (thermal) effects

Effect of buoyancy on Turbulence Intensities:

Table 6.2. Typical turbulence intensities near the ground level

Thermal stratification	i_y	i_z
Extremely unstable	0.40–0.55	0.15–0.55
Moderately unstable	0.25–0.40	0.10–0.15
Near neutral	0.10–0.25	0.05–0.08
Moderately stable	0.08–0.25	0.03–0.07
Extremely stable	0.03–0.25	0.00–0.03

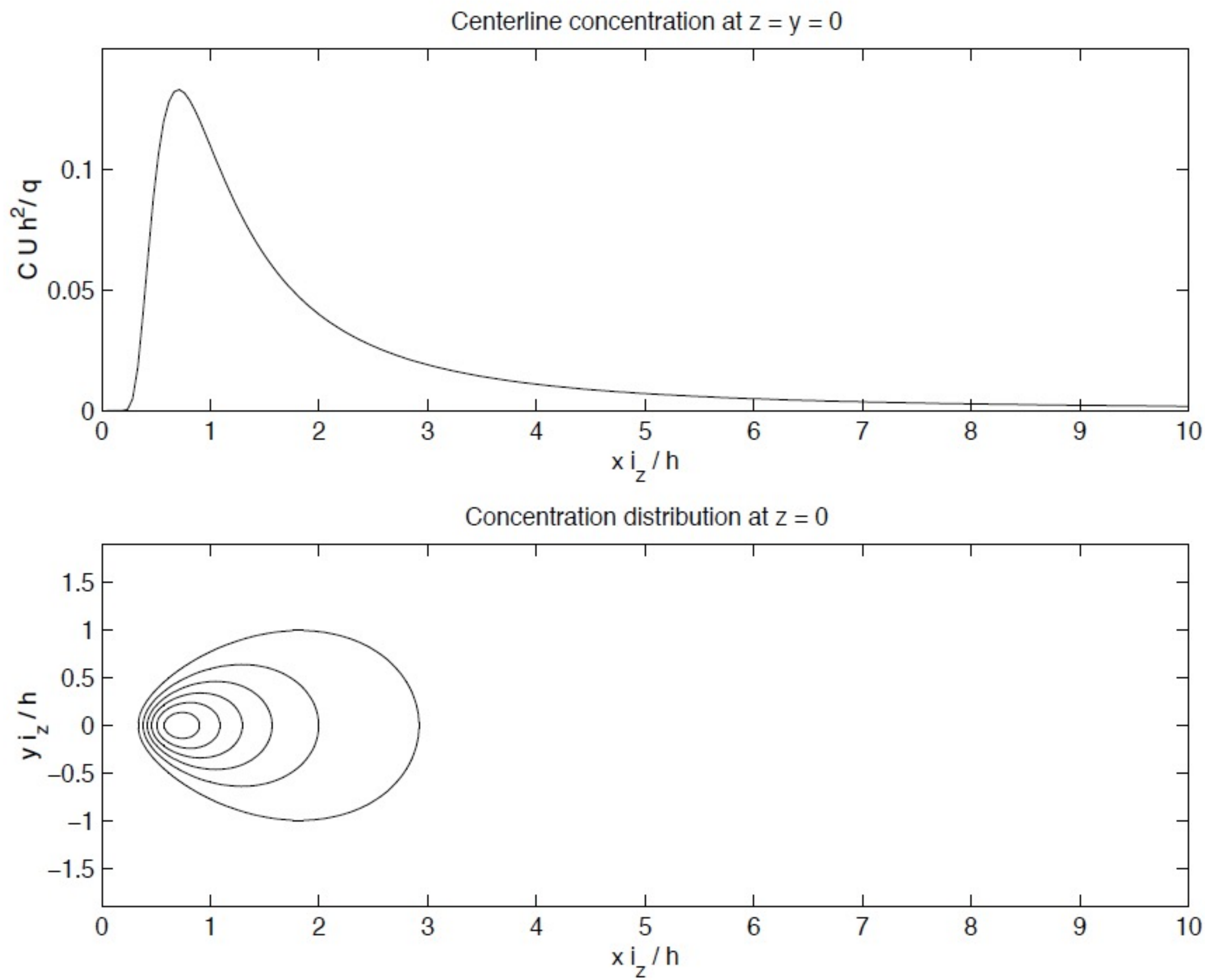
Example: Continuous point release

Solution for concentration at ground level *setting z=0*

$$C\left(x,\, y,\, 0\right) = \frac{\dot{m}}{\pi \bar{u} \sigma_y \sigma_z} \exp\left[-\frac{y^2}{2\sigma_y^2} - \frac{h^2}{2\sigma_z^2} \right]$$

Solution centerline of plume at ground level *setting y=z=0*

$$C\left(x,\, 0,\, 0\right) = \frac{\dot{m}}{\pi \bar{u} \sigma_y \sigma_z} \exp\left[-\frac{h^2}{2\sigma_z^2} \right]$$



Centerline concentration at z = y = 0
C U h²/q
0.1
0.05
0
0 1 2 3 4 5 6 7 8 9 10
x i_z / h
Concentration distribution at z = 0
y i_z / h
1.5
1
0.5
0
−0.5
−1
−1.5
0 1 2 3 4 5 6 7 8 9 10
x i_z / h