Environmental Transport Phenomena

Turbulent Flow

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Turbulent Flow Regime

I. Motivation

II. Description of Turbulence

III. Transport (Advection-Diffusion) Equations for Turbulent Flows

IV. Turbulent Diffusion

Reference: Book, Chapter 3

Turbulent Flows - Motivation

- **Most environmental flows** (e.g., water flow in rivers, lakes; air flow in the atmospheric boundary layer) **are turbulent**
- The **transport and mixing** of momentum, heat and pollutants in most environmental flows are dominated by turbulence

Turbulent Flows - Motivation

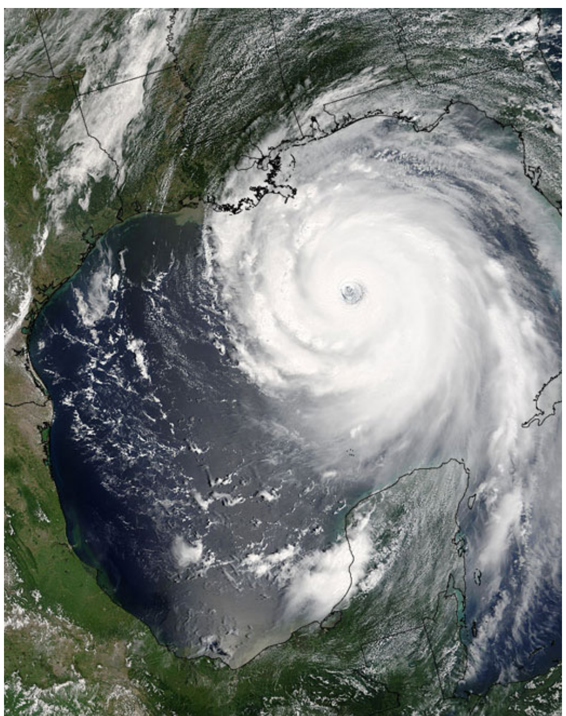
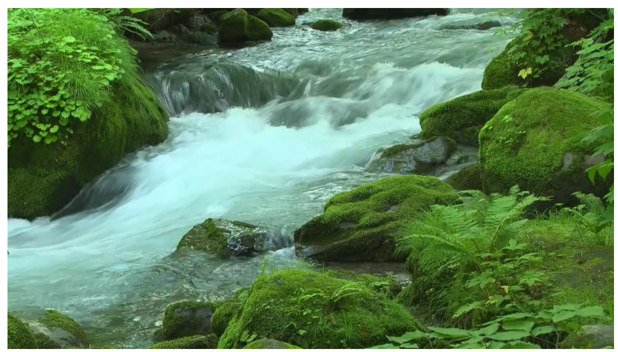
- Measuring and modeling turbulent **transport and mixing** is needed to understand and predict air quality, water quality, water movement in rivers and lakes, wind energy, weather, etc.

(Air pollution dispersion after a fire in a car recycling plant in Ecublens)

(Sediment-laden Rhone river entering lake Geneva)

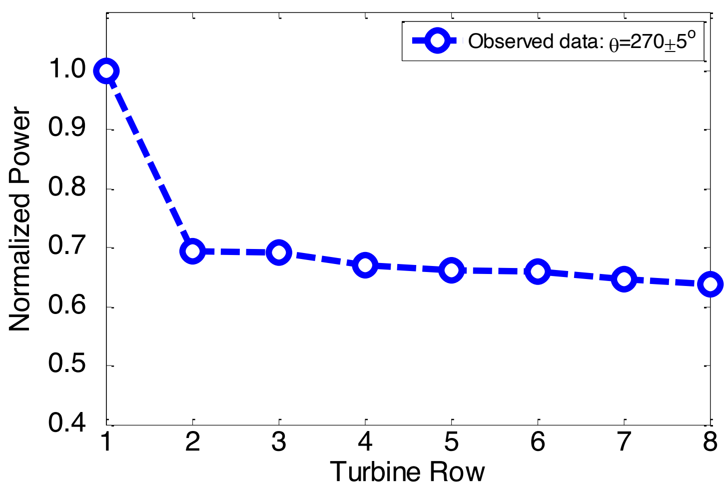
Turbulent Flows - Motivation



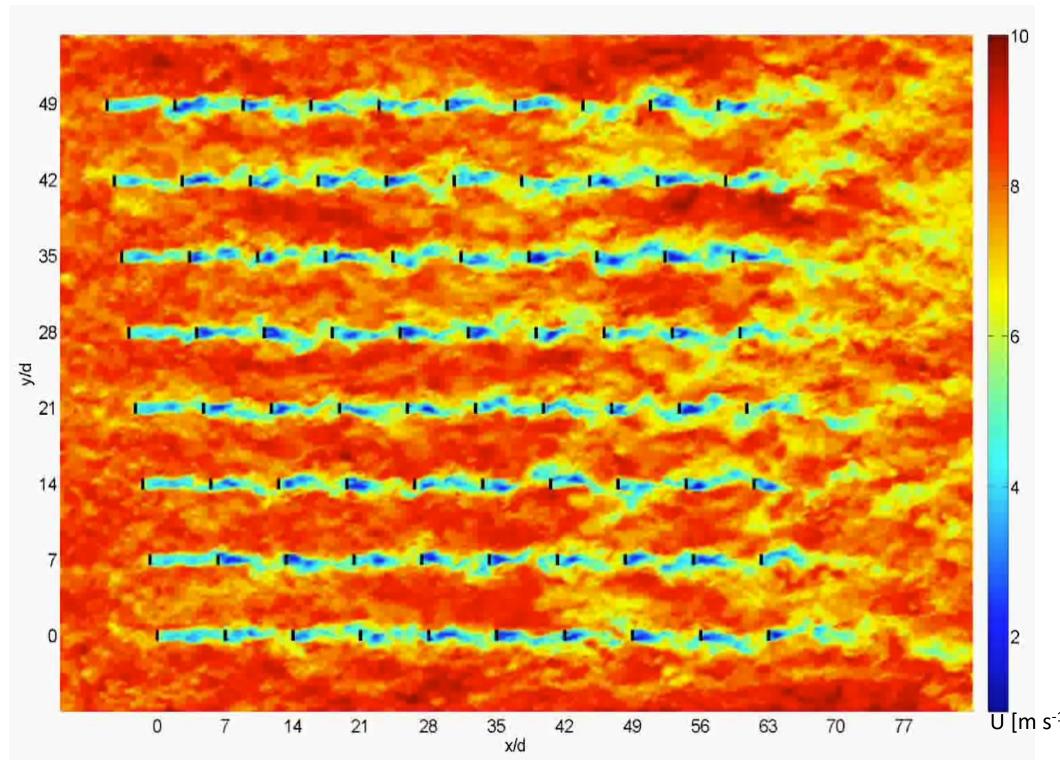





Turbulent Flows - Motivation

(Source: Barthelmie et al, 2007)



Large-eddy simulation (CFD) using the EPFL-WiRE LES code



The mixing power of Turbulence: A simple example

- **How long would it take for sugar at the bottom of a coffee cup to diffuse and dissolve uniformly in stagnant coffee?**

  In that case, coffee will diffuse only due to molecular diffusion (zero advection). The molecular diffusion time can be estimated using:

$$L^2 \approx 2D_m t \quad \rightarrow \quad \boxed{t \sim \frac{L^2}{2D_m}}$$

For sugar in water: $D_m \approx 10^{-9} m^2 s^{-1}$ → Assuming: $L \approx 0.05m$ → $\boxed{t \approx 15\ days!!!}$

(Typical values for gases in air: $D_m \approx 10^{-5} m^2 s^{-1}$)

- **<u>If turbulence is introduced</u> (e.g., by stirring the coffee), full mixing takes place in just a few seconds.**

  As we will see, it is common to model turbulence by using a 'turbulent/eddy diffusion coefficient' (eddy diffusivity)

$$\boxed{D_t >> D_m}$$

➢ **Exercise:** Estimate the value of D_t in this case

Reynolds Number *(recall from basic fluid mechanics)*

$$\mathrm{Re} = \frac{U \cdot L}{\nu}$$

$$\mathrm{Re} \approx \frac{Inertial\ Forces}{Viscous\ Forces}$$

U = Velocity scale (e.g., average velocity or r.m.s.) $[LT^{-1}]$

L = Integral length scale $[L]$: Scale of the largest eddies

ν = Kinematic viscosity $[L^2T^{-1}]$

Reynolds found approximately:
(for pipe flow)

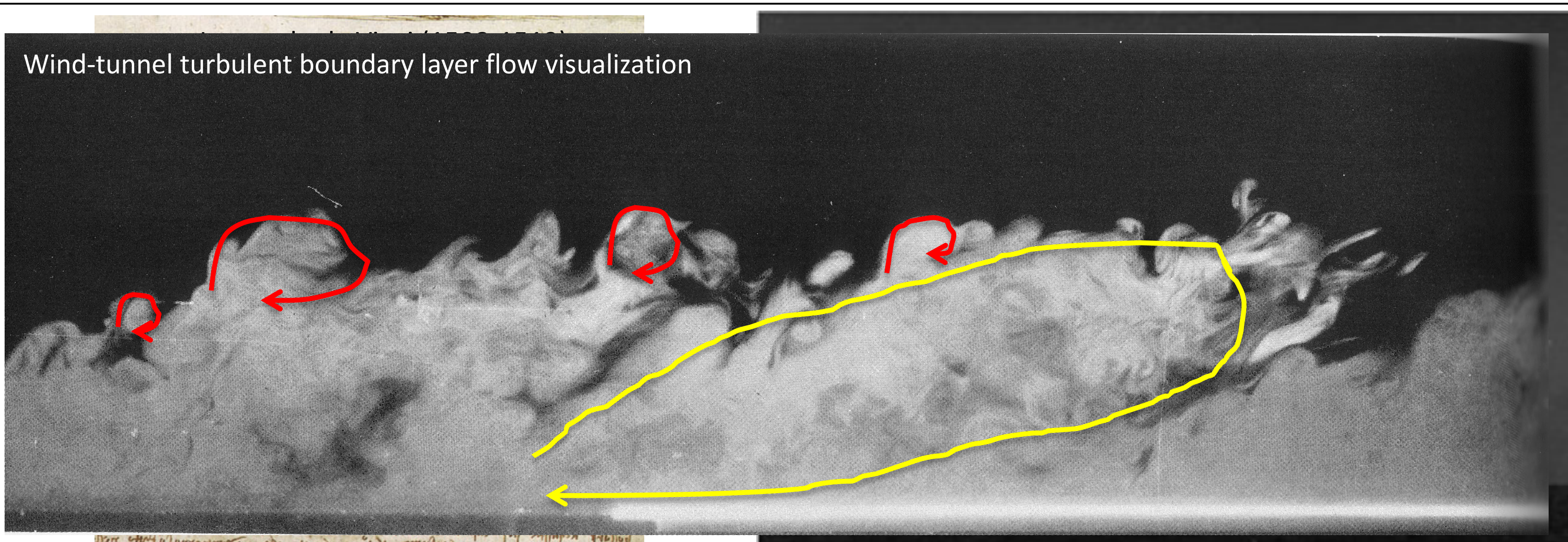
- **Re < 2300 : Laminar Flow**
- **Re > 2300 : Turbulent Flow**

For additional reading (optional):

http://www.princeton.edu/~asmits/Bicycle_web/transition.html

Turbulence

- Fluid flow characterized by **chaotic changes in flow velocity and pressure**
- **Unsteady**; **3-dimensional**; Random-like (but not really: coherent structures)
- Turbulence increases **MIXING and TRANSPORT (FLUX)** rates
- It can be visualized as consisting of irregular swirls of motion called **eddies**
- **Continuous spectrum of eddy sizes** – from integral scale (L_l) to Kolmogorov scale (L_K)
- Turbulence is considered as one of the unresolved problems in physics and mathematics

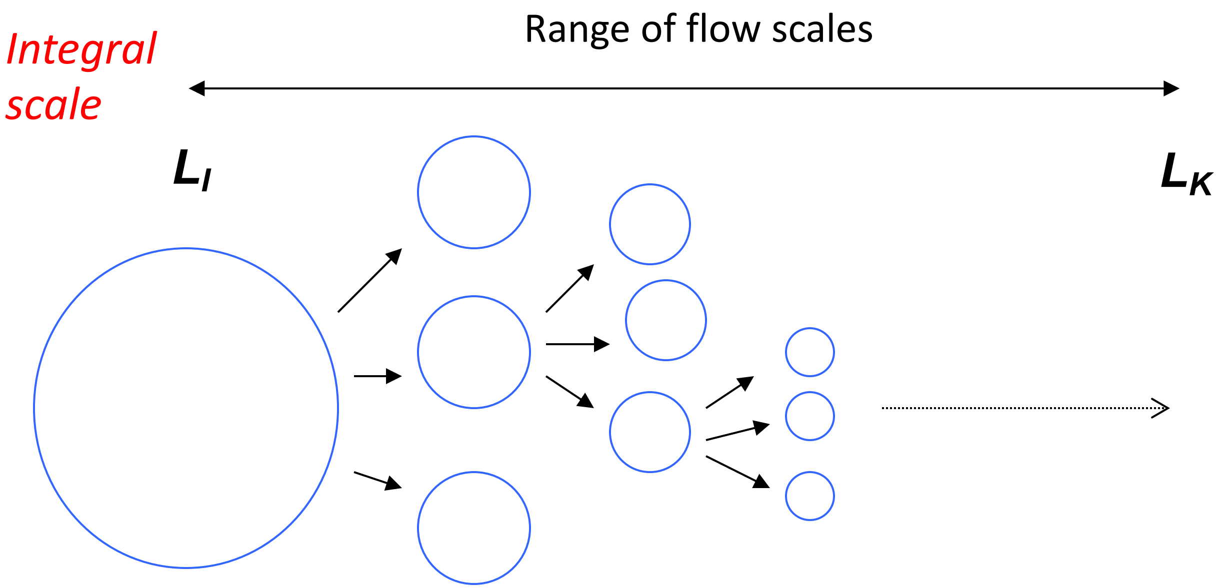


Description of Turbulence

Continuous range of eddy scales: The eddy (& energy) cascade

'Big whirls have little whirls that feed on their velocity, and little whirls have lesser whirls and so on to viscosity'

Richardson (1922)

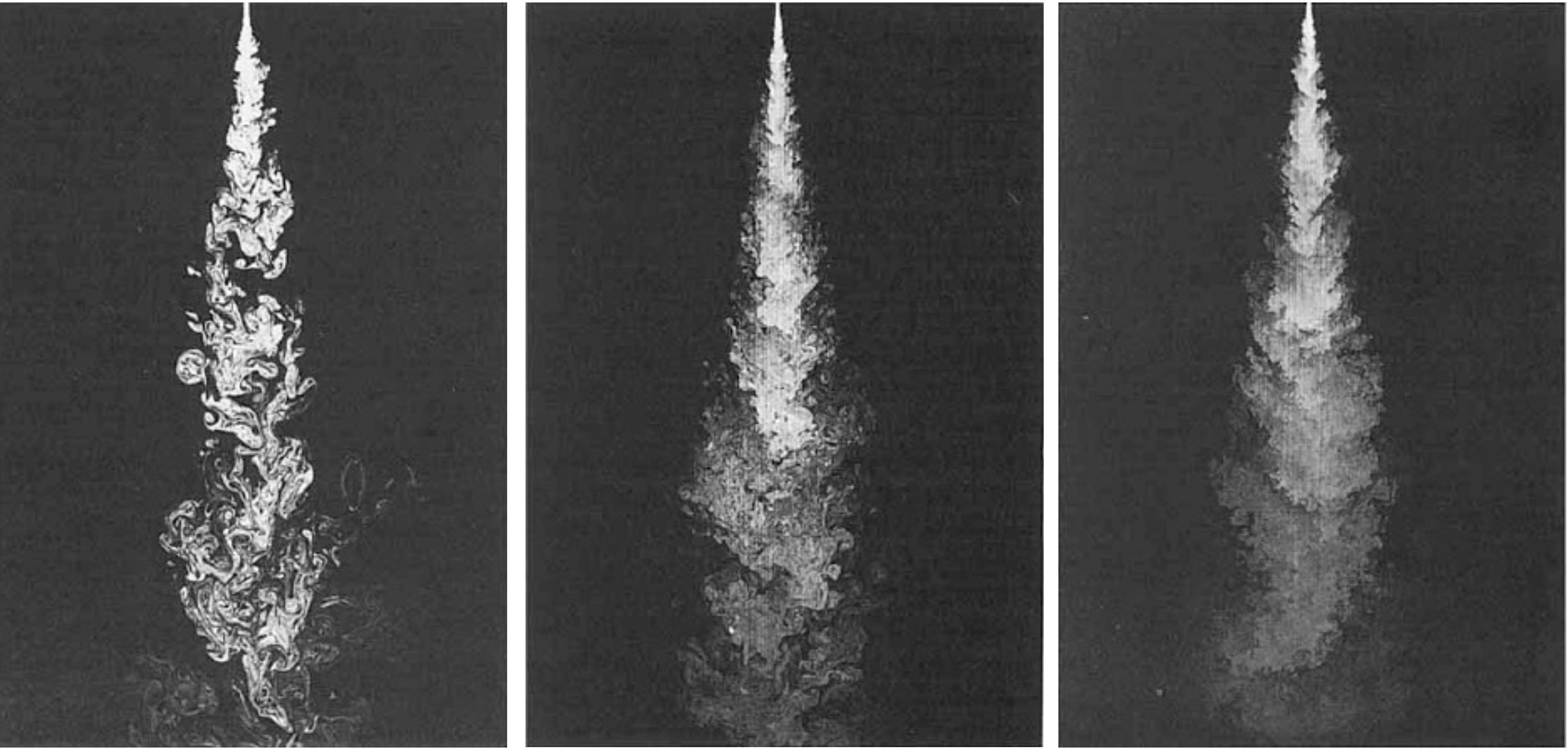


$$\frac{L_I}{L_K} \sim \mathrm{Re}^{3/4}$$

Energy production → ***(Energy cascade)*** → ***Energy dissipation***

(Inertial effects) *(Viscous effects)*

What Turbulent Jet Flow has higher Reynolds number?



Re ~ 1,500 Re ~ 5,000 Re ~ 20,000

Werner et al (1990, J Fluid Mech)

Reynolds decomposition

average — fluctuation

$$u_i = \overline{u}_i + u_i' \qquad (i = 1,2,3)^*$$

$$C = \overline{C} + C'$$

$^*u_1 = u$ (streamwise)

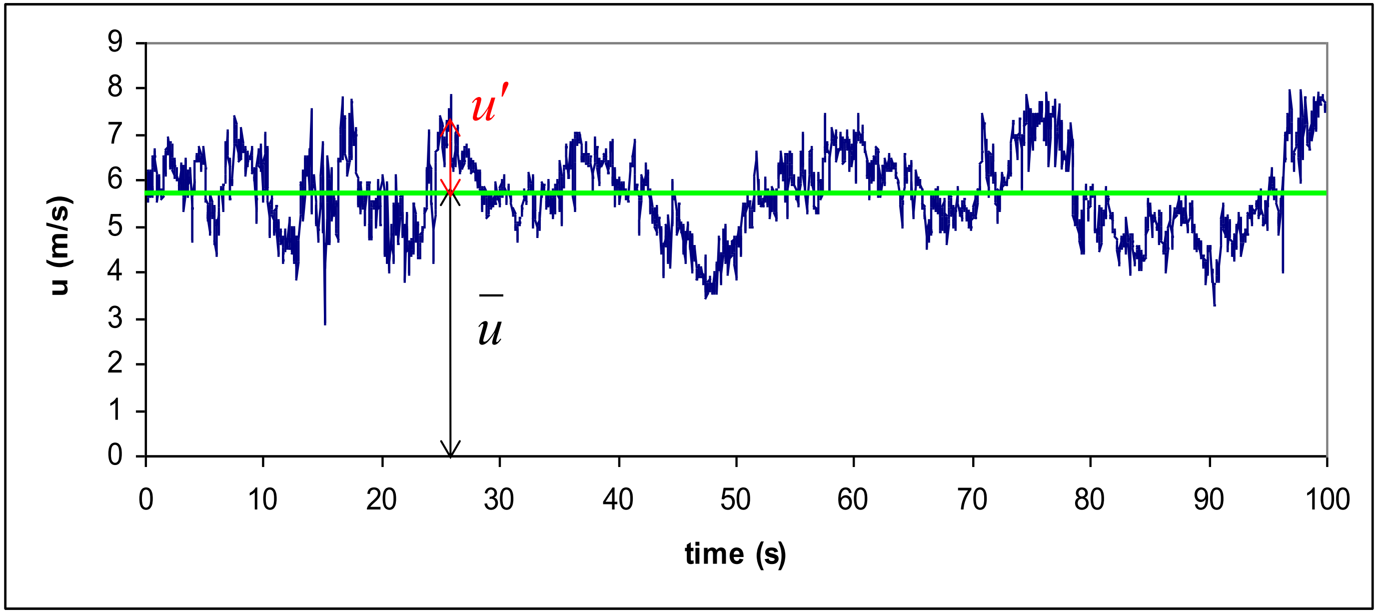
$^*u_2 = v$ (spanwise)

$^*u_3 = w$ (vertical)

$$\overline{u}_i = \frac{1}{T}\int_0^T u_i(t)\, dt$$

$$\overline{C} = \frac{1}{T}\int_0^T C(t)\, dt$$

with T>t$_I$



Wind speed measurement obtained with a sonic anemometer
(similar to those on the EPFL tower and also that of the homework)

How to calculate the Integral Time Scale (t_I) from data

• Turbulent eddies (**coherent structures**) produce **<u>auto-correlation</u>** in **space and in time**; that correlation exists up to distances and times as large as the largest eddies.

Example: High-resolution simulation (LES) of atmospheric boundary layer (ABL) flow:

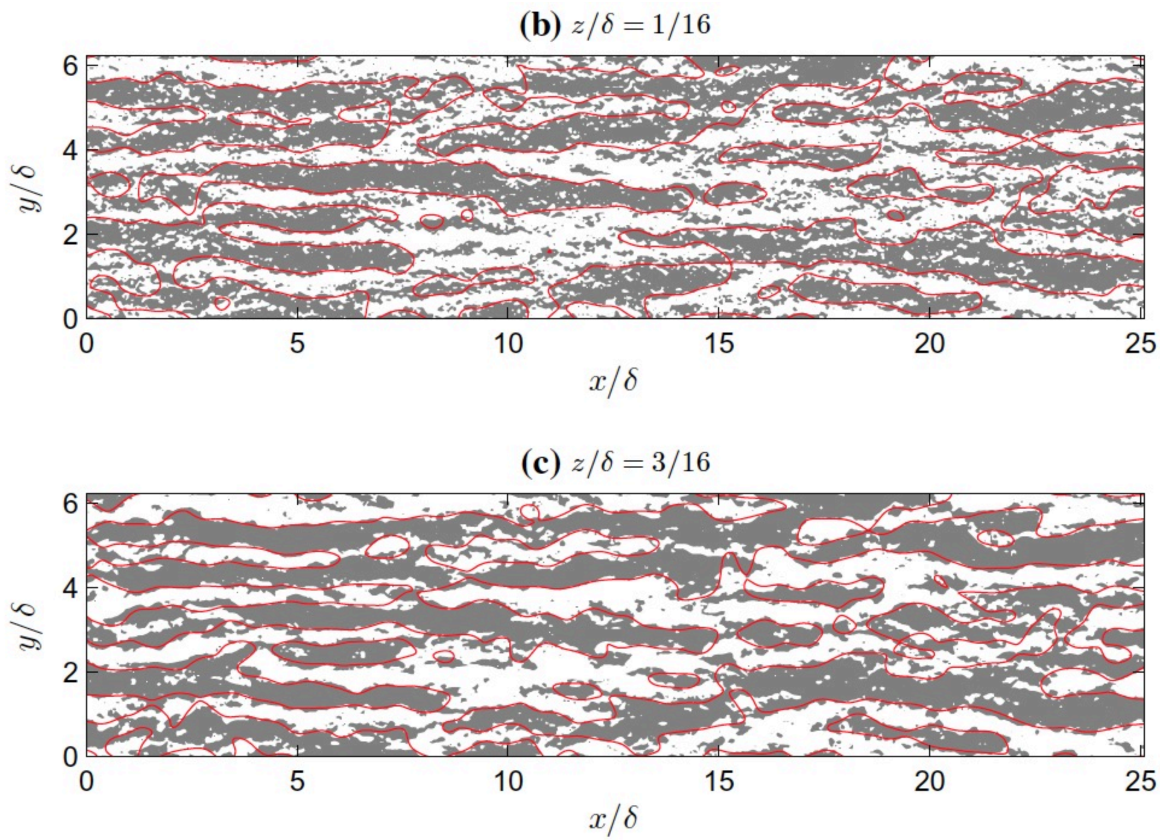


Fig. 2 Instantaneous fluctuations of the streamwise velocity component normalized by the friction velocity in the horizontal planes at three different heights: **a** $z/\delta = 1/32$; **b** $z/\delta = 1/16$; **c** $z/\delta = 3/16$. Positive and negative fluctuations are represented by *dark* and *white colours*, respectively. In each sub-figure, the *red contour line* is for the spatially-filtered field and corresponds to the value of zero

Grey: faster than average (high-speed coherent eddies)

White: slower than average (low-speed coherent eddies)

Note: the existence of elongated very large scales of motion (eddies) is evident

Note: δ is the depth of the flow

How to calculate the Integral Time Scale (t_I) from data

- Data taken at a fixed point over a period of time: *Time Series*
- Turbulent eddies (coherent structures) produce auto-correlation in the time series; that correlation exists up to distances as large as the largest eddies.

Autocorrelation Function:

$$\rho(\tau) = \frac{\overline{u'(t)\, u'(t+\tau)}}{\overline{u'(t)\, u'(t)}} = \frac{\overline{u'(t)\, u'(t+\tau)}}{\sigma_u^2}$$

$$\tau = \text{lag}$$

Integral time scale:

$$t_I = \int_0^{\infty} \rho(\tau)\, d\tau$$

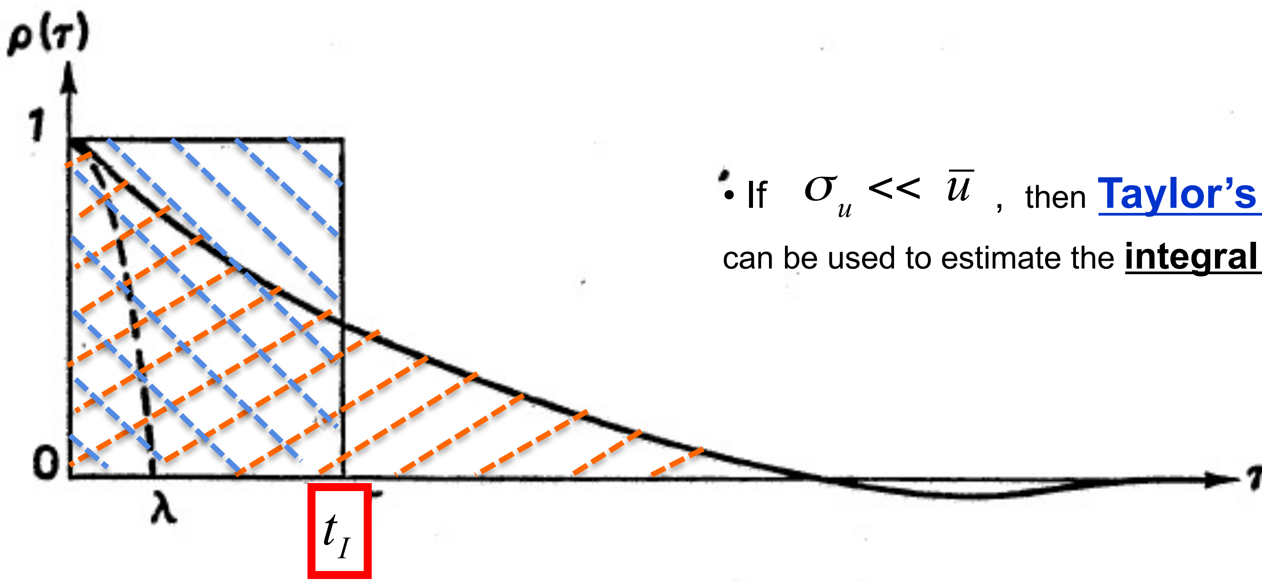


Figure 6.10. Sketch of an autocorrelation coefficient.

- If $\sigma_u << \bar{u}$, then Taylor's 'frozen flow' hypothesis can be used to estimate the **integral length scale** as:

$$L_I = \bar{u}\ t_I$$

Kinetic Energy (KE): Mean KE (MKE) and Turbulence KE (TKE)

(kinetic energy per unit mass) ---- Units: m^2/s^2

*Repeated index notation (Einstein notation)**

$$\frac{K.E.}{mass} = \frac{1}{2}\overline{u_i u_i} == \frac{1}{2}\overline{\left(\overline{u}_i + u_i'\right)\left(\overline{u}_i + u_i'\right)} = \frac{1}{2}\overline{u}_i\ \overline{u}_i + \frac{1}{2}\overline{u_i' u_i'}$$

$$\frac{M.K.E.}{mass} = \frac{1}{2}\overline{u}_i\ \overline{u}_i$$

Mean Kinetic Energy (MKE)

$$\frac{T.K.E.}{mass} = \frac{1}{2}\overline{u_i' u_i'}$$

Turbulence Kinetic Energy (TKE)

NOTE: TKE is generated at the largest eddy scales, transferred from larger to smaller eddies in the eddy cascade, and dissipated at the smallest (Kolmogorov) scales.

**Repeated index notation (Einstein notation)*

$$\frac{1}{2}\overline{u_i' u_i'} = \frac{1}{2}\left(\overline{u_1' u_1'} + \overline{u_2' u_2'} + \overline{u_3' u_3'}\right)$$

Turbulence Kinetic Energy (TKE)

(kinetic energy per unit mass) ---- Units: m^2/s^2

$$\frac{T.K.E.}{mass} = \frac{1}{2}\overline{u_i'u_i'} = \frac{1}{2}\left(\overline{u_1'u_1'} + \overline{u_2'u_2'} + \overline{u_3'u_3'}\right)$$

Turbulence Kinetic Energy (TKE)

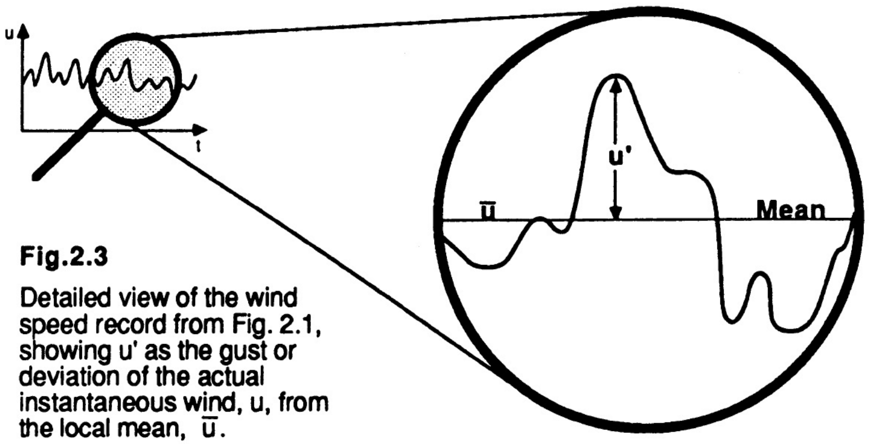


Fig.2.3

Detailed view of the wind speed record from Fig. 2.1, showing u' as the gust or deviation of the actual instantaneous wind, u, from the local mean, ū.

Energy dissipation = Dissipation Rate of TKE = $\varepsilon = \frac{\text{Dissipated TKE}}{\text{Time}}$ → $\left[\frac{L^2}{T^3}\right]$

EXERCISE: *Using dimensional analysis, show that:*

$$L_K \propto \frac{\nu^{3/4}}{\varepsilon^{1/4}}$$

Turbulent Advection-Diffusion Equation: Derivation

Starting with the Advection-Diffusion Equation (derived in previous lectures):

$$\frac{\partial C}{\partial t}+\frac{\partial}{\partial x_i}\left(u_i\ C\right)=\frac{\partial}{\partial x_i}\left[D_m\frac{\partial C}{\partial x_i}\right]$$

$$\frac{\partial C}{\partial t}+\frac{\partial}{\partial x_1}\left(u_1\ C\right)+\frac{\partial}{\partial x_2}\left(u_2\ C\right)+\frac{\partial}{\partial x_3}\left(u_3\ C\right)=\frac{\partial}{\partial x_1}\left[D_m\frac{\partial C}{\partial x_1}\right]+\frac{\partial}{\partial x_2}\left[D_m\frac{\partial C}{\partial x_2}\right]+\frac{\partial}{\partial x_3}\left[D_m\frac{\partial C}{\partial x_3}\right]$$

(1) Using Reynolds Decomposition:

$$\frac{\partial}{\partial t}\left(\bar{C}+C'\right)+\frac{\partial}{\partial x_i}\left(\left(\bar{u}_i+u_i'\right)\left(\bar{C}+C'\right)\right)=\frac{\partial}{\partial x_i}\left[D_m\frac{\partial}{\partial x_i}\left(\bar{C}+C'\right)\right]$$

(2) Rearranging:

$$\frac{\partial}{\partial t}\left(\bar{C}+C'\right)+\frac{\partial}{\partial x_i}\left(\bar{u}_i\bar{C}+\bar{u}_iC'+u_i'\ \bar{C}+u_i'\ C'\right)=\frac{\partial}{\partial x_i}\left[D_m\frac{\partial}{\partial x_i}\left(\bar{C}+C'\right)\right]$$

(3) Applying Reynolds Averaging to all the terms:

$$\frac{\partial}{\partial t}\overline{\left(\bar{C}+C'\right)}+\frac{\partial}{\partial x_i}\overline{\left(\bar{u}_i\bar{C}+\bar{u}_iC'+u_i'\ \bar{C}+u_i'\ C'\right)}=\frac{\partial}{\partial x_i}\left[D_m\frac{\partial}{\partial x_i}\overline{\left(\bar{C}+C'\right)}\right]$$

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Turbulent Advection-Diffusion Equation

$$\frac{\partial}{\partial t}\left(\overline{\overline{C}}+\overline{C'}\right)+\frac{\partial}{\partial x_i}\left(\overline{\overline{u}_i\overline{C}}+\overline{\overline{u}_i C'}+\overline{u_i'\,\overline{C}}+\overline{u_i'\,C'}\right)=\frac{\partial}{\partial x_i}\left[D_m\frac{\partial}{\partial x_i}\left(\overline{\overline{C}}+\overline{C'}\right)\right]$$

(4) Applying Reynolds averaging rules:

$$\overline{C'}=0;\ \overline{u_i'}=0;\ \overline{\overline{u}_i C'}=0;\ \overline{\overline{C}}=\overline{C}$$

(5) Simplifying:

$$\frac{\partial\overline{C}}{\partial t}+\frac{\partial}{\partial x_i}\left(\overline{u}_i\ \overline{C}\right)=-\frac{\partial}{\partial x_i}\left[\overline{u_i'\ C'}\right]+\frac{\partial}{\partial x_i}\left[D_m\frac{\partial\overline{C}}{\partial x_i}\right]$$

$$\overline{u_i'\ C'}\ \equiv\ \text{Turbulent Flux !}$$

- *Turbulent fluxes are responsible for **enhanced transport and mixing in turbulent flows***
- *They add more unknowns in the equations: **More unknowns than equations! (closure problem)***
- *To solve the equations one needs a **MODEL for the TURBULENT FLUX***

FLUX: *Transfer rate of a quantity per unit area per unit time*

Note: Kinematic flux is easy to measure!

Quantity	Flux		Kinematic Flux	
Heat	Q^*	$\left[\frac{J}{m^2 s}\right]$	$Q = \frac{Q^*}{\rho_{fluid} C_p}$	$\left[K \frac{m}{s}\right]$
Pollutant	$q^*_{pollut.}$	$\left[\frac{kg_{pollut.}}{m^2 s}\right]$	$q_{pollut} = \frac{q^*_{pollut}}{\rho_{fluid}}$	$\left[\frac{kg_{pollut}}{kg_{fluid}} \frac{m}{s}\right]$
Momentum	F^*	$\left[\frac{kg \cdot m \cdot s^{-1}}{m^2 s}\right]$	$F = \frac{F^*}{\rho_{fluid}}$	$\left[\frac{m}{s} \frac{m}{s}\right]$

Mean Fluxes

$$\overline{W} \cdot \overline{\theta}$$

$$\overline{W} \cdot \overline{q}$$

$$\overline{W} \cdot \overline{U}$$

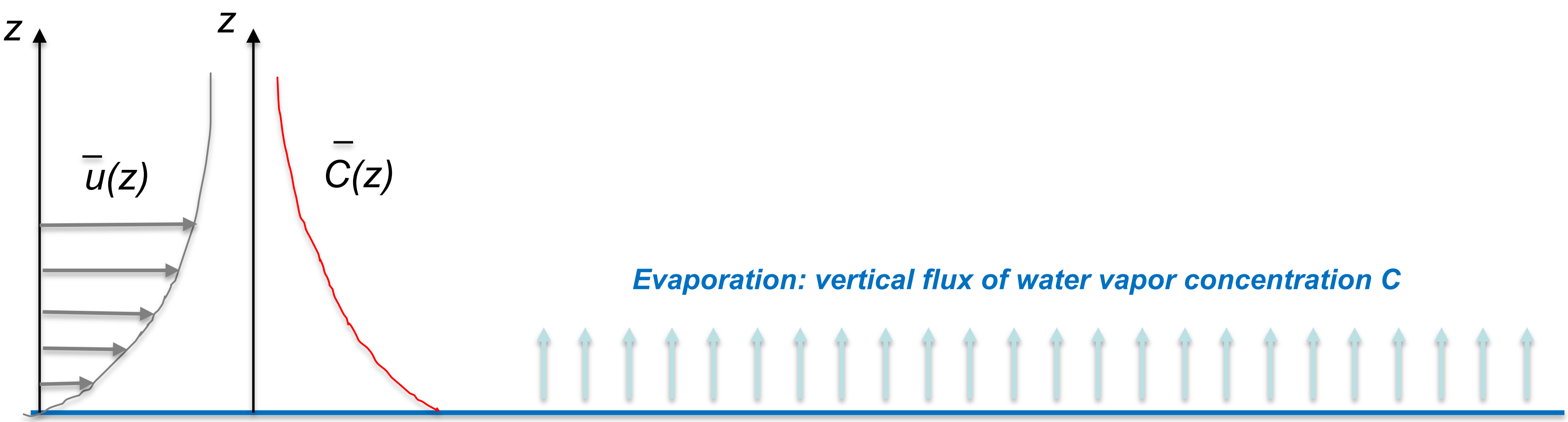
Turbulent (Reynolds) Fluxes

$$\overline{w'\theta'}$$

$$\overline{w'q'}$$

$$\overline{w'u'}$$

Turbulent flux example: Evaporative flux from lake surface



Reynolds decomposition

$$\overline{u_i C} = \overline{u_i}\,\overline{C} + \overline{u_i' C'}$$

Mean flux ***Turbulent flux***

Vertical direction: $\overline{wC} = \overline{w}\,\overline{C} + \overline{w'C'}$

$\overline{w}\,\overline{C} = 0$ ***(mean vertical flux)***

$\overline{w'C'} \neq 0$ ***(vertical turbulent flux!)***

Model: $\overline{w'C'} = -D_{t,z}\dfrac{\partial \overline{C}}{\partial z}$

Turbulence Model: Eddy-diffusion Model

Eddy-diffusion **MODEL**: $$\overline{u_i' C'} = -D_{t,i} \frac{\partial \overline{C}}{\partial x_i}$$

D_t = Turbulent (Eddy) Diffusion Coefficient

D_m = Molecular Diffusion Coefficient (see previous lectures)

$D_t >> D_m$!!!!! therefore, molecular diffusion is negligible

Other difference between D_m and D_t :

- D_m depends only on the fluid, not on the flow
- D_t depends on the local flow characteristics; hence, it is often difficult to predict (CHALLENGE for MODELING!)

Combining the turbulent advection-diffusion equation with the eddy-diffusion model:

Much smaller than turbulent term in very high Re flows

$$\frac{\partial \overline{C}}{\partial t} + \overline{u}_i \frac{\partial \overline{C}}{\partial x_i} = \frac{\partial}{\partial x_i}\left[D_{t,i} \frac{\partial \overline{C}}{\partial x_i} \right] + \frac{\partial}{\partial x_i}\left[D_m \frac{\partial \overline{C}}{\partial x_i} \right]$$

D_m = Molecular Diffusion Coefficient (see previous lectures)

$D_t >> D_m$!!!!! therefore, last term can be neglected

Turbulent Advection-Diffusion Equation with Eddy Diffusion Model

- **Note: Equation includes mixing in 3 dimensions *(i=1,2,3)***

$$\frac{\partial \overline{C}}{\partial t} + \overline{u}_i \frac{\partial \overline{C}}{\partial x_i} = \frac{\partial}{\partial x_i}\left[D_{t,i} \frac{\partial \overline{C}}{\partial x_i} \right]$$

↓

$$\frac{\partial \overline{C}}{\partial t} + \overline{u}\frac{\partial \overline{C}}{\partial x} + \overline{v}\frac{\partial \overline{C}}{\partial y} + \overline{w}\frac{\partial \overline{C}}{\partial z} = \frac{\partial}{\partial x}\left[D_{t,x} \frac{\partial \overline{C}}{\partial x} \right] + \frac{\partial}{\partial y}\left[D_{t,y} \frac{\partial \overline{C}}{\partial y} \right] + \frac{\partial}{\partial z}\left[D_{t,z} \frac{\partial \overline{C}}{\partial z} \right]$$

Turbulent Diffusion Coefficients: *(channel flow example)*

Dimensional analysis: Identify key variables

$$D_t \propto u_{scale} \, L_{scale} \qquad \left[\frac{L^2}{T}\right]$$

Integral length scale: Largest eddies (most energetic) with size limited by the smallest dimension (*h*)

$$L_{scale} \approx h$$

h =flow depth in a channel

Velocity scale: Turbulence generated by shear (e.g., near the surface)

Shear velocity (*u)** is an important parameter that captures the strength of shear

$$u_{scale} \approx u_*$$

$$u_* = \sqrt{\tau_o / \rho}$$

τ_o =surface shear stress

For uniform channel flow:

$$u_* = \sqrt{g \cdot h \cdot S}$$

S =slope

Note: Velocity distribution (profile) is different in the vertical (z) direction as compared with the transverse (y) direction → ***D_t*** **is** **NON-ISOTROPIC!!!** (**another difference** with ***D_m***)

Turbulent Diffusion Coefficients in Channels **(Fischer et al., 1979)**

Vertical Mixing

Vertical Diffusion Coefficient: $D_{t,z}$ can be derived from velocity profile

In a **homogeneous** open channel flow**: Logarithmic velocity profile**

$$D_{t,z} \approx 0.067\ h\ u_*$$

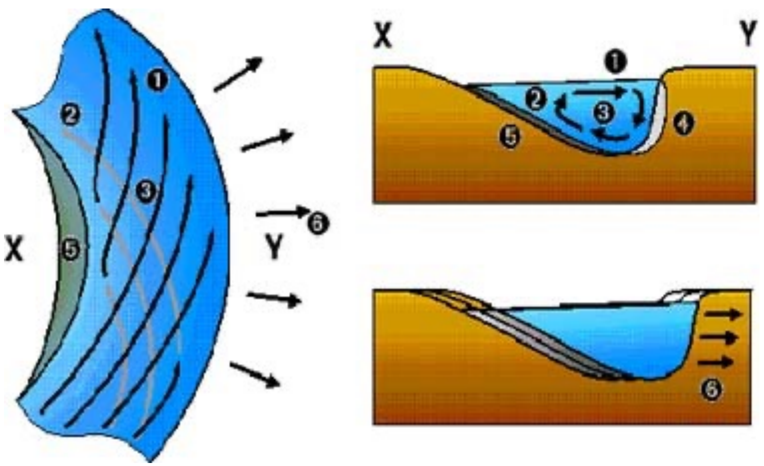
Transverse Mixing

- Empirical (for straight channel with constant cross-section): $D_{t,y} \approx 0.15\ h\ u_*$

- However, in rivers $D_{t,y}$ is larger due to:
  * Non-uniform cross-sectional depth
  * Meandering

$$D_{t,y} \approx 0.6\ h\ u_*$$

Turbulent Diffusion – Meandering effect on $D_{t,y}$



Turbulent Diffusion Coefficients in Rivers (Fischer et al., 1979)

Longitudinal Mixing

- If the flow would be uniform, in principle: $D_{t,x} \approx D_{t,y}$

- However, **in practice**, ***$D_{t,x}$* is larger due to**:

  - Non-uniformity of vertical velocity profile
  - Other non-uniformities like:
    - Curves
    - Non-uniform depth
    - Dead zones

$$D_{t,x} > D_{t,y}$$

- *Note: In 1-D problems (e.g., rivers):* ***Turbulent dispersion ($D_L > D_{t,x}$)*** ***(will see in detail in Week 10)***