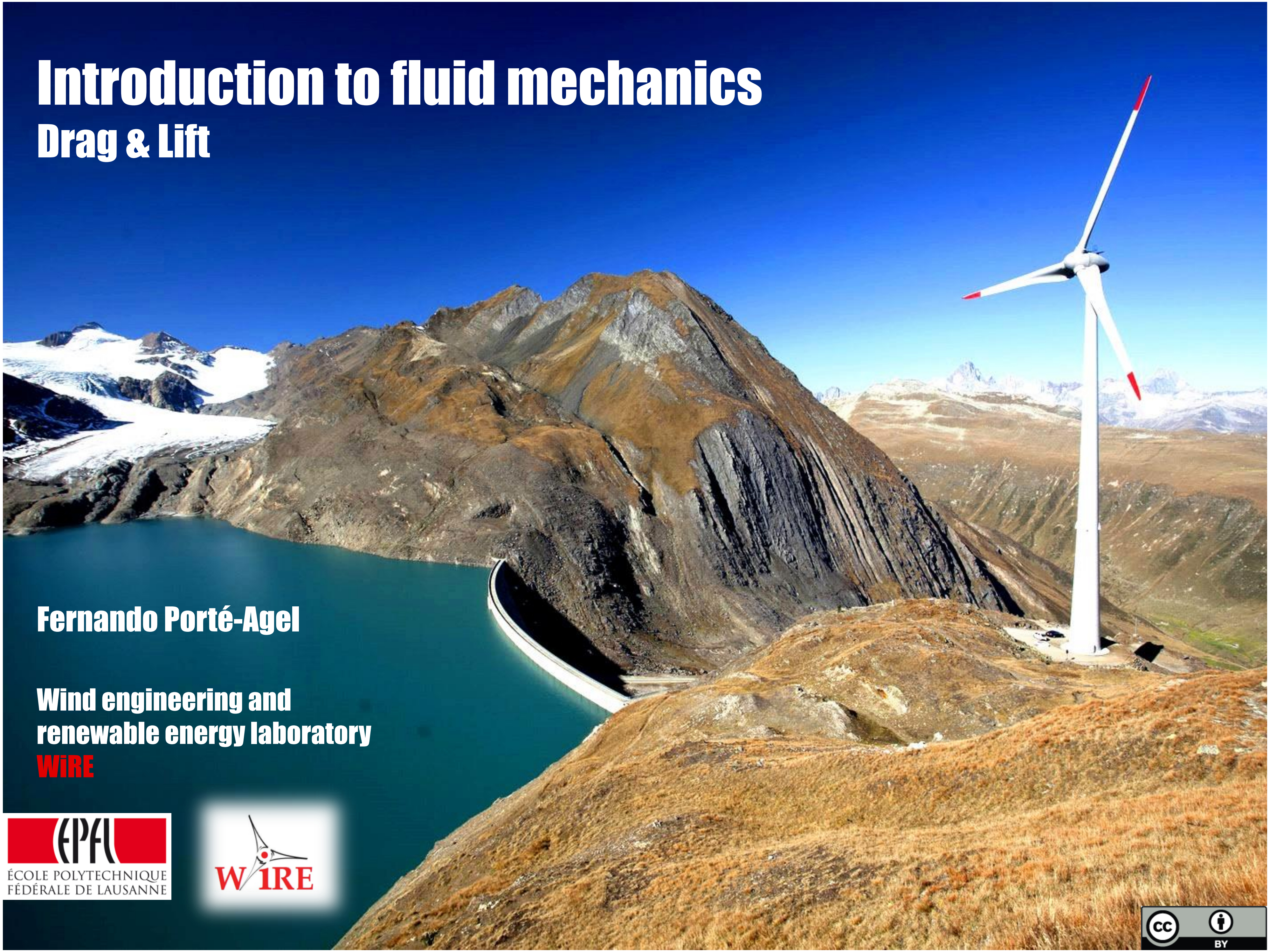
Introduction to fluid mechanics

Drag & Lift

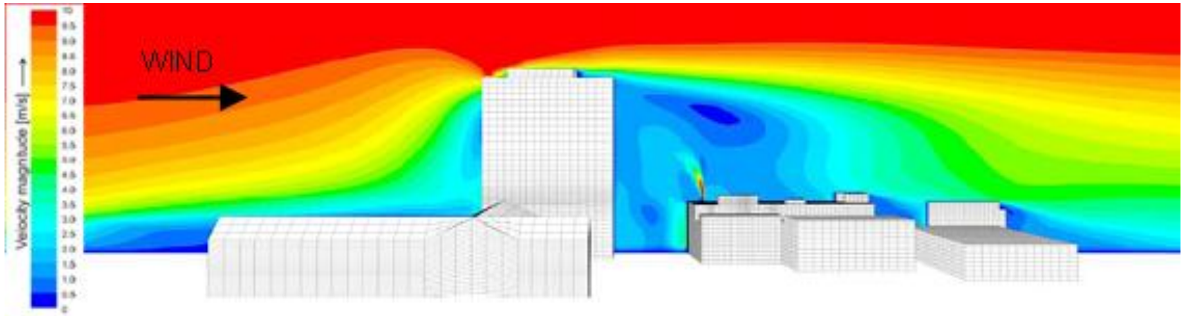


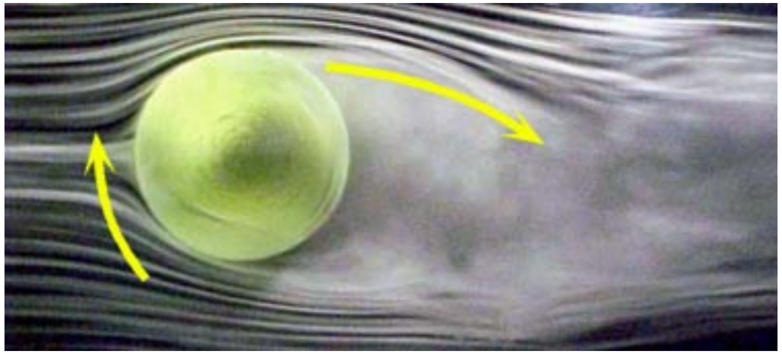
Fernando Porté-Agel

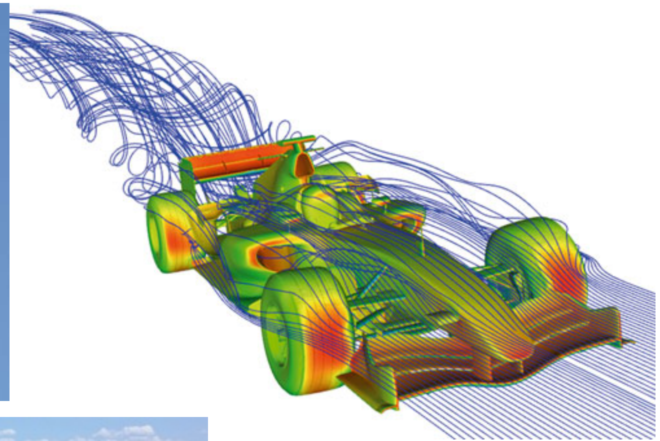
Wind engineering and renewable energy laboratory
WiRE

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Introduction



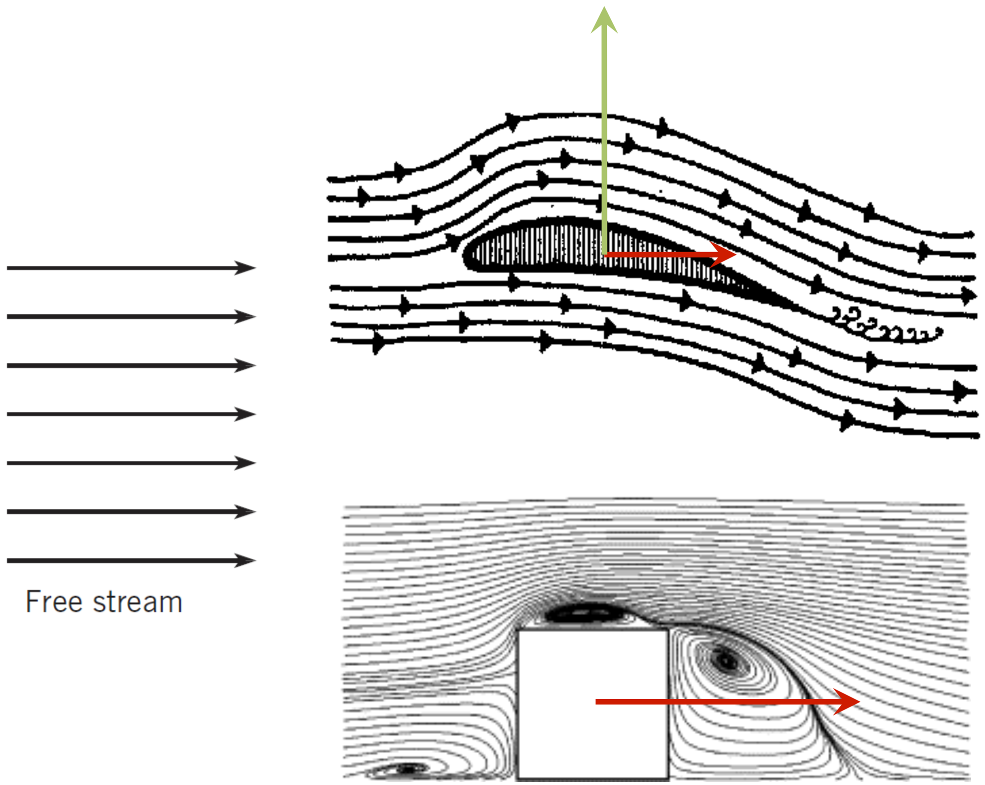




Introduction

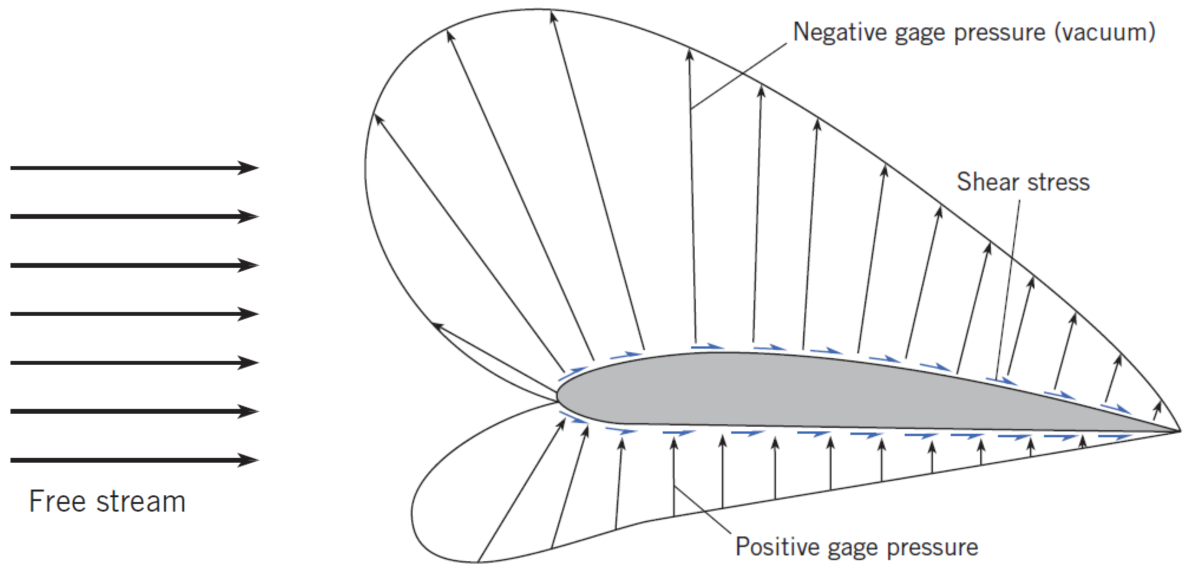
- Drag: force parallel to the free-stream velocity
- Lift: force perpendicular to the free-stream velocity

> - All bodies immersed in a viscous flowing fluid will be subjected to a **drag force**
> - Airfoils are 2D shapes that are aerodynamically efficient, this is, they are able to generate **high lift forces** with respect to the inevitable drag forces



Drag, lift, pressure and shear

- The lift and drag are the result of pressure forces and viscous forces acting on the surface of a body
  - Pressure forces are normal to the surface
  - Viscous shear forces are tangential to the surface
  - Integrating them over the total surface and projecting them with respect to the free-stream yields the resultant drag and lift



Drag, lift, pressure and shear

- Differential lift force:

$$dF_L = -p\ dA\sin\theta - \tau\ dA\cos\theta$$

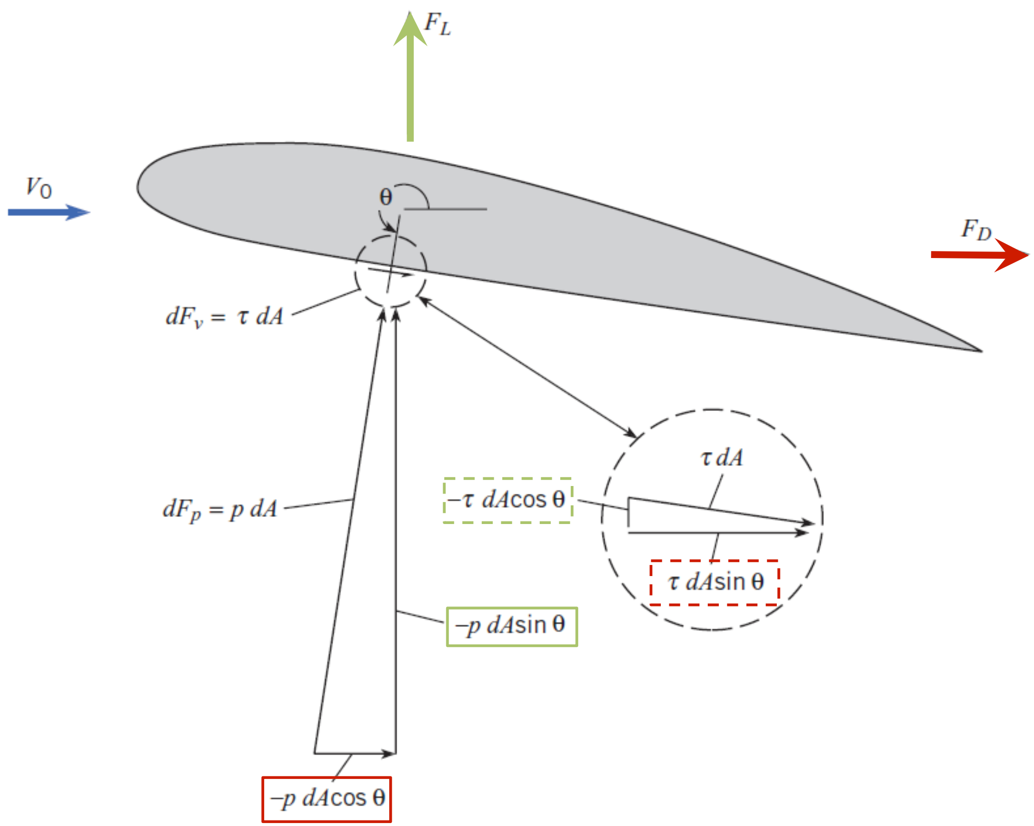
- Differential drag force:

$$dF_D = -p\ dA\cos\theta + \tau\ dA\sin\theta$$

- Integrating:

$$F_L = \int (-p\sin\theta - \tau\cos\theta)\, dA$$

$$F_D = \int (-p\cos\theta + \tau\sin\theta)\, dA$$



- **Drag can be divided into**:

$$F_D = \underbrace{\int (-p\cos\theta)\, dA}_{\textbf{Form drag}} + \underbrace{\int (\tau\sin\theta)\, dA}_{\textbf{Friction drag}}$$

Drag coefficient

- The drag force can be estimated by:

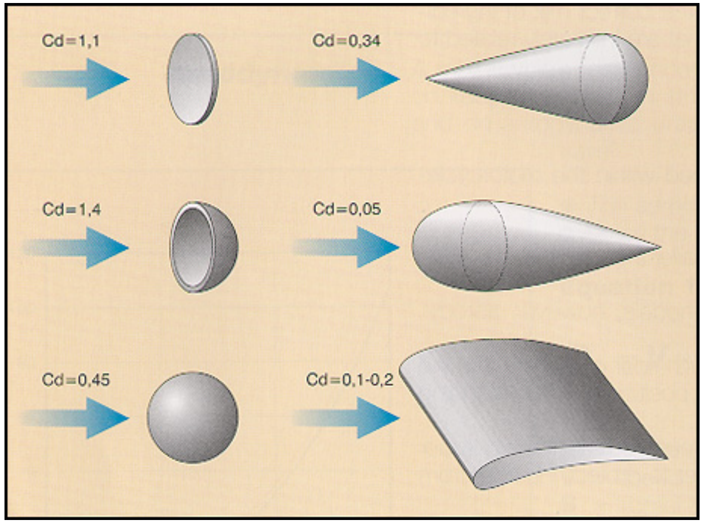
$$F_D = C_D\, A\left(\frac{\rho V_0^2}{2}\right)$$

  - Proportional to:
    - Density of the fluid ρ
    - Free-stream velocity V_o^2
    - Reference area A: projected area of the body in the direction of the free-stream
    - Drag coefficient

$$C_D \equiv \frac{F_D}{A(\rho V_0^{\,2}/2)} = \frac{\text{(drag force)}}{\text{(reference area)(kinetic pressure)}}$$

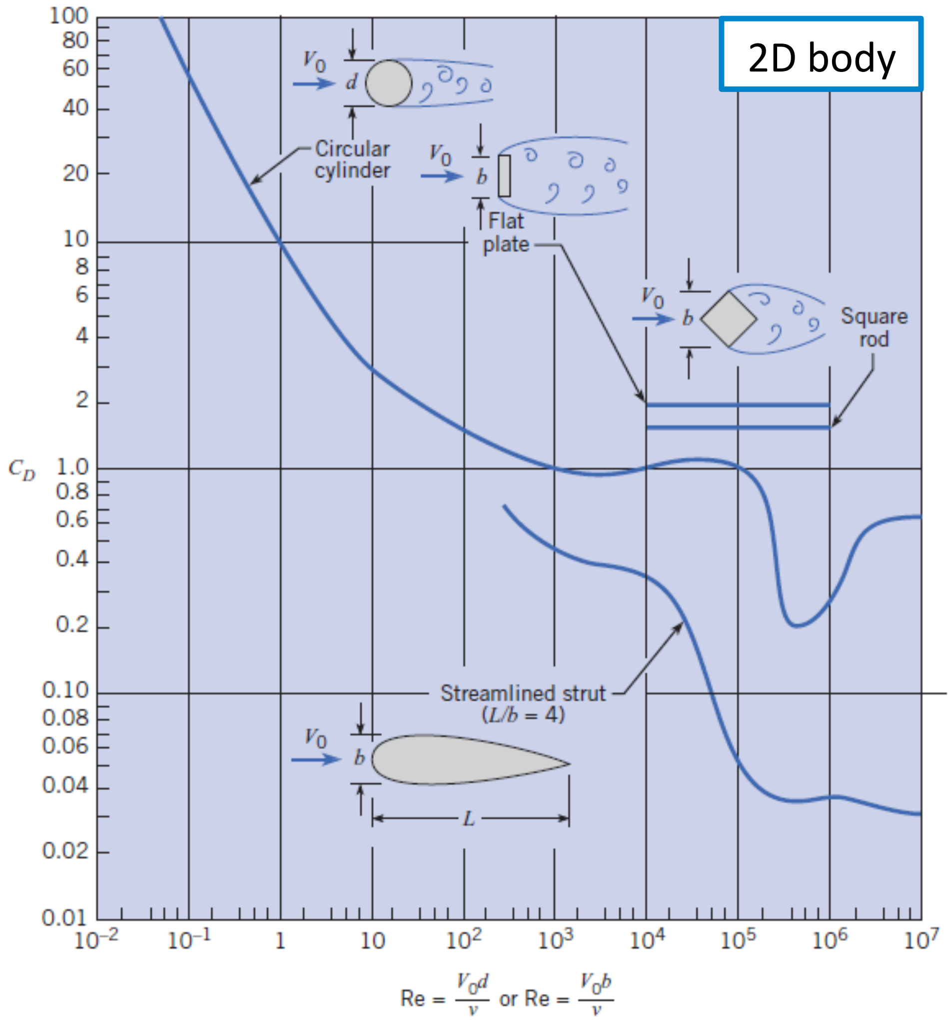
- Drag coefficient:
  - Depends on:
    - Body shape
    - Roughness
    - Reynolds number
    - Mach number

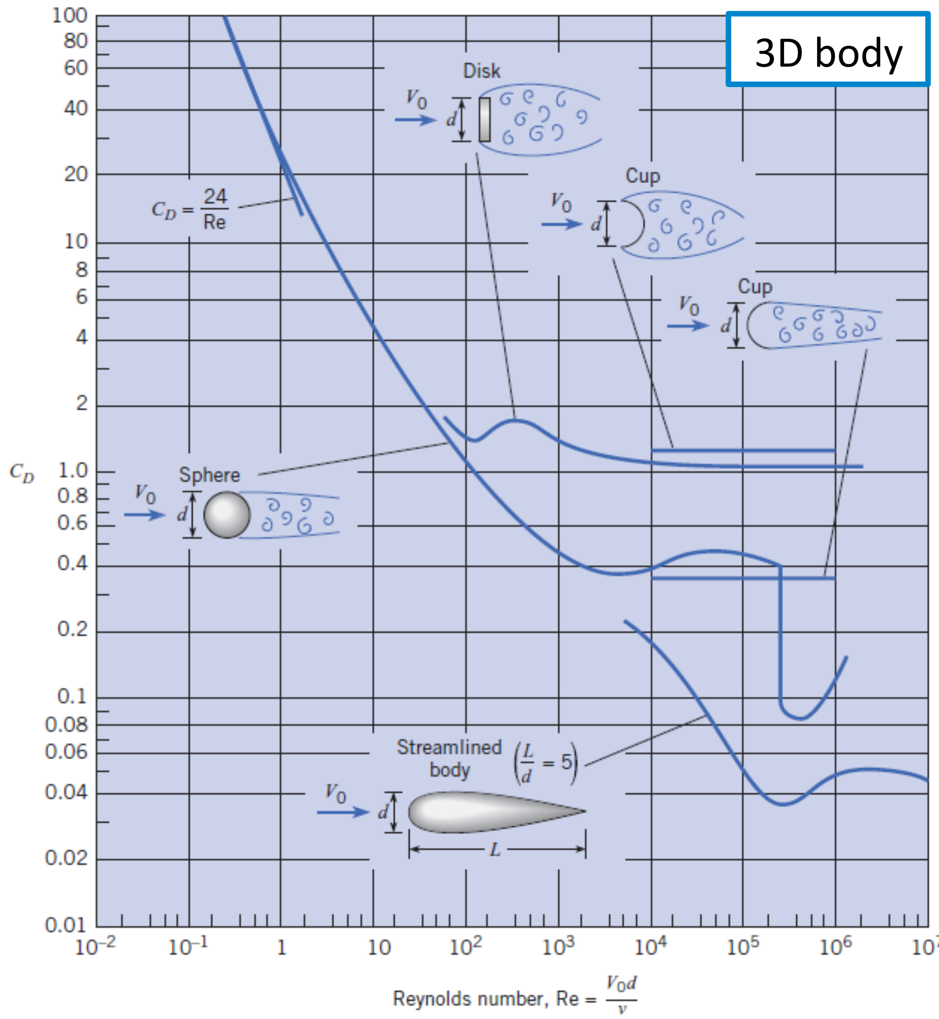
RECALL: Non-dimensional analysis!



Drag coefficient

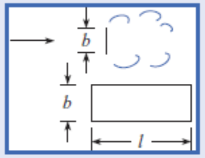
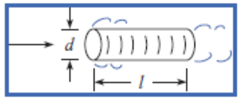
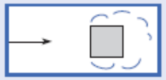
$$F_D = \boxed{C_D} A \left(\frac{\rho V_0^2}{2} \right)$$

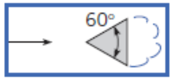
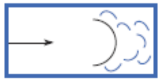
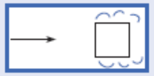




Drag coefficient

Table 11.1 APPROXIMATE C_D VALUES FOR VARIOUS BODIES

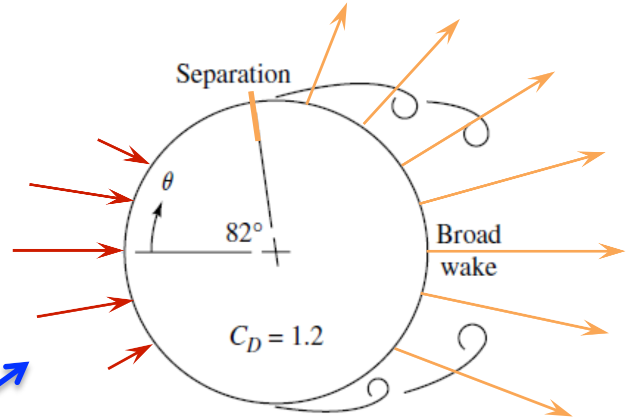
	Type of Body	Length Ratio	Re	C_D
	Rectangular plate	$l/b = 1$ $l/b = 5$ $l/b = 10$ $l/b = 20$ $l/b = \infty$	$>10^4$ $>10^4$ $>10^4$ $>10^4$ $>10^4$	1.18 1.20 1.30 1.50 1.98
	Circular cylinder with axis parallel to flow	$l/d = 0$ (disk) $l/d = 0.5$ $l/d = 1$ $l/d = 2$ $l/d = 4$ $l/d = 8$	$>10^4$ $>10^4$ $>10^4$ $>10^4$ $>10^4$ $>10^4$	1.17 1.15 0.90 0.85 0.87 0.99
	Square rod	∞	$>10^4$	2.00

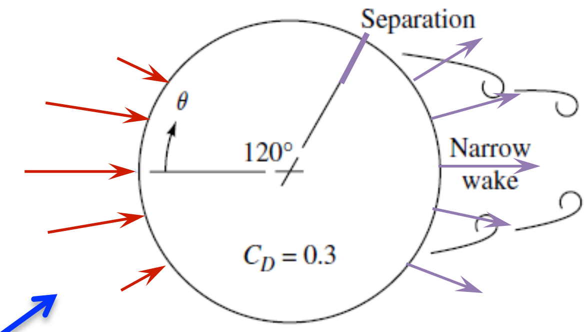
	Type of Body	Length Ratio	Re	C_D
	Square rod	∞	$>10^4$	1.50
	Triangular cylinder	∞	$>10^4$	1.39
	Semicircular shell	∞	$>10^4$	1.20
	Semicircular shell	∞	$>10^4$	2.30
	Hemispherical shell		$>10^4$	0.39
	Hemispherical shell		$>10^4$	1.40
	Cube		$>10^4$	1.10
	Cube		$>10^4$	0.81
	Cone—60° vertex		$>10^4$	0.49
	Parachute		$\approx 3 \times 10^7$	1.20

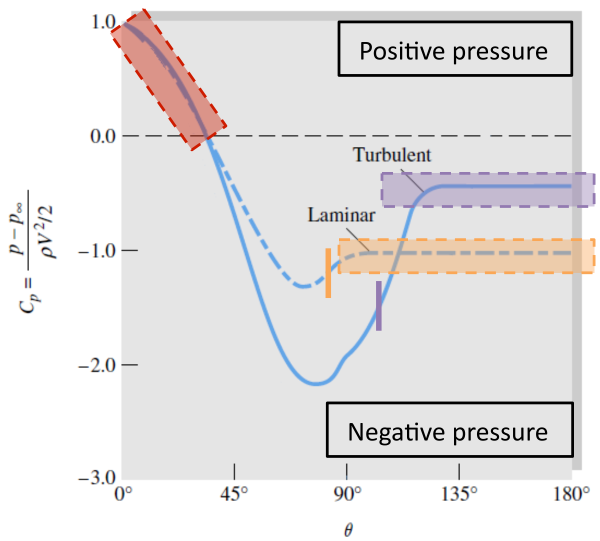
$$F_D = C_D A\left(\frac{\rho V_0^2}{2}\right)$$

Drag coefficient – effect of *Re*

- Smooth circular cylinder flow regimes:
  - Regime I: $Re < 1000$
    - Laminar BL, laminar wake
    - C_D decreases with Re
  - Regime II: $10^3 < Re < 10^5$
    - Laminar BL, turbulent wake
    - C_D constant
    - Subcritical
  - Regime III: $10^5 < Re < 5 \cdot 10^5$
    - Turbulent BL, turbulent wake
    - C_D drops abruptly
    - Supercritical

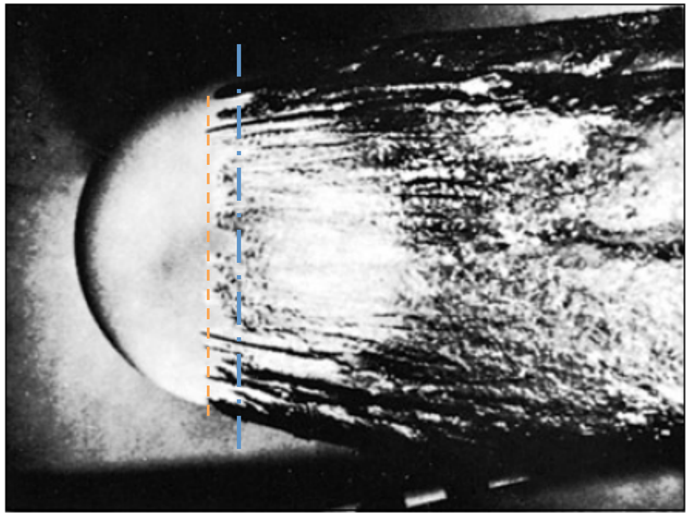




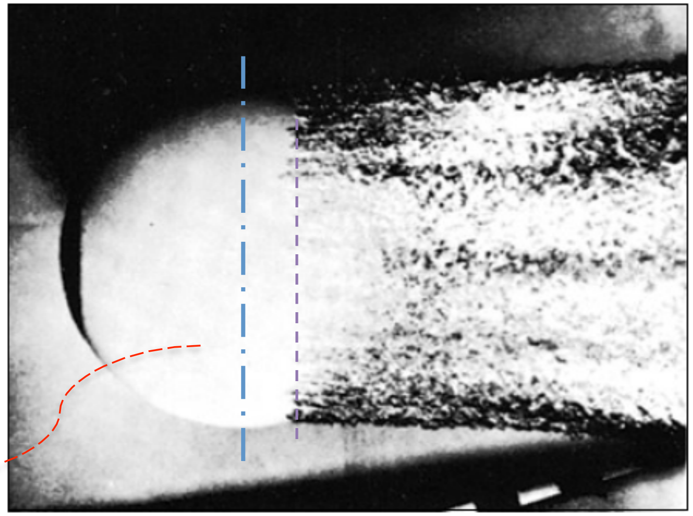


Drag coefficient – effect of *Re*

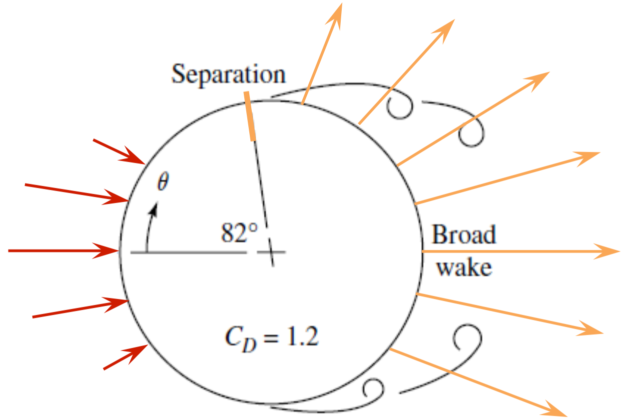
Subcritical:
- higher C_D
- broad wake

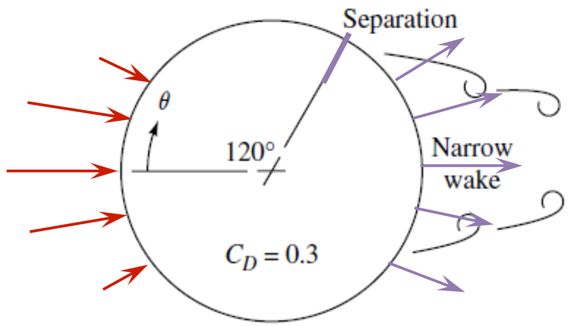


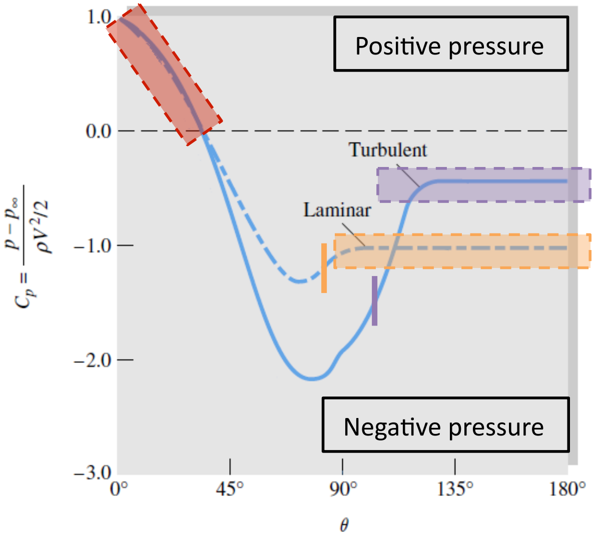
Supercritical:
- lower C_D
- narrow wake



Turbulent boundary layer around the ball
→ It remains attached to the surface longer distances (it separates further away)

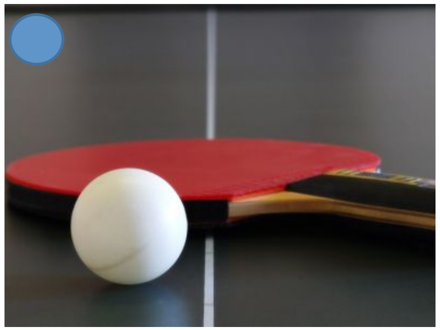


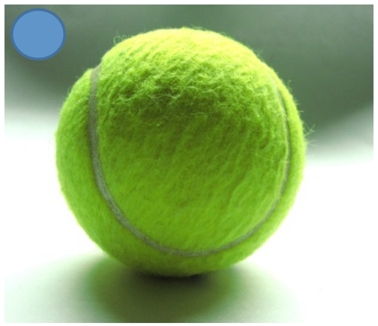


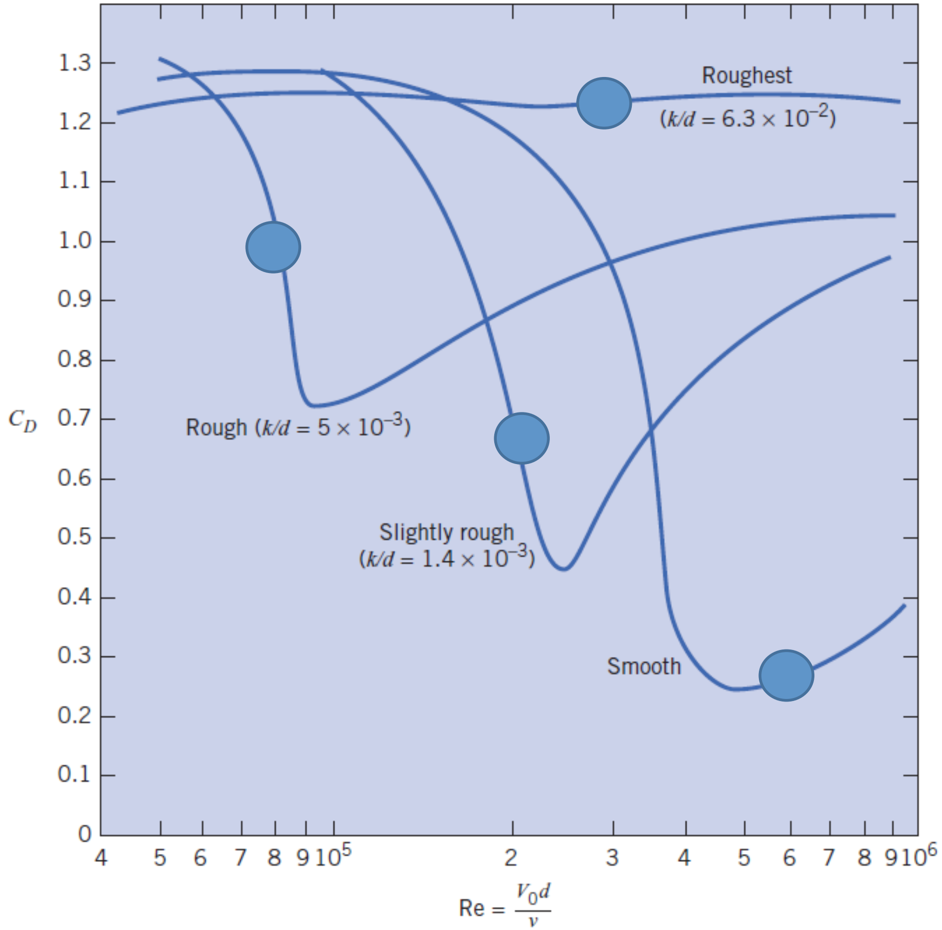


Drag coefficient – roughness effect

- Roughness induces early transition to turbulent boundary layer:
  – Earlier critical Re
  – Lower form (pressure) drag
  – Higher friction drag







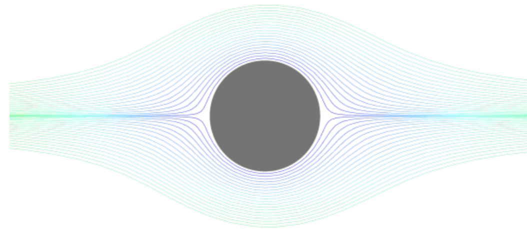
Drag coefficient – roughness effect

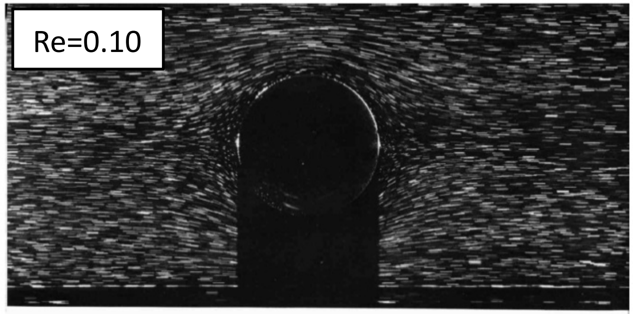
- Question: Why aren't all motorcycle helmets like golf balls?

Drag coefficient

- C_D of a sphere:
  - $Re < 0.5$ exact solution – Stokes flow

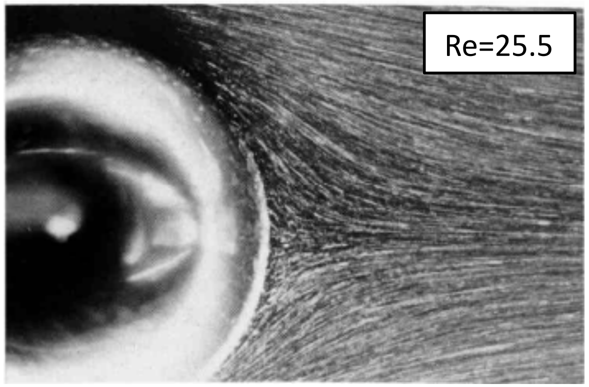
$$C_D = \frac{24}{\mathrm{Re}}$$

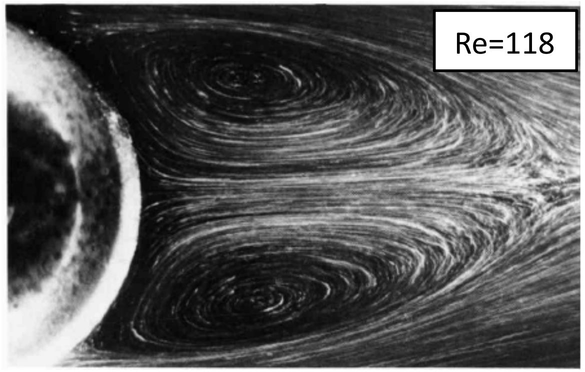


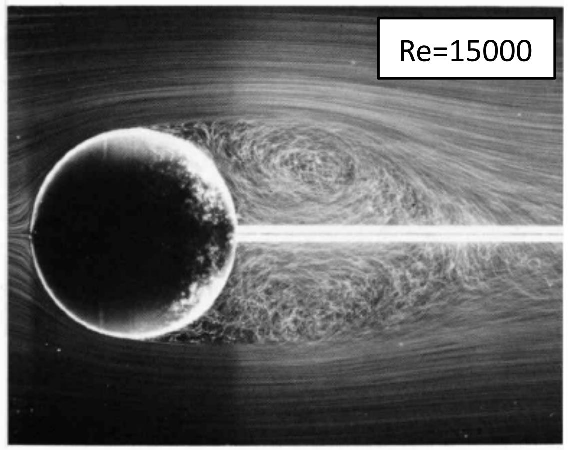


  - $0.5 < Re < 3 \cdot 10^5$ empirical correlation

$$C_D = \frac{24}{\mathrm{Re}}\left(1 + 0.15\mathrm{Re}^{0.687}\right) + \frac{0.42}{1 + 4.25 \times 10^4 \mathrm{Re}^{-1.16}}$$







Drag coefficient

- Exercise:

EXAMPLE 11.2 DRAG ON A SPHERE

What is the drag of a 12 mm sphere that drops at a rate of 8 cm/s in oil ($\mu = 10^{-1}\ \text{N}\cdot\text{s/m}^2$, $S = 0.85$)?

Problem Definition

Situation:

1. A sphere ($d = 0.012$ m) is falling in oil.
2. Speed of the sphere is $V = 0.08$ m/s.

Find: Drag force (in newtons) on the sphere.

Assumptions: Sphere is moving at a steady speed (terminal velocity).

Properties:

Oil: $\mu = 10^{-1}\ \text{N}\cdot\text{s/m}^2$, $S = 0.85$, $\rho = 850\ \text{kg/m}^3$.

Plan

1. Calculate the Reynolds number.
2. Find the coefficient of drag using Fig. 11.8.
3. Calculate drag force using Eq. (11.5).

Solution

1. Reynolds number

$$\text{Re} = \frac{Vd\rho}{\mu} = \frac{(0.08\ \text{m/s})(0.012\ \text{m})(850\ \text{kg/m}^3)}{10^{-1}\ \text{N}\cdot\text{s/m}^2} = 8.16$$

2. Coefficient of drag (from Fig. 11.8) is $C_D = 5.3$.

3. Drag force

$$F_D = \frac{C_D A_p \rho V_0^2}{2}$$

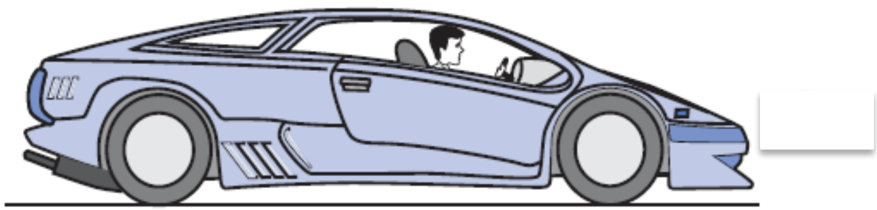
$$F_D = \frac{(5.3)(\pi/4)(0.012^2\ \text{m}^2)(850\ \text{kg/m}^3)(0.08\ \text{m/s})^2}{2}$$

$$= \boxed{1.63 \times 10^{-3}\ \text{N}}$$

Drag coefficient

- Drag and rolling resistance:
  - Equilibrium of forces:

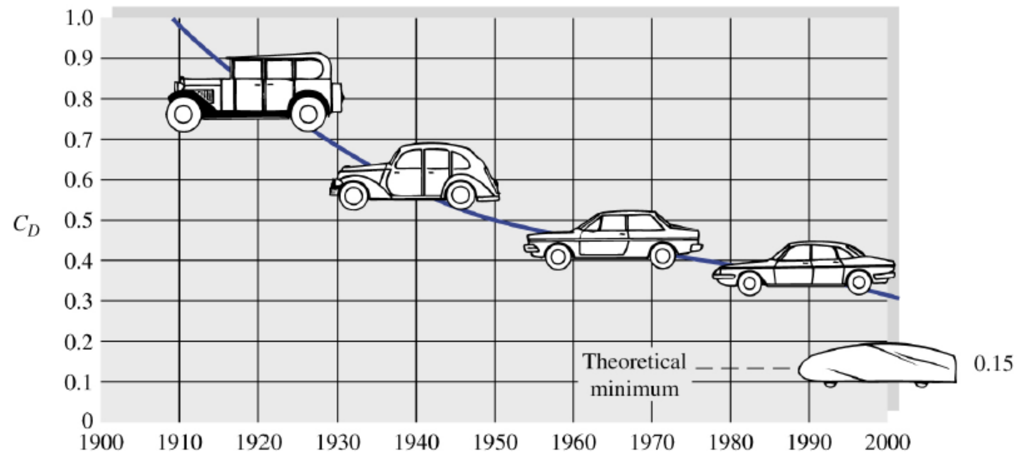
$$F_{\text{Drive}} = F_{\text{Drag}} + F_{\text{Rolling resistance}}$$



  - Rolling resistance:

$$F_{\text{Rolling resistance}} = F_r = C_r N$$

    - Cr – friction coefficient
    - N – normal force



  - Power equation:

$$P = FV = F_{\text{Drive}} V_{\text{Car}} = (F_{\text{Drag}} + F_{\text{Rolling resistance}}) V_{\text{Car}}$$

Drag coefficient

- Exercise:

EXAMPLE 11.3 SPEED OF A BICYCLE RIDER

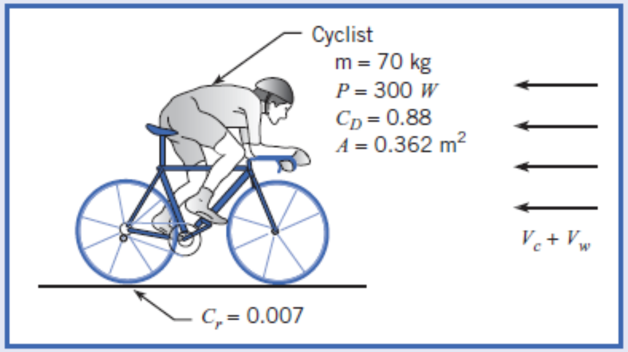
A bicyclist of mass 70 *kg* supplies 300 watts of power while riding into a 3 m/s head wind. The frontal area of the cyclist and bicycle together is 3.9 ft^2 = 0.362 m^2, the drag coefficient is 0.88, and the coefficient of rolling resistance is 0.007. Determine the speed V_c of the cyclist. Express your answer in mph and in m/s.

Problem Definition

Situation: A bicycle rider is cycling into a head wind of magnitude $V_w = 3$ m/s.

Find: Speed (m/s and mph) of the rider.

Sketch:



Assumptions:

1. The path is level, with no hills.
2. Mechanical losses in the bike gear train are zero.

Properties: Air (20°C, 1 atm), Table A.2: $\rho = 1.2$ kg/m^3.

Plan

1. Relate bike speed (V_c) to power using Eq. (11.11).
2. Calculate rolling resistance.
3. Develop an equation for drag force using Eq. (11.5).
4. Combine steps 1 to 3.
5. Solve for V_c.

Solution

1. Power equation
   - The power from the bike rider is being used to overcome drag and rolling resistance. Thus,

$$P = (F_D + F_r)V_c$$

2. Rolling resistance

$$F_r = C_r N = C_r mg = 0.007(70\text{ kg})(9.81\text{ m/s}^2) = 4.81\text{ N}$$

3. Drag force
   - V_0 = speed of the air relative to the bike rider

$$V_0 = V_c + 3\text{ m/s}$$

   - Drag force

$$F_D = C_D A\left(\frac{\rho V_0^2}{2}\right) = \frac{0.88(0.362\text{ m}^2)(1.2\text{ kg/m}^3)}{2} \times (V_c + 3\text{ m/s})^2$$

$$= 0.1911(V_c + 3\text{ m/s})^2$$

4. Combine results:

$$P = (F_D + F_r)V_c$$

$$300\text{ W} = (0.1911(V_c + 3)^2 + 4.81)V_c$$

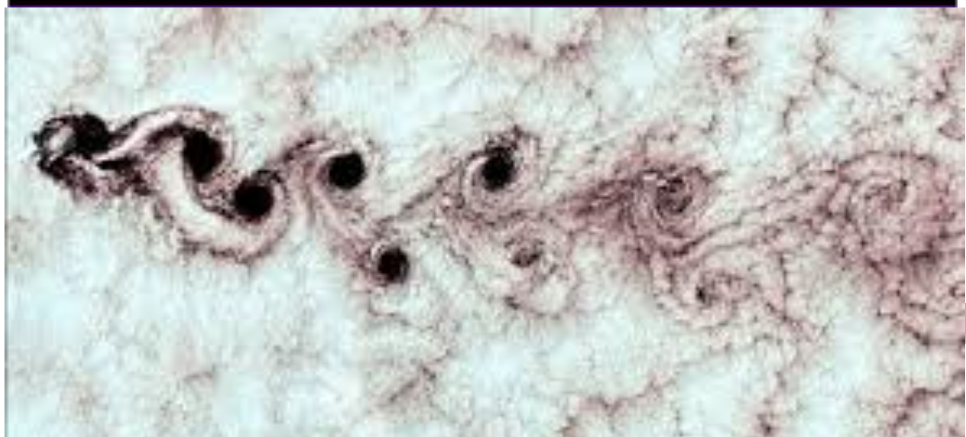
5. Since the equation is cubic, use a spreadsheet program as shown. In this spreadsheet, let V_c vary and then search for the value of V_c that causes the right side of the equation to equal 300. The result is

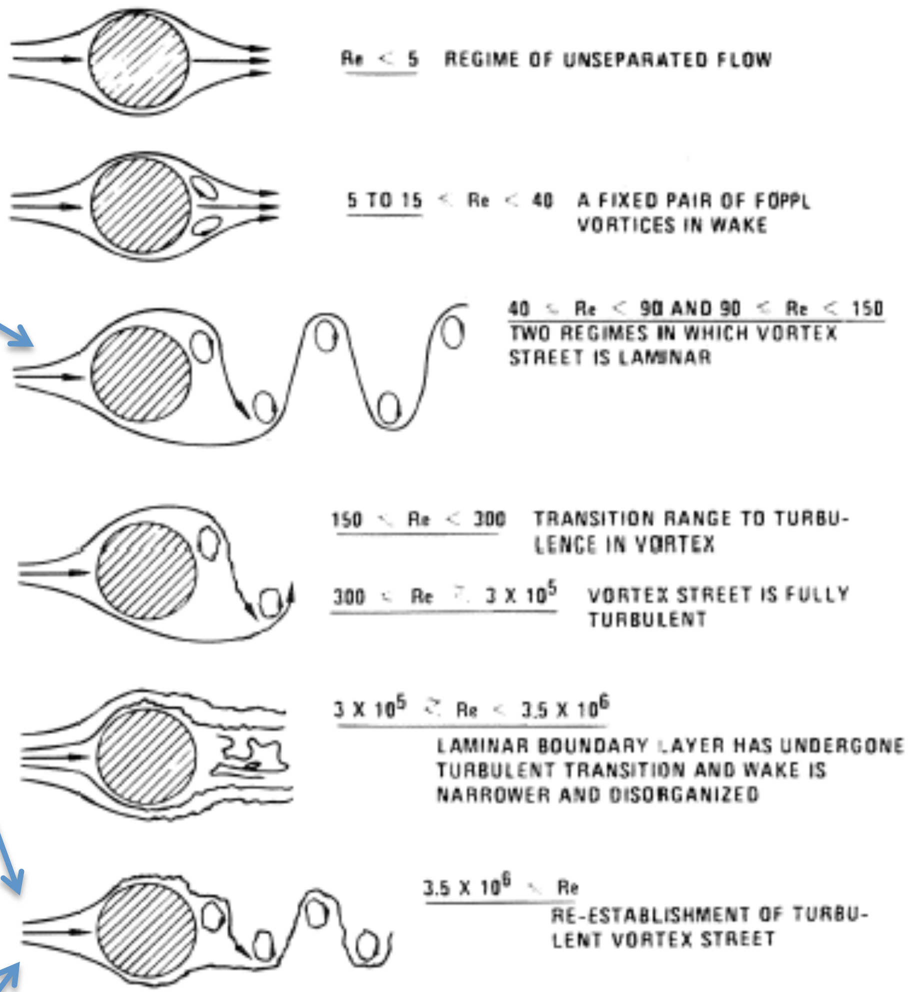
$$V_c = \boxed{9.12\text{ m/s} = 20.4\text{ mph}}$$

V_c	RHS
(m/s)	(W)
0	0.0
5	85.2
8	223.5
9	291.0
9.1	298.4
9.11	299.1
9.12	299.9
9.13	300.6

Vortex shedding (von Karman vortex street)

- Flow past a cylinder shows an oscillatory behavior at certain Reynolds number ranges in the manner of alternative detaching vortices.
- The flow can be indistinctly laminar or turbulent.





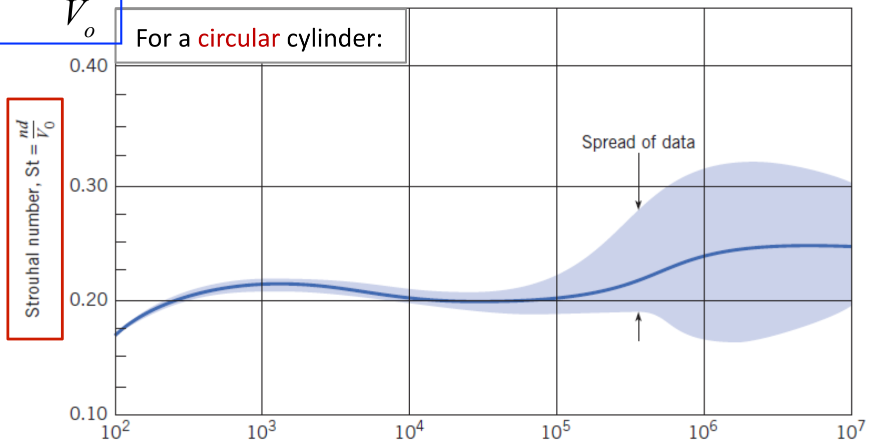
Vortex shedding

- Flow past a cylinder shows an oscillatory behavior at certain Reynolds ranges in the manner of alternative detaching vortices.
- The flow can be indistinctly laminar or turbulent.
- Strouhal number:

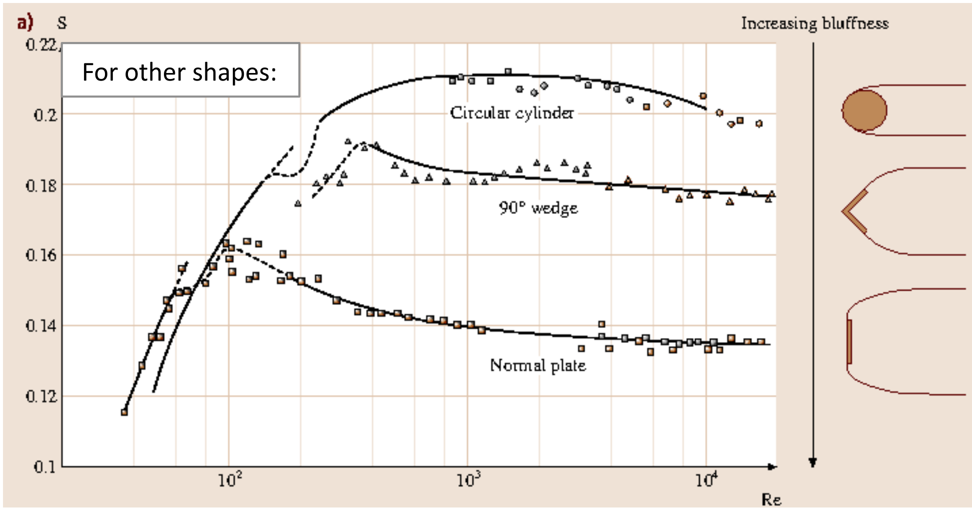
frequency (n) – length (d) – velocity (V_0)

$$St \equiv \frac{n \cdot d}{V_o}$$

$$St = \frac{n \cdot d}{V_o}$$



$$\mathrm{Re} = \frac{V_o \cdot d}{\nu}$$



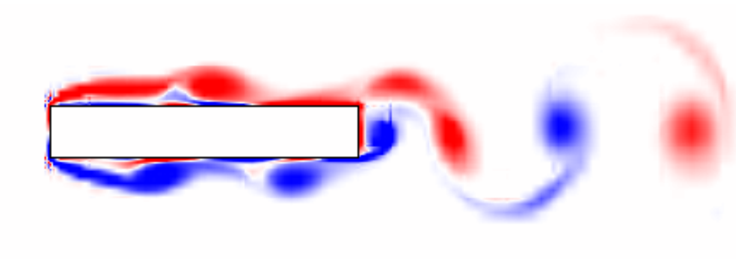
$$\mathrm{Re} = \frac{V_o \cdot d}{\nu}$$

Vortex shedding

- It causes cyclic changes in pressure on the 'cylinder' and therefore induces **vibrations** **if the structure is not completely stiff**.

- Example:

  Rectangular cylinder,
  Tacoma Narrows bridge,
  collapsed in 1940

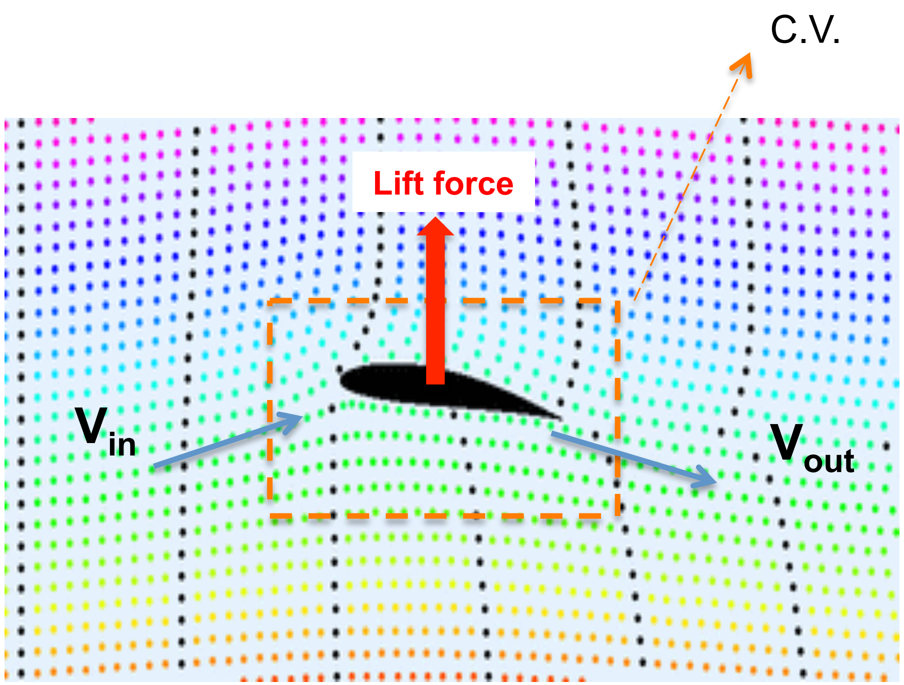


Lift force

Why is lift generated in an airfoil?

Two consistent explanations based on conservation equations:

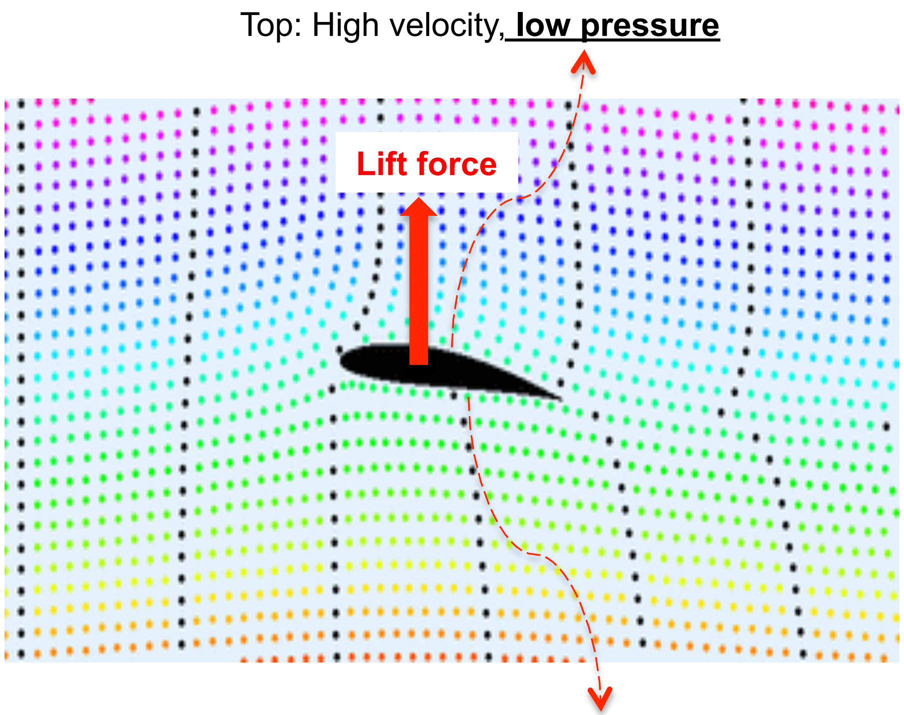
Momentum conservation equation:



Change of momentum in the vertical direction because the **airfoil deflects the air downward**

➔ **Upward force (LIFT)**

Bernoulli equation:

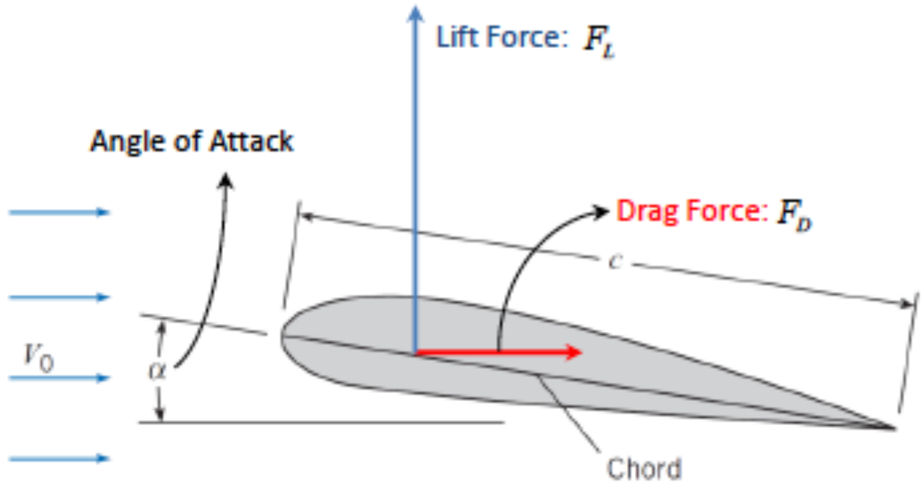


Larger pressure at bottom ➔ Upward force (**LIFT**)

Lift Coefficient

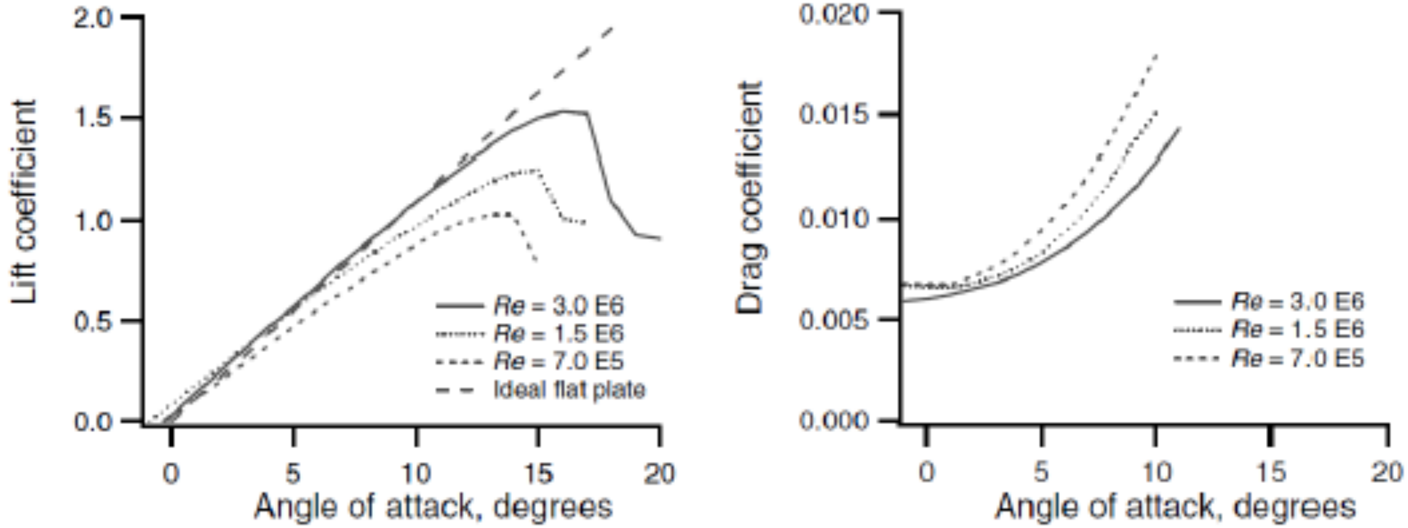
$$C_L \equiv \frac{F_L}{A\left(\rho V_0^2 / 2\right)}$$

$$F_L = C_L A\left(\frac{\rho V_0^2}{2}\right)$$



- C_L and C_D for airfoils are functions of:

1. **Airfoil type (geometry)**
2. **Reynolds number**
3. **Angle of attack**

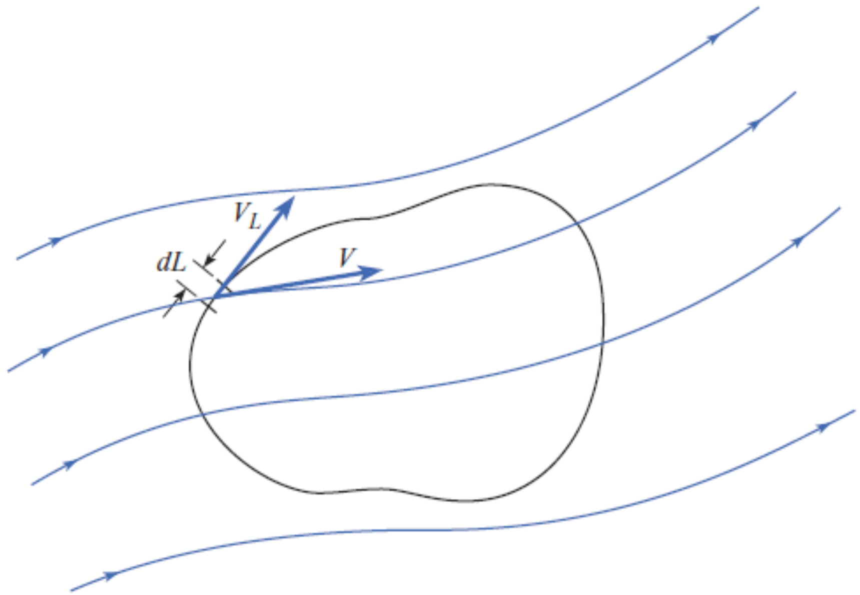


Lift and drag coefficients for the NACA 0012 symmetric airfoil

Circulation

- Average rate of rotation of fluid particles in an area bounded by a closed curve.
- It is mathematically defined by the following path integral:

$$\Gamma = \oint V_L \, dL$$

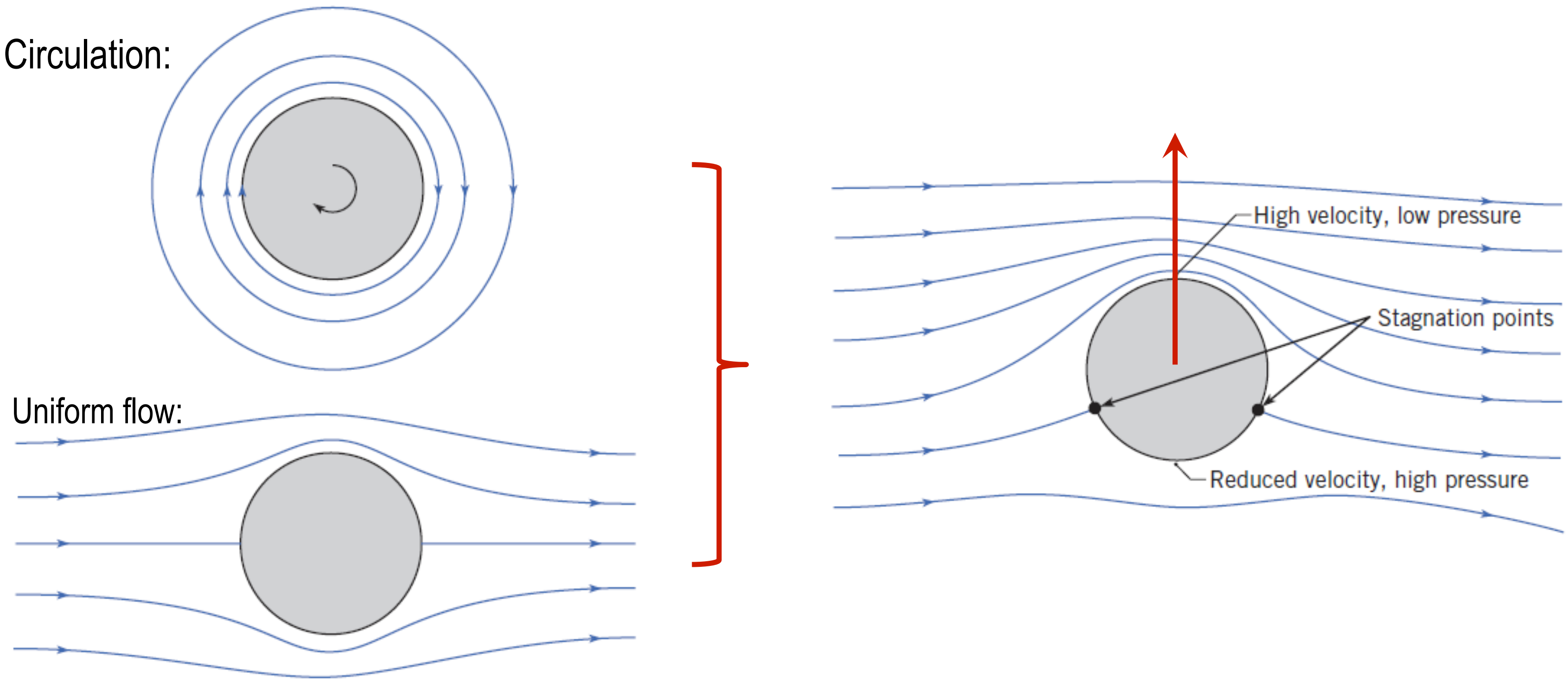


- Sign convention:
  - Clockwise – positive
  - Counterclockwise – negative

* Opposite to the mathematical definition of line integral

Circulation

- **Circulation around a body creates a force perpendicular to the incoming flow: lift.**
- One of the simplest ways to create circulation is rotating a sphere or a cylinder around its axis
- Note: also circulation (and therefore lift) is not zero around an airfoil with an angle of attack.



Circulation

- The spin of a moving sphere produces a lift and therefore it deflects its trajectory from straight to curve.
- It is called Magnus effect and it is commonly used in ball sports.