Linear System Theory

Lecture 4

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Outline

1. Recap: [redacted]
2. Algorithm: [redacted]
3. 3.1 [redacted] decoupling zeros
   3.2. [redacted] " "
   3.3 [redacted] decoupling zeros
   3.4 [redacted]
   3.5 [redacted]
4. Examples.

1. Recap system description [redacted]

Polynomial [redacted]

$$P(s) = \left[\text{[redacted]} \right] \qquad P\left[\text{[redacted]} \right] = \left[\text{[redacted]} \right]$$

in state-space description $T(s) =$ [redacted] with $s = \lambda$

and considering only $T(s)$ we can "compute" the invariant polynomials of $T(s)$.

Strict system equivalence

Equivalence throug

are square matrices nonsingular

whose inverse

$| \quad | \in$ (polynomial)

2. Smith Form

By (resp.)

an $n \times n$ polynomial matrix $A(s)$

can be brought ( )

<u>Definition</u>: Elementary unimodular matrices

$T_1 = \begin{bmatrix} [redacted] \end{bmatrix}$ [redacted] $\in$ [redacted]

$\in$ [redacted]

$$T_2 = \begin{bmatrix} 1 & & & & & & & \\ & 1 & & & & & 0 & \\ & & 1 & & & & & \\ & & & 1 & & & & \\ & & & & 1 & & & \\ & & & & & \ddots & & \\ & & & & & & 1 & \\ & 0 & & & & & 1 & 1 \end{bmatrix} = \text{[redacted]}$$

$$T_3 = \begin{bmatrix} 1 & & & 0 \\ 0 & \ddots & & \\ & & \ddots & \\ 0 & & 0 & 1 \end{bmatrix}$$

single entry where [redacted] appears

otherwise identity matrix

[redacted]

([redacted]).

$S_1 =$ [redacted], $S_2 =$ [redacted] $S_3 =$ [redacted]

Smith decomposition:

$$P(s) = \begin{bmatrix} P_{11}(s) \cdots & & \\ \vdots & & \\ \vdots & & \\ & & P_{nn}(s) \end{bmatrix} \xrightarrow{\text{Smith Form}} \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

Algorithm:

STEP1: Find the polynomial in s

in the 1^{st} column

$\leq$

STEP2: Bring the

$(\quad) (\quad)$

STEP3: Use on the

elements of the 1^{st} column

them with

$= \quad +$

$\leq$

Use $T_3 = \begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix}$

Repeat STEPS 1-2-3 until the first column looks like

$\begin{bmatrix} c_{11} & \\ & \end{bmatrix}$ [redacted]

<u>STEP 4</u> do STEPS 1-2-3 [redacted] ______________

using matrices [redacted]

until [[redacted]]

<u>STEP 5</u> do steps 1 to 4 with [redacted]

→ until [[redacted]] is obtained.

Definition the resulting ( )

is called the

of the original polynomial matrix.

$$P = \begin{bmatrix} T & U \\ -V & W \end{bmatrix}$$

3. <u>decoupling zeros, zeros, poles, i-d. z., o-d. z., i-o-d. z.</u>

3.1 consider the matrix

( )

3.1. i-d. zeros

input decoupling = i-d.

and compute its Smith Form using the algorithm.

collect all the zeros of the diagonal entries in the Smith Form.

βs are the

3.2 <u>output-decoupling zeros</u> (o-d. z.)

Consider ( ) in form

Perform on ( ) a Smith decomposition using the algorithm to achieve

( )

and compute the zeros of every to get the set of output-decoupling zeros :

3.3 <u>input-output decoupling zeros</u> (i.-o.-d. z.)

Remove the { } from

to get

i.e. apply the unimodular matrices obtained to bring the ( ) to Smith form by inverting them

extend

([redacted]) is therefore obtained

⟼ compute the Smith Form of ([redacted])

and compute the zeros of the diagonal entries of the resulting Smith-Form.

$\longrightarrow$ { [redacted] }

Remove the { [redacted] } from the { [redacted] }

to find the input-output decoupling zeros. (i-o-d z.).

{ [redacted] } = { [redacted] } − { [redacted] }

3.4 Zeros

Simply compute the Smith-Form of [redacted]

Let the zeros of the diagonal entries of the Smith form be { [redacted] }

3.5 Poles

$\{\text{[redacted]}\} = \{\text{[redacted]}\} - \{\{\text{[redacted]}, \text{[redacted]}\} - \{\text{[redacted]}\}\}$

$\{\text{[redacted]}\}$ i.d.

$\{\text{[redacted]}\}$ o.d. $\{\text{[redacted]}\}$ zeros

$\{\text{[redacted]}\}$ i.o.d $\{\text{[redacted]}\}$ poles of $G(s)$.

$P(s) =$ [redacted]

$\{\eta_i\} = \{\text{[redacted]}, \text{[redacted]}, \text{[redacted]}, \text{[redacted]}\}$

$\{\beta_i\} = \{\text{[redacted]}\}$ $\{\delta_i\} = \{\text{[redacted]}\}$

$\{\gamma_i\} = \{\text{[redacted]}, \text{[redacted]}, \text{[redacted]}\}$ $\{\alpha_i\} = \{\text{[redacted]}\}$.

$\{\theta_i\} = \{\text{[redacted]}, \text{[redacted]}\}$