EE-585 – Space Mission Design and Operations

Dr Thibault Kuntzer

Ecole Polytechnique Fédérale de Lausanne

Week 09 – 15 Nov 2024

Today's outline

Planetary flybys and slingshots

Lunar trajectories I: getting to the Moon

Lunar trajectories II: orbiting the Moon

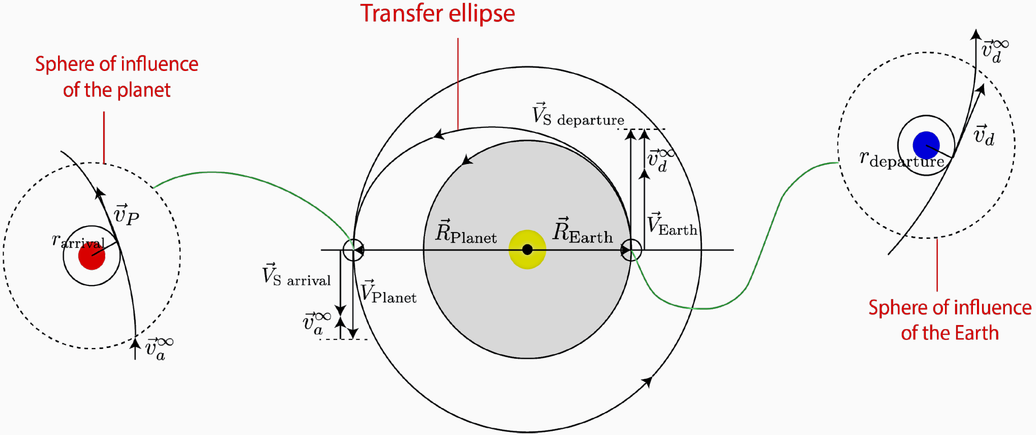
Lunar trajectories III: returning to the Earth

Spacecraft propulsion

Non-impulsive manoeuvres

Planetary flybys and slingshots

Reminder: strategy for interplanetary transfer

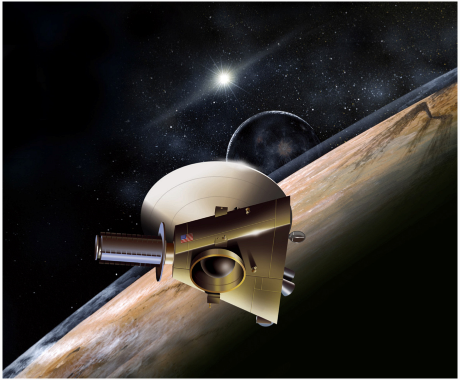


Gravity assist – a.k.a. slingshot – of New Horizons

Launched on 16 Jan 2006.
Gravity assist by Jupiter on 28 Feb 2007.
Pluto flyby on 14 July 2015.

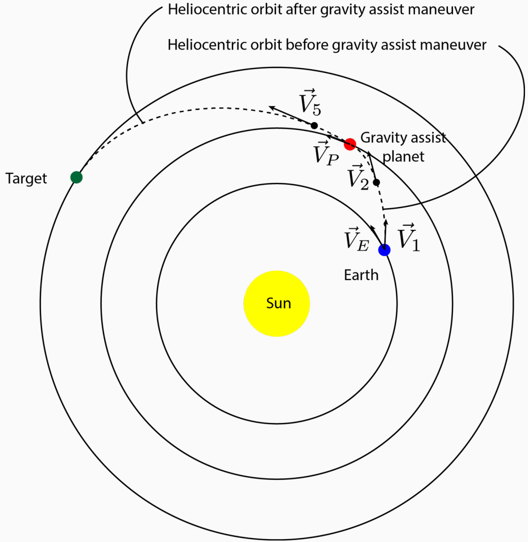
When a spacecraft on an planetocentric hyperbolic trajectory approaches a planet, it comes within its sphere of influence (SOI), passes its hyperbolic periapsis and exits the SOI, the gravity of that planet alters its heliocentric velocity $\vec{V}$ in amplitude $|\vec{V}|$ and direction $\vec{V}$.

The amount by which the spacecraft speeds up or slows down is determined by the geometry of the approach, passing behind or in front of the planet.



Credits: Credits: NASA, Southwest Research Institute, Johns Hopkins University Applied Physics Laboratory

Definitions



where:

$\vec{V}_P$: Heliocentric velocity of the gravity-assist-planet.

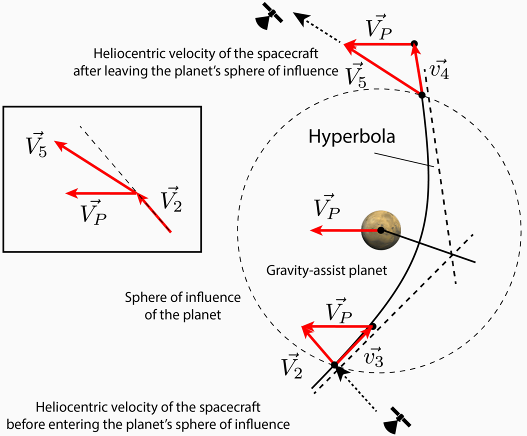
$\vec{V}_E = \vec{V}_\oplus$: Heliocentric velocity of Earth.

$\vec{V}_1$: Heliocentric velocity of the spacecraft after leaving Earth.

$\vec{V}_2$: Heliocentric velocity of the spacecraft entering the gravity-assist planet's sphere of influence.

$\vec{V}_5$: Heliocentric velocity of the spacecraft leaving the gravity-assist planet's sphere of influence.

Slingshot manoeuvre profile



$\vec{V}_2$: Heliocentric velocity of the spacecraft entering the planet's sphere of influence (SOI).

$\vec{V}_P$: Heliocentric velocity of the planet.

$\vec{v}_3 = \vec{v}_a^{\infty}$: Planetocentric velocity of the spacecraft entering the planet's SOI.

$\vec{v}_4 = \vec{v}_d^{\infty}$: Planetocentric velocity of the spacecraft leaving the planet's SOI.

$\vec{V}_5$: Heliocentric velocity of the spacecraft leaving the gravity-assist planet's SOI.

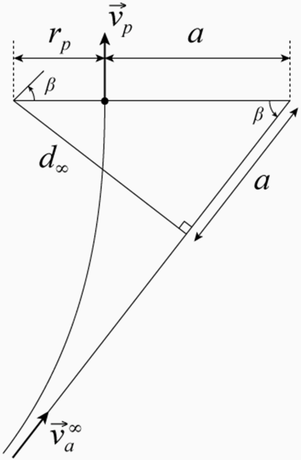
with

$$|\vec{v}_a^{\infty}| = |\vec{v}_3| = |\vec{v}_4| = |\vec{v}_d^{\infty}|$$

and, in this example,

$$|\vec{V}_5| = |\vec{V}_P + \vec{v}_4| > |\vec{V}_P + \vec{v}_3| = |\vec{V}_2|$$

Reminder: determination of important parameters



Conservation of total energy $\rightarrow v_p^2 = (v_a^\infty)^2 + v_{Er_p}$

As $a \cong \frac{\mu}{(v_a^\infty)^2}$

$a^2 + b^2 = c^2 = (a + r_p)^2$

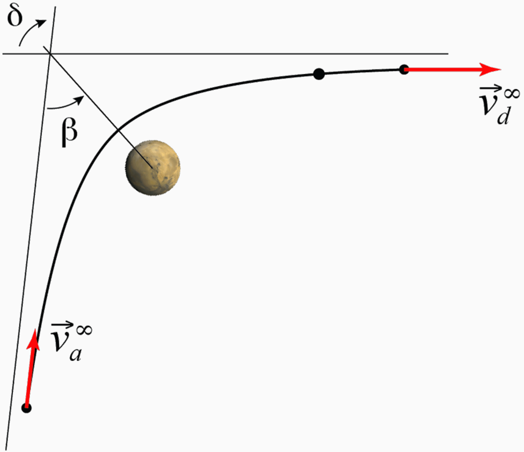
and $b = d_\infty$,

$$r_p = -\frac{\mu}{(v_a^\infty)^2} + \sqrt{\frac{\mu^2}{(v_a^\infty)^4} + d_\infty^2}$$

and

$$\cos\beta = \frac{a}{a + r_p} = \frac{a}{c}$$

Slingshot manoeuvre profile



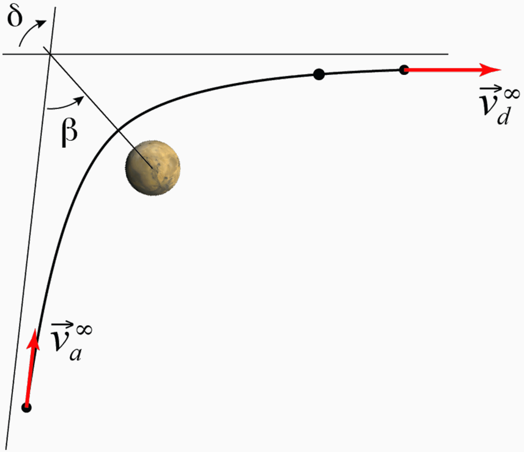
In the vicinity of a planet, the trajectory of a spacecraft is hyperbolic with a peri-apsis at distance r_p to the center of the planet.

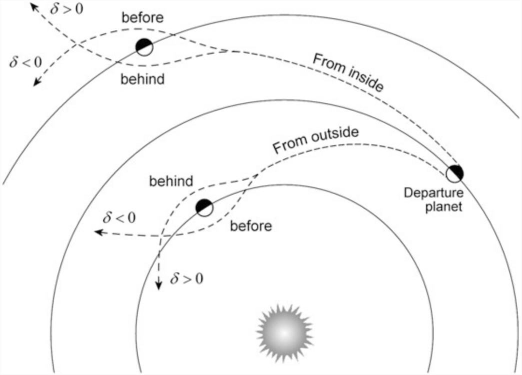
δ is the angle between the directions of $\vec{v}_a^\infty$ and $\vec{v}_d^\infty$.

$$\cos\left(\frac{\delta}{2}\right) = \cos\beta = \frac{a}{a + r_p}$$

$\rightarrow$ A slingshot manoeuvre – aka gravity assit – can be designed to increase or decrease the satellite's heliocentric velocity. You have to play with mainly arrival angle β and the angle between $\vec{V}_P$ and $\vec{v}_a^\infty$ to design the manoeuvre.

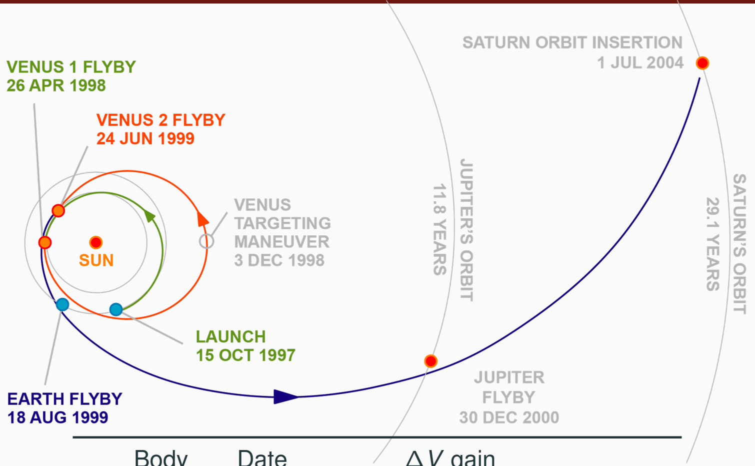
Slingshot type depends on the angle δ





Credits: U. Walter, *Astronautics, The Physics of Space Flight*, 3rd Ed.

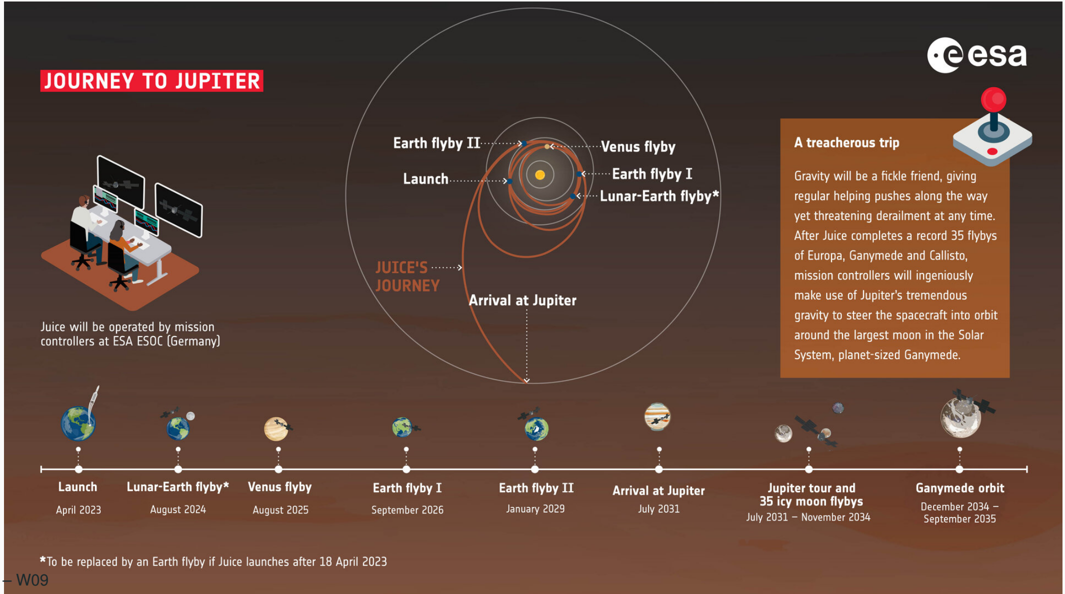
Case study: Cassini-Huygens (4 gravity assists)



	Body	Date	ΔV gain
1	Venus	26 Apr 1998	7 km/s
2	Venus	24 Jun 1999	sent C.-H. towards Earth
3	Earth	18 Aug 1999	5.5 km/s
4	Jupiter	30 Dec 2000	2 km/s

Credits: Credits: PD, USGOV

ESA's Juice: 8 years to destination with 4 gravity assists



ESA's Juice: August 2024 Moon-Earth flyby

Credits: ESA/Juice/JMC

The flyby of the Moon increased the heliocentric speed by 0.9 km/s, guiding it towards Earth.

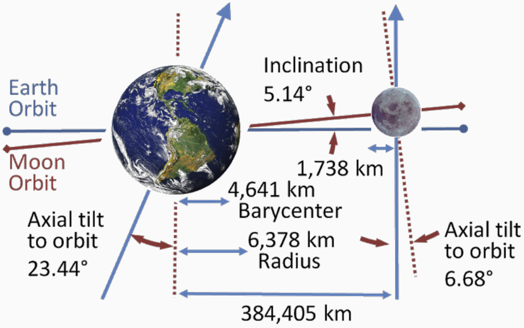
The flyby of Earth reduced the heliocentric speed by 4.8 km/s, guiding Juice onto a new trajectory towards Venus.

Overall, the lunar-Earth flyby deflected Juice by an angle of 100° compared to its pre-flyby path.

In the month before the flyby, spacecraft operators gave Juice slight nudges to put it on exactly the right approach trajectory. Overall, flybys will have saved 100-150 kg of fuel.

Lunar trajectories I: getting to the Moon

The Earth-Moon system



Credits: SeriousScience.org/NASA

The ⊕-☽ barycenter orbits the Sun in 1 year (by definition).

⊕-☽ system revolve around the center of mass in 27.31 days, with up to 7 hours variations because of solar perturbations.

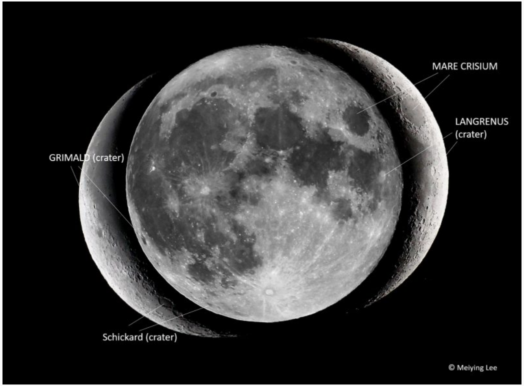
$d_{⊕☽} \sim 384,400\,\text{km} \sim 40\%$ of Earth's Sphere of influence

Eccentricity $e \approx 0.055$. During a lunar month, the distance to ⊕ varies by $\sim 7.5 R_{⊕}$

Inclination of Moon's orbit $i \approx 4.98 - 5.3°$, on average $5.14°$

The inclination w.r.t. Earth's equator varies between $23.44 \pm 5.14 = 18.3° - 28.6°$.

Lunar libration



The Moon period of revolution around the Earth is the same as period of rotation around its axis $\rightarrow$ always same face turned towards Earth.

We can see $\approx$ 59% of the Moon's surface $\rightarrow$ "lunar libration". Three causes:

- Inclination of the orbit
- Eccentricity of the orbit
- Non-sphericality of the bodies

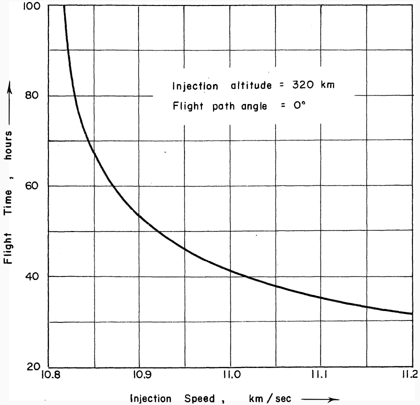
Simple Earth-Moon trajectories

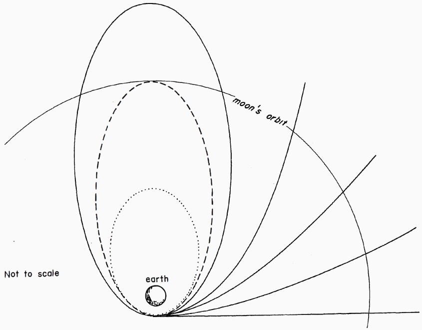
We could take all the perturbations into account and simulate a trajectory. Because of the Moon's motion, plans would heavily depend on the time of departure $\rightarrow$ computationally intensive.

Let's make simplifying assumptions:

- $e_{☽} = 0$, $r_{☽} = 384,400\,\text{km}$
- Spacecraft trajectory is coplanar with Moon's orbit $\rightarrow$ this usually the actually selected geometry to avoid a plane change.
- Neglect the Moon's gravitational field (*we will relax this soon!*)

Moon injection trajectories



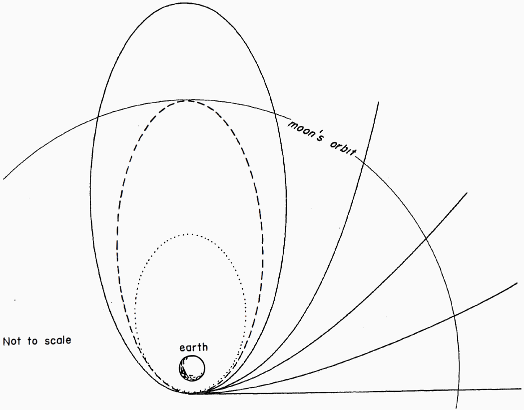


Credits: Bate et al., *Fundamentals of astrodynamics*

Significant reduction in time of flight is possible with only modest increases in injection speed.

Flight time for Apollo lunar landing missions is $\sim$ 72 hours.

Analysis of possible trajectories



Credits: Bate et al., *Fundamentals of astrodynamics*

Minimum injection speed is 10.82 km/s $\Longrightarrow$ $t_{\text{ToF}} \approx 120$ hours, this is the maximum possible flight time.

Minimum eccentricity is $e = 0.966$

$v_{\text{apo}} = v_{☽} = 0.188$ km/s. ($v_{☽} = 1$ km/s $\rightarrow$ impact on the leading = eastern edge). Spacecraft going faster will impact on the side of the Moon facing us.

Patched-conic approximation for Lunar trajectories

Moon's gravitational well is significant! $\rightarrow$ let's remove the massless Moon assumption.

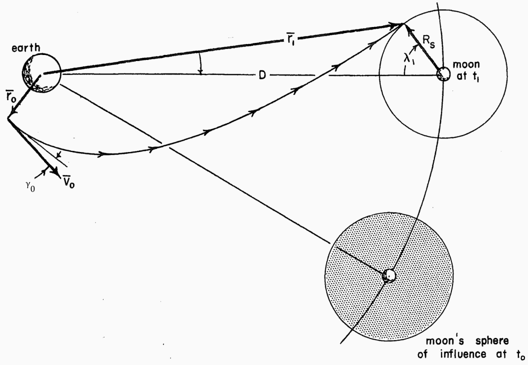
Geocentric motion becomes a selenocentric motion upon entering the Moon's sphere of influence.

As in the interplanetary trajectories ($\rightarrow$ week 8), the transition from geocentric motion to selenocentric motion is a gradual process which takes place over a finite arc of the trajectory where both Earth and Moon affect the path equally.

Good enough for approximations of preliminary mission analysis.

$R_{\text{SOI},☽} = 66,300\,\text{km} \approx 1/6 d_{\oplus☽} = 8.5^\circ$ as seen from Earth

Translunar injection point and trajectory



Credits: Bate et al., *Fundamentals of astrodynamics*

There are 4 quantities that completely specify the geocentric phase:

r_0, v_0 state vector of the translunar injection point (TLI)

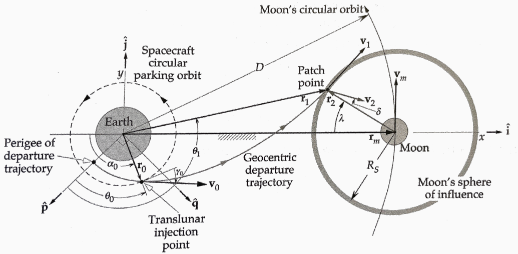
γ_0 the flight path angle at TLI's burn

λ_1 is the point at which geocentric trajectory crosses the Moon's SOI.

The time of flight (ToF) from TLI to SOI can be computed from the eccentric anomalies, but from the previous slides, ToF$\sim$ 40 – 120 hours.

$\rightarrow$ Fast computation (i.e. 2 body problem) needed to iterate over the 4 quantities to find optimal trajectory.

Condition at the patch point and lunar hyperbolic orbit



Credits: Curtis, *Orb. Mech. for Eng. Students*

From the law of cosines the radius r_1 at lunar SOI arrival is: $r_1 = \sqrt{D^2 + R_{\text{SOI}}^2 - 2DR_{\text{SOI}} \cos \lambda_1}$

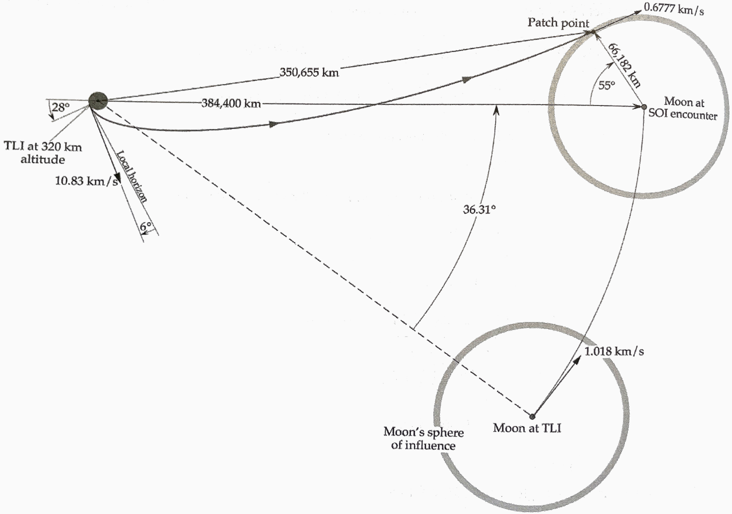
The speed is $v_1 = \sqrt{2\left(\epsilon + \frac{\mu}{r_1}\right)}$ with the energy of the orbit $\epsilon = \frac{v_0^2}{2} - \frac{\mu}{r_0}$

α_0 angle from perigee to TLI on lunar trajectory. θ_1 geometric sweep angle.

Subscript 2 is relative to the Moon. At the SOI, $r_2 = R_{\text{SOI}}$. The selenocentric velocity $\vec{v}_2 = \vec{v}_1 - \vec{v}_m$ ($v_m = 1.018$ km/s).

The eccentricity of the arrival orbit $e_2 > 1$, i.e. hyperbolic trajectory. The perilune r_{p_2} can be computed from the procedure from last lecture (or from the angular momentum h_2, e_2 and $r \sim r(h_2, e_2)$).

Example: departing from the Earth



Parking orbit at 320 km altitude.

When $\alpha_0 = 28^\circ$, the probe is launched into a translunar trajectory with a flight path angle of $\gamma_0 = 6^\circ$

$\implies v_0 = 10.8\,\text{km/s}$

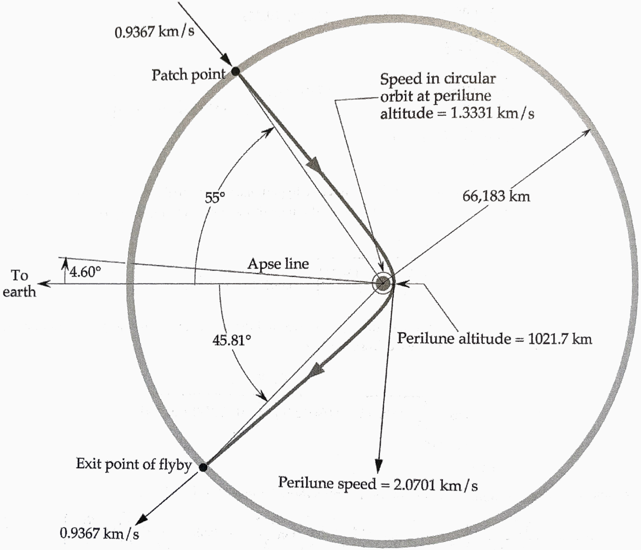
$\implies v_1 = 0.7\,\text{km/s}$

$\implies t_{\text{ToF}} = t_1 - t_0 = 66.5\,\text{h}$

Moon lead angle at TLI of $\omega_m t_{\text{ToF}} = 36.3^\circ$ (i.e. phase angle)

Credits: Curtis, *Orb. Mech. for Eng. Students*

Example: flyby from patch point



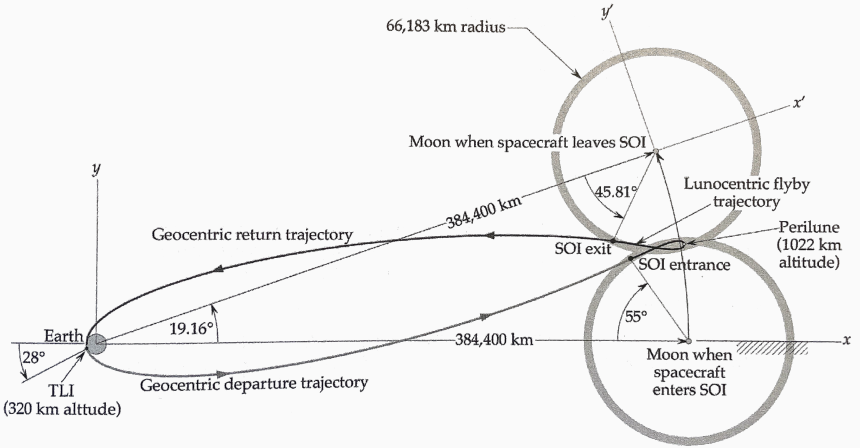
ToF to perilune = 17.5 hours

$\vec{v}_2 = \vec{v}_1 - \vec{v}_{☽} \implies v_2 = 0.96\,\text{km/s}$

alt. $z_{p_2} \approx 1020\,\text{km}$

$\Delta v_2 = -0.73\,\text{km/s}$ to circularise

Example: complete coplanar ballistic trajectory



Earth-fixed reference frame, drawn to scale.

Credits: Curtis, *Orb. Mech. for Eng. Students*

At the exit of the lunar SOI, the geocentric $v_3 = 0.65\,\text{km/s}$,

$e_3 = 0.97 \implies z_3 = 6090\,\text{km} < R_\oplus \rightarrow$ **free-return trajectory: only one Δv at TLI**.

Lunar trajectories II: orbiting the Moon

Orbits within the sphere of influence

At **low altitude** ($z_{☽} \lesssim 100\,\text{km} \implies T \sim 2.1$ hours), the orbits are **significantly perturbed** by the inhomogeneous mass distribution.

There are **frozen low altitude orbits** (27°, 50°, 76°, and 86°) in which a spacecraft can stay in a low orbit indefinitely, discovered in 2001 (!)

Mountains on Moon can reach ~ 6.1 km height $\rightarrow$ be careful.

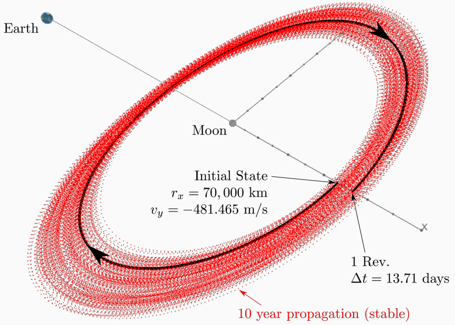
High lunar orbits are **significantly perturbed** by the Earth's gravitational influence (even if within the SOI). Integration of the three-body model integration is necessary.

Orbits in cislunar space: DRO

The Lagrange points (→ week 3) of the Earth-Moon system can provide stable orbits in the lunar vicinity, such as halo orbits and distant retrograde orbits.

Distant Retrograde Orbits (DROs) move in the retrograde direction around the Moon, appearing as large quasi-elliptical inplane orbits. The gravitational perturbations are significant from both bodies.

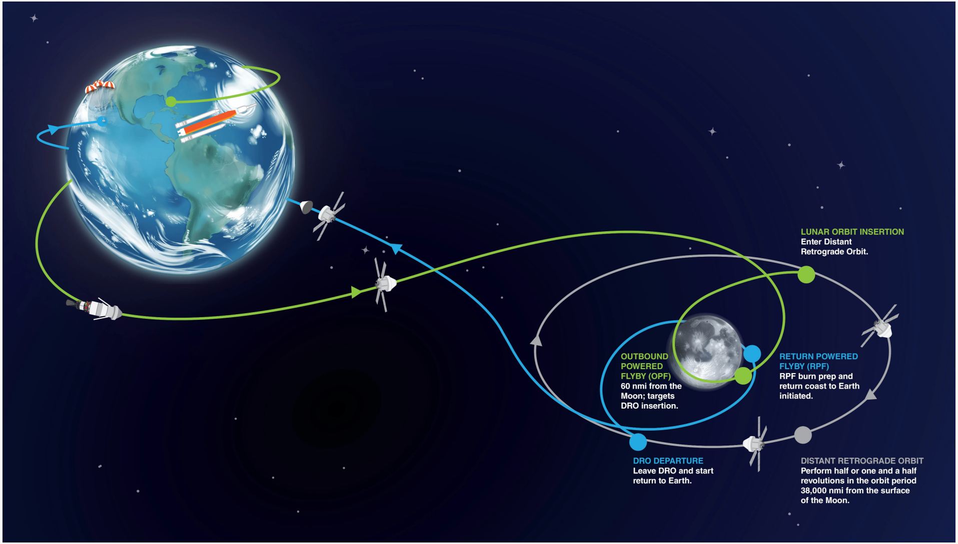
This is the orbit chosen for the Artemis 1 mission (launch date 16 Nov 2022, duration 25.4 days).



Credits: F. Dawn, *Trajectory Design Considerations for Exploration Mission 1*, 2018

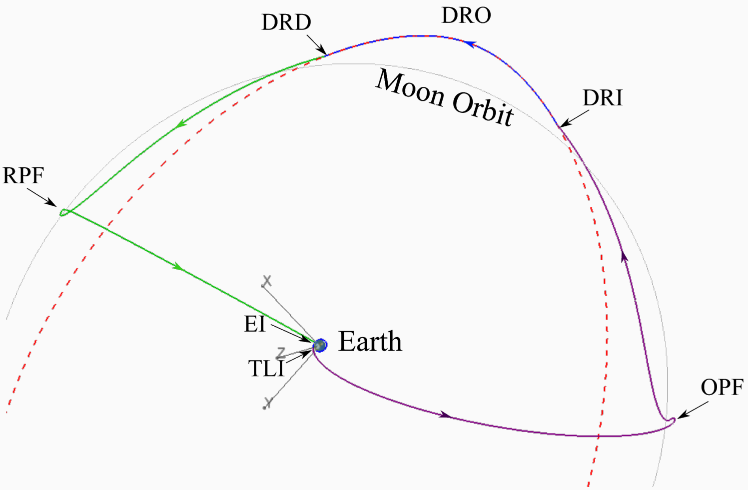
In a realistic force model, the orbits are quasi-periodic and the line of nodes (x-axis) crossing distance oscillates over time, but remain very stable. Little Δv is required for station-keeping.

Artemis I mission profile – 22 Nov 2022 - 11 Dec 2022



Credits: NASA

Artemis I mission profile in inertial frame (J2000)



Credits: F. Dawn, *Trajectory Design Considerations for Exploration Mission 1*, 2018

Artemis II mission profile – Not Earlier Than (NET) Sep 2025 - 10 days

Credits: NASA

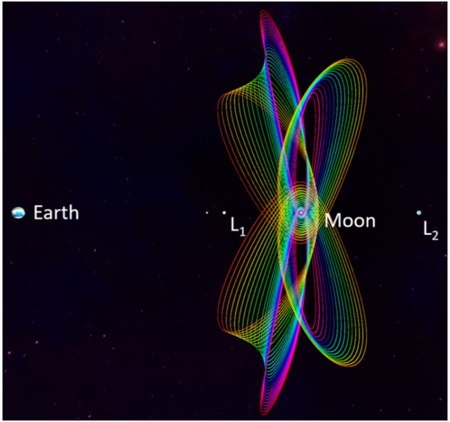
Free-return trajectory. Perilune $z_p = 7,400\,\text{km}$ from surface

Orbits in cislunar space: Near Rectiline Halo Orbits (NRHOs)

Near Rectiline Halo Orbits (NRHOs) are out-of-plane orbits (whereas DROs are inplane).

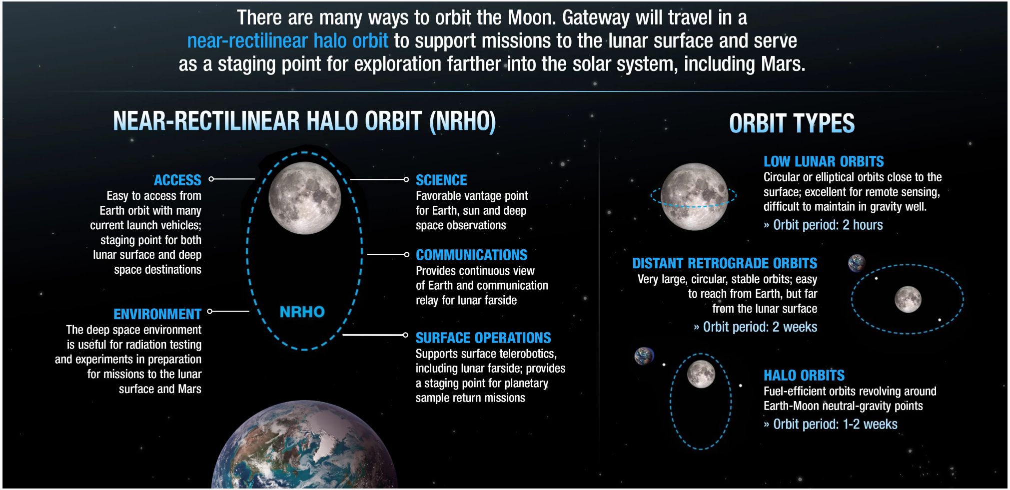
Perilune z_p in the range 1850-17,350 km altitude, period 1-2 weeks.

Low Δv requirements for station keeping (down to $\sim 0.2 - 25$ m/s for 1 year)



Credits: D. Davis et al., *Orb. Maint. and Nav. of Human S/C at cislunar NHROs*, 2017

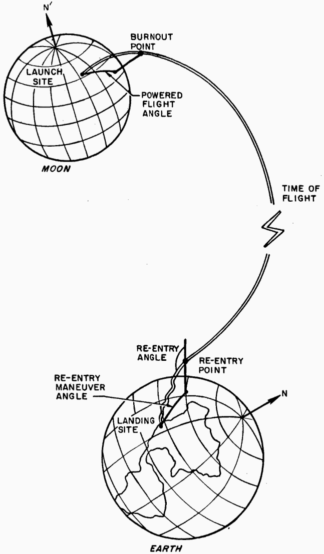
Lunar orbiting station: Gateway (NET 2025)



Credits: NASA

Lunar trajectories III: returning to the Earth

Independent parameters to the problem



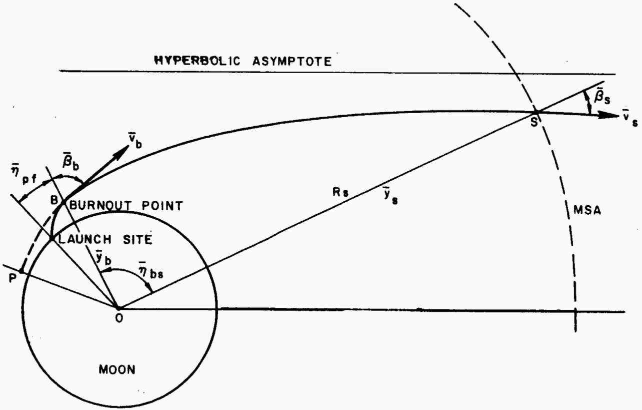
Independent for Moon-to-Earth trajectory are:

1. the launch site latitude
2. the launch site longitude
3. the burnout altitude
4. the landing site latitude
5. the re-entry flight path angle
6. the re-entry altitude
7. the combination of day of launch, landing site longitude and the total time of flight.

To solve for all parameters, we need an iterative back-propagation approach: starting with landing and making our way to Moon launch.

This discussion follows P. A. Penzo, *An Analysis Of Moon-To-Earth Trajectories*, 1961

Moon phase trajectory



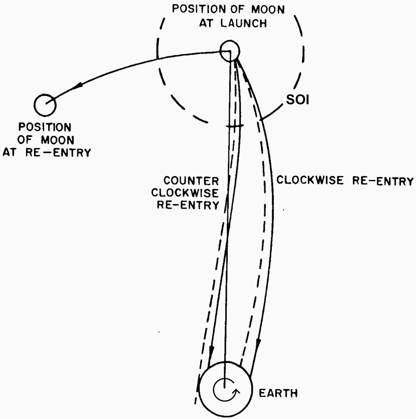
MSA = Moon sphere of action = SOI

There is a powered flight from launch site to burnout point, then ballistic trajectory.

Treatment is similar to the Earth's departure for an interplanetary transfer $\rightarrow$ hyperbolic trajectory.

Selenocentric velocity at SOI will always be to the east of the Moon-Earth line.

Errors made using the patched-conic approximation



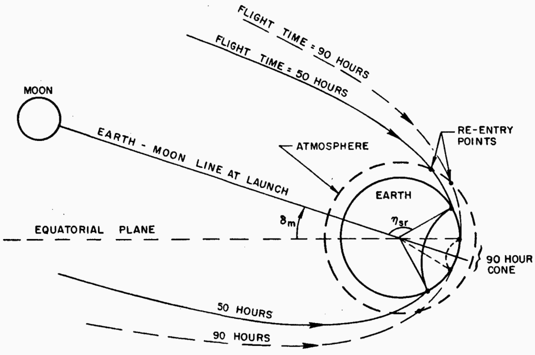
Dashed line: exact trajectory

The effect of the Moon on the Earth phase trajectory is 2-3$\times$ larger than the effect of the Earth on the Moon phase.

This choice of parameters makes the patched-conic approx. more accurate: faster trajectories, steeper re-entry angles.

Actual re-entry point is east of the desired point. This comes from the fact the Moon is moving east (as seen from Earth) $\rightarrow$ the Moon pulls the trajectory eastwards.

Allowable touchdown cones



Cones for fixed re-entry angle and two flight times.

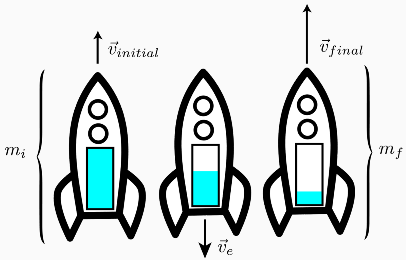
The maximum allowable latitude will be reached for trajectories passing the north pole whereas minimum latitude will be for a trajectory passing over the south pole.

For a given landing site latitude, ToF, re-entry flight path, there are limited possible declinations of the Moon δ_m (which is equivalent to days of the lunar month) $\rightarrow$ trajectories and landing site restricted by Earth-Moon system geometry.

Longitude of landing site depends on launch time, ToF, re-entry angle (hence re-entry time).

Spacecraft propulsion

Tsiolkovsky equation or rocket equation



$$\Delta v = v_e \ln\left(\frac{m_i}{m_f}\right)$$

where:
Δv is the change of velocity induced by the propulsion system,
v_e is the exhaust velocity of the gas in the propulsion system,
m_i, m_f are the initial and final mass.

Valid in free space. Gravitational field-induced and perturbations-induced Δv will be added to the propulsion-induced Δv.

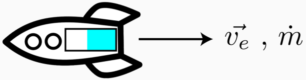
Thrust and acceleration

Thrust of the propulsion system (static case):

$$F = v_e \dot{m} \quad \text{with } \dot{m} \text{ mass flow and } v_e \text{ exhaust velocity}$$

The resulting acceleration of the spacecraft:

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{F}{m} = v_e \frac{\dot{m}}{m}$$



Integrating between the initial and final conditions:

$$\boxed{\Delta v = v_e \ln\left(\frac{m_i}{m_f}\right)}$$

Tsiolkovsky equation

Thrust and acceleration

Other form of the Tsiolkovsky equation:

$$\boxed{\Delta v = I_{sp} g_0 \ln\left(\frac{m_i}{m_f}\right)}$$

where
g_0 is Earth's gravity acceleration, 9.81 m/s^2,
I_{sp} is the specific impulse, measured in seconds, that is

$$I_{sp} = \frac{\text{Thrust}}{\text{Sea-level weight flow of consumption}} = \frac{F}{\dot{m} g_0} = \frac{v_e}{g_0}$$

Specific Impulse I_{sp}

I_{sp} is the specific impulse, measured in seconds, that is

$$I_{sp} = \frac{\text{Thrust}}{\text{Sea-level weight flow of consumption}} = \frac{F}{\dot{m} g_0} = \frac{v_e}{g_0}$$

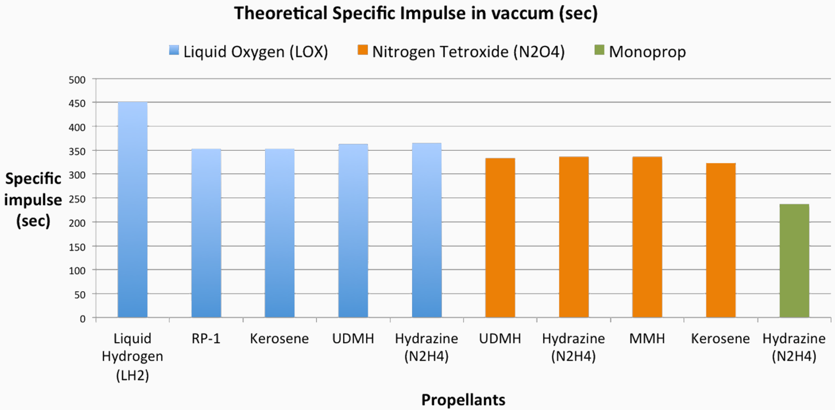
It is an important performance parameter for a thruster/propellant combination.

I_{sp} gives how many seconds a given propellant, with a given engine, can accelerate its own initial mass at 1 g. The longer it can accelerate its own mass, the more Δv it delivers to the whole system.

For a given thrust F, if $I_{sp} \nearrow$, $\dot{m} \searrow$, i.e. less propellant need to reach the same Δv.

A large $I_{sp} \implies$ large thrust F if and only if the mass flow $\dot{m}$ is large.

Kind of chemical propulsion



Credits: NASA

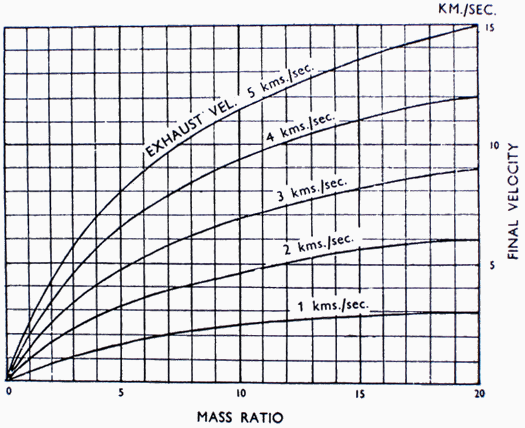
Cold gas propulsion: $I_{sp} = 50 - 75$ s, solid motor: $I_{sp} = 280 - 300$ s

Chemical propulsion is typically used for launch, orbit insertion and large & fast manoeuvres. The highest value I_{sp} is for liquid hydrogen as propellant and liquid oxygen as oxidizer. It will give $I_{sp} = 450$ s for the specific impulse, which means $v_e \approx 4.5$ km/s.

Final velocity VS Mass ratio

The typical final velocity for a rocket to reach LEO is $v_f \approx 10$ km/s and make up for losses (e.g. gravitational and drag).

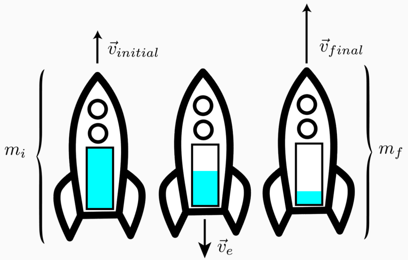
For a LH2/LOX mix ($v_e \approx 4.5$ km/s), the mass ratio is about 10 $\Longrightarrow$ about 90% of the total mass at lift off is propellant mass.



Credits: *Ascent to Orbit*, Arthur C. Clarke

Mass of propellant needed

$$\Delta v = I_{sp} g_0 \ln\left(\frac{m_i}{m_f}\right) \implies \begin{cases} m_p = m_i \left[1 - \exp\left(-\frac{\Delta v}{I_{sp} g_0}\right)\right] \\ m_p = m_f \left[\exp\left(\frac{\Delta v}{I_{sp} g_0}\right) - 1\right] \end{cases}$$

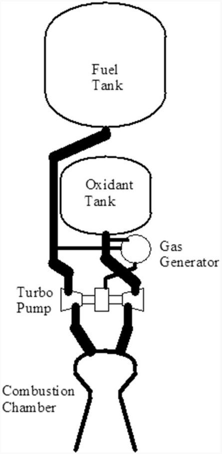


where
Δv is the change of velocity induced by the propulsion system,
m_i, m_f are the initial and final mass [kg],
m_p is the propellant mass consumed to produce the given Δv [kg],
g_0 is Earth's gravity acceleration, 9.81 m/s^2,
I_{sp} is the specific impulse [s].

Liquid propellant rocket engine – Viking 5C example

This motor was used on the Ariane 4 launcher with hypergolic propellant, UH25 which was unsymmetrical monomethylhydrazine as fuel, and nitrogen tetroxide as oxidizer.

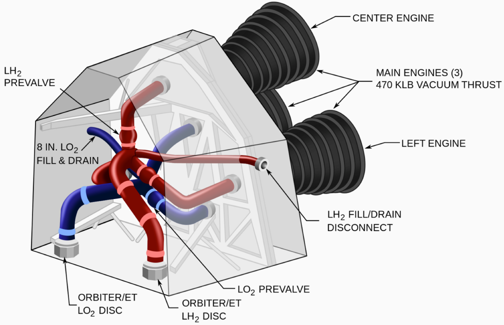
The specific impulse of this system was about 350-380 sec, less than a cryogenic motor using LH2 and LOX.

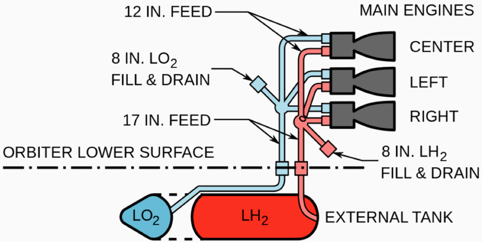


Credits: Wikipedia, Sanjay Acharya

Space Shuttle Main Engines (SSME)

SSME – Fuel and oxidizer supply





Credits: NASA, JSC, Booster Systems Briefs, October 1, 1984

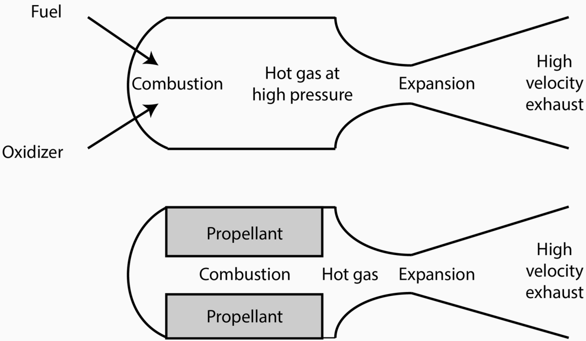
The external tank of the space shuttle contained 630 t of liquid-oxygen. The liquid-hydrogen tank contained about 100 t of LH2.

Liquid and solid propellant rocket motors

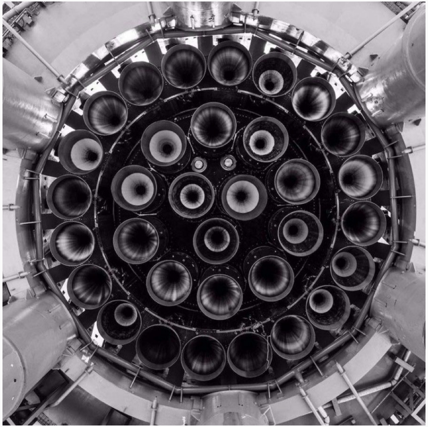
In addition to the 3 orbiter-attached SSMEs, the Space Shuttle had 2 solid rocket boosters (SRBs) with a central cavity in order to increase the surface of the burning propellant generating thrust.

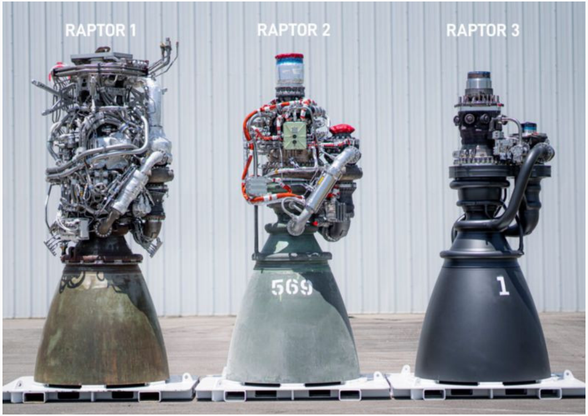
Same principle for Ariane 5 and 6 solid propellant boosters.

Credits: ESA



Starship – 33 Raptor engines

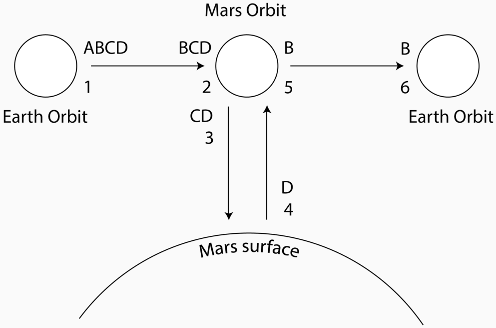




Credits: SpaceX

Cryogenic liquid CH_4 and LOX, a mixture known as methalox. Thrust of Raptor 3 ≈ 2.75 MN, $I_{sp} = 327$ s. Design for re-used with minimal maintenance.

Propellant needed for a mission to Mars



Vehicles

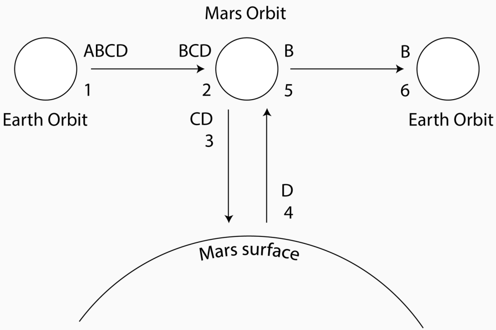
A	Discarded Booster
B	Earth-Mars Transfer Vehicle
C	Mars Lander
D	Ascent Vehicle

Maneuvers

1	LEO to MTO
2	MTO to LMO
3	LMO to Surface
4	Surface to LMO
5	LMO to ETO
6	ETO to LEO

Based on Tsiolkovsky equation, about 90% of the initial mass Earth's surface is propellant, if the mass of the payload is increased, the propellants needed to leave the gravity well of the Earth have to be considerably increased. The need for high I_{sp}, or altermative propulsion system for Mars exploration is clear!

Propellant needed for a mission to Mars



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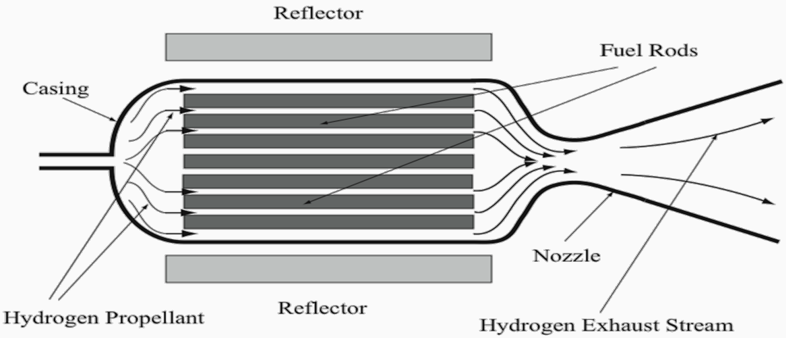
A mission to Mars and back involves (at the very least) 6 Δv manoeuvres.

The propellant for manoeuvre 6, for instance, has to be carried to Mars and back. The payload for manoeuvre 1 includes all the propellant needed for the other manoeuvrers.

Reducing propellant is a major requirement; Producing propellant on the surface of Mars is an option.

Nuclear rocket principle



Hot fuel rods heat hydrogen propellant.

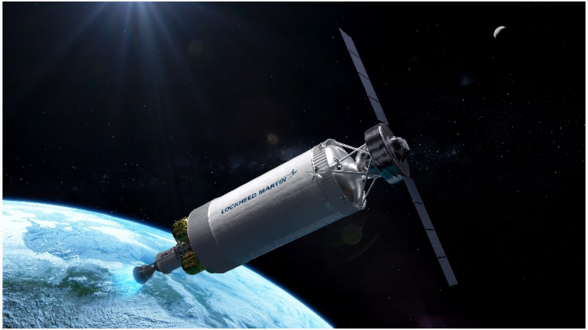
The hot hydrogen expands in the nozzle as in a chemical rocket motor.

Nuclear engine: the revival of an old idea?

Lockheed Martin to develop first nuclear-powered Demonstration Rocket for Agile Cislunar Operations (DRACO) rocket for a demo as early as 2026.

The spacecraft will be loaded with 2,000 kg of hydrogen and 100 kg of low-enriched HALEU uranium.

Will include a fail-safe poison wire which can absorb neutrons and prevent a chain reaction in case the launch fails and the reactor falls in the ocean.



Credits: Lockheed Martin

The reactor will not be turned on until it reaches a nuclear-safe orbit between 700 and 2,000 km altitude.

Hoping for $I_{sp} \gtrsim 800$ s. Would reduce our travel time to Mars from seven months down to a few months.

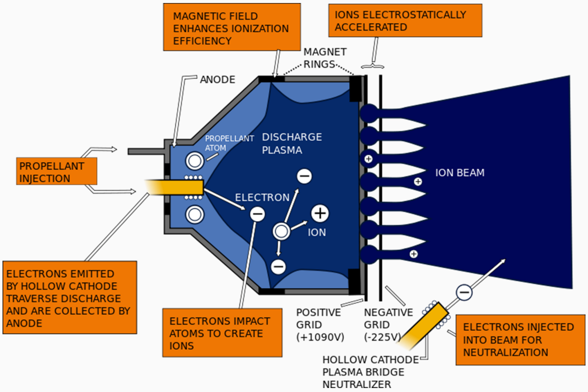
Electric or ion propulsion

Ionization of the propellant material, and acceleration with an electric field.

Higher exhaust velocities than with a liquid-fuelled or solid propellant rocket engine.

There is very high ejection velocity and high efficiency, but low thrust, of the order of a fraction of a Newton, because the mass flow is very small.

Such a system can be used for propulsion in space, but not for leaving the Earth's surface.



Credits: Wikipedia, Chabacano, retrieved from NASA

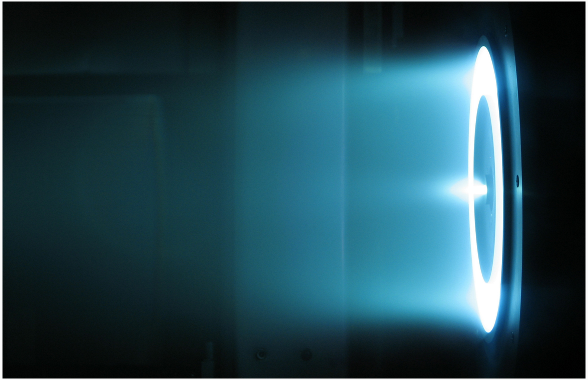
Hall thruster as an example of electric propulsion

Electric propulsion = acceleration of charged particles (ions) by an electric field or electric and magnetic fields.

Typical $I_{sp} = 1000 - 5000$ s $\Leftrightarrow$ exhaust velocities of 10 to 500 km/s.

Thrust values of typically $10^{-2} - 10^{-1}$ N.

This a 6 kW Hall thruster in operation at the NASA Jet Propulsion Laboratory



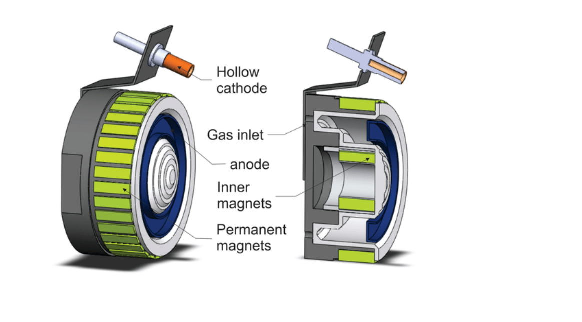
Credits: NASA/JPL

Want to know more about propulsion? $\rightarrow$ *Course ENG-510 / Space propulsion*

Prevalence of electric propulsion

Electric propulsion is a very common choice for station-keeping and even some positioning activities.

Virtually all manoeuvrable small satellites use electric propulsion, mostly for fuel efficiency reasons, however, they must be able to generate large electric fields → high demand on the power generation subsystem.



Skematics of a Hall thruster

Credits: Moraes et al., 2014

Example: all 7,000+ Starlink satellites use electric propulsion. Early models used krypton-fueled ion thrusters. SpaceX switched to argon Hall thruster because it is $\sim 100\times$ cheaper (and $1.5 \times I_{sp}$).

Non-impulsive manoeuvres

Motivation for non-impulsive manoeuvres

So far, we assumed that the duration of the burn Δv was much shorter than the orbital period, i.e. $t_b \ll T_{\mathrm{orb}}$, almost instantaneous.

This assumption holds for most Δv conducted with chemical propulsion subsystems as the thrust is high enough.

For electric propulsion, the duration of the burns $t_{b,\mathrm{EP}}$ are much longer than the same Δvs with chemical propulsion $t_{b,\mathrm{EP}} \gg t_{b,\mathrm{CP}}$

The equation of motion

During the burn, the equation of motion must be rewritten with an external force $\vec{F}$:

$$\ddot{\vec{r}} = -\mu \frac{\vec{r}}{r^3} + \frac{\vec{F}}{m} = \text{gravity} + \text{thrust}$$

Let's assume for simplicity that the thrust is always aligned with the velocity vector (it's usually not for long and complex manoeuvres).

$$\vec{F} = T \frac{\vec{v}}{v}$$

and remembering that the mass of propellant decreases with time,

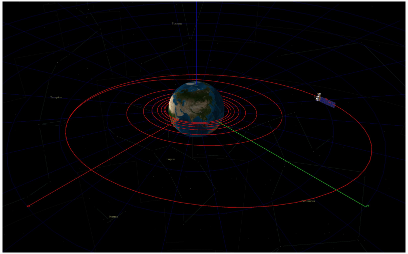
$$\ddot{\vec{r}} = -\mu \frac{\vec{r}}{r^3} + \frac{T}{m} \frac{\dot{\vec{r}}}{\dot{r}} \qquad \dot{m} = -\frac{T}{I_{sp} g_0}$$

This generally does not accept analytical solutions. The equations can be rewritten as a system of linear equations with 7 components (state vector + mass) and numerically integrated.

Example: spiralling up to GEO

Assuming very low thrust, it can be shown that a spiralling trajectory solves the equation of motion.

Example of a 1 t satellite in a 300 km altitude circular parking going to GEO with no inclination change needed. With a Hohamnn transfer:



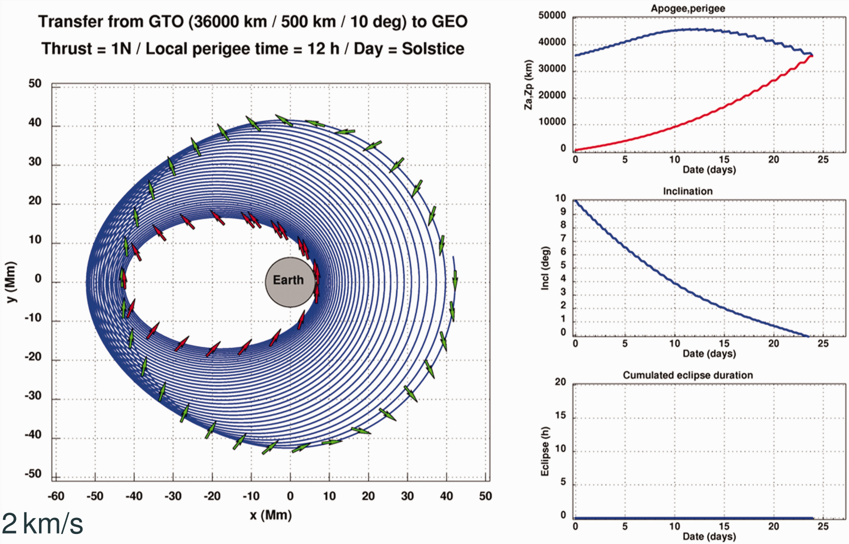
$$\Delta v_{\text{tot}} = 3.89\,\text{km/s} \qquad t_{\text{Hoh}} = 5.3\,\text{h} \qquad I_{\text{sp}} = 300\,\text{s} \implies m_p = 734\,\text{kg}$$

With an electric propulsion with $I_{\text{sp}} = 5000$ s and constant (tangential) thrust $T = 2\,\text{mN}$:

$$\Delta v_{\text{tot}} = 4.65\,\text{km/s} \qquad t_{\text{EP}} = 6173\,\text{h} = 257\,\text{d} \implies m_p = 90.6\,\text{kg}$$

Electric propulsion is much more efficient, but requires much more time! Final situation in a slightly elliptical orbit that need to be circularised $\rightarrow$ a non-tangential burn is needed.

Example: Minimum-time low-thruster transfer



$\Delta v \approx 2.2\,\text{km/s}$

Credits: Cerf M., *Fast solution of minimum-time low-thrust transfer with eclipses*, 2018

Interactive quiz

→ EchoPoll platform

- You can scan a QR code or go to the link
- EchoPoll is the EPFL-recommended solution
- You do not have to register, just skip entering a username and/or email address