EE-585 – Space Mission Design and Operations

Dr Thibault Kuntzer

Ecole Polytechnique Fédérale de Lausanne

Week 08 – 08 Nov 2024

Oral examination

Oral examination: Monday 20 Jan to Friday 24 Jan with Saturday 25 Jan as backup

Around mid-December, you will have the opportunity to book a slot (first come, first served basis)

Today's outline

The deep space environment

Sphere of influence & patched conics approximation

Departure from a planet

Arrival to a planet

Aerodynamic braking manoeuvres

Rendezvous opportunities

Non-Hohmann trajectories & departure opportunities

Summary

The deep space environment

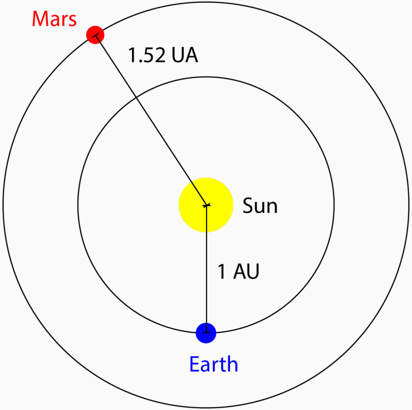
The Astronomical Unit (AU)

Astronomical unit = average distance Sun - Earth.

The orbit of the Earth around the Sun is slightly elliptical, eccentricity $e = 0.017$.

At perihelion on 3 Jan, the Earth is about 147 million km to the Sun, at aphelion, on 3 July, the distance is 152 million km to the Sun.

Mars has a 1.52 AU average distance to the Sun, which means that its distance to Earth varies from 0.52 AU to 2.52 AU on average. Its orbit has a large eccentricity of $e = 0.094$.



$$1\text{AU} = 149.5978707 \cdot 10^6 \text{ km}$$

The Solar System

Omitted dwarf planets: Haumea, Makemake + $\gtrsim$ 4 large transneptunian objects. Credits: NASA,JPL

Orbital characteristics of planets

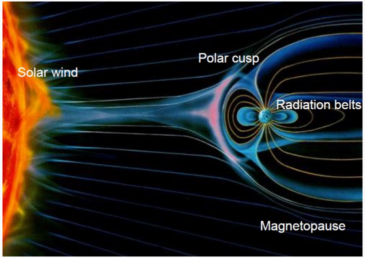
Planet	Semi-major axis a [AU]	Perihelion r_p [10^6 km]	Orbital eccentricity e	Orbital inclination i [deg]	Orbital velocity V [km/s]
Mercury	0.39	46.0	0.205	7.0	47.4
Venus	0.72	107.5	0.007	3.4	35.0
Earth	1.00	147.1	0.017	0.0	29.8
Mars	1.52	206.6	0.094	1.9	24.1
Jupiter	5.20	740.5	0.049	1.3	13.1
Saturn	9.65	1353.6	0.057	2.5	9.7
Uranus	19.20	2741.3	0.046	0.8	6.8
Neptune	30.04	4444.5	0.011	1.8	5.4

http://nssdc.gsfc.nasa.gov/planetary/factsheet/

The deep space environment

The term "deep space" is not well defined, but it usually means beyond Earth's influence. Travelling beyond GEO is often considered in deep space.

Deep space is exposed to unabated solar winds and cosmic rays $\rightarrow$ radiation is higher than within the Earth's magnetosphere



Credits: NASA

In deep space (but in our solar system), the Sun dominates $\rightarrow$ the spacecraft must be radiation hardened and be able to handle the effects of violent CMEs.

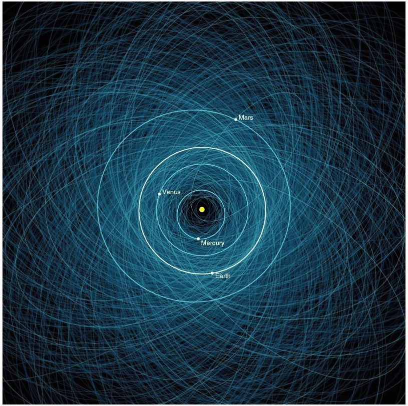
Planetary environments – like around Jupiter – can be much harsher in terms of radiation than during the interplanetary transfer phase.

Near-Earth Objects (NEOs) and asteroids (NEAs)

The density of objects in the solar system is very low, but there are many objects out there ($\gtrsim 10^6$ larger than 1 km in the solar system).

NEOs are celestial bodies, such as asteroids and comets, which orbits that cross Earth's trajectory, posing a potential collision risk with our planet.

Planetary defence is the field of finding and potentially deviating objects with an unacceptable probability of collision. Demonstration in 2022 with NASA's DART mission that nudged a double system (and follow-up with ESA's HERA mission).



Credits: NASA

Recent collisions – Tunguska in Siberia in 1908

Estimated size of the object 60 to 190 m, largest impact on Earth in recent history. 3-10 megaton of TNT ($\sim 10^3\times$ Hiroshima). It rates at 8 on the Torino scale.

About 80 million trees knocked down over an area of 2,150 square kilometres

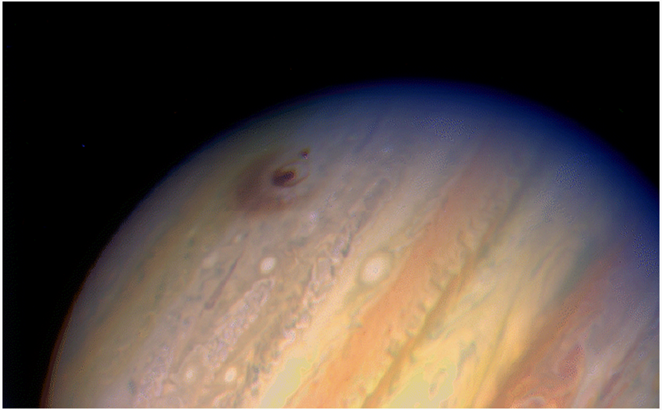
Explosion heard in Western Europe.

Credits: PD, Wikipedia, CYD

Recent collisions – Comet Shoemaker-Levy 9 impact on Jupiter in 1994

Comet Shoemaker-Levy 9 is an example of object captured by Jupiter.

The comet had been in an orbit around Jupiter for a while, then its nucleus was fragmented by tidal forces, and the fragments plunged into the planet between 16 July and 22 July 1994.



Credits: WFPC2, HST, NASA, H. Hammel (MIT)

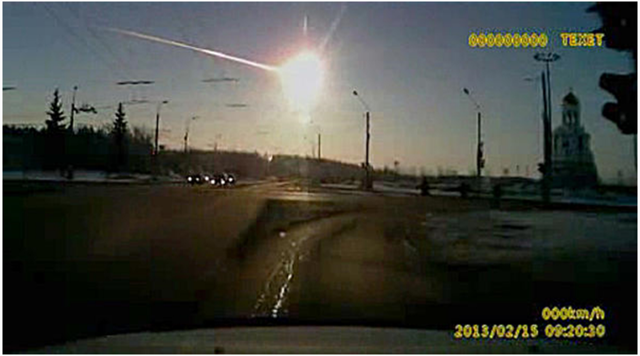
Recent collisions – Meteor impact at Chelyabinsk, 15 Feb 2013

Probable size about 20 m.

Initial velocity of entry 19 km/s
$\Longrightarrow$ 0.5 megaton TNT.

Very shallow entry angle.
Exploded in the atmosphere.

Significant destruction and injuries.



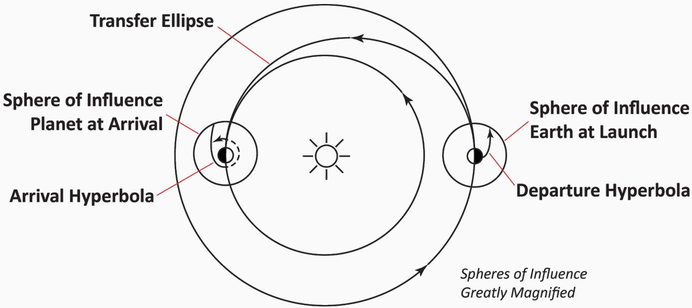
Credits: Wikipedia, Aleksandr Ivanov

Sphere of influence & patched conics approximation

Interplanetary trajectories - Strategy to solve the problem

To plan for and execute a mission to another planet, we consider the Sun, the planet of departure (the Earth), the planet of destination, and the spacecraft.

It is a four-body problem (with 1 body almost massless) that we divide into three segments, each of them a two-body problem.



Credits: Charles D. Brown, *Elements of Spacecraft Design*, AIAA

1. Departure phase (planetocentric 1)
2. Cruise phase (heliocentric)
3. Arrival phase (planetocentric 2)

Symbol convention for position and velocity

Uppercase variables, e.g. R, V, are heliocentric, i.e. described in the frame of reference of the Sun.

$$\vec{R}_{\text{S/C}} = \vec{r}_{\text{S/C}} + \vec{R}_P$$
$$\vec{V}_{\text{S/C}} = \vec{v}_{\text{S/C}} + \vec{V}_P$$

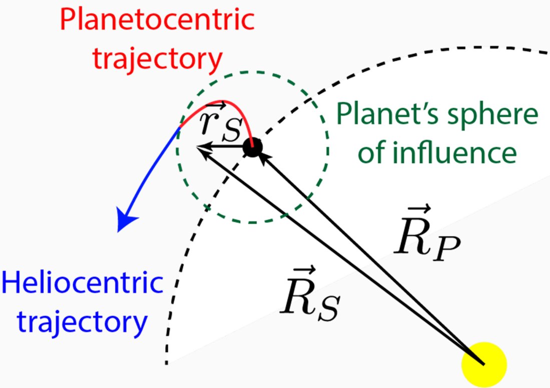
Heliocentric movement of the S/C

=

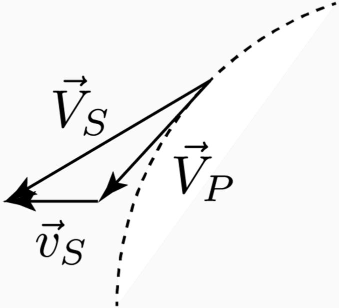
Planetocentric movement of the S/C + Heliocentric movement of the planet

Lowercase variables, e.g. r, v, are planetocentric, i.e. described in the frame of reference of the planet.

Symbol convention for position and velocity



P for Planet
S for Spacecraft



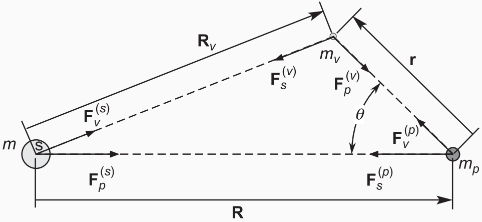
Sphere of influence (1/2)

The sphere of influence is a region in which the gravitational influence of other bodies can be neglected.

The acceleration on the vehicle (v) in heliocentric frame is:

$$\ddot{\vec{R}}_v = \vec{A}_\odot + \vec{P}_p$$

where $\vec{A}_\odot = \vec{F}_s^{(v)}/m_v$ is the gravitational acceleration on the vehicle due to the Sun ($\odot$) and $\vec{P}_p = \vec{F}_p^{(v)}/m_v$ is the gravitational perturbation of the secondary object, the planet.



Credits: Curtis, *Orb. Mech. for Eng. Students*

Similarly, in the planetocentric frame:

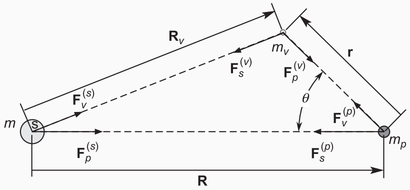
$$\ddot{\vec{r}} = \vec{a}_p + \vec{p}_\odot$$

where $a_p = \mu_p \frac{1}{r^2}$ and $p_\odot = \mu_\odot \frac{r}{R^3}$

Sphere of influence (2/2)

For the motion relative to the planet, $p_\odot/a_p$ is a measure of the deviation from a Keplerian orbit (Keplerian motion $\implies p_\odot/a_p = 0$).

For the motion relative to the Sun, $P_p/A_\odot$ is the measure of the planet's influence on the orbit of the vehicle.



Credits: Curtis, *Orb. Mech. for Eng. Students*

If

$$p_\odot/a_p < P_p/A_\odot$$

then the perturbing effect of the Sun on the vehicle's orbit around the planet is less than the perturbing effect of the planet on the vehicle's orbit around the Sun.

This leads to

$$\frac{r}{R} < \left(\frac{m_p}{m_s}\right)^{2/5}$$

where R is the distance between the two massive objects (e.g. Sun and Earth).

Radius of the sphere of influence

Sphere of influence = region around each body inside which the motion of a spacecraft can be considered to be two-body Keplerian.

The radius of the sphere of influence r_{SOI} is therefore:

$$\boxed{r_{\mathrm{SOI}} = R\left(\frac{\mu_{\mathrm{planet}}}{\mu_{\mathrm{Sun}}}\right)^{\frac{2}{5}}}$$

where R is the distance between the two massive objects (e.g. Sun and Earth).

Spheres of influence in the solar system

The concept of the sphere of influence is usable for the motion of a spacecraft from the Earth to another planet.

For the Moon, the sphere of influence is calculated with $R =$ distance Earth-Moon. $r_{\text{SOI}} = 66,200$ km = 38 Moon radii.

For all planets beyond Mercury, r_{SOI} is really large and the boundary of the SOI will often be considered as being a location of zero potential energy with respect to the central body ($= \infty$).

Planet	r_{SOI} (10^6 km)
Mercury	0.111
Venus	0.616
Earth	0.924
Mars	0.577
Jupiter	48.157
Saturn	54.796
Uranus	51.954
Neptune	80.196
Moon	0.0662

Earth's sphere of influence

This is, to scale, Earth's SOI with respect to the Sun:

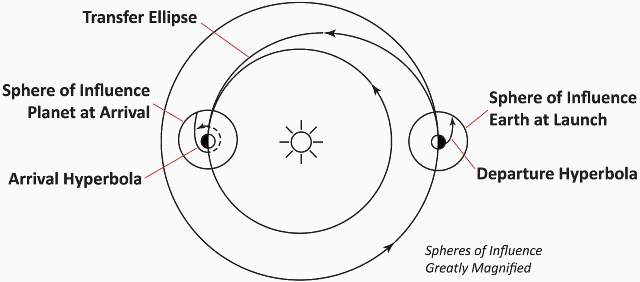
Credits: Curtis, *Orb. Mech. for Eng. Students*

$r_{\mathrm{SOI}_\oplus} = 0.924 \cdot 10^6\,\mathrm{km} = 144 R_\oplus = 0.6\% d_{\odot\oplus} \rightarrow$ interplanetary voyages are on truly large scales.

Sphere of influence (SOI) and patched conics approximation

As long as the S/C is within the SOI of the Earth, its motion with respect to the Earth is a two-body problem with the Earth as a central body. We can ignore the Sun's gravitational influence.

When the S/C leaves $\mathrm{SOI}_{\oplus}$, $r_{\mathrm{SOI}} = 10^6$ km, it comes on a heliocentric elliptical trajectory (Hohmann transfer) towards the destination planet, either larger than the Earth's orbit (outer planets), or smaller (inner planets). The gravitational influence of both the departure and the destination planets is negligible on this heliocentric arc.



Credits: Charles D. Brown, *Elements of Spacecraft Design*, AIAA

At the end its elliptic heliocentric arc, in the vicinity of the destination planet, the S/C enters the SOI of the destination planet, then we can ignore the Sun and determine the S/C's trajectory as a two-body problem with the destination planet as the only attracting body.

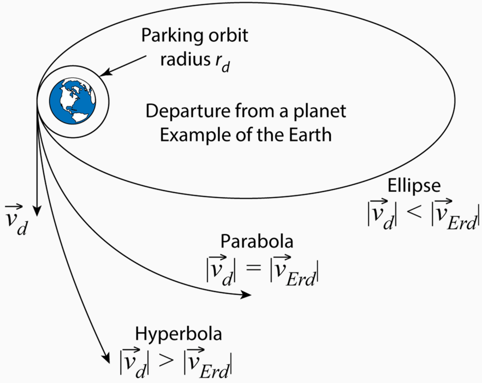
Departure from a planet

Departure from a planet

An interplanetary journey can start with a parking orbit around Earth (direct-ascent trajectories are also possible).

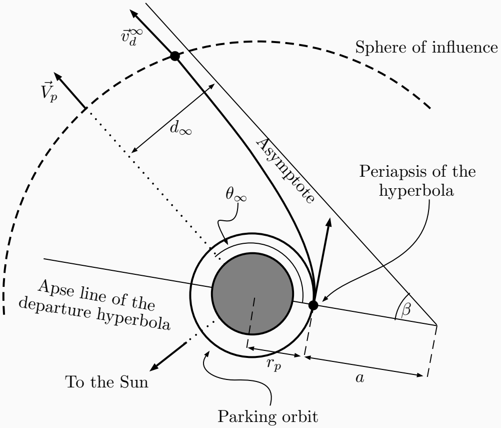
The velocity is increased to reach the **departure velocity**, $\vec{v}_d$, which is always larger than the escape velocity for the altitude of the parking orbit, $\vec{v}_{\mathrm{E}r_d}$. The escape velocity is about 11.2 km/s at the surface. It decreases with the altitude as $v_{\mathrm{E}r_d} = \sqrt{2\mu/r}$.

On a planetocentric hyperbolic departure orbit, at a large distance from the Earth, the spacecraft comes to a constant velocity called the **hyperbolic excess velocity** $\vec{v}_d^{\infty}$.



$\vec{v}_d$ is the departure velocity
$\vec{v}_{\mathrm{E}r_d}$ is the escape velocity from the parking orbit

Reaching the sphere of influence



The conservation of total energy implies that:

$$\left.\frac{v^2}{2} - \frac{\mu}{r}\right|_{\text{at departure, i.e. } r_p = r_d} = \left.\frac{v^2}{2} - \frac{\mu}{r}\right|_{\text{at SOI}\approx\infty}$$

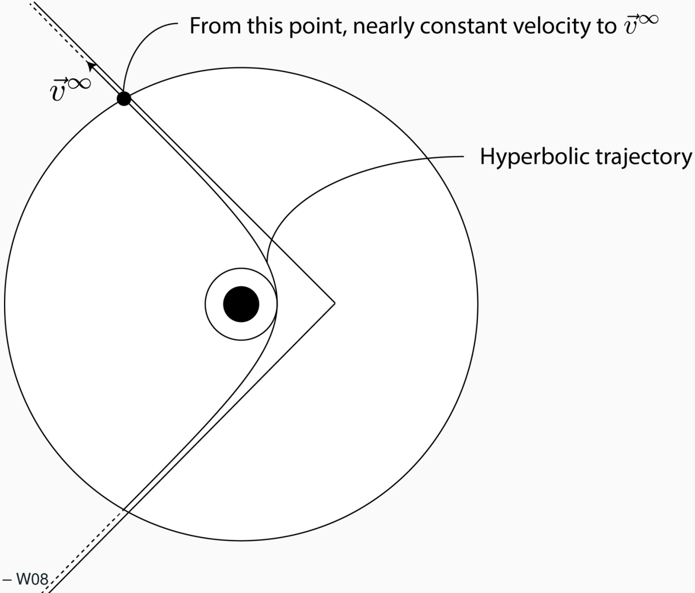
$$\frac{v_d^2}{2} - \frac{\mu}{r_p} = \frac{(v_d^\infty)^2}{2}$$

$$\implies v_d^2 = (v_d^\infty)^2 + \frac{2\mu}{r_p}$$

As $v_{Er_d} = \sqrt{\frac{2\mu}{r_p}}$, $r_d = r_p$,

$$\boxed{v_d^2 = (v_d^\infty)^2 + v_{Er_d}^2}$$

Departure from a planet



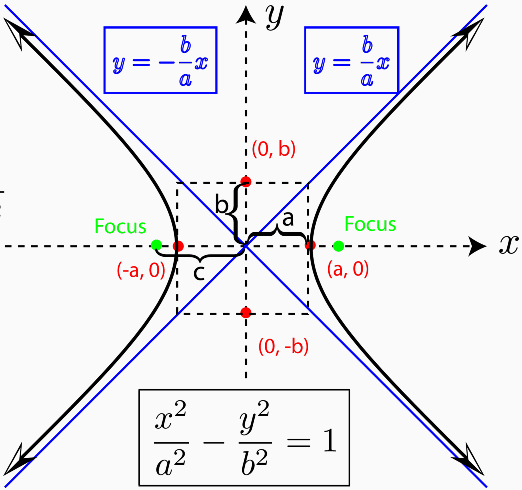
v^{∞} is the hyperbolic excess velocity and

$(v^{\infty})^2 = C_3$ the characteristic energy, i.e. twice the kinetic energy at ∞.

Hyperbola basics

$$c = \sqrt{a^2 + b^2}$$

$$\boxed{c = ae}$$



Vis-Viva equation – Velocities on elliptical and hyperbolic orbits

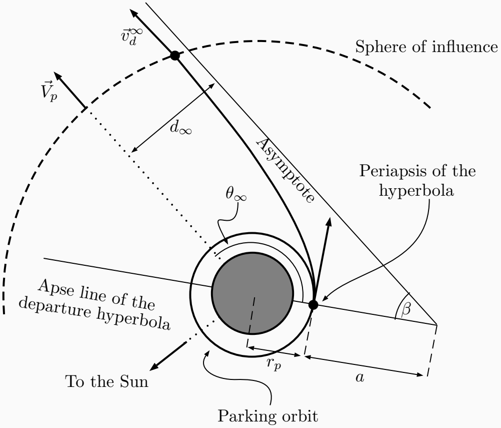
$$\text{Ellipse} \quad v = \sqrt{\frac{2\mu}{r} - \frac{\mu}{a}} \quad \epsilon = -\frac{\mu}{2a}$$

$$\text{Hyperbola} \quad v = \sqrt{\frac{2\mu}{r} + \frac{\mu}{a}} \quad \epsilon = +\frac{\mu}{2a}$$

The energy per unit mass ϵ on an elliptical or hyperbolic trajectory is only dependent on the mass of the central object μ, and on the value of the semi-major axis a, and not on the eccentricity e.

Note that as the trajectory is unbounded with respect to the planet, $\epsilon > 0$.

Velocity on the departure hyperbola



$$\boxed{v = \sqrt{\frac{2\mu}{r} + \frac{\mu}{a}}}$$

$$a \cong \frac{\mu}{(v_d^\infty)^2}$$

$$e = \frac{a + r_d}{a} = \frac{a + r_p}{a} = \frac{c}{a} > 1$$

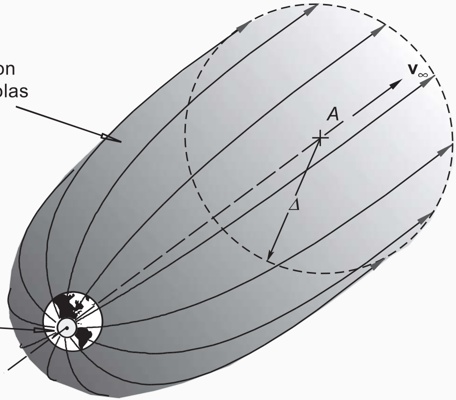
$$\theta_\infty = \arccos\left(-\frac{1}{e}\right)$$

$$\beta = \arccos\left(\frac{1}{e}\right) = \arccos\left(\frac{1}{1 + \frac{r_p (v_d^\infty)^2}{\mu}}\right)$$

In this configuration, the Hohmann transfer is to an outer planet. To a inner planet, $\vec{v}^\infty$ should be in the anti-direction of $\vec{V}_p$.

β gives the orientation of the apse line of the hyperbola to the planet's heliocentric velocity vector.

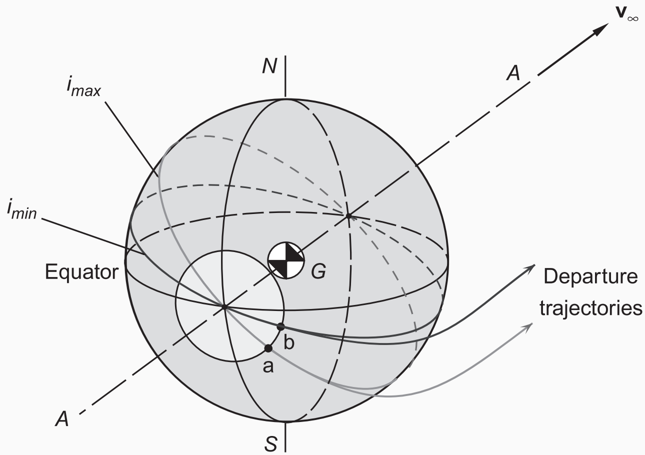
Locus of possible departure trajectories for a given $\vec{v}_d^{\infty}$ and r_p



Credits: Curtis, *Orb. Mech. for Eng. Students*

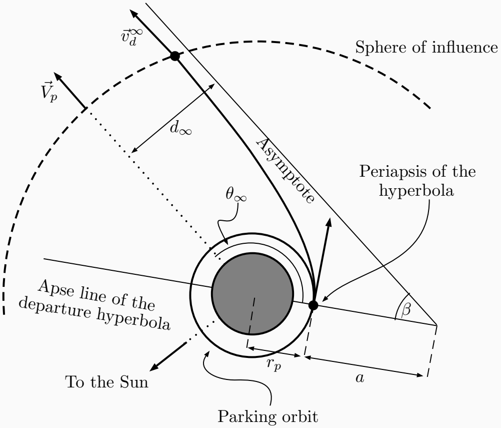
Inclined parking orbits and departure trajectories

Possible departure points are reached once per orbit, *a*, *b* even on differently inclined orbits, i_{min}, i_{max}.



Credits: Adapted from Curtis, *Orb. Mech. for Eng. Students*

Heliocentric velocity right after crossing the sphere of influence



Right after leaving the sphere of influence of the planet, in the heliocentric frame of reference, the departure speed V_D is

$$\vec{V}_D = \vec{V}_P + \vec{v}^\infty$$

where V_P is the orbital of the planet.

Sensitivity analysis (1/2)

The aphelion of the Hohmann transfer R_2 between the planet is described by:

$$R_2 = \frac{h^2}{\mu_\odot} \frac{1}{1 + e \cos \Theta}$$

where $h = R_1 V_D$ is the angular momentum, $V_D = V_{p,d} + v_d^\infty$ and $\cos \Theta = -1$ as $\Theta = 180°$.

Computing the change in V_D due to variations in δr_p and δv_p of the burnout position (= periapsis) and speed respetively leads to

$$\frac{\delta R_2}{R_2} = f(\delta r_p, \delta v_p, R_1, V_D, r_p, v_p)$$

Sensitivity analysis (2/2)

Plugging in values for an Earth-Mars transit,

$$\frac{\delta R_2}{R_2} = 3.127 \frac{\delta r_p}{r_p} + 6.708 \frac{\delta v_p}{v_p}$$

This expression shows that a 0.01% variation (1.1 m/s) in the burnout speed v_p changes the target radius R_2 by 0.067% or 153,000 km ($\approx 45 R_{\text{Mars}}$)!

Likewise, an error of 0.01% (0.31 km) in burnout radius r_p produces an error of over 70,000 km ($\approx 20 R_{\text{Mars}}$).

These small errors can be corrected by mid-course burns.

Arrival to a planet

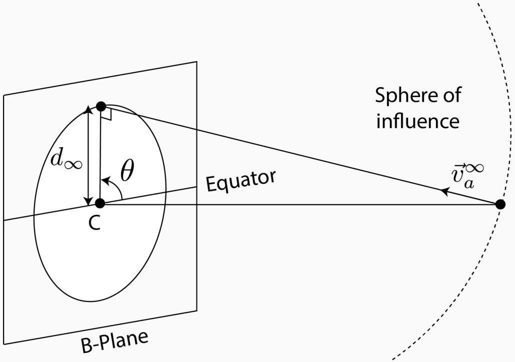
Definitions

On its journey to the planet of destination, the spacecraft is on an elliptical heliocentric trajectory, until it gets close to that planet. The only massive body considered is the Sun.

Then only the motion of the spacecraft with respect to the destination planet is considered as hyperbolic trajectory inside the sphere of influence of this arrival planet.

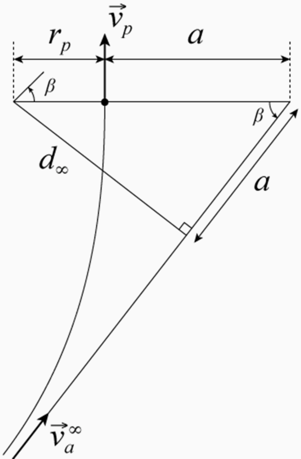
Flight controllers steer the spacecraft and choose d_∞, the impact parameter, and θ to accomplish the mission objective, either a flyby (and possible orbit insertion) or a direct landing on the surface of the planet.

Arrival at velocity $\vec{v}_a^\infty = \vec{V}_{S/C} - \vec{V}_P$ at the sphere of influence of the destination planet.



On the arrival hyperbolic orbit

Procedure is very similar to the departure phase. The spaceraft enters the sphere of influence of the destination planet on an hyperbolic trajectory.



Conservation of total energy $\rightarrow v_p^2 = (v_a^\infty)^2 + v_{Er_p}$

As $a \cong \frac{\mu}{(v_a^\infty)^2}$, $a^2 + b^2 = c^2 = (a + r_p)^2$ and $b = d_\infty$

$$r_p = -\frac{\mu}{(v_a^\infty)^2} + \sqrt{\frac{\mu^2}{(v_a^\infty)^4} + d_\infty^2}$$

Note that the impact parameter d_∞ is the only free parameter in the above.

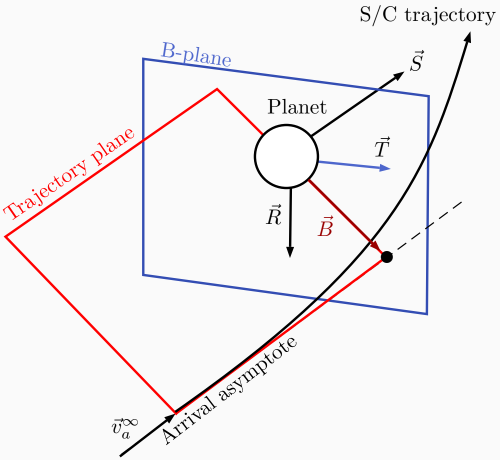
We also have: $\cos\beta = \frac{a}{a+r_p} = \frac{a}{c}$

B-Plane

The B-Plane is a planar coordinate system that allows easy targeting for the hyperbolic trajectory.

The impact parameter d_∞ is the B-vector which lies at the intersection of the B-plane and the trajectory plane. It shows where the hyperbolic asymptote intersects the B-plane.

$\vec{S}$ is parallel to the arrival asymptote, i.e. $\vec{v}_a^\infty$
$\vec{T}$ is normal to the planet and to $\vec{S}$ and typically chosen to be in the plane of the ecliptic
$\vec{R} = \vec{S} \times \vec{T}$



This is often generalised and used to compute a plane of reference for collision assessment around the Earth.

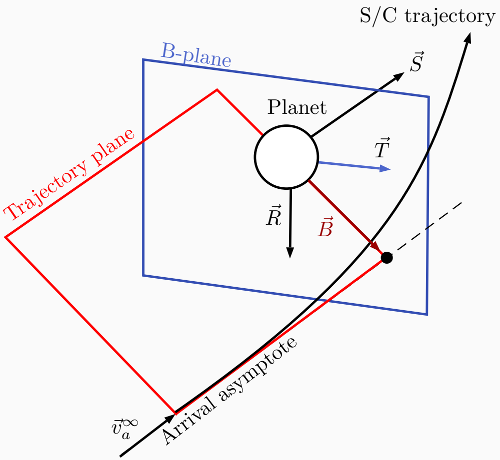
B-vector and mid-course corrections

$\vec{B} = b\hat{B} = d_\infty \hat{B}$, where b is the semi-minor axis of the arrival hyperbola, also called impact parameter.

The trajectory of the spacecraft is chosen to target the B-vector (described using $\vec{T}, \vec{R}$).

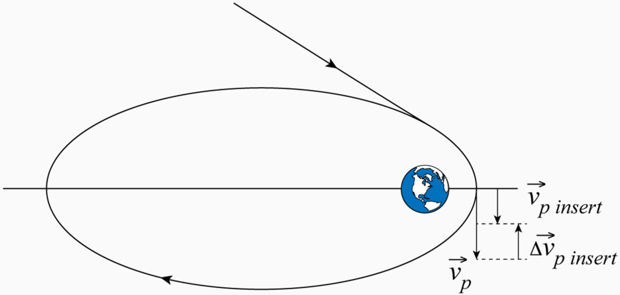
This is also used to compute the necessary mid-course corrections burns, ΔVs, while still in the heliocentric and elliptical transfer phase $\rightarrow$ Taylor expansion of the reference parameters (B_T, B_R)

$$\begin{bmatrix} \Delta B_T \\ \Delta B_R \\ \Delta t_{\text{Time of flight}} \end{bmatrix} = \left[\frac{\partial \vec{B}}{\partial \vec{V}}\right] \Delta \vec{V}$$



On the arrival hyperbolic orbit

The braking manoeuvre $\Delta\vec{v}_{p,\text{insert}}$ is the difference between the velocity at perigee of the elliptic orbit $\vec{v}_{p,\text{insert}}$ and the velocity at perigee of the hyperbolic orbit $\vec{v}_p$.



$$\left|\Delta\vec{v}_{p,\text{insert}}\right| = \left|\vec{v}_p\right| - \left|\vec{v}_{p,\text{insert}}\right|$$

$$= \sqrt{\frac{2\mu}{r_p} + (v_a^\infty)^2} - \sqrt{\frac{2\mu}{r_p} - \frac{\mu}{a_\text{insert}}}$$

where the subscript p stands for periapsis and a_insert is the semi-major axis of the elliptical orbit at insertion.

Aerodynamic braking manoeuvres

Aerodynamic braking manoeuvres

Aerocapture transfers the spacecraft from a hyperbolic approach trajectory to an elliptical orbit around the target planet. Further loss of energy will occur at every subsequent crossing of the periapsis (through aerobraking).

Aerobraking transfers the spacecraft from an initial elliptical orbit to a less energetic (i.e. lower apoapsis) elliptical orbit. Involves relatively small Δv.

Aeroentry transfers the spacecraft from either a hyperbolic, parabolic or elliptical approach orbit to the planet surface.

The 3 techniques use braking through the atmosphere of a planet for capture by the planet, or to change the trajectory, or cause a full entry in the atmosphere of the planet, to a touch down.

Aeroentry is followed by the deployment of a parachute (e.g. Apollo CM, Soyuz, Orion), or transition to atmospheric flight for a winged spacecraft (Shuttle).

A thermal shield is needed to avoid overheating during the braking manoeuvre.

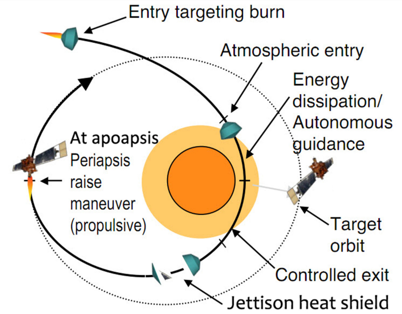
Aerocapture example

Aerocapture is a technique used to reduce velocity of a spacecraft, arriving at a body on a hyperbolic trajectory, in order to bring it in an orbit with an eccentricity of less than 1.

This approach requires significant thermal protection and precision closed-loop guidance during the manoeuvre.

Aerocapture has never been used in practice.

If the spacecraft is too close to the planet and the high-density layers of the atmosphere, with a higher Δv than expected, the entry into the planet may be uncontrolled.



Credits: NASA, In-Space Propulsion Technology (ISPT), Michelle M. Munk & Tibor Kremic, March 24, 2008

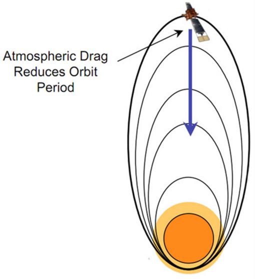
If the spacecraft is too high, it may have a too small Δv, and not be able to reach a velocity under the escape velocity. It may still escape the planet $\rightarrow$ a good knowledge of the atmosphere is important.

Aerobraking example

Aerobraking is a process that reduces the apoapsis of an elliptical orbit by flying the vehicle through the atmosphere at periapsis.

The resulting drag slows the spacecraft at periapsis which reduces the apoapsis.

Aerobraking is used when a spacecraft requires a low orbit after arriving at a body with an atmosphere.



Credits: NASA, In-Space Propulsion Technology (ISPT), Michelle M. Munk & Tibor Kremic, March 24, 2008

Example of the Apollo Command Module coming back from a 3 days journey from the Moon.

The velocity of entry of the Command Module with the three crew members on board in the high atmosphere was very high, about 11 km/s.

Credits: NASA

Rendezvous opportunities

Planetary phase angles

To determine the launch date, we need to know:

- The angular separtion of the departure and arrival planets, that is their phase angle
- The transit time, $T_{\text{Hoh}}/2$

As we assumed circular orbits for the planets,

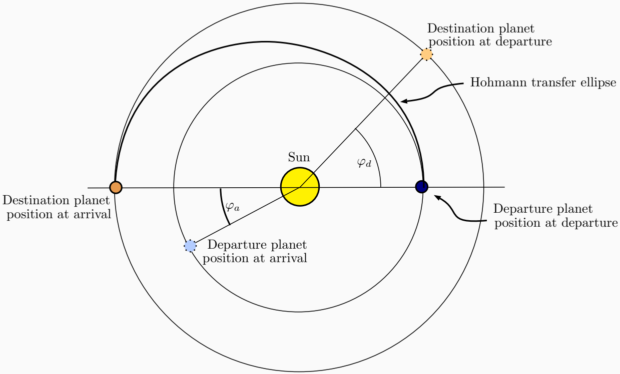
$$n = \frac{2\pi}{T}$$

The true anomaly, θ, is

$$\theta = \theta_0 + nt$$

w.r.t. a reference true anomaly θ_0. For the 2 planets,

$$\varphi = \theta_{aP} - \theta_{dP}|_t = \varphi_0 + (n_a - n_d)t$$



Synodic period

How long do I need to wait for the phase angle to become φ_0 again? This is the synodic period.

That is, $\varphi_0 = \varphi_0 - 2\pi$ if the departure planet has a larger orbital radius than at arrival ($+2\pi$ for the opposite case). We need to solve for T_{syn}

$$\varphi_0 \pm 2\pi = \varphi_0 + (n_a - n_d) T_{\text{syn}}$$

This yields:

$$T_{\text{syn}} = \frac{2\pi}{|n_a - n_d|} = \frac{T_d T_a}{|T_d - T_a|}$$

For Earth-Mars, the synodic period is 2.13 yr

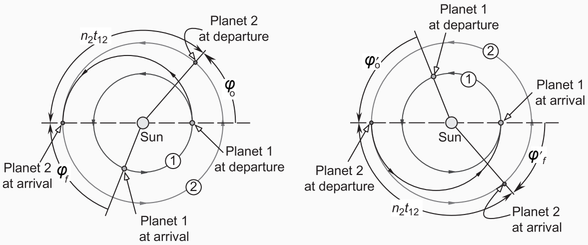
Phasing angle at departure and arrival

The period for the transfer is

$$T_{da} = \frac{\pi}{\mu_\odot}\left(\frac{R_d + R_a}{2}\right)^{3/2}$$

During T_{da}, arrival planet moves by $n_a T_{da}$.

The initial phase angle is $\varphi_d = \pi - n_a T_{da}$
The final phase angle of departure planet: $\varphi_a = \pi - n_d T_{da}$.



$\varphi_f = \varphi_a$, $\varphi_d = \varphi_0$, Credits: Curtis, *Orb. Mech. for Eng. Students*

At the start of the return trip, the relative phase angle is φ'_d.

As $T_{da} = T_{ad} \implies \varphi'_d = -\varphi_a$

Return and waiting time

The time required for the phase angle to reach its proper value is the waiting time t_{wait}. Starting the clock when the spacecraft arrives at planet 2,

$$\varphi = \varphi_a + (n_a - n_d)t$$

The waiting time is

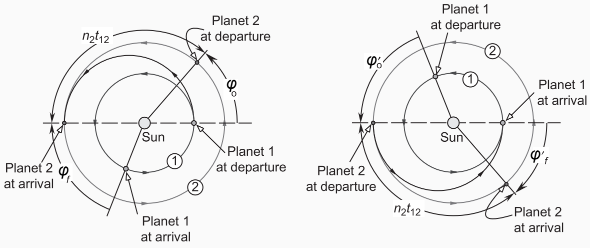
$$-\varphi_a = \varphi_a + (n_a - n_d)t_{\text{wait}}$$

Therefore

$$t_{\text{wait}} = \frac{-2\varphi_a - 2\pi N}{n_a - n_d} \quad \text{if } n_d > n_a$$

$$t_{\text{wait}} = \frac{-2\varphi_a + 2\pi N}{n_a - n_d} \quad \text{if } n_d < n_a$$

where $N = 0, 1, 2, \ldots$ is chosen to make t_{wait} positive.



$\varphi_a = \varphi_a$, $\varphi_d = \varphi_0$, Credits: Curtis, *Orb. Mech. for Eng. Students*

Example: min. waiting time for a Martian return trip to Earth

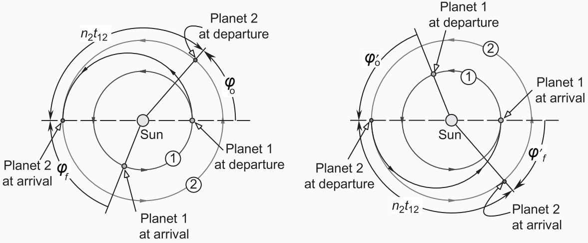
Assumption: Hohmann transfer

The transit time $T_{ad} = 258.8$ days

$$n_d = n_\oplus \approx 0.017'202 \text{ rad/day}$$
$$n_a = n_{\text{Mars}} \approx 0.009'133 \text{ rad/day}$$

At the end of the outbound trip, $\varphi_f = -75°$.

With $N = 1$, $\boxed{t_{\text{wait}} = 453.8 \text{ days}}$



$\varphi_f = \varphi_a$, $\varphi_d = \varphi_0$, Credits: Curtis, *Orb. Mech. for Eng. Students*

The total time for the round trip is
$t_{\text{roundtrip}} = T_{da} + t_{\text{wait}} + T_{ad} = 258.8 + 453.8 + 258.8 = 2.66$ yr

The phase angle for the outbound trip is $\varphi_d = \pi - n_{\text{Mars}} T_{ad} \approx 44.6°$, this occurs every synodic period, that is 2.13 yr.

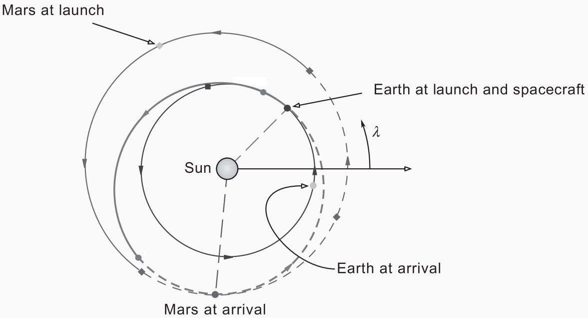
Non-Hohmann trajectories & departure opportunities

Lambert's problem, again

So far, Hohmann transfer was assumed, but non-Hohmann transfer are possible.

→ Solve Lambert's problem!

In this case, Mars Global Surveyor which was launched on 7 Nov 1996 took 309 days to reach Mars ($\sim$ 50 days more than Hohmann transfer).



Credits: Adapted from Curtis, *Orb. Mech. for Eng. Students*

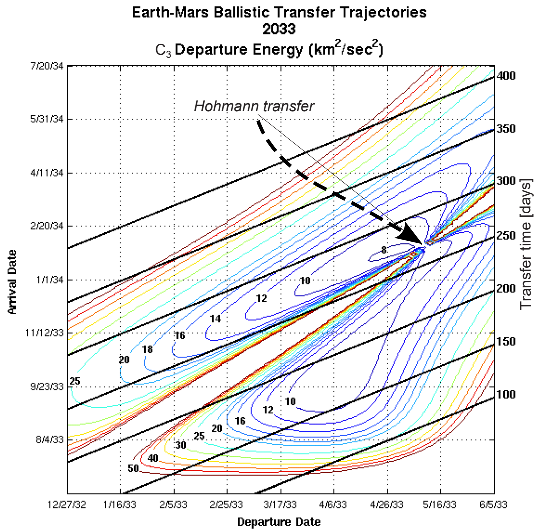
→ This gives some margin in the departure phase angle and thus in launch window.

Porkchop plots

Porkchop plots show contour lines of equal characteristic energy C_3 (in km/s).

They summarise all of the possible solutions to Lambert's problem.

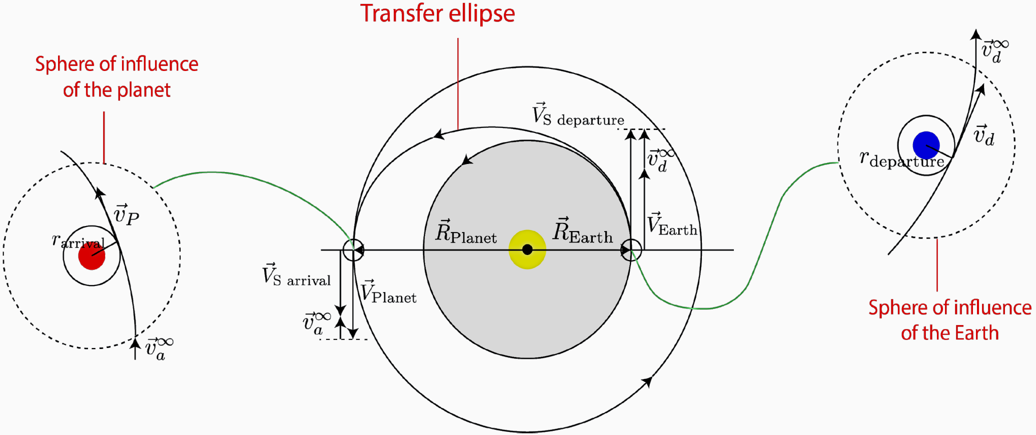
They allow to evaluate whether a launch window exists for any given time.



Credits: NASA

Summary

Strategy for interplanetary transfer



Data for interplanetary trajectories departing Earth

		Venus	Mars	Jupiter	Saturn
Semi-major axis of the planet	R_a [AU]	0.723	1.523	5.204	9.554
Sidereal period of the planet	T_a [years]	6.615	1.880	11.865	29.531
Periodicity of the Hohmann transfer	T_{syn}[years]	1.596	2.137	1.092	1.035
Transfer orbit periapsis radius	R_p [AU]	0.723	1	1	1
Transfer orbit apoapsis radius	R_a [AU]	1	1.523	5.204	9.554
Duration of the Hohmann transfer	t_{Hoh} [years]	0.399	0.708	2.730	6.061
Heliocentric Earth departure velocity	V_D [km/s]	27.29	32.73	38.58	40.08
Earth departure excess velocity	v_d^∞ [km/s]	−2.50	2.94	8.79	10.29
Arrival helicentric velocity	V_a [km/s]	37.74	21.49	7.42	4.19
Destination planet heliocentric velocity	V_P [km/s]	35.02	24.13	13.06	9.64
Arrival excess velocity	v_a^∞ [km/s]	2.71	−2.65	−5.64	−5.45

Interactive quiz

→ EchoPoll platform

- You can scan a QR code or go to the link
- EchoPoll is the EPFL-recommended solution
- You do not have to register, just skip entering a username and/or email address