EE-585 – Space Mission Design and Operations

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Today’s outline

Orbital manoeuvres: Hohmann transfer and plane change

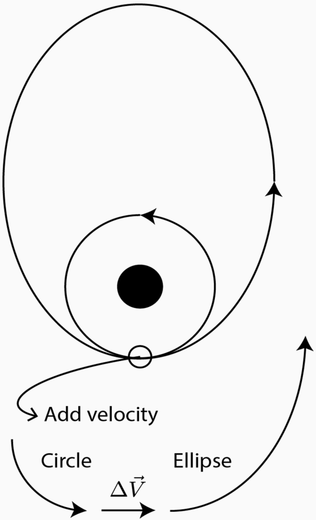
The Earth viewed from space and inversely

Orbit determination

Positioning & station-keeping

Orbital manoeuvres: Hohmann transfer and plane change

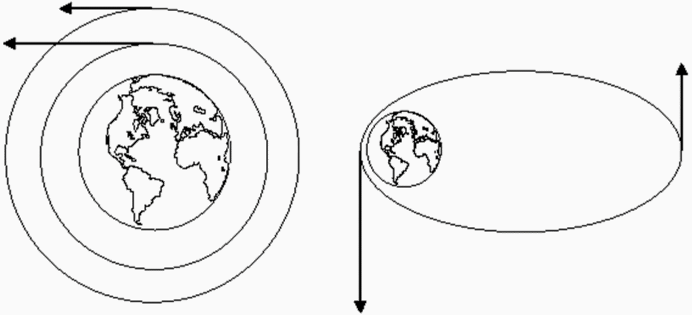
Manoeuvres in-orbit



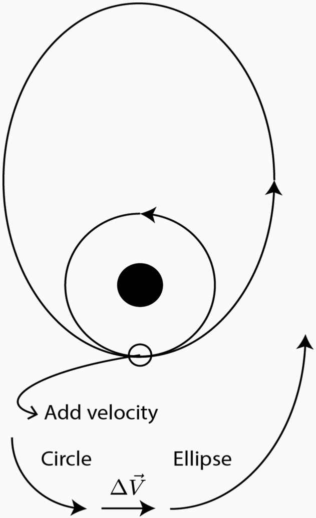
- A manoeuvre in orbit is an application of external force on the spacecraft $\rightarrow$ vectorial change of the velocity vector $\Delta\vec{v}$ causing a change of the orbital elements.
- We will use indifferently the terms manoeuvre, “Delta-V” or “Burn”.
- We will focus first on instantaneous manoeuvres, that is the Δv occurs during a very short time frame Δt, that is $\Delta t \ll T$ the orbtial period.
- Continuous manoeuvres can exert a thrust on the spacecraft for long period of time.

Reminder: orbital velocity and altitude

- As orbital altitude increases, $z \nearrow$, orbital period increases , $T \nearrow$, and orbital velocity decreases, $v \searrow$.
- Velocity is greatest at perigee and least at apogee.



Single impulse orbital change



Assume a satellite is on a circular orbit at $z_i = 200$ km. The operator wants to change an elliptical orbit of $z_i = z_p = 200 \times z_a = 500$ km.

Initial velocity $v_i = \sqrt{\frac{\mu}{R_\oplus + z_i}} \approx 7780$ m/s

Semi-major of the elliptical orbit $a = R_\oplus + 350$ km

Vis Viva equation $v = \sqrt{\frac{2\mu}{r} - \frac{\mu}{a}}$:

$$v_f(z_p) = \sqrt{\frac{2\mu}{R_\oplus + z_p} - \frac{\mu}{a}} \approx 7870 \text{ m/s}$$

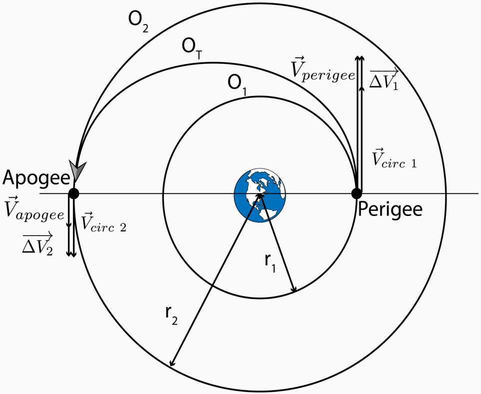
Velocity differential Δv:

$$\boxed{\Delta v = v_f(z_p) - v_i} \approx 90 \text{ m/s}$$

Posigrade Δv to go to a higher orbit.

Hohmann Transfer

The Hohmann transfer is a very common method of transfer from one circular orbit to another, around the same central body. The transfer orbit is tangent to both the initial orbit and the destination orbit.

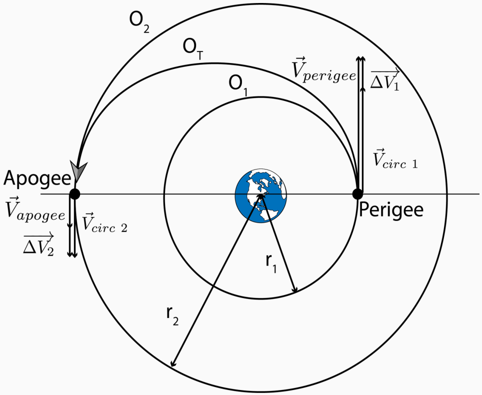


Again, let's start with the Vis Viva equation:

$$
\begin{aligned}
v_{\text{perigee}} &= \sqrt{\frac{2\mu}{r_1} - \frac{2\mu}{r_1 + r_2}} \\
&= \sqrt{\frac{2\mu(r_1 + r_2) - 2\mu r_1}{r_1(r_1 + r_2)}} \\
&= \sqrt{\frac{2\mu r_2}{r_1(r_1 + r_2)}} \\
\implies \Delta v_{\text{perigee}} &= \sqrt{\frac{2\mu r_2}{r_1(r_1 + r_2)}} - v_{\text{circ},1}
\end{aligned}
$$

Similarly, $v_{\text{apogee}} = \sqrt{\frac{2\mu r_1}{r_2(r_1+r_2)}} \implies \Delta v_{\text{apogee}} = v_{\text{circ},2} - v_{\text{apogee}}$

Hohmann Transfer



First burn: increase of the apogee

$$\Delta v_1 = \sqrt{\frac{2\mu r_2}{r_1(r_1 + r_2)}} - \sqrt{\frac{\mu}{r_1}}$$

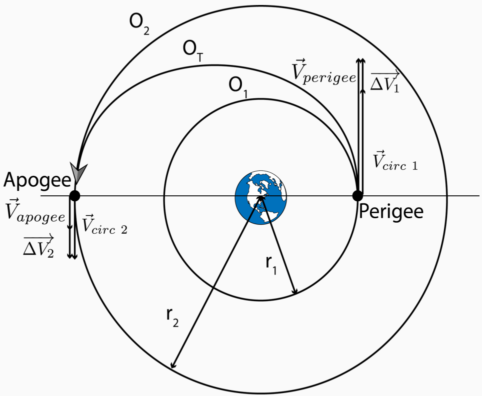
and second burn: circularisation

$$\Delta v_2 = -\sqrt{\frac{2\mu r_1}{r_2(r_1 + r_2)}} + \sqrt{\frac{\mu}{r_2}}$$

The two Δvs are posigrade for a transfer to a higher orbit, and retrograde for a transfer to a smaller orbit.

The Hohmann transfer is the most efficient transfer because the changes in velocity are used entirely for changes in kinetic energy.

Transit time and energy considerations



The transit time from the lower orbit to the higher circular one is

$$t_{\text{transit}} = T_T/2 = \pi\sqrt{\frac{a_T^3}{\mu}} = \pi\sqrt{\frac{(r_1 + r_2)^3}{8\mu}}$$

The Hohmann transfer is the most efficient transfer with 2 burns.

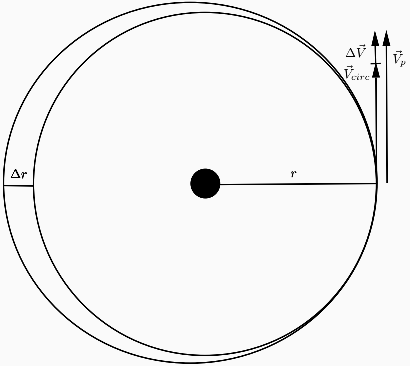
Reminder: the specific energy of an orbit is

$$\epsilon = -\frac{\mu}{2a}$$

So, the energy of the orbit increases $\epsilon_1 < \epsilon_T < \epsilon_2$ from its initial to final orbit because $r_1 < a_T = \frac{r_1+r_2}{2} < r_2$.

The energy remains $\epsilon < 0$ (bound to the Earth) but approaches 0.

Hohmann Transfer – Case of small Δv



If Δr is small, e.g. $\Delta r / r < 10^{-3}$, we can linearise the equation for Δv.

The semi-major axis of the elliptical orbit is

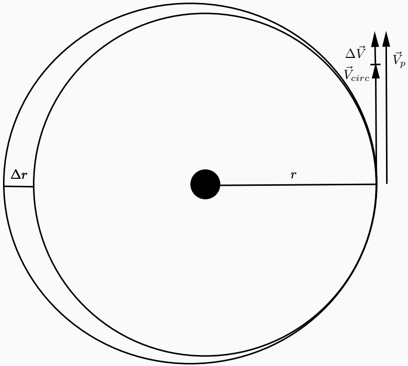
$$a = \frac{r + (r + \Delta r)}{2} = r + \frac{\Delta r}{2}$$

The velocity at perigee of the elliptical orbit is

$$v_p = \sqrt{\frac{2\mu}{r} - \frac{\mu}{a}} = \sqrt{\frac{\mu}{r}}\sqrt{2 - \frac{1}{1 + \frac{\Delta r}{2r}}} \approx \underbrace{\sqrt{\frac{\mu}{r}}}_{= v_{\text{circ}}}\sqrt{2 - \left(1 - \frac{\Delta r}{2r}\right)} \approx v_{\text{circ}}\sqrt{1 + \frac{\Delta r}{2r}} \approx v_{\text{circ}}\left(1 + \frac{\Delta r}{4r}\right)$$

using $\sqrt{1 + x} \approx 1 + \frac{x}{2} \; \forall x | x \ll 1$

Hohmann Transfer – Case of small Δv



Recalling that $v_p \geq v_{\text{circ}}$, for a slightly elliptical orbit we have $v_p = v_{\text{circ}} + \Delta v$,

$$\frac{v_p}{v_{\text{circ}}} = \frac{v_{\text{circ}} + \Delta v}{v_{\text{circ}}} \approx 1 + \frac{\Delta r}{4r} \implies \boxed{\frac{\Delta r}{r} \approx 4\frac{\Delta v}{v}}$$

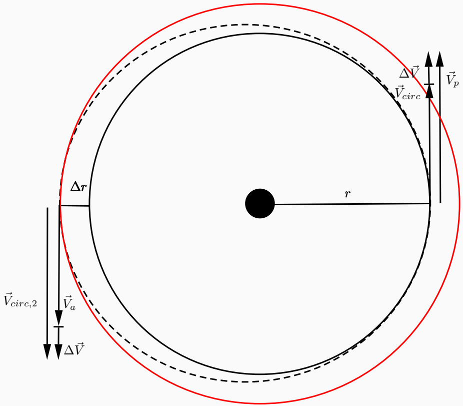
For LEO, where $r \sim R_\oplus + 500$ km and $v \sim 7.7$ km/s,

$$\boxed{\Delta r \approx 3.5\Delta v}$$

LEO approximation

with Δr in km and Δv in m/s.

Full transfer from one circular orbit to another



For the full transfer from black to red orbit is twice the LEO approximation:

$$\Delta v_{\text{total}} \approx 2\Delta v = 2 \times \frac{\Delta r}{r} \times \frac{v}{4} = \frac{1}{2}\frac{v}{r}\Delta r \approx 0.57\Delta r_{\text{circ}}$$

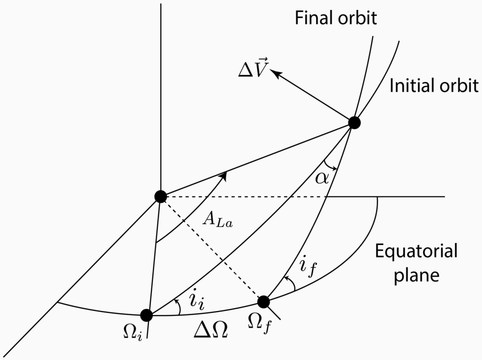
or, rearranging to get Δr_{circ},

$$\boxed{\Delta r_{\text{circ}} \approx \frac{3.5}{2}\Delta v_{\text{total}}}$$

with Δr in km and Δv in m/s.

Change of orbital plane

$$\Delta\vec{v} \rightarrow \alpha \rightarrow \Delta\Omega \leftrightarrow \Delta i$$



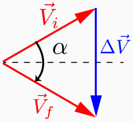
A manoeuvre with an out-of-plane component, typically a $\Delta\vec{v}$ perpendicular to the orbital plane will cause a change of several orbital parameters, in particular Ω (RAAN) and i.

As illustrated here, the $\Delta\vec{v}$ perpendicular to $\vec{v}$ causes a change of direction α in the instantaneous velocity vector. Changes in Ω and i involve spherical trigonometry calculations which will not be detailed here.

An orbital plane change is best performed at equator crossing (simplicity and efficiency).

ΔV needed for a change of orbital plane

The simplest way to perform a plane change is to burn at one of the two crossing points of the initial and final planes.



$$|\vec{V}_i| = |\vec{V}_f|$$

Δv can be computed by the law of cosines

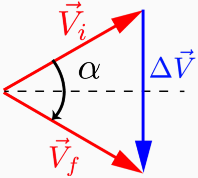
$$\Delta v = \sqrt{v_i^2 + v_f^2 - 2 v_i v_f \cos\alpha}$$

if the magnitude of the velocity don't change (i.e. pure rotation of the the orbital plane), no other orbital parameter change:

$$\Delta v = v\sqrt{2(1-\cos\alpha)} = v\sqrt{2 \cdot 2\sin^2\left(\frac{\alpha}{2}\right)}$$

using the trigonometric identity $\cos\alpha = 1 - \sin^2\frac{\alpha}{2}$

ΔV needed for a change of orbital plane



$$|\vec{V}_i| = |\vec{V}_f|$$

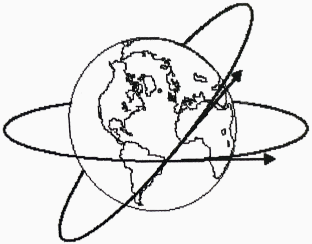
Change of orbital plane best done at equator crossing $\rightarrow$ no change to other orbital parameters

$$\boxed{\Delta v = 2 v_i \sin\left(\frac{\alpha}{2}\right)}$$

In LEO, $v \sim 7.7$ km/s $\implies$ plane change with an out-of-plane $\Delta\vec{v}$ is expensive!
The specific energy does not change with pure plane change manoeuvres.

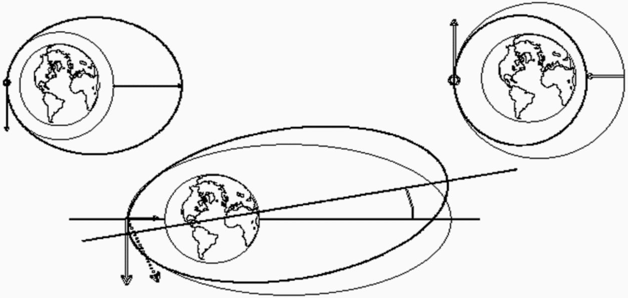
Effects of out-plane burn on the orbit

- Plane change must be performed at intersection of two orbits (at the nodal crossing).
- There are 2 opportunities per orbit.
- Plane change requires a large amount of propellant compared to in-plane manoeuvres.



Effects of in-plane burns on the orbit

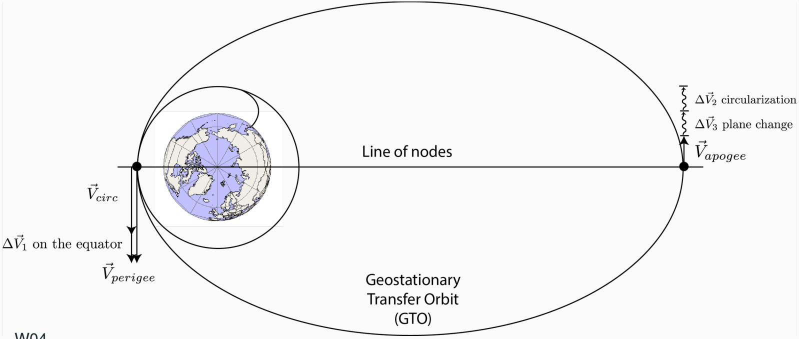
- **Posigrade** burns **increase** altitude 180° from the burn point.
- **Retrograde** burns **decrease** altitude 180° from the burn point.
- **Radial** burns **shift** the semi-major axis without significantly altering other orbital parameters.



Strategy to reach the geostationary orbit

A satellite bound for GEO is usually launched into an inclined parking orbit in LEO or directly on its transfer orbit. E.g. $i = 7^\circ$ for Kourou in French Guiana and 28.5° from Flordia.
Three burns are required to reach the GEO conditions:

1. A first Hohmann burn to get into the transit orbit.
2. A second Hohmann burn to circularise the orbit in GEO.
3. A plane change manoeuvre to bring i to 0.



Reminder, for GEO:

$z \approx 36'000$ km

$i = 0^\circ$

$e = 0$

Δv savings with a combined manoeuvre (1/2)

When should the plane changed be performed?

There are 3 options:

1. First, make the plane change and then Hohmann transfer.
2. A plane change at apogee and then a circularisation.
3. A combined manoeuvre that changes both the plane and circularises at apogee.

Δv savings with a combined manoeuvre (1/2)

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1. First, make the plane change and then Hohmann transfer.
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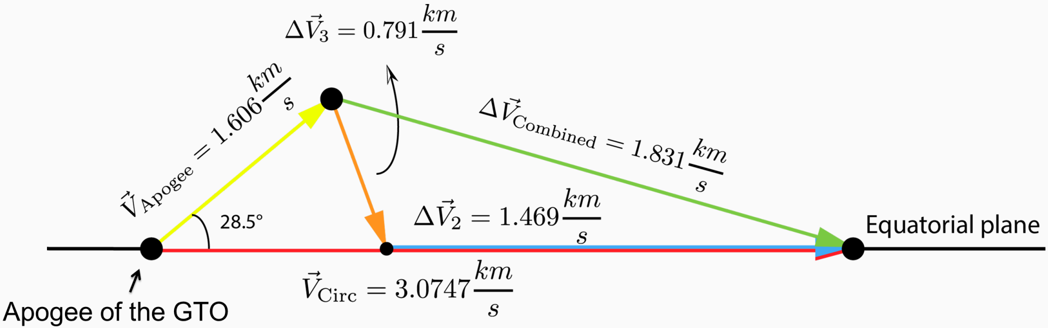
3. A combined manoeuvre that changes both the plane and circularises at apogee.

$\rightarrow$ since the plane change is $\Delta v = 2 v_i \sin\left(\frac{\alpha}{2}\right)$ and at apogee, $v_{\text{apo}} < v_{\text{peri}}$, it is better to change the orbital plane high in the gravitational well, when v is low.

$\rightarrow$ Not option 1

Example: combined maneuver at the apogee of the transfer orbit for insertion into GEO for a launch from Kennedy Space Center, Florida (Lat. 28.5°).

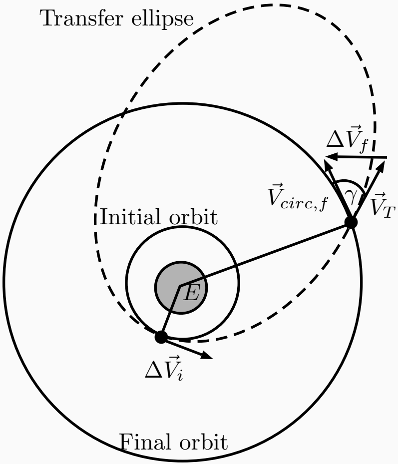


Separate manoeuvres: $|\Delta \vec{v}_{\text{tot, apo}}| = |\Delta \vec{v}_2| + |\Delta \vec{v}_3| = 2.26$ km/s

Combined manoeuvres: $|\Delta \vec{v}_{\text{tot, apo}}| = |\Delta \vec{v}_2 + \Delta \vec{v}_3| = 1.83$ km/s

$\rightarrow$ Option 3: a combined manoeuvre at apogee is more efficient.

One-tangent burn manoeuvres



If the transit time needs to be shorter than the Hohmann transfer or the satellite should intercept the target orbit at a given true anomaly of the second orbit, then one-tangent burn is the solution.

Recipe to analyse the one-tangent manoeuvre:

- Compute e_T and a_T, eccentric anomaly E

$\rightarrow$ Get the time of flight

- Compute $v_{T,\text{peri}}$ of the transfer orbit and $v_{i,\text{circ}}$

$\rightarrow$ Get $\Delta\vec{v}_i$

- Compte the flight path angle γ, v_T, $v_{f,\text{circ}}$

$\rightarrow$ Get $\Delta\vec{v}_f$

The time of flight might be shorter, but $|\Delta\vec{v}_i| + |\Delta\vec{v}_f| > \Delta V_{\text{Hohmann}}$

Oberth effect or the powered flyby effect

The impact of a burn Δv on the specific energy depends on the instantaneous velocity v.

The increase in kinetic energy

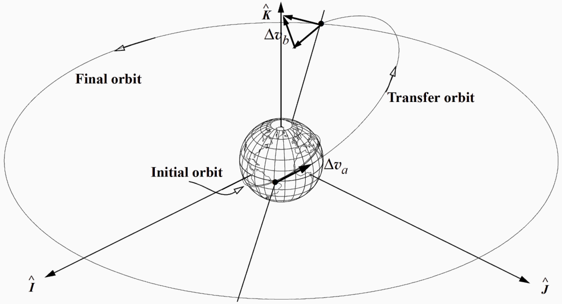
$$\Delta \mathrm{KE} = (v + \Delta v)^2/2 - v^2/2$$

is largest when v is largest (thus at periapsis, i.e. deep in the gravitational well).

In certain cases, it might be worth doing a 3-burn manoeuvre to go from a low circular orbit to a very high circular orbit as the total Δv spent is lower than a Hohmann transfer $\rightarrow$ bi-elliptic manoeuvres.

In planetary flybys, this effect can give a very significant boost (! $\neq$ slingshot manoeuvres).

Lambert's problem: finding complex transfer trajectories



Credits: Vallado, *Fundamentals of Astrodynmaics and Applications*, 4th edition

To change multiple orbital parameters with a transfer orbit (combined manoeuvre or rotation of line of apsides, RAAN change, . . .), the best is to solve the so-called Lambert problem.

The classical formulation of Lambert's problem finds the orbit between two points (a, b) as a function of the time of flight.

For transfer orbits, the Δv_a and Δv_b can be computed from the expected initial and final velocity vectors.

Lambert's problem can find the minimum-energy solution. It also handles multiple revolution on the transfer orbit.

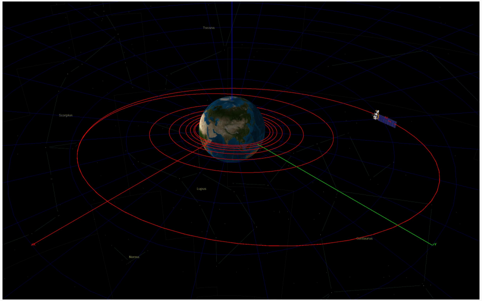
Many solutions (and algorithms) to the Lambert's problem exist.

A few remarks on orbital manoeuvres

1. A Hohmann transfer is the most efficient 2-burn manoeuvre.
2. Plane change...
   - are costly, avoid if possible.
   - should be made at equator crossing to rotate the plane only.
   - should be made at apogee (you might consider raising the apogee before).
3. Make use of the perturbations to change the parameters of the orbit.
4. On a very non-circular orbit ($e \neq 0$), the most efficient way to change the energy of the orbit is to make a Δv at perigee.

A quick outlook: Continuous manoeuvre

If the Δv is not instantaneous, but continuous, we have an additional acceleration to the gravitational force in the budget of forces $\rightarrow$ the trajectory is a spiral. The distance between each rotation will depend on the type of acceleration imparted to the spacecraft.

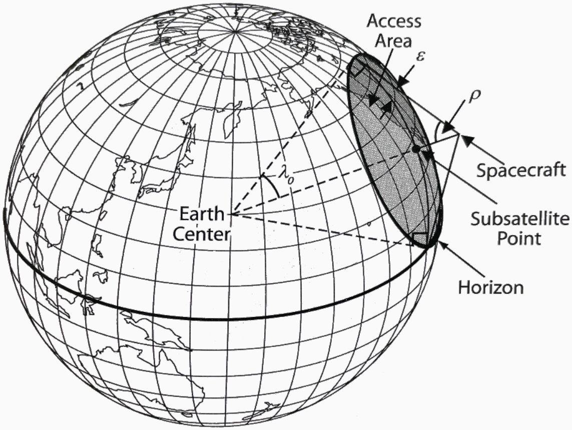


This approach can be used for propulsions that have a continuous thrust.

It's an approximation of the impact of the drag on the orbit (although here it is not to scale and the drag depends on the instantaneous distance to the centre of the Earth.

The Earth viewed from space and inversely

Earth geometry (1/3)



Credits: SMAD

What is the angular radius, ρ, of the (spherical) Earth as soon from a satellite?
The line from the satellite to Earth's horizon is $\perp$ to Earth's radius, thus

$$\sin \rho = \cos \lambda_0 = \frac{R_\oplus}{R_\oplus + z}$$

where λ_0 is the angle from the satellite's horizon to the spacecraft as measured from the Earth's centre.

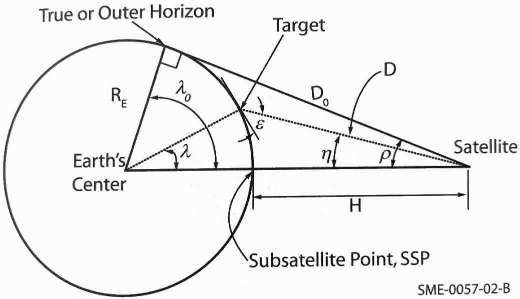
Altitude [km]	400	500	800	1000	2000	20'000	36'000	Moon (400'000)
ρ [deg]	70	58	63	60	50	14	8.7	1.0

Earth geometry (2/3)

Maximum distance to the horizon D_{max} is

$$D_{\text{max}} = \sqrt{(R_\oplus + z)^2 - R_\oplus^2} = R_\oplus \tan \lambda_0$$

That might be fine for some applications, but Earth is non-spherical, so more complex modelling might be required (or numerical computations).



Credits: SMAD

In most applications, we cannot look at/communicate with satellite at too low elevation angle ε because there might be terrain (e.g. mountains) or too much loss due to the atmosphere.

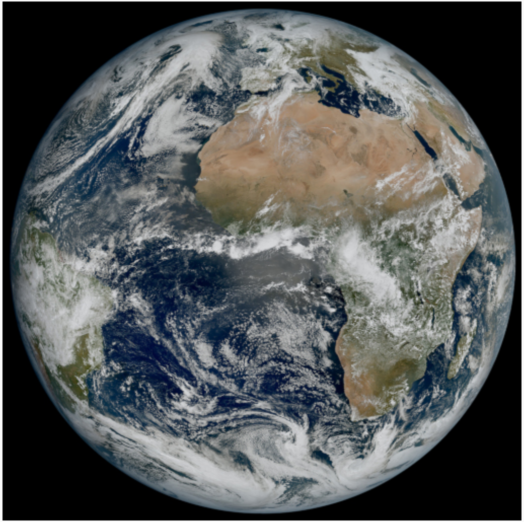
$$\sin \eta = \cos \varepsilon \sin \rho$$

$$\eta + \lambda + \varepsilon = 90^\circ$$

$$D = R_\oplus \sin \lambda / \sin \eta$$

Earth geometry (3/3)

ISS seen from Space Shuttle Endeavour on 20 Feb 2010 Credits: NASA



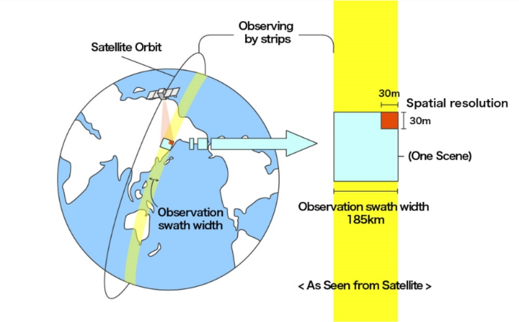
First EUMETSAT MTG i1 image on 31 Mar 2023

Credits: EUMETSAT

Swath

Typical ε_{min} to communicate with a satellite is $5°$.

For Earth observation, $\varepsilon_{\text{min}} \sim 40 - 50°$.



Credits: www.restec.or.jp

The minimum elevation angle, ε_{min}, defines the maximum swath, that is the maximum area a satellite can observe.

It is often given as a width (in km) of, e.g. the largest possible image.

Apparent motion of satellite from the Earth's surface in LEO

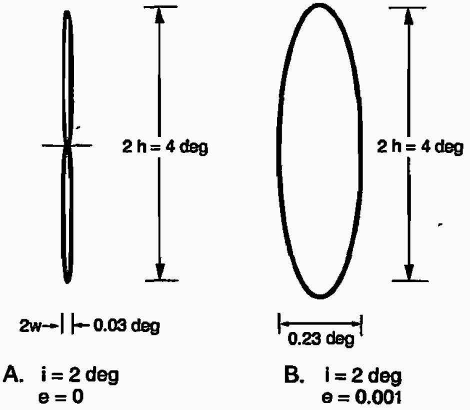
Even for a spherical Earth and circular orbit, the apparent motion of a satellite across the sky is not simple to describe.

In circular LEO (such that the rotation of the Earth during the satellite's transit can be neglected), this is possible to compute the maximum angular rate $\dot{\Theta}_{max}$ at which the satellite moves with respect to the observer. Typically $\dot{\Theta}_{max} \sim 10 - 15^{\circ}/\text{min} \rightarrow$ this is fast for tracking.

Time in view of the station depend (heavily) on ε_{min} and how far the observer is far from the subsatellite ground track. For LEO, the useable pass duration is $\sim 7 - 12$ minutes.

$\rightarrow$ For general solutions, use simulations. Naive algorithm: predict the position for the observer and satellite and compare to the terrain around the observer.

Apparent motion of satellite from the Earth's surface in GEO



Credits: SMAD

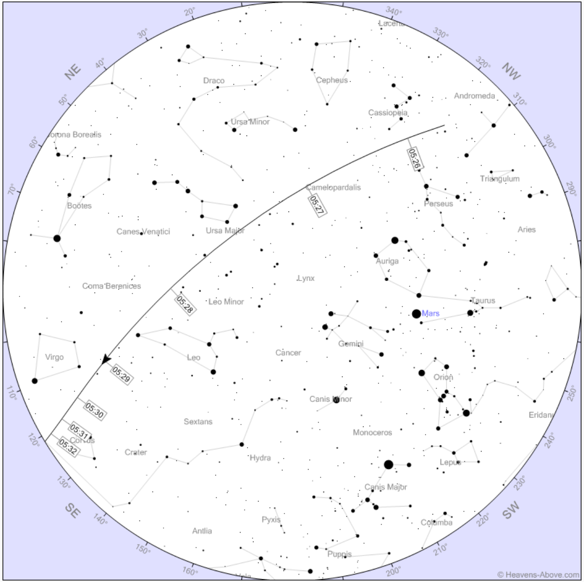
At satellite in GEO is fixed with reference to the star $\rightarrow$ rotation period = sidereal period.

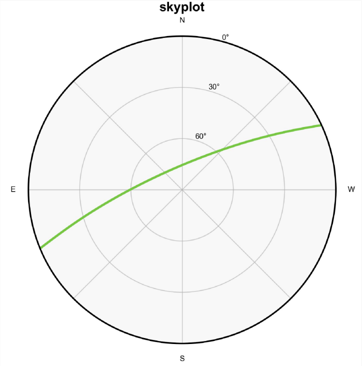
Small variations may occur due to the observer's location.

Non-perfect GEO conditions (i.e. $e, i \neq 0$) $\implies$ apparent motion in latitude (i) and longitude (e and i).

The "slot" in GEO are agreed at the international level by the ITU (see next lectures) $\rightarrow$ size of slot in longitude $\pm 0.1^\circ$.

Apparent motion of satellite: sky plot of a pass





A skyplot shows the trajectory of a satellite across the sky as viewed from an observer on the surface.
Note that the apparent motion across the sky is not uniform.

Orbit determination

Keeping track of a satellite's position

Knowing the state vector of the satellite to a good enough accuracy is crucial for mission operations

- pass prediction
- observability of targets
- timing of burns
- relative motion with respect to other satellites (rendezvous or avoidance $\rightarrow$ next lecture)

The orbit parameters determined at time t_0 can be propagated to some future (or past) time t_1, but there are errors in the perturbations and the initial knowledge of the orbit $\rightarrow$ need a measurement, this is orbit determination.

We will start by focussing on ground-based measurements before looking at space-based methods.

Initial orbit determination

Initial orbit determination describes the process of estimating an orbit or state based solely on measurements and without a priori information.

You have a set of measurements of the position and its epoch of an object. (Initial) orbit determination algorithms transform the measurements into a representation of the trajectory, i.e. orbital parameters.

Initial orbit determination is the same for asteroids, planets, satellites, but does not model orbit perturbations.

There are many possible algorithms that solve slightly different problems. It mainly depends on the observables.

1. 2 or 3 position vectors, $\vec{r}_i$ with their epoch
2. 3 angular positions, $(\alpha_i, \delta_i, t_i), i = 1, 2, 3$ (α right ascension, δ declination)

The position of the site must be known to a good accuracy. Not accounting for the Earth's equatorial bulge results in errors on the order of 15 km!

Angle-only methods

This is the most basic information: α right ascension, δ declination at different time, but no range information (i.e. distance).

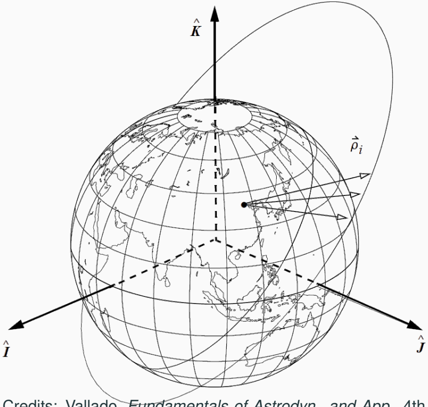
The methods require 3 sets of angular measurements to provide the six independent quantities required for an orbit.

There are several methods: Laplace, Gauss, double-r iteration, Gooding, ...
The most commonly used is Gooding.

The methods may need several iterations before converging. Some methods tend to output biased parameters (e.g. hyperbolic orbits instead of elliptical)

Angle-only methods are still commonly used, because optical observations of satellite do not yield the range!

Gibbs Method from three position vectors



Credits: Vallado, *Fundamentals of Astrodyn. and App.*, 4th edition

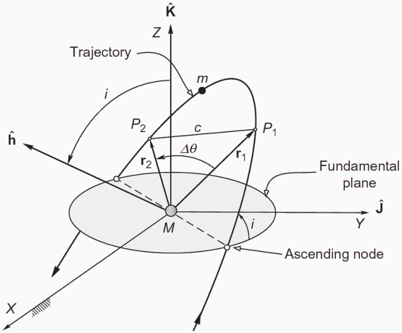
Suppose that three observations are made of an objects that yield the geocentric position vector, $\vec{r}_1, \vec{r}_2$ and $\vec{r}_3$ at t_1, t_2, t_3.

Gibbs method computes $\vec{v}_1, \vec{v}_2, \vec{v}_3$. This method is solely based on vector analysis.

The Keplerian parameters can be computed from any state vector $(\vec{r}_i, \vec{v}_i, t_i)$.

The conservation of angular momentum $\Longrightarrow$ all position vectors lie in the same plane and by performing vector analysis only you get an expression for $\vec{v}_i = f(\vec{r}_1, \vec{r}_2, \vec{r}_3)$.

Lambert's problem, revisited



Credits: H. Curtis, *Orb. Mech. for Eng. Students*, 4th Ed.

Suppose that 2 observations are made of an objects that yield the geocentric position vector, $\vec{r}_1, \vec{r}_2$ at t_1, t_2. What is the orbit?

This is exactly the same problem we encountered to find a transfer trajectory between two arbitrary orbits.

The solution space includes all orbit types, and the elliptical transfers may also include multi-revolution cases.

Ground observatories: radar, satellite laser ranging

Radar data return angle and range data.

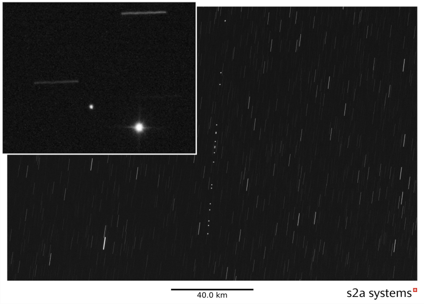
Main limitation is distance to objects $\rightarrow$ best for LEO

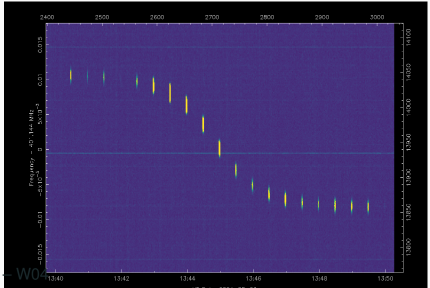
Credits: LeoLabs

SLR is measures the two-way time-of-flight Δt of ultra-short laser pulses emitted from a ground station (GS) to a satellite, which ideally has retro-reflectors, and reflected back to the GS.

Angle and range data, but SLR needs good a priori knowledge. Credits: UNIBE

Ground observatories: optical observations and Doppler





Optical images return angle only data.

Typical observation techique: sky staring and detecting fast moving objects in a series of images.

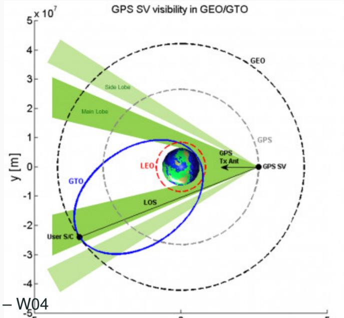
Credits: s2a Systems, 1st batch of G60 Chinese SATCOM tracked

Doppler effect on the frequency of the radio signal received by a ground station. $\rightarrow$ works only with active satellites.

Spacecraft-based methods: GNSS, space-based observatories

Some satellites can observe other objects to compute the relative position and generate precise orbits. Better observations because no atmosphere.

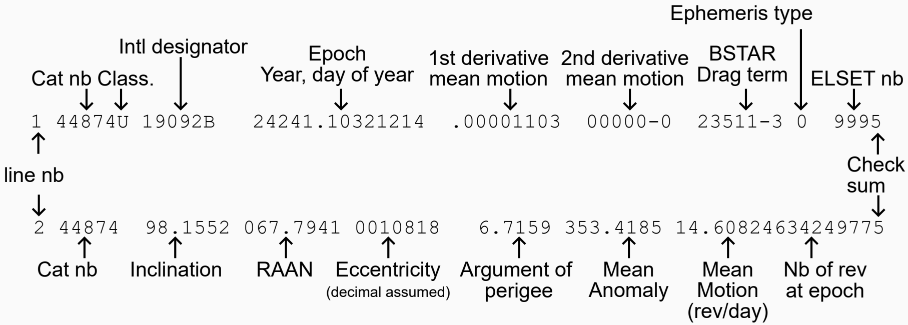
Credits: Vyoma



GNSS signals can be used to triangulate the position (and with a few points speed) of an object. Has been shown to work up to GEO. Could be pushed to the Moon.

Credits: ESA

Two-line element set (ELSET) and other formats



The two line element set (TLE, 2LE, sometimes 3LE with an initial comment line) are the most common way of disseminating the orbital data.

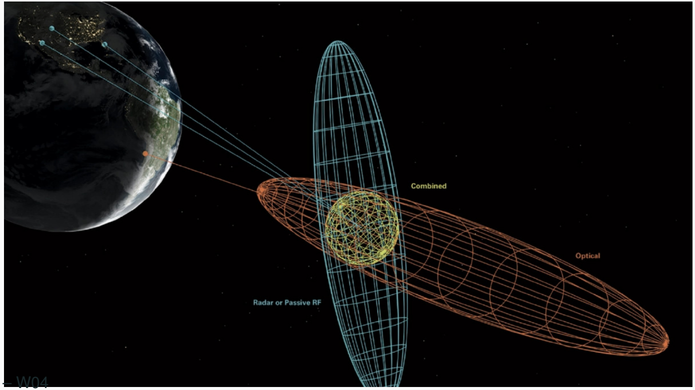
Precision limited by the format and there is a limit on the epoch (`YYDDD.fffffff`), no indication on the errors.

Other formats: Orbit Mean-Element Message (OMM, json/xml extension of TLE), ephemeris (state vectors), comprehensive with covariances or additional information.

Orbit determination & data fusion

Data from the different sensors can be processed independently or together ("data fusion"). This requires an analysis of the different sources of errors and a rigorous combination. Not easy.

Fusing the data once the orbital data is disseminated is also possible → active area of research.



Credits: AGI Orbit Determination Tool Kit (ODTK)

Location of the observatories

The number of observatories is exploding, but the number of objects is too. Orbit determination (and error reduction) is demanding hardware and processing capacity.

Observatories are located all over the World. High-North stations are interesting because of the many SSO passes and relative proximity to population centres.

There is a hole in Central Asia and South-East Asia.

Orbit determination is needed for safe space mission operations (for manoeuvring but also for collision avoidance).

Positioning & station-keeping

Launch and early orbit phase (LEOP)

$\rightarrow$ *launch will be discussed in a later lecture*

LEOP (also "commissioning") starts after separation with the launch vehicle. It contains – but is not limited to – solar panel deployment, system test and checks, "first light" and calibration of instruments and positioning to the final operating orbit.

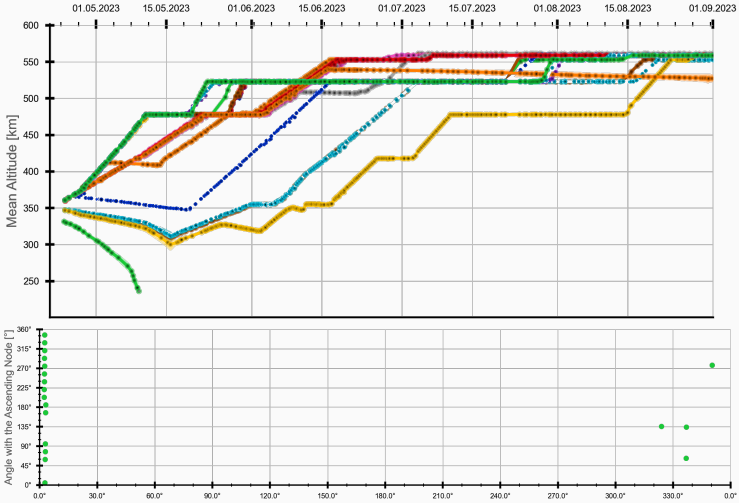
This is a critical phase of the mission. The operating team works on an extended schedule and there might be additional contacts with the ground stations.

LEOP, especially with extended orbital changes, can be long. Typically weeks to months.

Going from LEO to GEO on a Geo-transfer orbit (GTO) is an example of positioning.

End of mission disposal (orbit raising to graveyard orbit or, e.g., perigee lowering) are part of a dedicated end of life phase.

Example: Starlink group 6 batch 2



Data: Space-Track.org

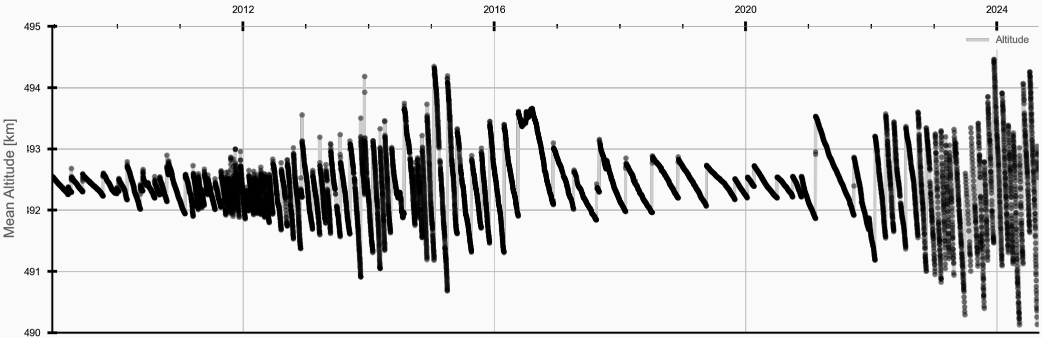
21 satellites launched 19 Apr 2023. Injected at $\sim$ 300 km in a train – all at the same time. Positioning needs to:

- Check that the platform is fine
- Check that the payload is fine
- Raise the orbit to $\sim$ 550 km
- Distribute the satellites in the orbital plane (using a differential RAAN drift)

1 failed early (light green), a second one at the end of the positioning manoeuvre (orange).

Station-keeping in LEO

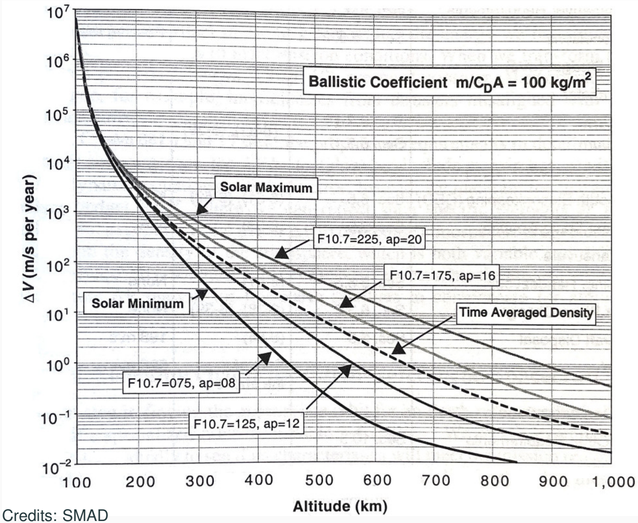
The Δv budget needed for station-keeping – that is maintaining the orbit within the operational parameters – depends on the mission objectives, altitude and the Sun's activity.



WorldView 1 (optical Earth observation) operated by Maxar – Data: Space-Track.org

The example above shows a long-lived satellite that has almost gone through two solar cycles. Both the amplitude, Δa, and the separation between the manoeuvres, Δt, are varied to keep the satellite at $z \approx 492.5$ km.

Typical station-keeping in LEO



Credits: SMAD

The Δv budget is given in m/s/yr to compensate for atmospheric drag.

For other BCs, $\sim \Delta v \cdot \text{BC}/100 \rightarrow$ if BC is $10\times$ lower, you need $10\times$ the Δv.

The main problem in LEO is to predict the solar activity.

Expect much longer lifetime if you launch in early quiet activity period.

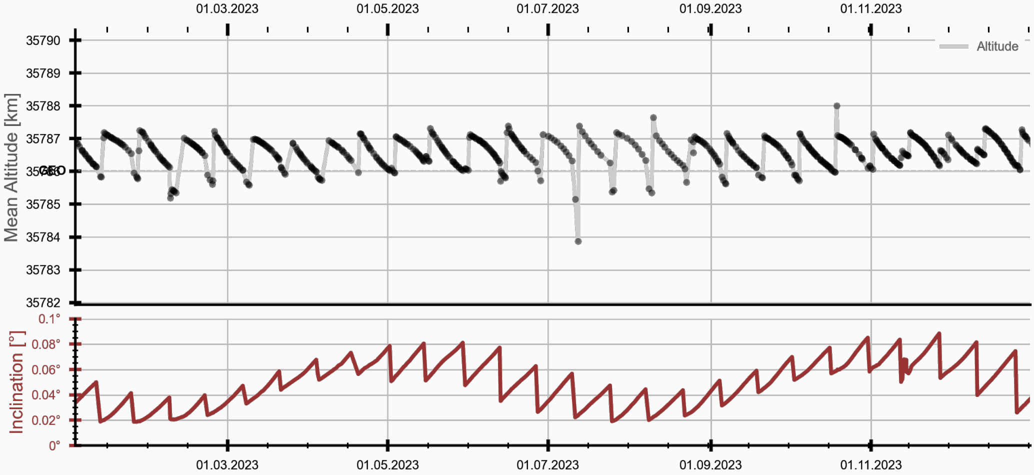
Station-keeping in GEO

In GEO, frequent station-keeping manoeuvres are required to maintain the longitudinal slot. There are two main effects:

- lunar & solar perturbations lead to $\Delta i \sim 0.85°$/year
  $\implies \sim$ 45 m/s/yr "North-South"
  Maximum inclination of $\sim 15°$ reached after 26.5 years.

- Earth's non-circularity & solar radiation lead to $\Delta a, \Delta e$ (respectively)
  $\implies \sim$ 2 m/s/yr "East-West"
  Depends on longitude because there are 2 stable equilibrium points + 2 unstable equilibrium points. Typically manoeuvre every $\sim$ 2 weeks.

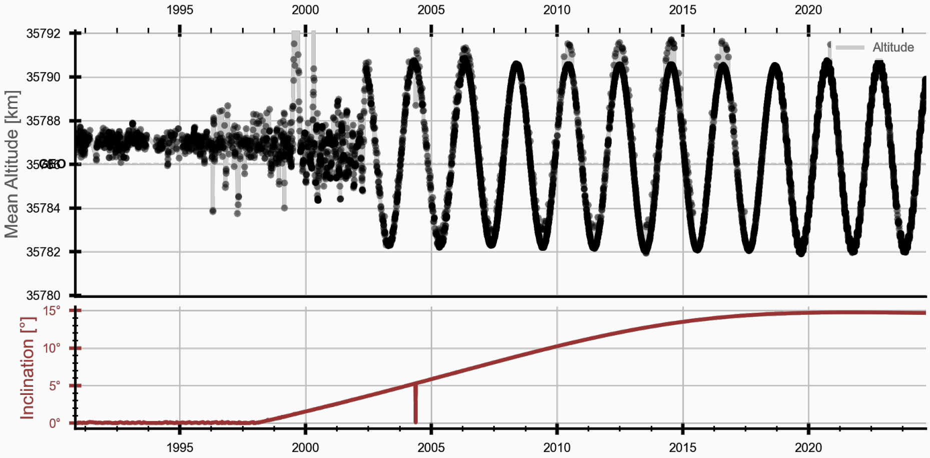
Station-keeping in GEO



Eutelsat 9B (SATCOM) – Data: Space-Track.org

Station-keeping in GEO

End of operations (mid 2002) and no disposal in graveyard orbit.



INSAT 1D (SATCOM, weather) – Data: Space-Track.org

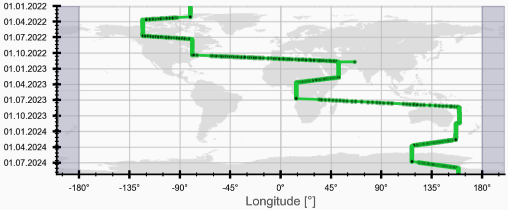
Satellite re-positioning in GEO

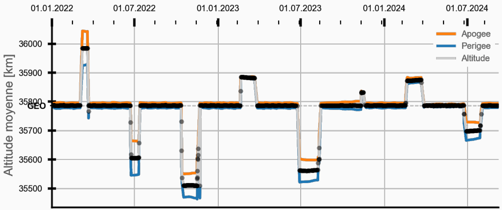
To relocate a satellite from one GEO slot to another, the semi-major axis can be lowered ($\rightarrow$ relocation eastwards) or increased ($\rightarrow$ westards) during a weeks- to months-long coasting time.

The change in longitude in $^\circ$/d, $\Delta\dot{L}$, is

$$\Delta\dot{L} = 360^\circ\left(1 - \frac{T}{T_S}\right) = 360^\circ\left(1 - \frac{2\pi\sqrt{a^3/\mu}}{T_S}\right)$$

with $T_s = 86164.0905$ s, the sidereal day.





Δv budget

A typical Δv budget must account for the following activities:

- positioning
- station-keeping/operational activities ($\Delta a, \Delta e, \Delta i, \Delta\Omega, \dots$)
- if applicable: mission phase transition (e.g. if there are different operational altitudes)
- if applicable: attitude control ($\rightarrow$ see subsequent lecture)
- disposal (re-entry, orbit raising to disposal orbit)
- margins!

Additional Δv means additional storage and mass $\rightarrow$ linked to the mass and structure budgets.

The satellite lifetime can be constrained by Δv reserves or reliability of or consumables for the payload.

Interactive quiz

→ EchoPoll platform

- You can scan a QR code or go to the link
- EchoPoll is the EPFL-recommended solution
- You do not have to register, just skip entering a username and/or email address