EE-585 – Space Mission Design and Operations

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Ecole Polytechnique Fédérale de Lausanne

Week 03 – 27 Sep 2024

Today's outline

The Sun's activity and space weather

Orbital regimes & ground track

Non-keplerian perturbations I: Atmospheric drag and orbital lifetime

Non-keplerian perturbations II: Earth non-circularity and sun-synchronous orbits

Non-keplerian perturbations III: restricted three-body problem

Non-keplerian perturbations IV: Solar radiation pressure

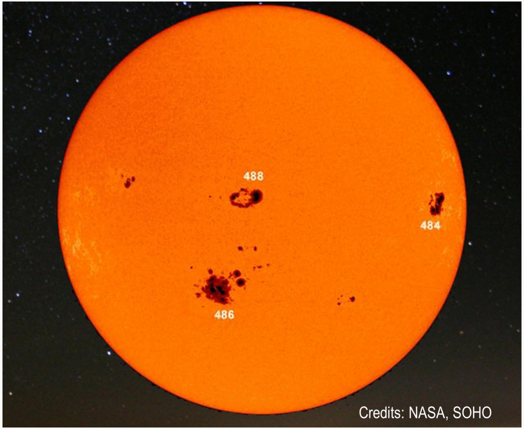
The Sun’s activity and space weather

Sunspot groups on the surface of the Sun

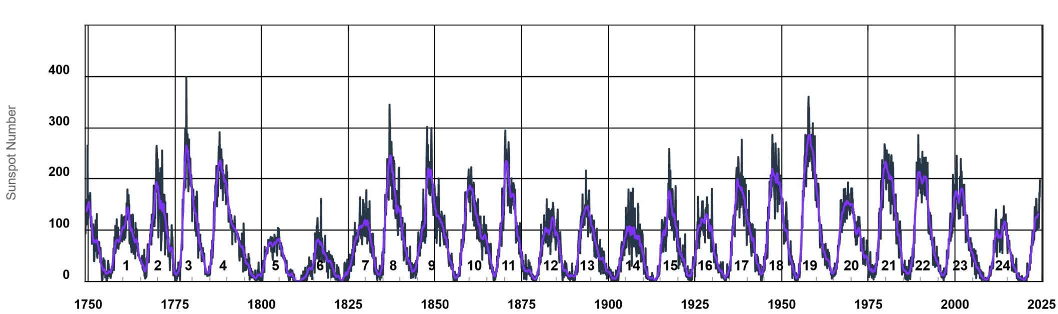
The Sun is a G-type dwarf star. It rotates every 27 days. Its appearance and radiation varies with time in various parts of the electromagnetic spectrum.

They are created by changes in the magnetic flux of the Sun.

Sunspots have been observed since 800 BC. Annual values series go back to 1700. Daily values since 1818.



Solar cycle



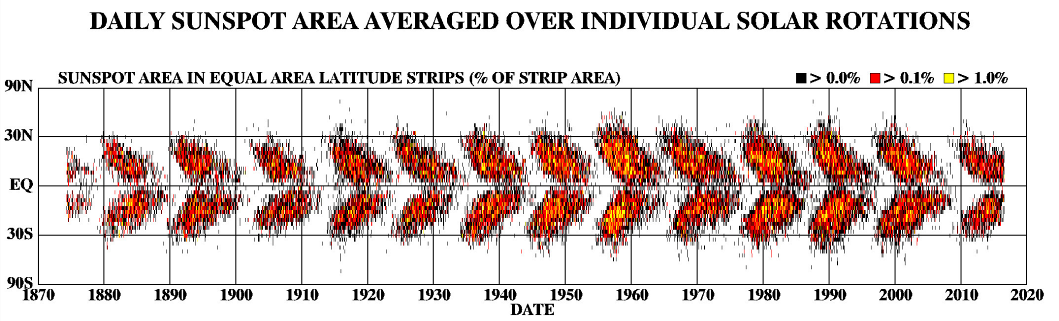
Credits: Space Weather Prediction Center, NOAA

There is a solar cycle of the sunspot numbers with a period of $\sim$ 11 yr.

Not symmetrical rise and fall. min $\rightarrow$ max takes $\sim$ 4.8 yr and max $\rightarrow$ min $\sim$ 6.2 yr.

Solar cycle	21	22	23	24	25	26
Sunspot Maximum	1979	1990	2001	2012	2024-5?	$\sim$ 2035
Sunspot Minimum	1985	1996	2007	2018	2029?	$\sim$ 2040

Sunspot location on the Sun – Butterfly diagram



Credits: NASA, Marshal Space Flight Center, Solar Physics

The latitude of sunspots depends of the phase of the solar cycle. At maximum, the latitude is relatively high, $\sim 25 - 30°$, and then it decreases as the solar minimum approaches.

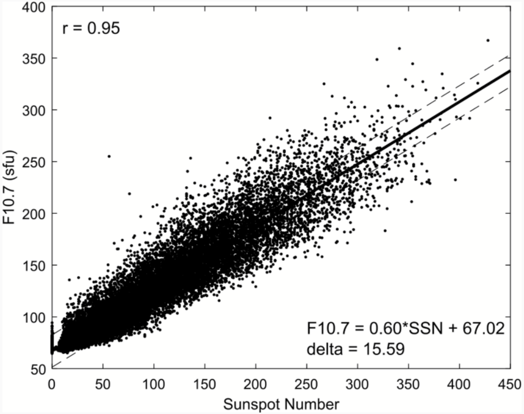
Solar radio flux and sunspot number

The number of sunspots is difficult to measure precisely (e.g. weather).

A better proxy for the phase of the solar cycle is the solar radio flux.

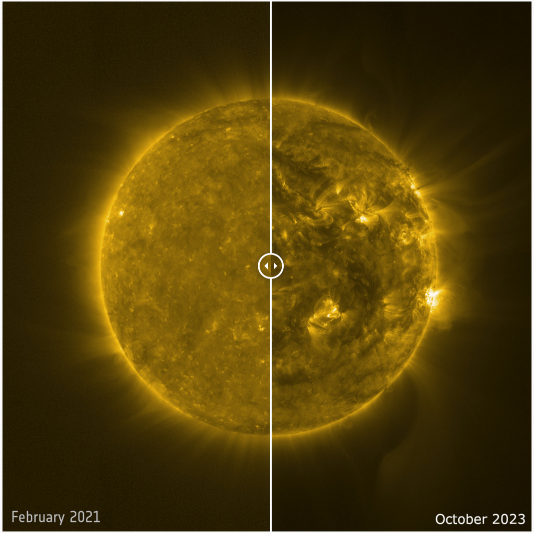
At the wavelength of 10.7 cm, the intensity of a hydrogen line changes depending on where we are in the solar cycle.

The F10.7 index can be measured reliably from the Earth's surface, in all types of weather. Reported in "solar flux units", (s.f.u.), between $\sim 50 - 300$ s.f.u.



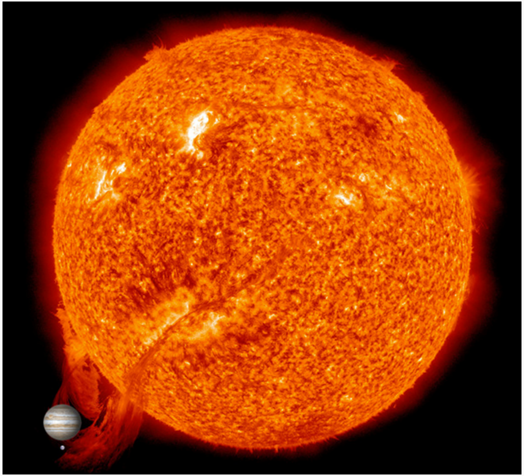
Credits: Okoh, D. & Okoro, E., 2019

The active sun



Credits: ESA & NASA/Solar Orbiter/EUI Team

Solar prominences



Credits: NASA

The Sun is an active star. Its surface is granular with prominences: flares and coronal mass ejections.

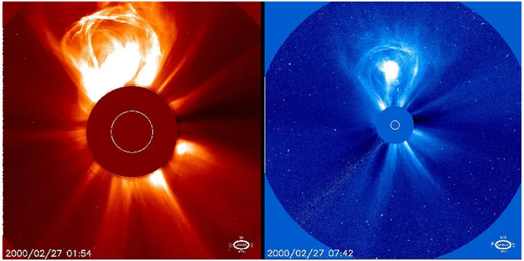
A solar prominence is a large, bright feature extending outward from the Sun's surface, often in a loop shape. While the corona consists of extremely hot ionised gases, which do not emit much visible light, prominences contain much cooler plasma, similar in composition to that of the lower chromosphere.

The image represents solar prominences with images of Jupiter and the Earth for size comparison.

Space weather

Solar wind continuously flows outward from the Sun and consists mainly of protons and electrons. Density and speed vary.

Coronal Mass Ejections (CMEs) are large expulsions of plasma and magnetic field from the Sun's corona. If they are Earth-directed, they arrive in from 15-18h to several days.



Credits: SOHO ESA & NASA

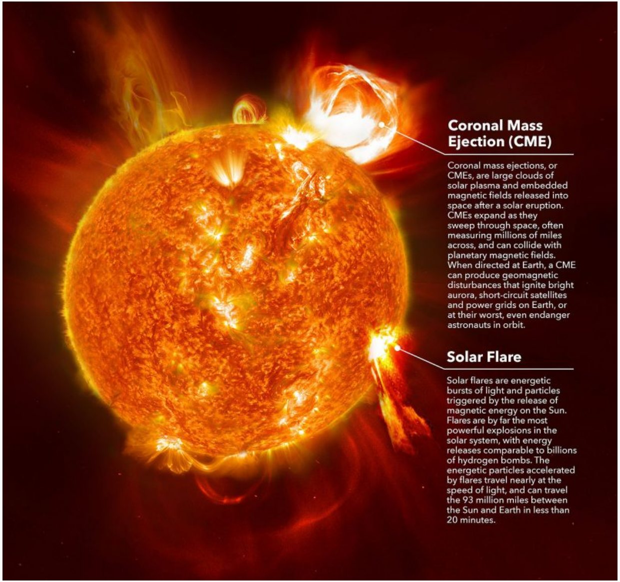
Solar flares are large eruptions of electromagnetic radiation (X-ray and UV radiation) from the Sun lasting from minutes to hours.

The intensity and speed of solar wind and CMEs depend on the solar activity.

→ This is creates space weather.

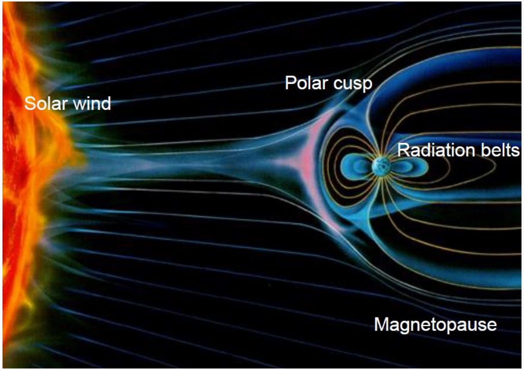
(Much more information on NOAA's Space Weather Prediction Center → spaceweather.gov)

CMEs vs solar flares



Credits: NASA

Interaction of the solar wind with the geomagnetic field



The geomagnetic field is significantly distorted due to the solar wind, made of charged particles flowing in all directions from the Sun.

The magnetopause is the boundary between the magnetosphere and the flow of particles from the solar wind, which is mainly protons and electrons .

The charged particles follow the Earth's magnetic field lines in the equatorial regions of the Earth. In the region of the magnetic poles, these particles can get into the low atmosphere and produce Northern Lights and Southern Lights.
In the anti-Sun direction, the geomagnetic field lines are open to the inteplanetary medium. Credits: NASA

Impact of space weather

- Electric Power Transmission → geomagnetic induced current
- High Frequency Radio + satellite Communications → variability of the ionsphere can block the propagation of radio waves (ionsphere = 80-600 km where UV + solar radiation ionize molecules → lots of electrons)
- GNSS → ionsphere propagation perturbation
- Satellite drag → see next slides
- Satellite electronics → see next slides
- Auroras → see next slides
- Climate → solar activity affect the total solar irradiance, but this change in TSI is too small to have a major impact on the Earth's climate.

Indicators of space weather

- **Geomagnetic Storms (G)** major disturbance of Earth's magnetosphere result by shocks with long periods high solar wind $\rightarrow$ induced currents, increased drag, surface charging, HF radio blackouts, auroras. K_p index measures disturbance of Earth's magnetic field.
- **Solar Radiation Storms (S)** $\rightarrow$ CME + flare eject accelerate protons which ionizes the atmosphere. $\rightarrow$ radiations levels harmful to high-altitude humans, satellite electronics + solar array damages, HF radio blackouts. Measure is proton flux.
- **Radio Blackouts (R)** $\rightarrow$ Large solar X-ray flares change Earth's ionosphere $\rightarrow$ high and low frequency blackouts, GNSS errors. Measure is X-ray flux.

The NOAA scale ranges from 0 (none) to 5 (extreme)

Northern/Southern lights

Northern and Southern Lights – ”auroras“ – are produced from the excitation and de-excitation of nitrogen and oxygen atoms, by electrons flowing from the solar wind.

Typically auroras are produced at and above the altitude of the airglow, $z \sim 100 - 400$ km.

Credits: NASA

Aurora intensity and latitudes

Auroras are usually only visible at high latitudes $\gtrsim 50-55°$.

Solar wind changes in intensity with time. Extreme events can generate auroras visible down to $\sim 35°$.

Credits: K. Chay

Reminder: Radiation effects on spacecraft systems

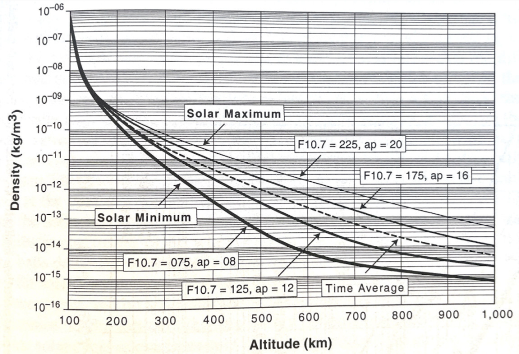
- Single event effects (SEE)
  - High energy particle travelling through a semiconductor leaves an ionised track behind.
  - May cause a highly localised effect.
- Single Event Upset (SEU) is a change of state caused by one single ionizing particle $\rightarrow$ bit flip $\rightarrow$ reboot
- Single Event Latchup (SEL) is a short circuit $\rightarrow$ lost it
- Measurements taken during solar storms or passes through the SAA will have more noise $\rightarrow$ is it acceptable for the mission?

Effects on the atmospheric density

The Solar activity also has a significant effect on the density of the Earth's atmosphere at a given altitude.

This effect is large for the altitudes $\lesssim 1,000$ km $\rightarrow$ it has an effect on the lifetime of satellites in LEO.

At solar maximum, for a given altitude, the density will be higher $\rightarrow$ the drag more important $\rightarrow$ lifetime reduced.



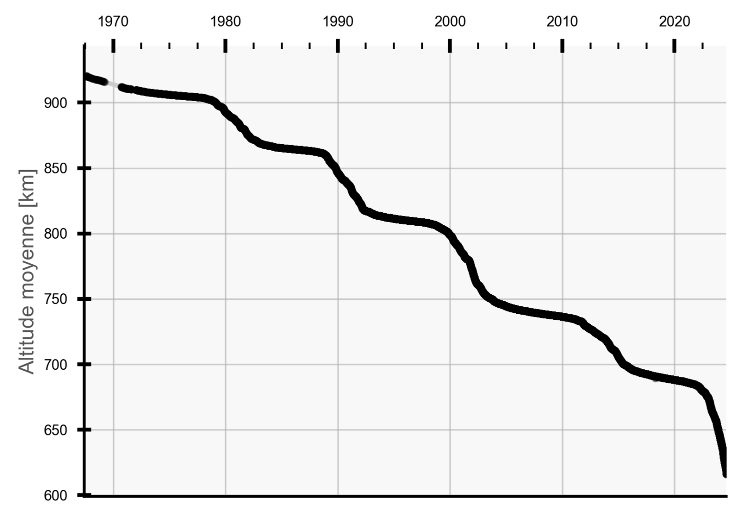
Credits: J. R. Wertz & W. J. Larson, *SMAD*, 3rd Edition, 2010

Effects of the solar activity on the drag

Surcal 150B was launched in 31 MAY 1967. It is a 41-cm diameter calibration sphere.

Periods of high solar activity are clearly identifiable, as the decay rates are much higher.

Data: Space-Track.org

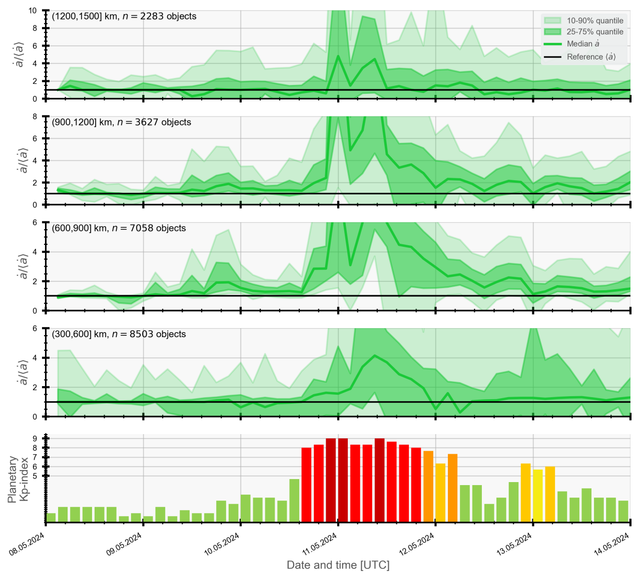


Extreme solar storm and the atmospheric drag

G5 event on the 11 May 2024 changed the mean decay rate $\dot{a}$ on LEO objects by a factor of $\sim 2-5$ for several days with respect to the typical decay rate $\langle \dot{a} \rangle$.

Prediction of trajectory is difficult.

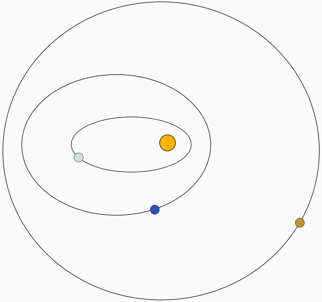
Credits: APCO / Swiss govt SSA report, 2024; Data from Space-Track.org



Orbital regimes & ground track

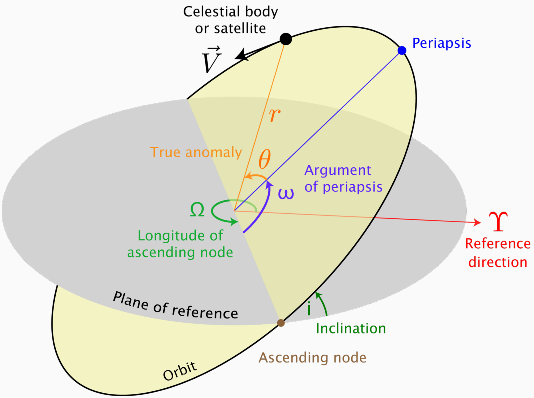
Reminder: Kepler's laws

Two-body system, with $M \gg m_{\text{S/C}}, \implies F = G\frac{Mm}{r^2} = \frac{\mu m}{r^2}$



1. Orbits are sections of conics.
2. Equal area are swept in equal times.
3. $T = 2\pi\sqrt{\frac{a^3}{\mu}}$

Reminder: classical orbital elements



Credits: Adapted from Wikipedia, Lasunnkty

e eccentricity, a semi-major axis

i inclination of the orbital plane

Ω longitude or Right Ascension of the Ascending Node (RAAN) in the plane of reference

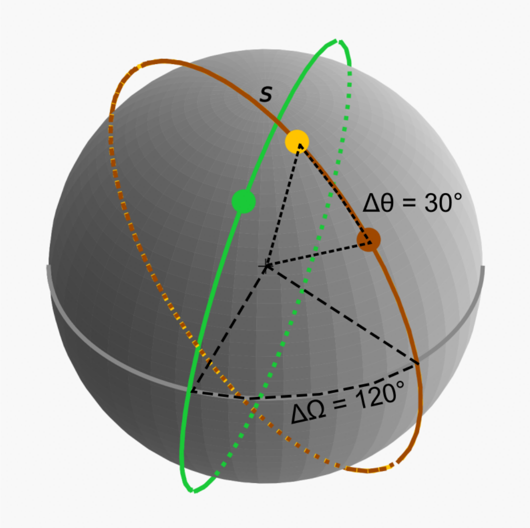
ω argument of periapsis

T_p time of periapsis transit

Current time t allowing a determination of the exact position of the celestial body or satellite.

The spacecraft is passing from the southern celestial hemisphere to the northern on a point on the plane of reference called the ascending node. The descending node is on the other side, when going from N to S.

Inclination i, RAAN Ω and true anomaly θ

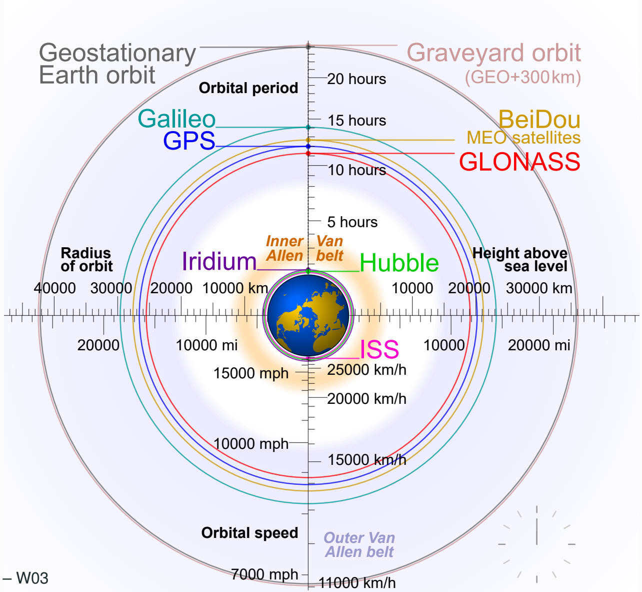


The sketch shows three satellites that have the same a, e, i.

$\Delta\Omega$ between the green and brown/yellow orbits is $120°$

Yellow and brown are on the same orbit ($=$ same orbital plane), but separated by $30°$

Mostly used orbits



LEO Low Earth orbit

MEO Medium Earth orbit

GEO Geostationary Earth orbit

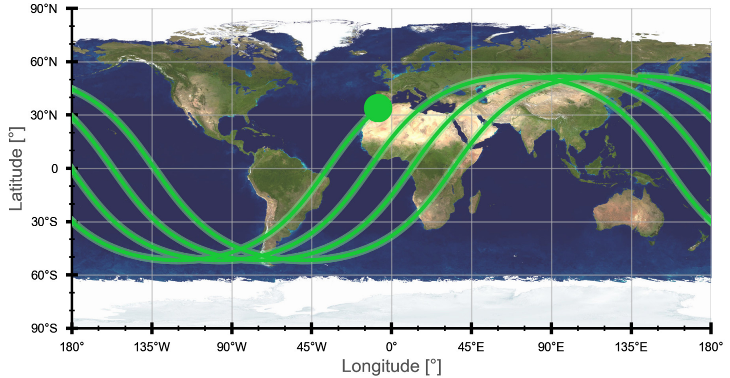
Credits: Wikipedia

Low Earth orbit (LEO)

- Low Earth orbit (LEO) are orbits at altitudes below 2000 km ($T \lesssim 127$ min) and low eccentricity.
- Typical altitudes are $z \sim 400 - 500$ km $\rightarrow T \sim 90$ min and $v \sim 7.7$ km/s
- Orbits at $\lesssim 350$ km altitude are called very low Earth orbit (VLEO)
- Uses: all, including human space flight. Most common use is satellite telecommunication (SATCOM).
- Relatively easy to reach, small altitude (low latency, high optical resolution)
- Main perturbations: atmospheric drag, Earth's non-sphericality

Satellite ground track

The ground track is the projection of the satellite's trajectory on the surface of the Earth.



International Space Station, alt $\sim$ 415 km, $i = 51.64°$

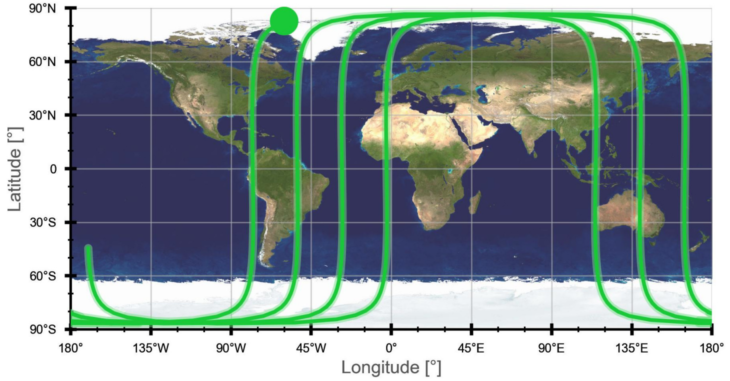
For LEO, the successive equatorial crossings (at the ascending nodes) $\sim 22.5°$ apart $(= T = 90 \text{ min} \times \frac{360°}{24\text{h}} = 1.5\text{h} \times 15°/\text{h})$

Each equatorial crossing moves to the west.

The satellite is moving to the east (for $i < 90°$) $\rightarrow$ prograde orbit.

LEO example: Polar Earth orbit (PEO)

PEO are mainly used for Earth-observation (EO), weather & climate and SATCOM

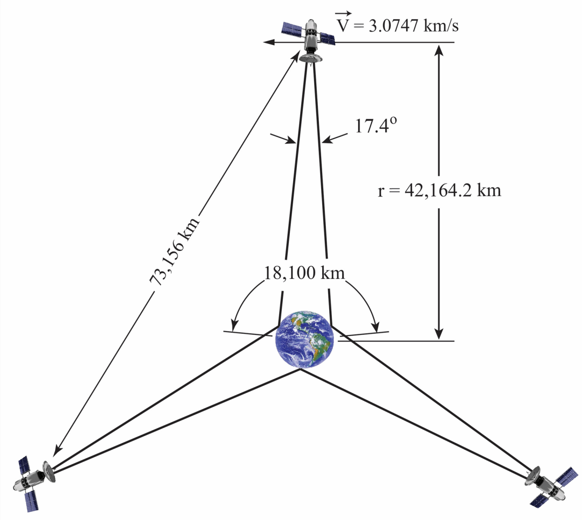


Iridium 181 (SATCOM), alt $\sim$ 750 km, $i = 86.7^\circ$

Medium Earth orbit (MEO)

- Medium Earth orbit (MEO) are orbits at altitudes z, $2000 < z < 35786$ km ($T < 1$ day) and low eccentricity.
- Typical altitudes are $z \sim 20'000$ km $\rightarrow T \sim 1/2$ day and $v \sim 3.7$ km/s.
- Uses: Mostly global navigation satellite services (GNSS: GPS, GLONASS, Galileo) and SATCOM.
- Main perturbations: solar radiation, radiation belts

Geosynchronous and geostationary orbits (GEO, GSO)



From a SATCOM and weather perspective, a satellite that is fixed in the sky is very interesting.

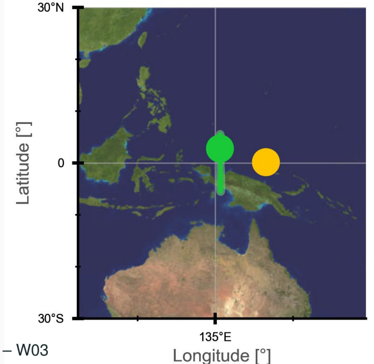
T = 1 sidereal day = 23h 56min 4.09s = 1436 minutes, $T = 2\pi\sqrt{\frac{r^3}{\mu}}$

$\implies$ $r_{\text{GEO}} = 42'164.2$ km or altitude 35'786 km ($\sim 36'000$ km),
$v \sim 3.1$ km/s.

Geostationary is $e = 0$, $i = 0$
Geosynchronous, $a = r_{\text{GEO}}, i \neq 0$

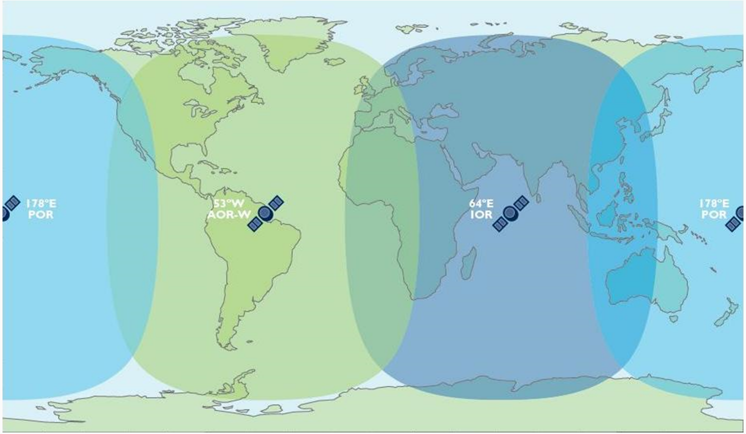
Geosynchronous and geostationary orbits (GEO, GSO)

- Uses: SATCOM, braodband (TV, radio), weather
- Coverage is limited to impossible at high latitudes.
- To increase coverage, inclination is increased ($i < 15°$)
- Main perturbations: solar radiation, gravitational perturbations from other bodies



The ground track for a geostationary orbiting satellite is a point on the equator.
If $i > 0$ (here $i = 5.7°$), the object makes small figure of 8.

Geosynchronous and geostationary orbits (GEO, GSO)



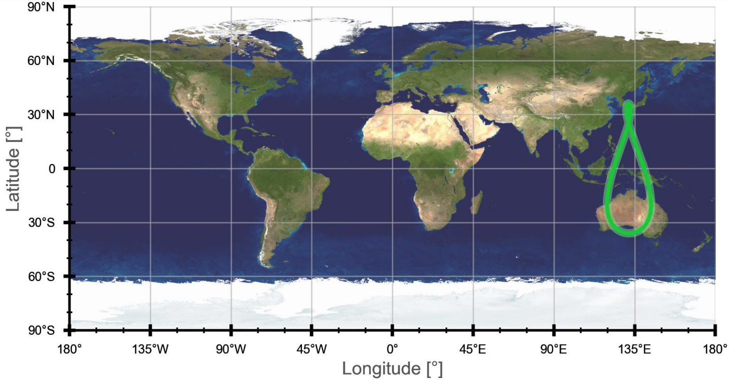
Credits: Inmarsat

The Inmarsat system blankets the Earth, connecting geostationary satellites for near total coverage.

This is a simplified picture. SATCOM satellites have beams that cover much smaller regions. E.g. Europe, Middle East, Southern Africa, . . .

Special GSO orbit: Quasi-Zenith Satellite System (QZSS)

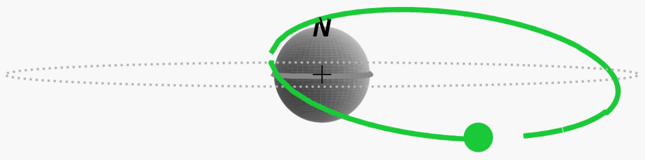
QZSS is a four-satellite constellation for regional navigation systems above Japan



The QZSS constellation is designed such that there is 1 satellite in GEO and three in these high-e and i orbits such that there are always 2 satellites above the horizon in Japan.

Typical orbits: alt $\sim z_{\text{GEO}}$, $i \sim 37^\circ$, $e \sim 0.07 \implies z_{\text{GEO}} \pm 3100$ km.

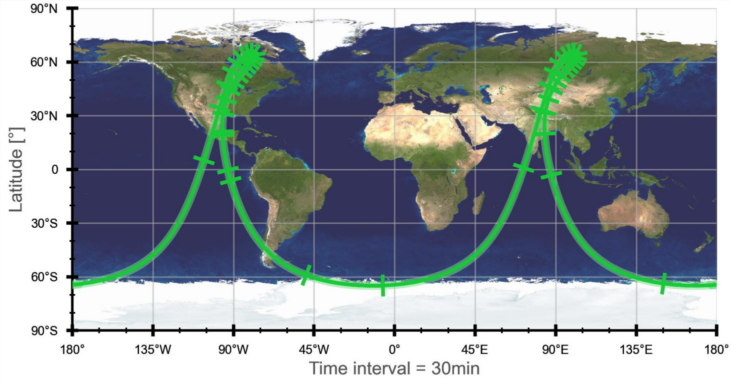
Highly elliptical orbits (HEO) and Geostationary transfer orbit



- Typically low perigee and very high apogee.
- Uses: SATCOM and transfer to GEO.
- Main perturbations: solar radiation, radiation belts, gravitational perturbations from other bodies

HEO example: Molniya

Molniya orbits were first used by USSR to guarantee SATCOM links at high latitudes



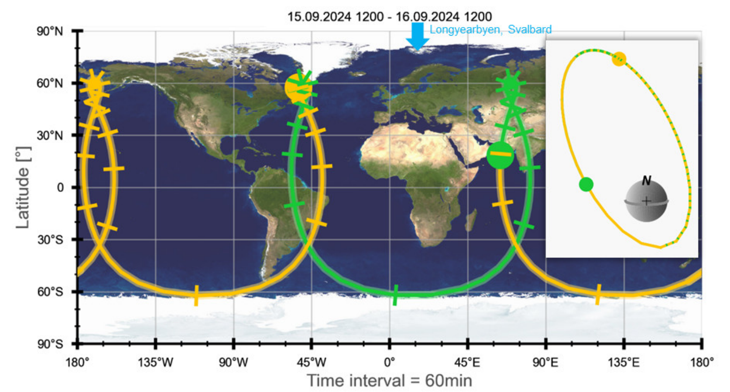
2nd Kepler law $\rightarrow$ more time is spent at apogee than perigee.

A constellation (= several satellites operated as a group) of ≥ 3 satellites with well chosen orbital parameters ensures that there constant coverage.

Typical orbits: $i = 63.4^\circ$, $T = 1/2$ sidereal day, $e \sim 0.7 \implies$ low perigee/high apogee.

HEO example: Molniya-like (1/2)

Two satellites in HEO to garantee coverage in the Artic.



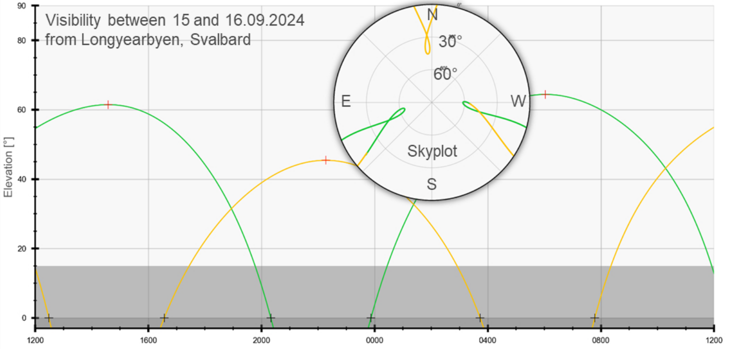
2nd Kepler law $\rightarrow$ more time is spent at apogee than perigee.

The ground antenna must follow the satellites.

$i = 62.4°$, $T = 2/3$ sidereal day, $e \sim 0.53 \implies$ low perigee/high apogee.

HEO example: Molniya-like (2/2)

Two satellites in HEO to garantee coverage in the Artic.



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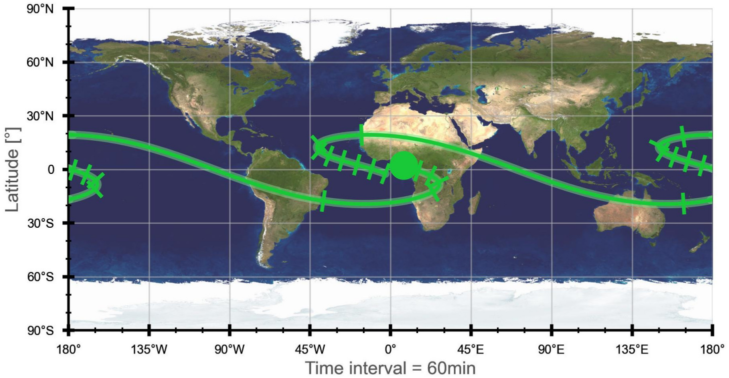
2nd Kepler law $\rightarrow$ more time is spent at apogee than perigee.

The ground antenna must follow the satellites.

HEO example: Geostationary transfer orbit (GTO)

Usually the insertion orbit after launch or first phase of positioning towards GEO



Near the apogee A,
$v_A < v_{\text{circ, GEO}}$
$\Longrightarrow$ retrograde motion (= westwards).

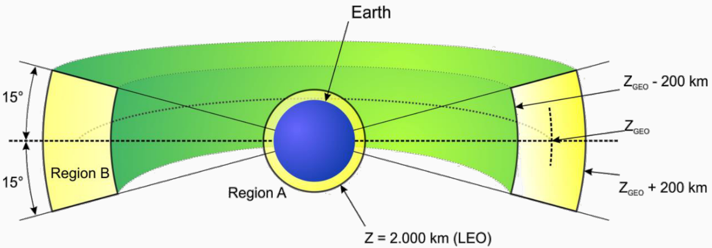
Near the perigee P,
$v_P \gg v_{\text{circ, LEO}}$
$\Longrightarrow$ large prograde motion.

Typical orbits: Perigee $\sim$ 200 km, apogee z_{GEO}, $i > 0°$.

Beyond GEO altitude

- Graveyard orbit: $r_{\text{GEO}} + \sim 300$ km, end-of-life satellites are raised there to be passivated and disposed of ($\rightarrow$ mostly non-operational satellites).
- xGEO (or far Earth orbit) are orbits around the Earth at higher altitude than GEO. Mostly for scientific uses (e.g. TESS searching for exoplanet).
- Cis-lunar: around the Moon, not bound gravitationally to the Earth.

Protected regions



Credits: IADC

There are 2 protected regions around the Earth:

A LEO, $z < 2000$ km

B GEO, $z \pm 200$ km

By the Inter-Agency Space Debris Coordination (IADC).
Protected = no (as little as possible) debris generation.

There are perturbations around the Earth

Kepler's assumptions no longer hold, because...

- the residual atmosphere is generating a drag force which lowers the altitude.
- the Earth is non-spherical which creates non-uniform gravitational potential.
- there are other significant objects that exert a non-negligible gravitational force on satellites close to the Earth (Moon, Sun).
- the solar radiation induces a non-central perturbing force.
- tides (solid Earth and ocean) or relativistic effects.

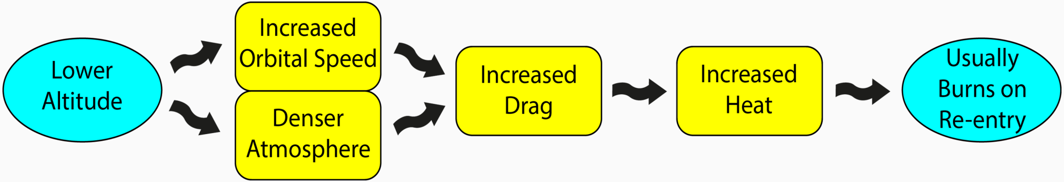
Non-keplerian perturbations I: Atmospheric drag and orbital lifetime

Orbit decay and limitation of orbital lifetime

The orbital decay is the reduction of the satellite's semi-major axis a.

This occurs up to altitudes of $\sim 600 - 700$ km.

Orbital decay is caused by the drag in Earth's residual atmosphere, $\rho_{\text{upper atmosphere}} \neq 0$ kg/m^3.



Drag equations and deceleration

The force due to the drag is

$$F_{\text{drag}} = \frac{1}{2}\rho V^2 C_D A_n$$

where

- ρ is the atmospheric density (kg/m^3)
- V is the velocity (m/s)
- C_D is the drag coefficient (-)
- A_n is the cross section (m^2)

The acceleration is

$$a_{\text{drag}} = \frac{1}{2}\rho V^2 \frac{C_D A_n}{m} = \frac{1}{2}\rho V^2 \times \frac{1}{BC}$$

where

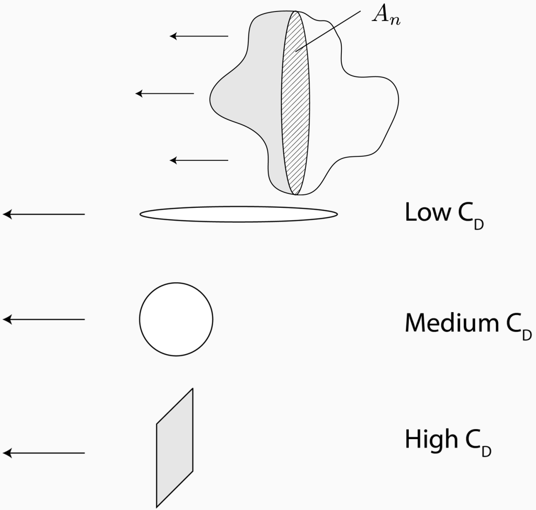
- BC is the ballistic coefficient (kg/m^2)

Ballistic coefficient

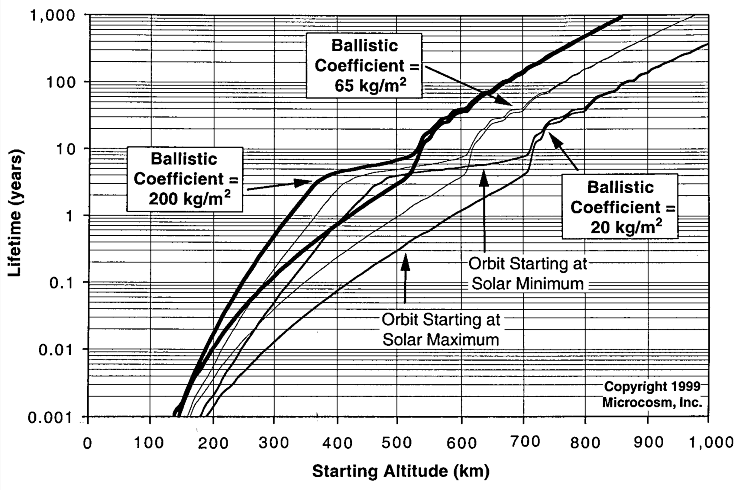
The ballistic coefficient measures the resistance to orbit decay caused by atmospheric drag.

$$BC = \frac{m}{C_D A_n} \quad \text{in kg/m}^2$$

The drag coefficient depends on shape, attitude and surface condition (specular and diffuse reflection, absorption). Typically, $C_D \sim 2.2$, but can vary from $2-4$



Lifetime and ballistic coefficient

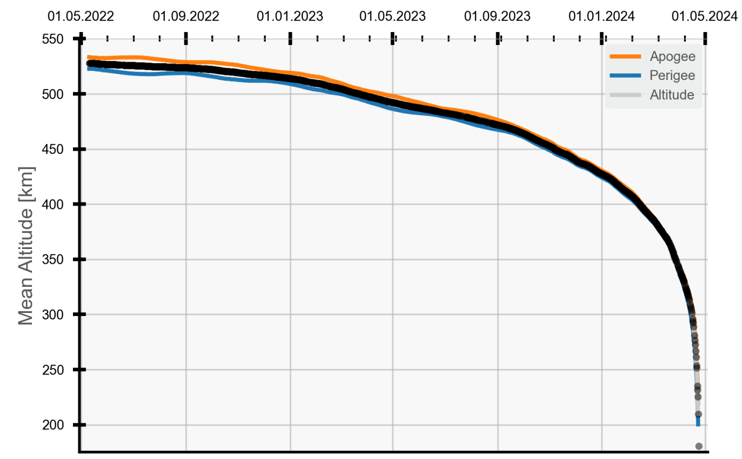


Credits: J. R. Wertz & W. J. Larson, *SMAD*, 1992

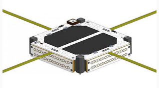
BC is inversely proportional to the drag deceleration.

A high *BC* indicates a small value of the deceleration and consequently a long lifetime

Orbital lifetime of SpaceBEE 146



Data: Space-Track.org



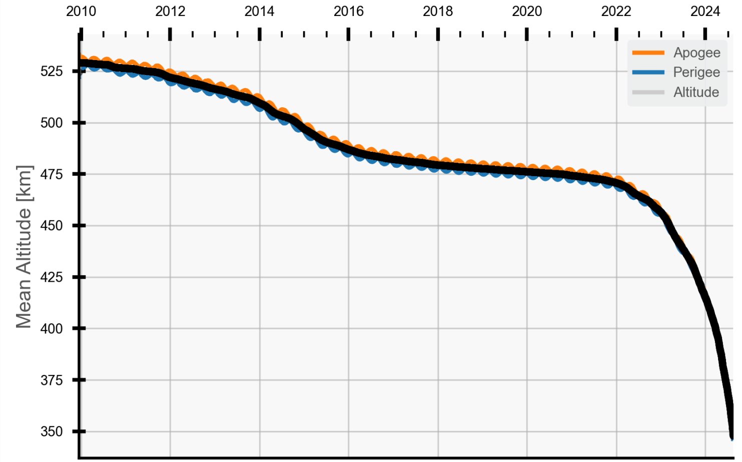
Credits: Swarm Technologies

SpaceBEE is a picosatellitea (0.25U cubesat, $10 \times 10 \times 2.5$ cm; 450 gr) to provide two-way satellite communications and data relay. The operator, Swarm Technologies, is owned by SpaceX.

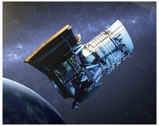
Launched 02 MAY 2022 on an Electron rocket (RocketLab) from New Zealand, re-entered 23 APR 2024.

Orbital lifetime: $\sim$ 2 years, $BC \sim 50 - 55$ kg/m^2.

Orbital lifetime of the Wide-field Infrared Survey Explorer (WISE)



Data: Space-Track.org



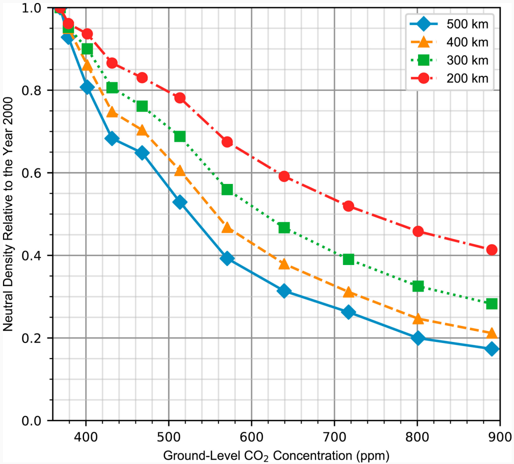
Credits: NASA/JPL-Caltech

NASA infrared astronomy space telescope (2.85 $\times$ 2 $\times$ 1.73 m, 661 kg)

Launched 14 DEC 2009 on an Delta II (ULA) from Vandenberg, California, decommissioned 01 AUG 2024.

Orbital lifetime: $>$ 14 years, $BC \sim 150 - 250$ kg/m^2.

Effect of climate change on the upper atmosphere



Credits: Brown, M. K., Lewis, H. G., Kavanagh, A. J., & Cnossen, I. (2021).

- Increasing CO_2 causes cooling in the upper atmosphere and a secular decrease in atmospheric density over time.
- Neutral density at 500 km altitude lowers by over 80% with a high ground-level carbon dioxide concentration of 890 ppm (RCP8.5 model).
- Meeting the 1.5 ℃ Paris Agreement target would limit the reduction in neutral density at 400 km since the year 2000 to around 28%.

→ Objects in LEO will have 30% longer lifetimes (1.5 ℃ assumption) compared to the year 2000.

→ Less manoeuvres to remain on orbit (”station-keeping“) but debris will linger for longer.

→ Space Sustainability Course EE-587.

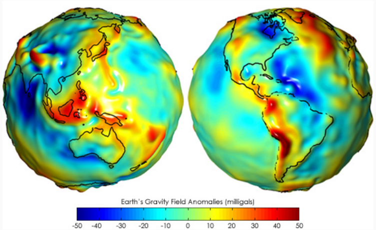
Effect of climate change on the upper atmosphere

	ISS	Starlink-60	MicroSat-R debris
Perigee [km]	417	371	272
Apogee [km]	420	373	845
BC [kg/m^2]	300	55	200
Ref. lifetime* [yr]	4.08	0.211	2.41
Lifetime at 468 ppm [yr]	5.40	0.279	3.06
Lifetime at 890 ppm [yr]	12.6	0.771	6.66

∗ for year 2000, at 379 ppm Credits: Brown, M. K., Lewis, H. G., Kavanagh, A. J., & Cnossen, I. (2021).

Non-keplerian perturbations II: Earth non-circularity and sun-synchronous orbits

Earth gravitational potential



Credits: Grace mission, NASA/DLR

The Earth is not a perfect sphere. A widely used form of the gravitational potential is

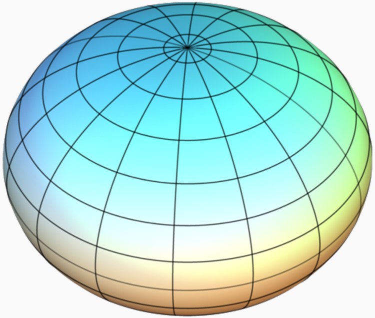
$$\Phi = \frac{\mu}{r}\left[1 - \sum_{n=2}^{\infty} J_n (R_\oplus/r)^n P_n \sin L\right]$$

where P_n are Legendre polynomials, L is the geocentric latitude and J_n are dimensionless coefficients.

A first approximation ($n = 2$) is an oblate sphere. The model can be improved by additional geopotential coefficients.

The rotation of the Earth's on its axis bulges the equator. This is modelled by the J_2 coefficient.

Equatorial bulge – J_2 perturbation



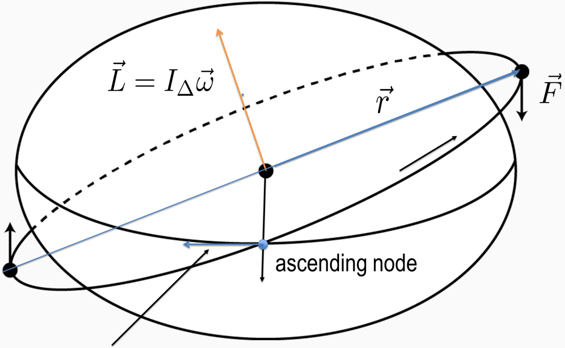
Credits: Wikipedia

The flattening of the Earth's surface is 1:298 (which corresponds to a radius difference of 21.38 km of the Earth radius 6378.13 - 6356.75 km).

This has two impact on the orbit:

1. Nodal regression (RAAN drift)
2. Rotation of the line of apsides (argument of perigee ω)

Nodal regression for LEO – Forces and torques



The equatorial bulge causes the gravitational force on a satellite to deviate from a purely central force, except at equator crossings, and for purely equatorial or polar orbits.

$$\vec{T} = \vec{r} \times \vec{F}$$
$$= \vec{r} \times \ddot{\vec{r}}$$
$$= \frac{\mathrm{d}\vec{h}}{\mathrm{d}t}$$

Nodal regression rate for low Earth orbits

The rate of changes of Ω and ω due to zonal coefficient J_2 are

$$\dot{\Omega}_{J_2} \approx -2.06474 \times 10^{14} \frac{\cos i}{a^{3.5}(1-e^2)^2} \qquad \text{RAAN drift}$$

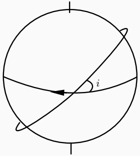
$$\boxed{\dot{\Omega}_{J_2} \approx -\text{cst} \cdot \frac{\cos i}{a^{3.5}}} \qquad \text{RAAN drift for circular orbit}$$

$$\dot{\omega}_{J_2} \approx 1.03237 \times 10^{14} \frac{4 - 5\sin^2 i}{a^{3.5}(1-e^2)^2} \qquad \text{Argument of perigee drift}$$

where $\dot{\Omega}$ and $\dot{\omega}$ are in degrees per mean solar day with a in kilometers.

Visualisation of the nodal regression and progression

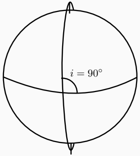
$i < 90^\circ$



$\dot{\Omega} < 0$

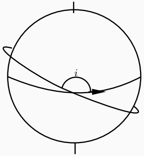
Nodal regression, ascending node $\rightarrow$ west

$i = 90^\circ$



$\dot{\Omega} = 0$

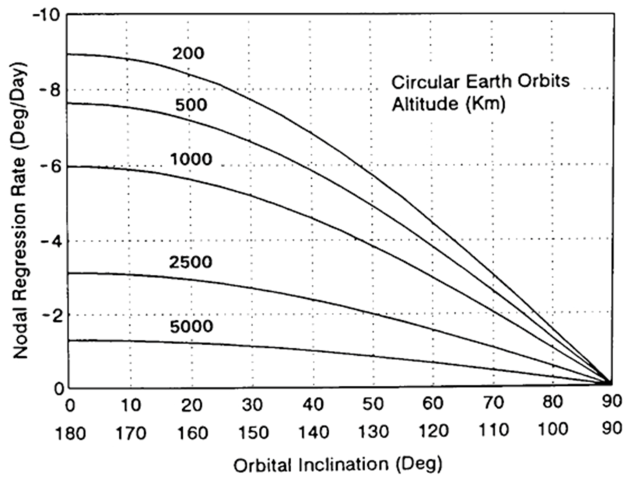
$i > 90^\circ$



$\dot{\Omega} > 0$

Nodal progression, ascending node $\rightarrow$ east

Nodal regression rate for low Earth orbits



Credits: J. R. Wertz & W. J. Larson, *SMAD*, 1992

The line of nodes drifts. . .

- westwards for $i < 90^\circ$ (posigrade or direct orbit) $\rightarrow$ nodal regression
- eastwards for $i > 90^\circ$ (retrograde orbit) $\rightarrow$ nodal progression

Sun-synchronous orbit

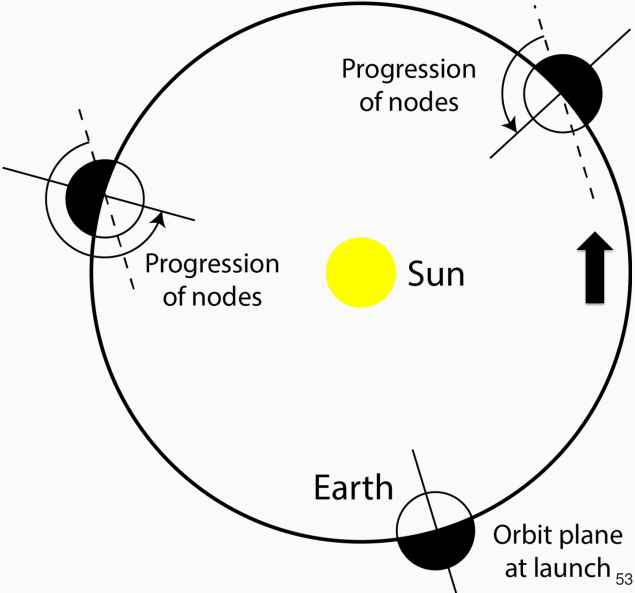
A Sun-synchronous orbit (SSO) is an Earth centred orbit which maintains a constant orientation with respect to the Sun during the year.

$\Longrightarrow$ that the line of nodes rotates to keep a constant angle to the vector of the Sun.

Requirements on $\dot{\Omega}$ for SSO:

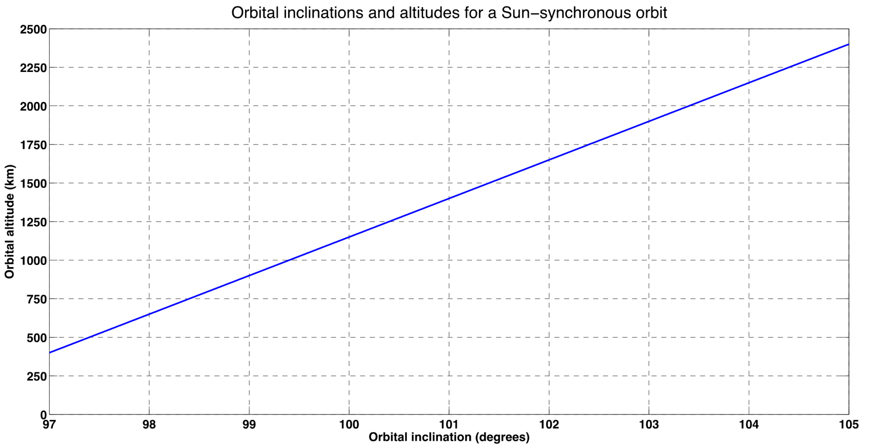
$$\dot{\Omega} = \frac{360^\circ}{365.242 \text{ days}} = 0.9856 \frac{\text{deg}}{\text{day}}$$

$\rightarrow$ nodal progression $\Longrightarrow$ $i > 90^\circ$



Orbital inclinations and altitudes for a Sun-synchronous orbit

The blue line shows the possible orbital parameters ($e = 0$) such that the SSO condition is fullfilled.



Typical SSO parameters: $97^\circ < i < 99.5^\circ, \quad 300 < z < 1000\text{km}, \quad e \sim 0$

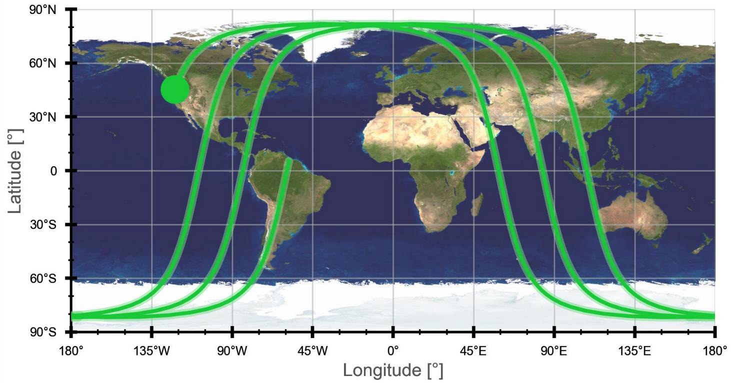
Use Cases for SSO

A satellite in a Sun-synchronous orbit passes the equator at the same (solar) time each day (e.g. a satellite's sun-synchronous orbit might cross the equator twelve times a day each time at 3:00 pm and 3:00 am local time). This is the equatorial crossing time (ECT).

SSO orbits are interesting because. . .

- satellites pass over the same region on the equator at the same illumination angle, however the solar illumination varies at non-zero latitudes.
- similar argument but for non-Earth observations: the angle between the plane of the orbit and the direction of the Sun is constant yielding a large field of view.
- the coverage is global.
- 6:00 am/6:00 pm ECT (i.e. dawn/dusk) SSO minimises the time spent in eclipses.

SSO ground track

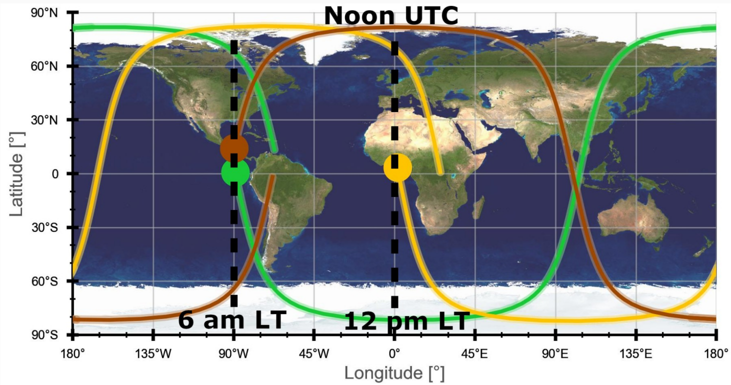


Example: alt, $z \sim 700$ km, $i = 98.2°$, ECT ↑6am/↓6pm.

$i > 90° \rightarrow$ retrograde motion.

SSO Equatorial Crossing Time

Satellites on SSO cross the equator always at the same local solar time (LT).



$i > 90° \rightarrow$ retrograde motion.

$\uparrow$ crossing of the ascending node
$\downarrow$ crossing of the descending node

Green: $\uparrow$6am/$\downarrow$6pm
Brown: $\uparrow$6pm/$\downarrow$6am
Yellow: $\uparrow$noon/$\downarrow$12am

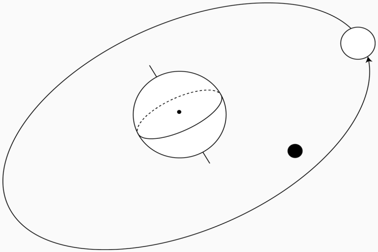
Example: when yellow object crossed the equator 1 orbit before, it was 12pm LT at Lake Victoria.

Non-keplerian perturbations III: restricted three-body problem

Restricted three-body problem

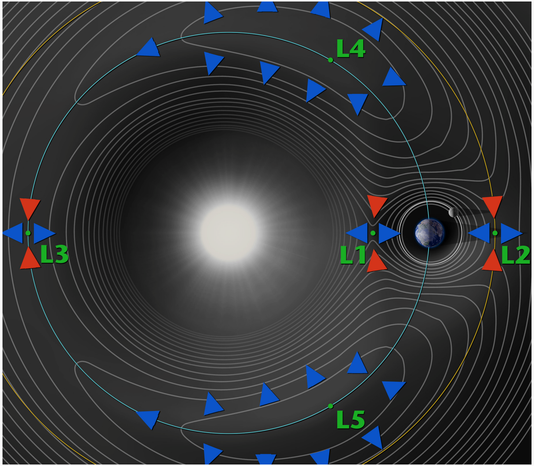
The assumption of only one significant massive object is relaxed. The restricted three-body problem describes the trajectory of a small mass in a system with two large bodies with the following assumptions:

1. The two main bodies (of mass m_1 and m_2) are on circular orbits around the centre of mass of the system.
2. The mass of the third body m is $m \ll m_1, m_2$.
3. The third body is in the plane of the orbit of the two main bodies.



This is valid for the Earth-Moon or Sun-Earth, Sun-Jupiter system.

Lagrange points



Credits: NASA, WMAP Science Team

There are 5 points at which the third mass m where the gravitational forces and the satellite's inertial acceleration are at equilibrium $\rightarrow$ Lagrange points.

The L4 or L5 points form a equilateral triangle with the two other masses. They are locally stable if m_1 is massive enough ($m_1 / m_2 \gtrsim 25$).

L1, L2, L3 are on a line. They are on saddle points of equilibrium. If perturbed, objects at these points will move away over time.

Red arrows indicate stability, blue arrows instability.

Interest of Lagrange points

- In the Sun-Earth system, the distance to the Earth of the L1 Lagrange point is about 1.5 million km, or 1% of the Sun-Earth distance = 0.01 astronomical unit (AU). The Lagrange point L2 is 1.5 million km away from the Earth in the anti-Sun direction.
- In the Sun-Jupiter system, there are asteroids trapped around L4 and L5, the so called Trojans (respectively the Trojan and Greek camps).
- In the Earth-Moon system, there are no objects around L4 or L5, because of the perturbation by the gravitational force of the Sun.
- Orbits can be achieved around L1, L2 or L3. They require limited station-keeping (Lissajous [usually aperiodic] or halo).
- Sun-observation missions can be placed in L1, like SOHO.
- Astronomical observatories are at L2 like James Webb Space Telescope or Euclid. They can continuously observe in the anti-sun direction.

Non-keplerian perturbations IV: Solar radiation pressure

Solar radiation pressure

- The total solar irradiance (recall: total energy of the photons emitted by the Sun integrated over the electromagnetic spectrum) at Earth's distance is $S_0 \sim 1368$ W/m^2.
- The momentum of a photon is its energy divided by the speed of light c.
- The perturbing acceleration is

$$\vec{p} = -\frac{S_0}{c}\frac{C_R A_n}{m}\hat{u}$$

  where C_R is the radiation pressure coefficient (1 if the surface is a blackbody, 2 if the incident radiation is reflected), m is the mass of the object and A_n is the absorbing cross-section and $\hat{u}$ the unit vector pointing towards the Sun.
- The solar radiation pressure dominates the drag deceleration at $\sim$ 800 km altitude $\rightarrow$ important for GEO orbits.

$\rightarrow$ EchoPoll platform

- You can scan a QR code or go to the link
- EchoPoll is the EPFL-recommended solution
- You do not have to register, just skip entering a username and/or email address