Exercise Session 3

Note Problems marked with a ($\star$) are complimentary exercises and will not be solved in class.

Problem 1 Multiple Choice Questions

A) ($\star$) A spacecraft is on a parking orbit at 1000 km above the surface of the Earth. At some point, the spacecraft is accelerated to a departure velocity of 11 km/s. What will be the speed on the surface of the sphere of influence and at infinity?

(1) 0.32, 0 km/s
(2) 1.18, 1.12 km/s
(3) 3.72, 3.60 km/s
(4) 6.15, 3.92 km/s

B) The MSL spacecraft, which carried Curiosity to Mars, had a mass of 4050 kg at 200 km altitude on a LEO parking orbit. Its orbital velocity was 7.78 km/s. To reach Mars, it had to accelerate to a departure velocity v_d of 11.50 km/s with a specific impulse of 320 s at a mass expulsion rate of 5 kg/s. What was the duration of the burn?

(1) 20 s
(2) 562 s
(3) 1832 s
(4) 102 s

C) ($\star$) A similar spacecraft has ion thrusters of specific impulse 3200 s. We want to perform a similar increase of velocity. Would the "burn" be shorter or longer?

(1) Longer
(2) Shorter

Why?

(1) The I_{sp} is much higher
(2) The thrust is much lower
(3) The exhaust velocity is much lower
(4) The thrust is much higher

D) ($\star$) A rocket is in free space, made of propellant only. Its initial mass is m and its thrust constant at a value mg_0. Its specific impulse is I_{sp}. It burns off completely in a time Δt. What is the value of Δt ?

(1) Its $\mathbf{I_{sp}}$ in seconds
(2) 9.81 s
(3) 42 s
(4) Cannot be determined with available data.

E) The Galileo mission which explored the Jovian system used the gravity assist of Venus during its complicated trip to Jupiter's neighbourhood. Its $\boldsymbol{v_a^\infty}$ at the entrance of the sphere of influence of Venus was perpendicular to the velocity vector of the planet and had an amplitude of 6.2 km/s. The gravity assist manoeuvre resulted in a deflection of Galileo's planetocentric velocity vector by 27°. The heliocentric arrival velocity at Venus was $V_{\text{before}} = 37.4$ km/s. Estimate the heliocentric velocity of the spacecraft at the departure from the sphere of influence of Venus.
Hint. Use the law of the cosine $c^2 = a^2 + b^2 - 2ab\cos(\gamma)$ to compute the heliocentric velocity after the slingshot.

(1) Cannot be determined with available data.
(2) 33.7 km/s
(3) 35.2 km/s
(4) 40.1 km/s

Problem 2 Interplanetary Transfer

We plan the interplanetary transfer of a spacecraft from the Earth to Jupiter. We assume that both orbits are circular and coplanar for simplicity.

A) Assume that the spacecraft follows a Hohmann transfer trajectory. Determine the departure hyperbolic excess velocity v_d^∞ needed to leave the influence of the Earth on our way to Jupiter and the arrival hyperbolic excess velocity v_a^∞ .

B) What is the departure velocity v_d to achieve this mission from a circular parking orbit around the Earth at 200 km altitude?

C) Find the spacecraft's energy (per unit mass) on the hyperbolic Earth escape trajectory.

D) What is the time needed for the trip from Earth to Jupiter, on an Hohmann transfer trajectory?

E) Upon arrival in the sphere of influence of Jupiter, the impact parameter is chosen so as to achieve, in the hyperbolic trajectory inside the Jupiter sphere of influence, a closest distance to the center of Jupiter equal to 100,000 km. What is the value of the braking impulse Δv_i so as to inject the spacecraft on a circular orbit around Jupiter at that altitude?

F) What is the phase angle between the Earth and Jupiter at Earth's departure?

G) How often does this launch windows occur?

Numerical values:
Radius of Jupiter's orbit $R_{\text{Jupiter}} = 5.204$ AU.
Mass of Jupiter $m_{\text{Jupiter}} = 1.8989 \cdot 10^{27}$ kg, mean radius $r_{\text{J}} = 69,911$ km.