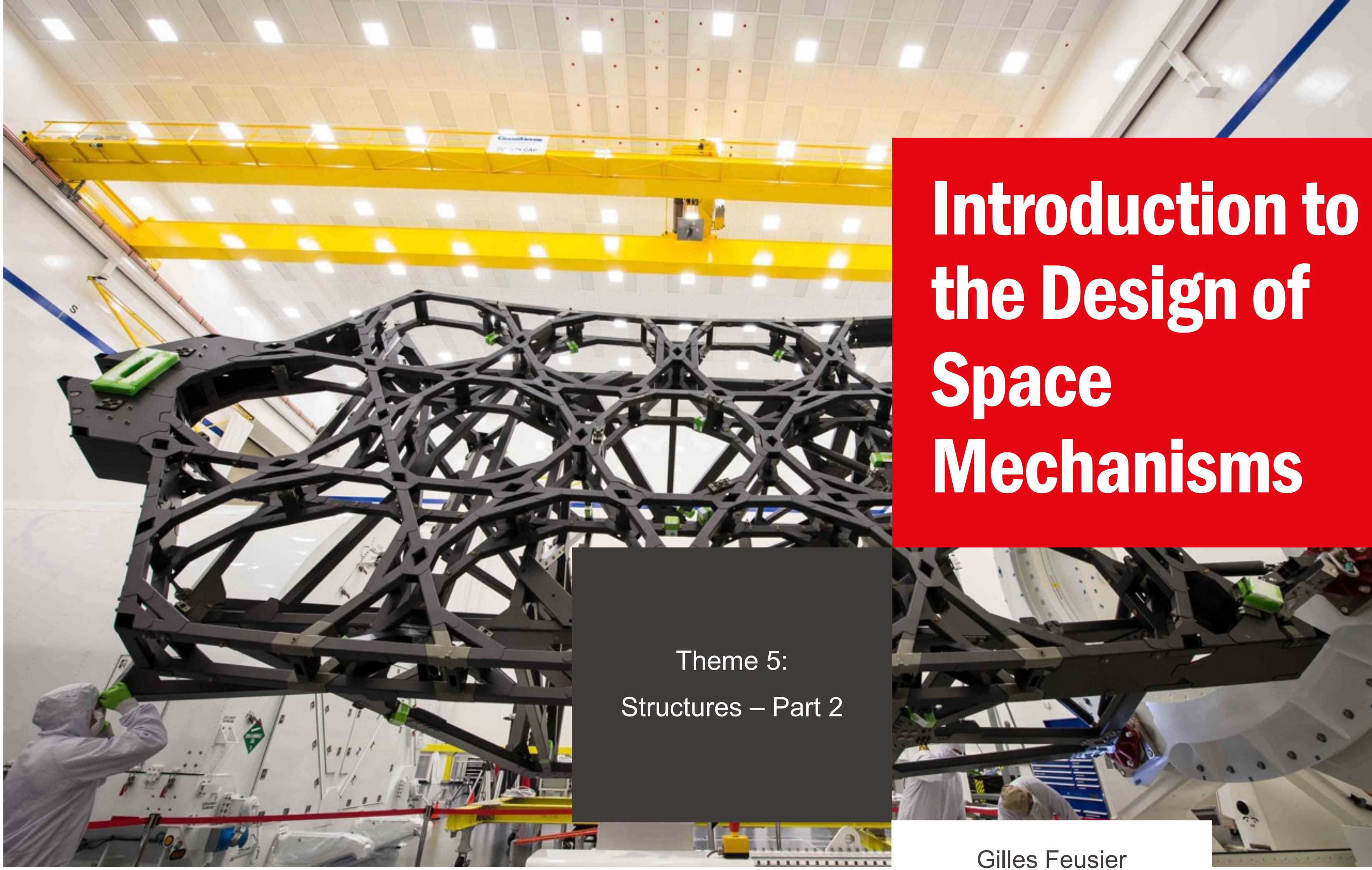


Introduction to the Design of Space Mechanisms

Theme 5:

Structures – Part 2

Gilles Feusier

Part 1 Summary

- Roles of structures
- Challenges of structures
  - Strength, including buckling
  - Mass
  - Deformations, including thermo-elastic deformations
- How to create structures, how to improve structures

Properties of Structures

- Support the load
  - Functional loads
  - Launch loads
    - Static acceleration
    - Vibrations
    - Shocks
    - Acoustic pressure
- Limited deformation under load
  - Elastic deformation
  - Permanent deformation
    - Plasticity
    - Creep
- Limited Thermo-Elastics deformation
- Adapted interfaces
- Adapted materials
  - Temperature range
  - Environment
- Mass constrains: reducing the mass

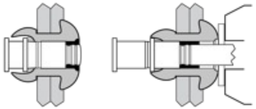
Assembly of Structures

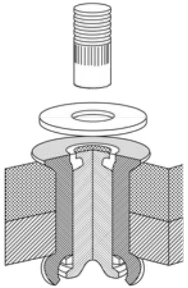
- Bolts
  - Stainless steel: e.g. A 286 / E-Z 6 NCT 25 (1.4944)
  - Titanium Ti6Al4V
  - Inconel 718
  - Preload (elastic, $\sigma_{pre}/\sigma_{yield} \cong$ 50% - 80%)
  - Coating (e.g. MoS_2)
  - Various standards: NF-L 22xxx, LN, ASNA, R-sat, …
  - ECSS-E-HB-32-23A Rev.1 Space engineering - Threaded fasteners handbook

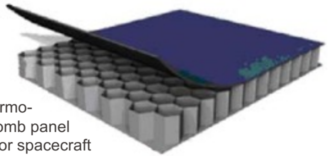
Source: Rabourdin.fr

- Rivets
- Welding
- Gluing

Source: Cherry Aerospace



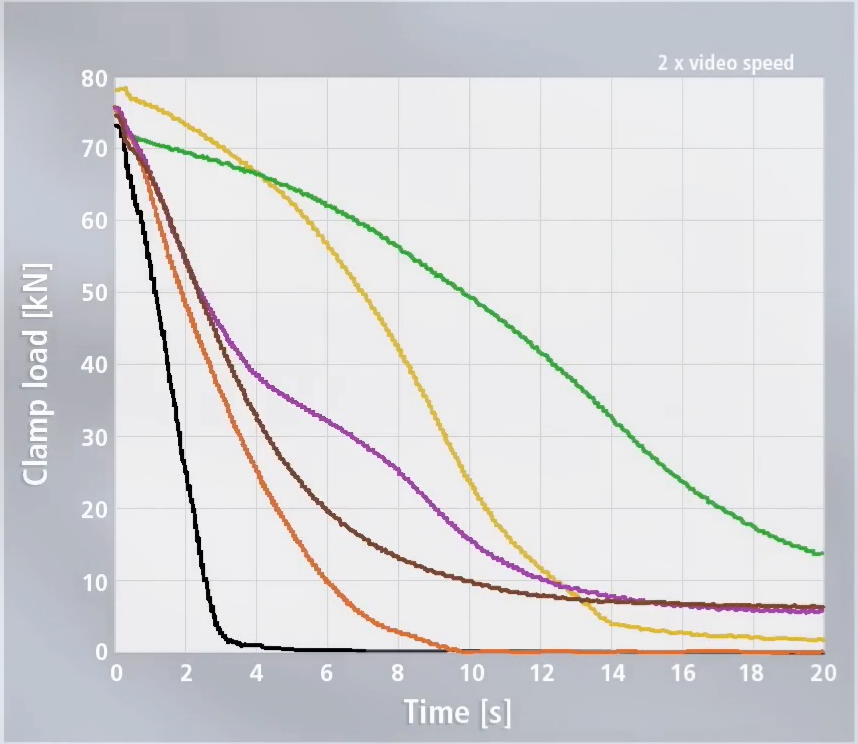




Source: Boudjemai et al. "Thermo-mechanical design of honeycomb panel with fully-potted inserts used for spacecraft design", 6th RAST, 2013

Source: G. Bianchi et al., "Optimization of Bolted Joints Connecting Honeycomb Panels", 1st CEAS, 2007

Why locking?



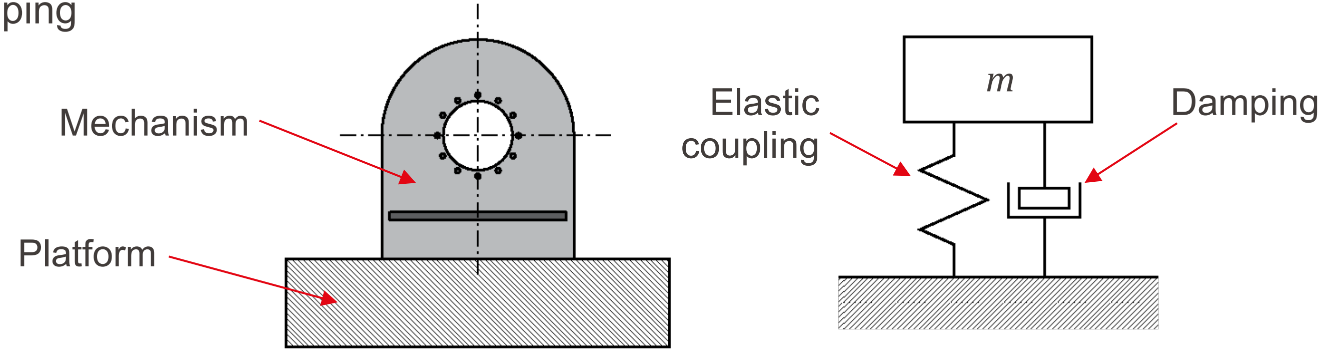
NORD-LOCK®

Vibration loads

- The mechanism is attached to a platform
  - The level of vibration is imposed by the platform

→ Specified spectral density

  - The mechanism react to the vibrations (resonator)
    - Eigenfrequencies
      - Several vibration modes
    - Amplification of the movement at certain frequencies (overload)
    - Damping



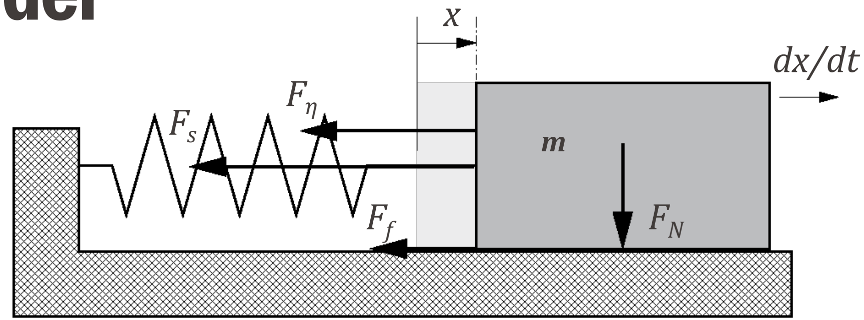
Vibration loads

- Sizing: worst case!
  - Highest load
  - Largest deformation
  - Worst case environmental conditions
    - If T_{max} is specified on orbit, but launch temperature is T_L, use T_L for the vibration load calculation.
    - This is not always obvious in the requirements

→ **Challenge the requirements!!!**

- Harmonic oscillator
  - Numerous references exist e.g.:
    - “Mécanique Vibratoire, Systèmes discrets linéaires”, Michel Del Pedro, Pierre Pahud, 1992, EPFL PRESS [5.3]
    - "Engineering Vibration", 4th Edition, Daniel J. Inman, University of Michigan, 2014, Pearson [5.4]
    - …

Harmonic oscillator: reminder



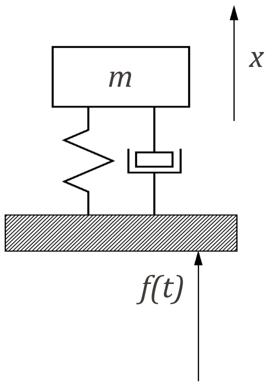
- Inertia: $F = m \cdot \frac{d^2x}{dt^2} = m \cdot \ddot{x}$
- Elastic force
  - **Linear stiffness**: ***k*** [N/m] $F_s = -k \cdot x$
    - Noticeable relationship: $k = \omega_0^2 \cdot m$

      Where $\boldsymbol{\omega_0}$: eigenfrequency of the undamped harmonic oscillator

      $\boldsymbol{m}$: oscillating mass
  - Stiffness may not be linear!
- Dissipative forces
  - **Friction coefficient**: $\boldsymbol{\mu}$ $F_f = \mu \cdot F_N$ [nondimensional]
- Damping
  - **Damping coefficient**: $\boldsymbol{c}$ $F_\eta = -c \cdot \frac{dx}{dt} = -c \cdot \dot{x}$ [N·s/m]
  - **Relative damping coefficient**: $\boldsymbol{\eta}$ $\eta = \frac{c}{c_{cr}} = \frac{c}{2 \cdot m \cdot \omega_0} = \frac{c}{2\sqrt{k \cdot m}}$ [nondimensional]

    c_{cr}: critical damping coefficient ω_0: eigenfrequency of the undamped oscillator
  - Damping can be non-linear (e.g. Coulomb damping/dry friction …)

Harmonic oscillator: reminder



- Equation of motion

$$m \cdot \ddot{x} + c \cdot \dot{x} + k \cdot x = f(t)$$

$$\Rightarrow \quad m \cdot \ddot{x} + c \cdot \dot{x} + \omega_0^2 \cdot m \cdot x = f(t)$$

where $f(t)$ is the external force imposed on the system

  - Note: the introduction of non-linear parameters, like static friction, requires a more complex treatment

- **Simple harmonic oscillator** solutions (free vibrations), i.e. neither driven ($f(t)$ = 0) nor damped (c = 0):

$$x = X \cdot \cos(\omega_0 \cdot t + \varphi)$$

Harmonic oscillator: reminder

- **Dissipative case ($c \neq 0$)**, but not driven ($f(t) = 0$):

$$x = A \cdot e^{r_1 \cdot t} + B \cdot e^{r_2 \cdot t}$$

$$r_1 = \omega_0 \cdot \left(-\eta + \sqrt{\eta^2 - 1}\right)$$

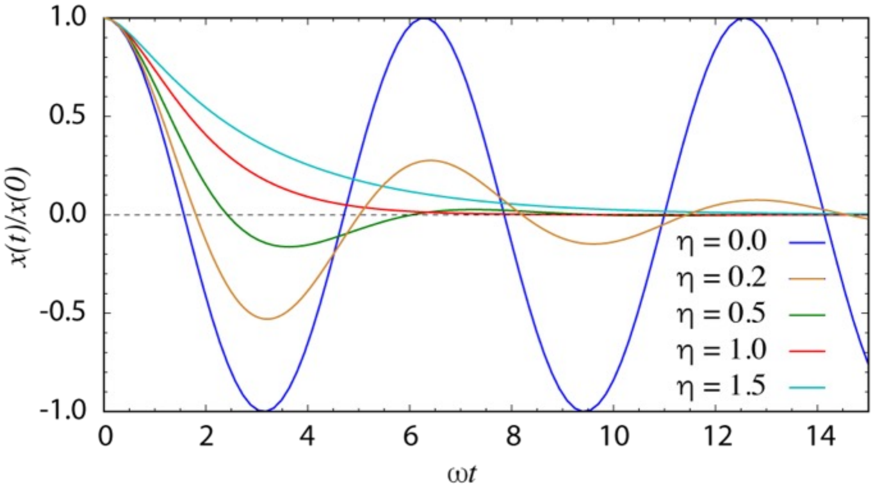
$$r_2 = \omega_0 \cdot \left(-\eta - \sqrt{\eta^2 - 1}\right)$$

A, B: constants depending on η

- $\eta > 1$: overdamped
- $\eta = 1$: critical damping
- $\eta < 1$: underdamped

Cf. previously (relative damping coefficient):

$$\eta = \frac{c}{2 \cdot m \cdot \omega_0}$$



Harmonic oscillator: reminder

- **Dissipative case ($c \neq 0$)** and **driven ($f(t) \neq 0$**, i.e. forced vibrations)
  - Sine applied external force F:

$$m \cdot \ddot{x} + c \cdot \dot{x} + \omega_0^2 \cdot m \cdot x = F \cdot cos(\omega \cdot t)$$

Note:
ω_0: eigenfrequency
ω: driving frequency

- The general solution becomes:

$$x = X \cdot cos(\omega \cdot t - \varphi)$$

with: $$X = \frac{\frac{F}{k}}{\sqrt{\left[1 - (\frac{\omega}{\omega_0})^2\right]^2 + 4 \cdot \eta^2 \cdot (\frac{\omega}{\omega_0})^2}}$$

$$\mathrm{tg}(\varphi) = \frac{2 \cdot \eta \cdot \frac{\omega}{\omega_0}}{1 - (\frac{\omega}{\omega_0})^2} = \frac{\omega \cdot c}{k - \omega^2 \cdot m}$$

with: $k = \omega_0^2 \cdot m$

Harmonic oscillator: reminder

- Dissipative case ($c \neq 0$) and driven ($f(t) \neq 0$) - continued

1) $$x = \frac{F}{k} \cdot cos(\omega \cdot t) \qquad \text{if} \quad \omega \ll \omega_0$$

2) $$x = \frac{F}{2 \cdot k \cdot \eta} \cdot cos(\omega_0 \cdot t + \frac{\pi}{2}) \qquad \text{if} \quad \omega = \omega_0$$

3) $$x = \frac{\omega_0^2 F}{\omega^2 k} \cdot cos(\omega \cdot t + \pi) = \frac{F}{m\omega^2} \cdot cos(\omega \cdot t) \qquad \text{if} \quad \omega \gg \omega_0$$

1) Spring controlled
2) Damper controlled
3) Mass controlled

Harmonic oscillator: reminder

- Dynamic amplification factor (overload)
  - The ratio between the dynamic amplitude X and the static one (elastic deformation under a static force F) is the amplification factor

$$\zeta = \frac{X}{F/k} = \frac{1}{\sqrt{(1-\beta^2)^2 + 4 \cdot \eta^2 \cdot \beta^2}}$$ [nondimensional]

with: $\beta = \frac{\omega}{\omega_0}$

(β relative angular frequency)

Reminder: ω is the driving frequency, ω_0 is the eigenfrequency of the undamped harmonic oscillator and η is the relative damping coefficient

  - More damping = less overload
  - Maximum amplification factor (resonance): $\zeta_{max} = \frac{1}{2 \cdot \eta \sqrt{1-\eta^2}}$

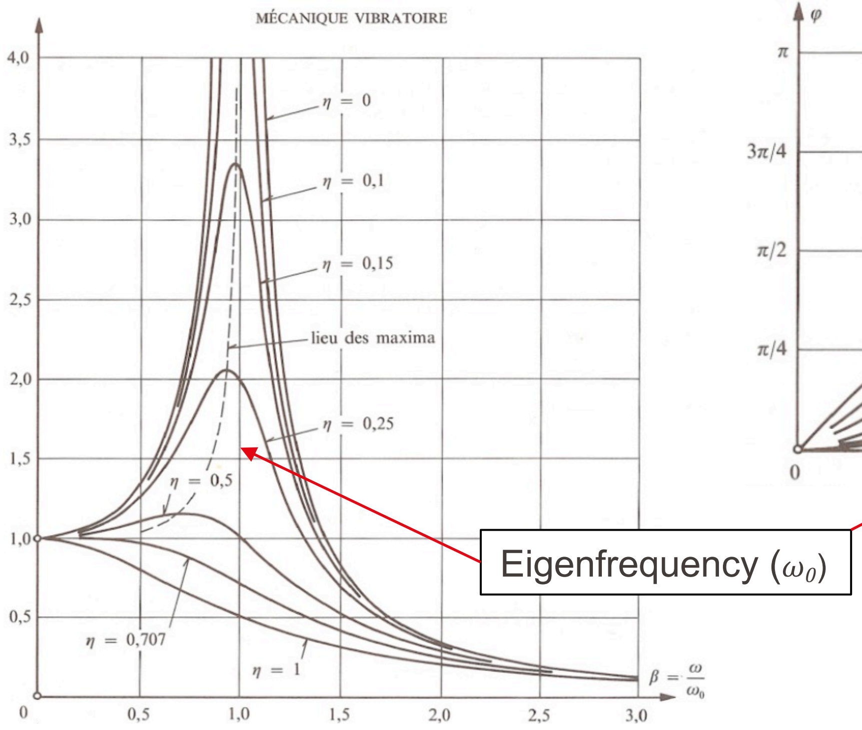
if $\beta \approx 1$: $\zeta \approx \frac{1}{2 \cdot \eta} \rightarrow \zeta_{max}$ if $\eta << 1$

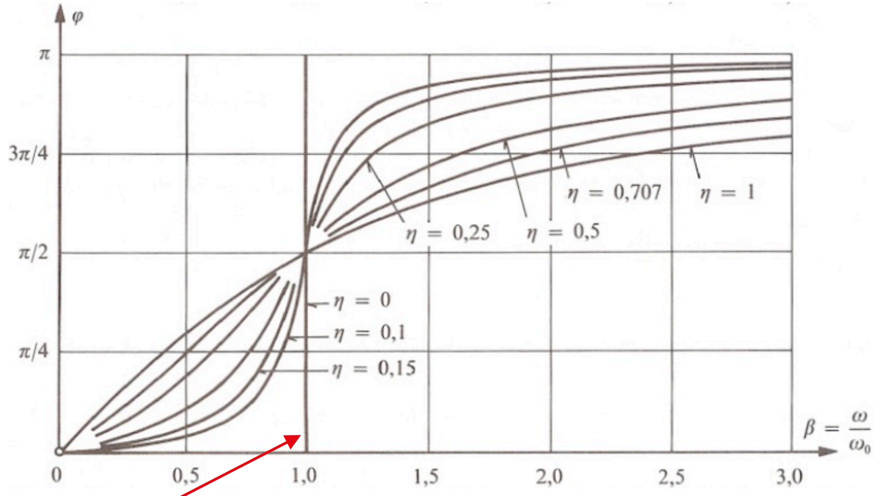
Note: $\frac{1}{2 \cdot \eta} = Q$

Harmonic oscillator: reminder

- Shape of the amplitude and phase of the vibration as a function of the relative angular frequency β and the relative damping coefficient η:

$$\zeta = \frac{X}{F/k}$$

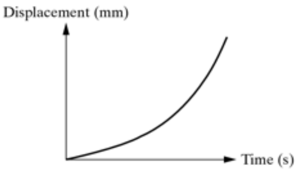




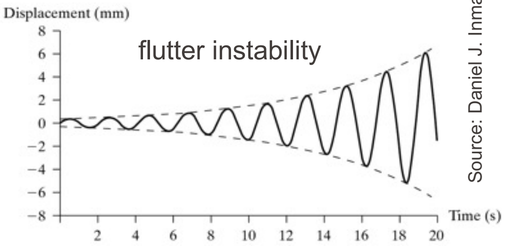
Source: Michel Del Pedro, Pierre Pahud, "Mécanique Vibratoire, Systèmes discrets linéaires", EPFL PRESS, 1992 [5.3]

Stability - Instability

- **Stable** oscillator: limited amplitude
  - $|x| = f(t) \rightarrow$ limited
  - Mass m, stiffness k and damping coefficient $c > 0$
- **Unstable** oscillator: diverging
  - $|x| = f(t) \rightarrow \infty$
  - Stiffness k or damping coefficient $c < 0$
  - Example:
    - Inverted pendulum (inverted pendulum maintained by a spring)
    - Wing of a plane (flutter instability)



divergent instability



Source: Daniel J. Inman "Engineering Vibration", 4th Edition, University of Michigan, 2014, Pearson

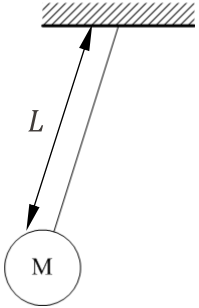
Useful formulas

- Parameters
  - M: concentrated mass (at center mass) [kg]
  - m: distributed mass (on the length or on the surface) [kg/m] or [kg/m^2]
  - L, a, b: characteristics lengths [m]
  - k: stiffness [N/m]
  - ρ: specific weight [kg/m^3]
  - g: terrestrial acceleration [9.81 m/s^2]
  - E: Young's modulus [N/m^2]
  - I: area moment of inertia (second moment of area) [m^4]

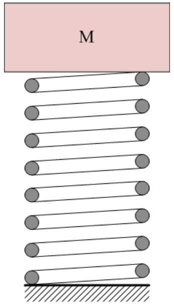
- Eigenfrequencies of:
  - Simple gravity pendulum
  - Loaded spring
  - Beams

Useful formulas

Source: Roark's Formulas for Stress and Strain (9th Edition), Budynas, R.G., Sadegh, A.M., Mc Graw Hill, 2020, ISBN 9781260453751

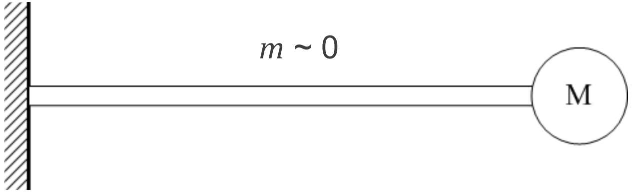


$$\omega = \sqrt{\frac{g}{L}}$$

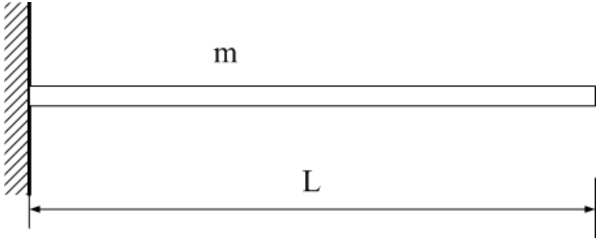


$$\omega = \sqrt{\frac{k}{M}}$$

$$\omega_0 = 15.4\sqrt{\frac{E \cdot I}{m \cdot L^4}} \quad \omega_1 = 50\sqrt{\frac{E \cdot I}{m \cdot L^4}} \quad \omega_2 = 104\sqrt{\frac{E \cdot I}{m \cdot L^4}}$$



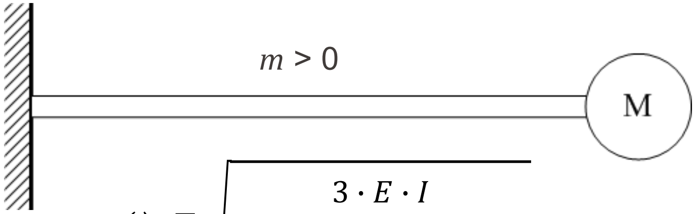
$$\omega = \sqrt{\frac{k}{M}}$$



$$\omega_0 = 3.52\sqrt{\frac{E \cdot I}{m \cdot L^4}}$$

$$\omega_1 = 22\sqrt{\frac{E \cdot I}{m \cdot L^4}}$$

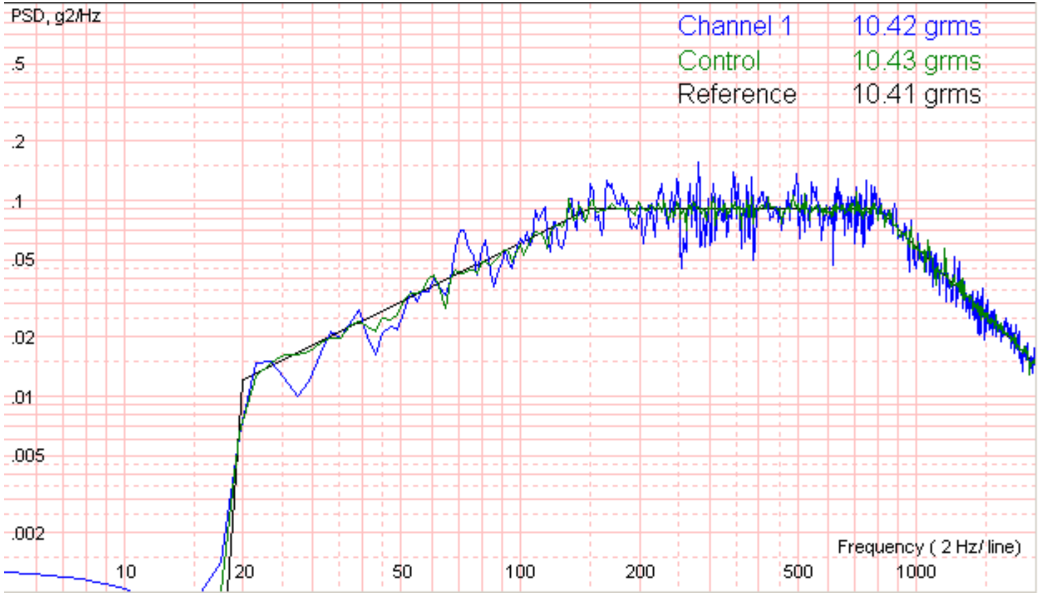
$$\omega_2 = 61.7\sqrt{\frac{E \cdot I}{m \cdot L^4}}$$

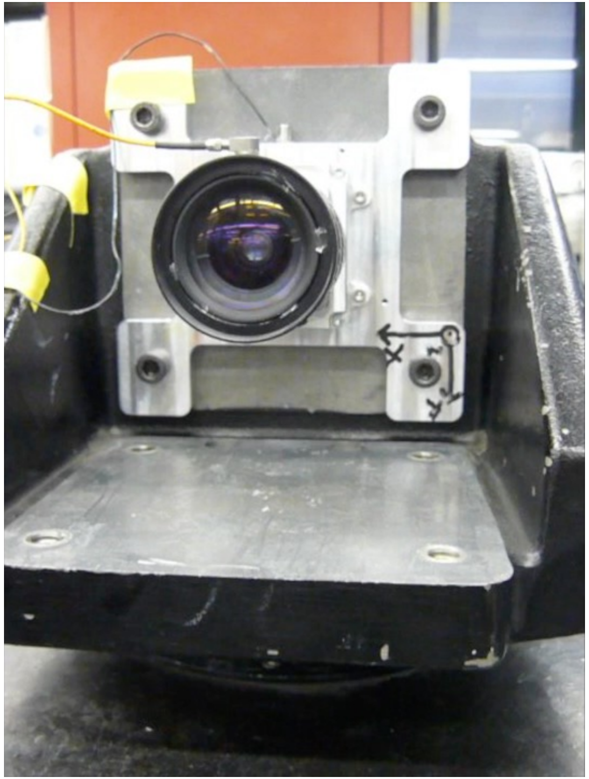


$$\omega_0 = \sqrt{\frac{3 \cdot E \cdot I}{M \cdot L^3 + 0.236 \cdot m \cdot L^4}}$$

Random vibrations

- General dissipative case: arbitrary driving force $F(t)$
  - Vibrations generated by the launcher:
  - $F(t)$ random
  - Vibration amplitude that varies with the frequency and characterized by its Acceleration Spectral Density (ASD)





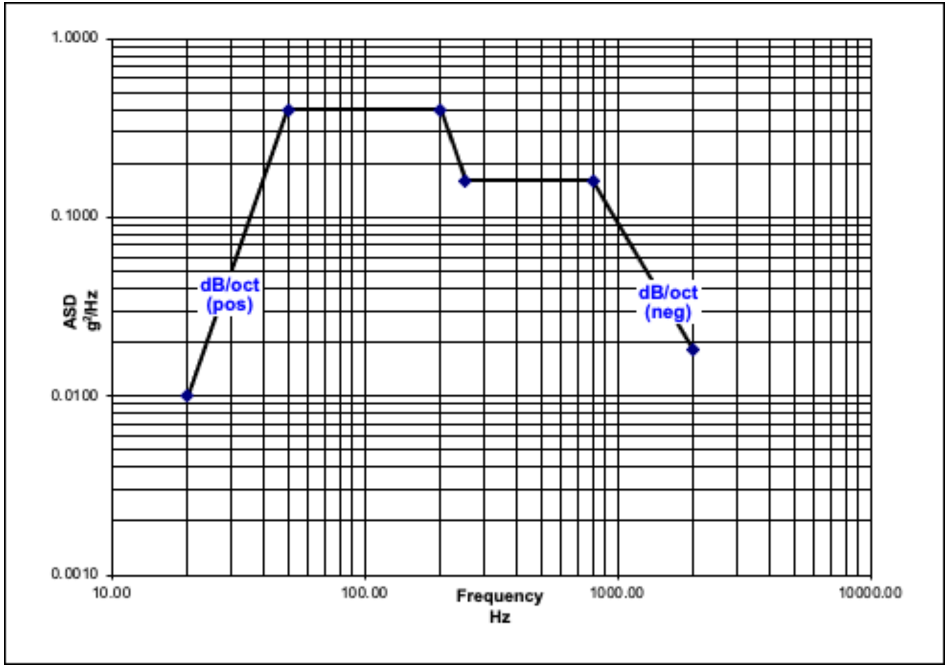
Source: Swiss Space Center


esa
Customer: TAS-I
Specimen: BepiColombo MCS PFM
SINE Sine_1
Control on: P3
Running
ets
ThalesAlenia Space

Random vibration - Example

Frequencies [Hz]	Amplitude (ASD) [g^2/Hz]	Slope [dB/octave]
20 to 50	0.01 to 0.4	12.12
50 to 200	0.4 to 0.4	0
200 to 250	0.4 to 0.16	-12.36
250 to 800	0.16 to 0.16	0
800 to 2000	0.16 to 0.018	-7.2

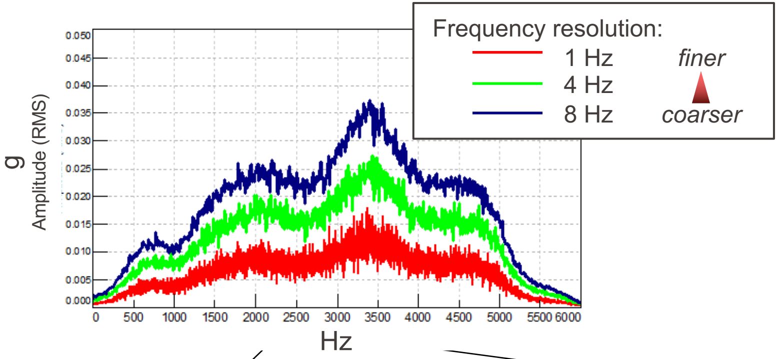
Envelope: 15.2 g_{rms}

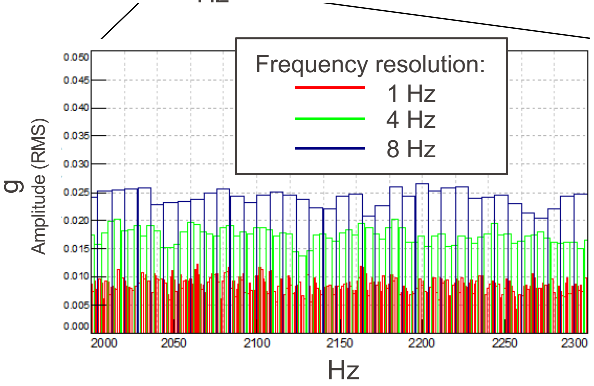


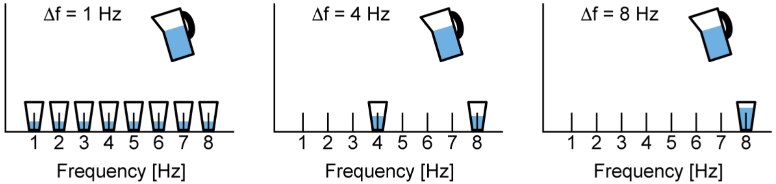
ASD: Acceleration Spectral Density, i.e. specified value

Source: Siemens

Random vibration – Why g²/Hz?



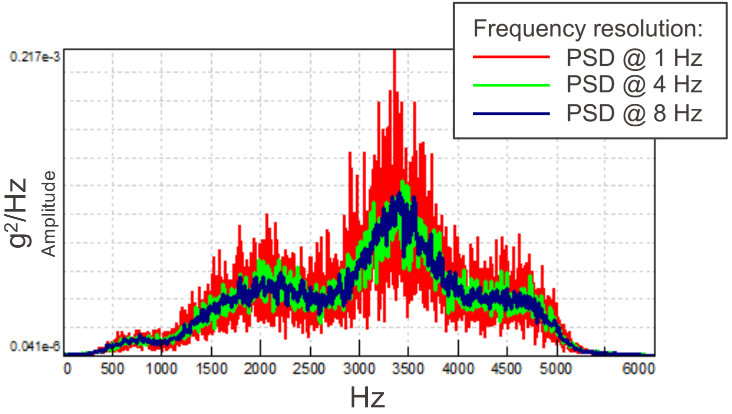


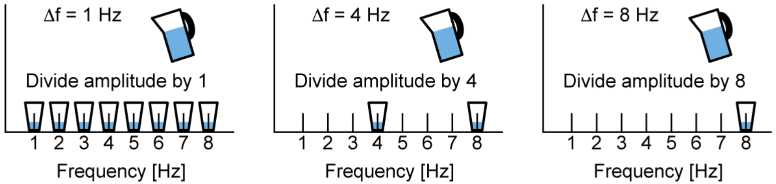


Source: Siemens (https://community.sw.siemens.com/s/article/what-is-a-power-spectral-density-psd)

PSD: Power Spectral Density, i.e. measured signal (e.g. accelerometer)

PSD: $\frac{(A_f)^2}{\Delta f}$





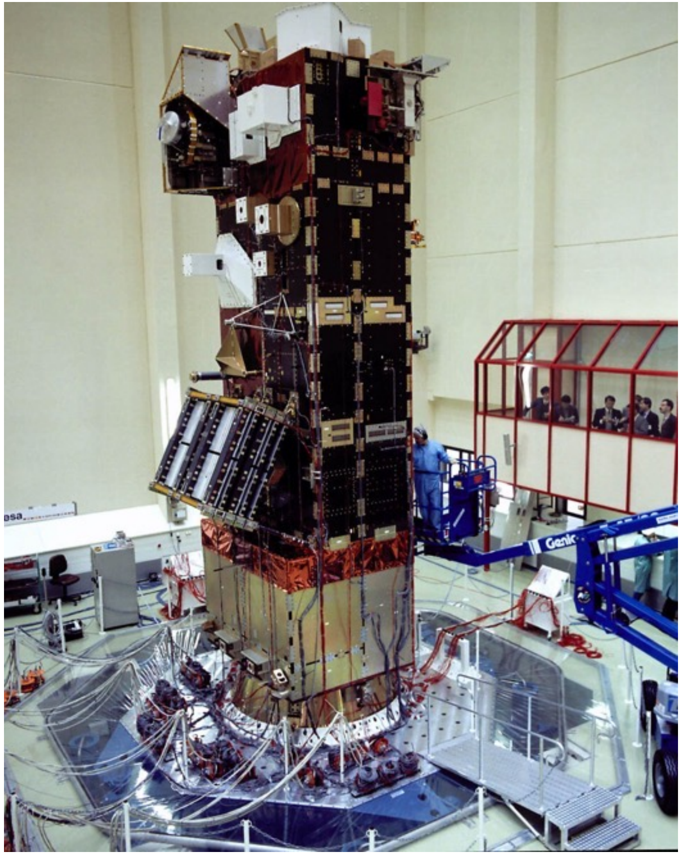
Note: if periodic signal, all energy distributed on specific spectral lines => amplitude depends on frequency resolution if using PSD! **PSD is not for periodic signal.**

Test equipment: shaker

Electrodynamic shaker

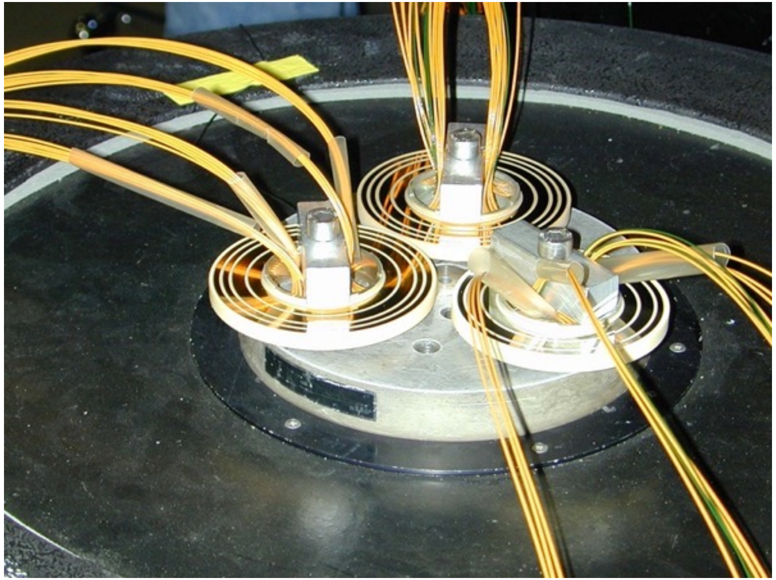
Source: Data Physics Corporation

HYDRA 6DOF ESA **Hydraulic shaker**



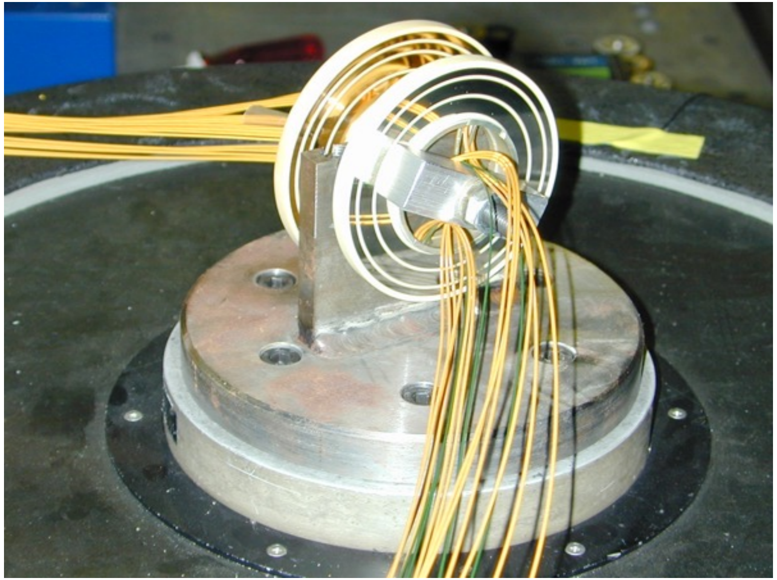
Source: ESA

Vibration Test: example of a test

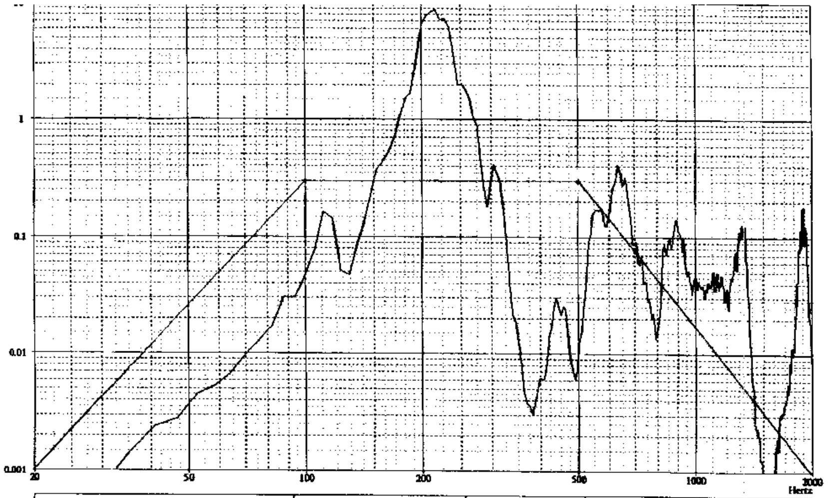


Source: Mecanex SA

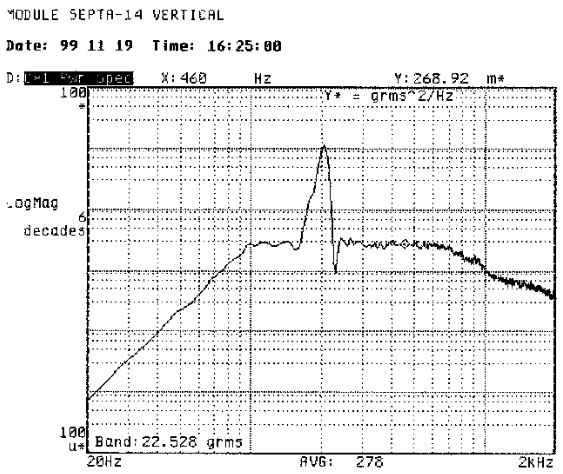
Vibration tests along two axes

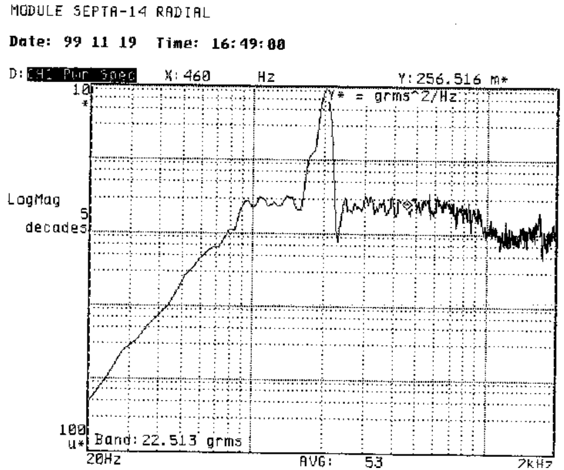


Vibration Test: example of a test



ESSAI DE VIBRATIONS ALEATOIRES			Materiel : SEPTA-14	Essai : 8.44g	Mesure : 7Y	Intespace
D.S.P.		N MOY : 100	Modele : EQM	0005Y	Valeur efficace : 23.47	
Resol : 5.9 Hz	HANNING	D.D.L. : 200	Commande : L31730	25.04.97 HC		





Random vibrations

- The vibration amplitude varies randomly as a function of time

→ $x(t)$ does not help a lot for the characterization

- Use of the **Fourier transform** in order to get a **function frequency**:

$$\ddot{X}(\nu) = H(\nu) \cdot \ddot{U}(\nu)$$

$$X(\nu) = \int_{-\infty}^{\infty} x(t) \cdot e^{-2\pi \cdot i \cdot v \cdot t} dt$$

$$U(\nu) = \int_{-\infty}^{\infty} u(t) \cdot e^{-2\pi \cdot i \cdot v \cdot t} dt$$

Fourier transforms of the induced $x(t)$ and injected $u(t)$ displacements

$$\ddot{X}(\nu) = -(2\pi \cdot v)^2 \cdot X(\nu)$$

$$\ddot{U}(\nu) = -(2\pi \cdot v)^2 \cdot U(\nu)$$

The second derivatives of the displacements (property of Fourier transform) give the **induced and injected accelerations**

Random vibrations

- $H(\nu)$: **frequency transfer** function linking together the amplitudes of the induced and injected accelerations.
  - Complex function which corresponds to the amplification factor ζ (cf. before), but for random amplitudes

- Root mean square (rms) of the **induced amplitude**:

$$\overline{x(t)}^2 = \frac{1}{T}\int_0^T x(t)\cdot x(t)^* dt$$

  - The **power spectral density $W_x(\nu)$** is calculated from this mean value by using the Fourier transform:

$$\overline{x(t)}^2 = \int_0^\infty \frac{2}{T}\cdot X(\nu)\cdot X(\nu)^* d\nu$$

with $$W_x(\nu) = \frac{2}{T}\cdot X(\nu)\cdot X(\nu)^*$$

Random vibrations

- In a similar way it is possible to define the **injected Acceleration Spectral Density (ASD)** $W_{\ddot{U}}(\nu)$:

$$\overline{\ddot{u}(t)}^2 = \int_0^{\infty} \frac{2}{T} \cdot \ddot{U}\,(\nu) \cdot \ddot{U}\,(\nu)^* d\nu$$

with $$W_{\ddot{U}}(\nu) = \frac{2}{T} \cdot \ddot{U}\,(\nu) \cdot \ddot{U}\,(\nu)^*$$ [g²/Hz]

- Root mean square of the induced acceleration using ASD and transfer function $H(\nu)$:

$$\ddot{x}_{rms} = \sqrt{\int_0^{\infty} H(\nu)^2 \cdot W_{\ddot{U}}(\nu) \cdot d\nu}$$

Random vibrations: Miles Formula

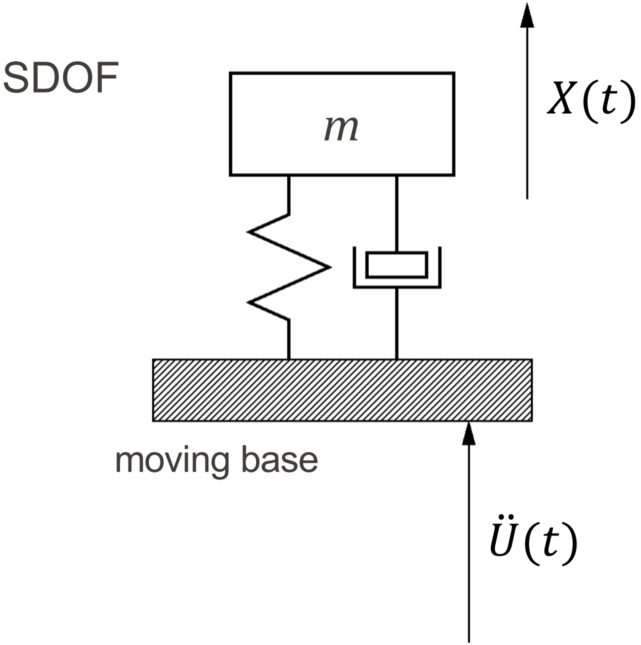
- **Miles Formula** (John W. Miles in Journal of the Aeronautical Sciences, 1954):
  - For systems with one dominant vibration mode of frequency ν, the root mean square of the acceleration is given through an approximation, the Miles formula (in [g]):

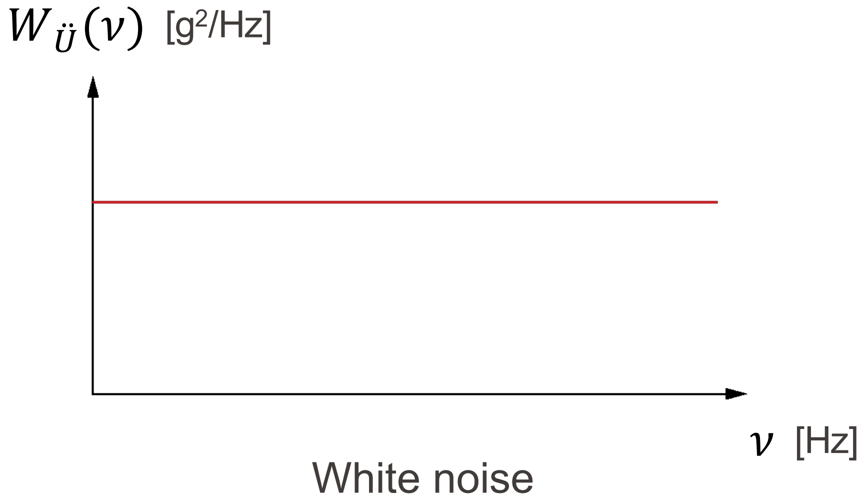
$$\ddot{x}_{rms} = \sqrt{\frac{\pi \cdot Q \cdot \nu \cdot W_{\ddot{U}}(\nu)}{2}} \quad \dashrightarrow \quad \ddot{x}_{peak} = n \cdot \sqrt{\frac{\pi \cdot Q \cdot \nu \cdot W_{\ddot{U}}(\nu)}{2}}$$

  - With $Q = \frac{1}{2 \cdot \eta}$ Q factor, i.e. max. amplification factor (overload)
  - n: envelope factor (normally 3 for Raleigh distribution 3σ)
    - Acceleration may be much higher! For gaussian distribution of the amplitudes:
      - Amplitude 1σ corresponds to $\ddot{x}_{rms}$
      - Acceptable amplitude for dimensioning: 3σ (i.e. 99.7% of all amplitudes)
  - This formula is important for the pre-dimensioning of mechanisms

Random vibrations: Miles Formula

- **Miles Formula**





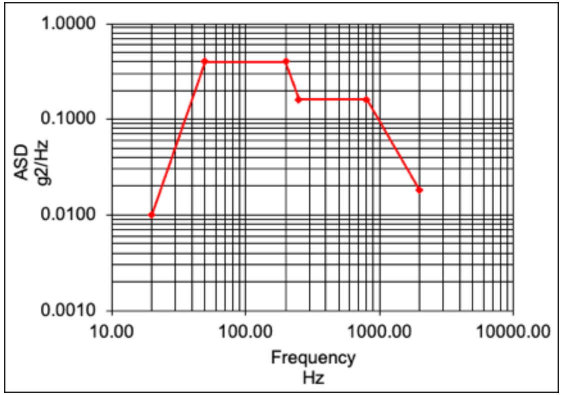
SDOF: Single Degree Of Freedom oscillator

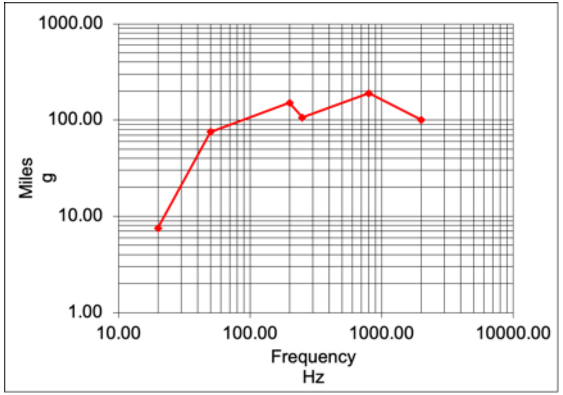
EPFL

Use of Miles formula

$$Miles = 3 \cdot \sqrt{\frac{\pi \cdot Q \cdot \nu \cdot ASD(\nu)}{2}}$$

- The ASD envelope shall be known through the mechanism requirements
- Define a value for Q (hypothesis, in general 10 to 20)
- If an eigenfrequency of the mechanism is known, use this frequency in the Miles formula. If not, search the worst case (cf. example here)
- The amplitude of Miles is expressed in [g = 9.81m/s^2]





Miles formula

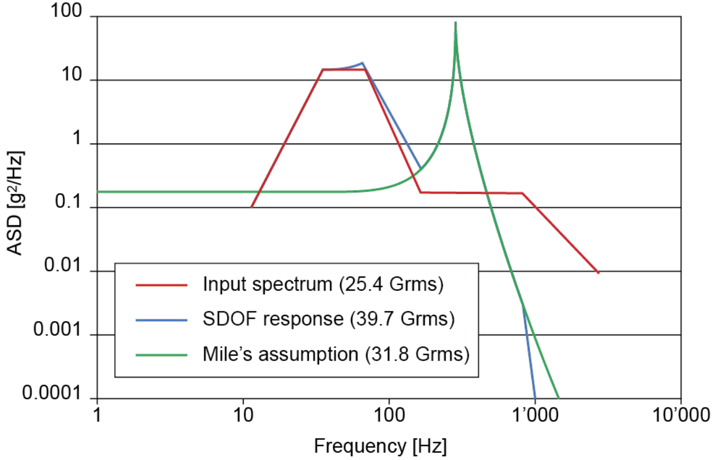
▪ Use

- **Design**: to estimate the loads due to random vibration (3σ)
- **Test**: estimate the overall RMS acceleration at resonant peak of interest

▪ Not use

- Does not work in reverse: accelerations cannot be determined during random vibration testing using Miles' Equation
- Does not give an equivalent static load: $G_{rms}(\nu_{res}) \cdot m \neq F_{static}$
- May not be conservative for a shaped input spectrum

  ⚠ *input spectrum with high ASD levels at* $\nu < \nu_{resonance}$



Vibrations: units

- Acceleration: [m/s²] or [g] (= 9.81 m/s²)
- Amplitude of the sine vibrations:
  - Normally: [g]
  - For very low frequency (< 20 Hz): [mm]

$$x = x_0 \cdot \cos(\omega \cdot t)$$

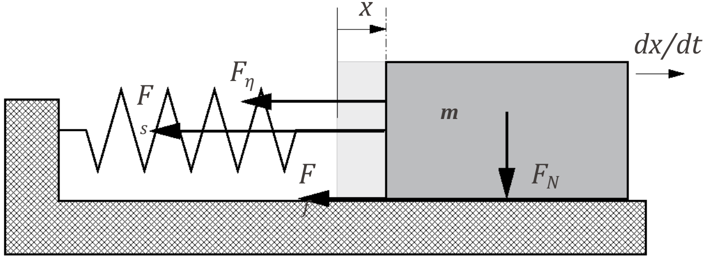
$$\ddot{x} = -\omega^2 x_0 \cdot \cos(\omega \cdot t)$$

$$\left|\frac{\ddot{x}}{x}\right| = \omega^2$$

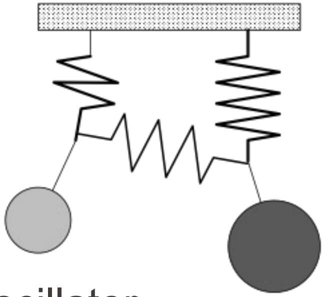
- Acceleration/Power Spectral Density (ASD/PSD): [g²/Hz]
- Variation of ASD/PSD: [dB/oct] or [dB/decade]
  - 1 octave: doubling of frequency
  - 1 decade: 10x the frequency
  - Define straight lines in the log scale representation of ASD/PSD

$$\#\,Octave = \frac{\log\left({}^{\upsilon_H}/_{\upsilon_L}\right)}{\log(2)}$$

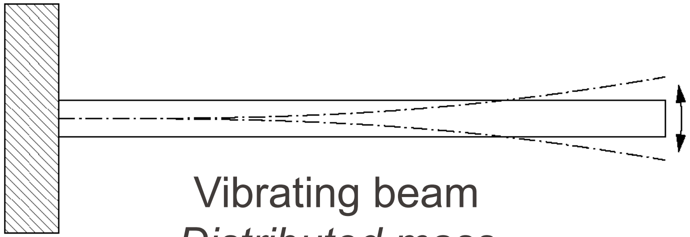
Vibration of deformable structures



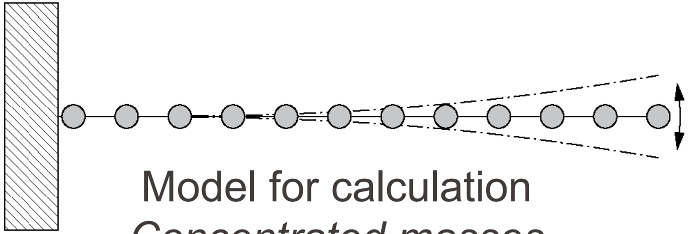
Simple oscillator



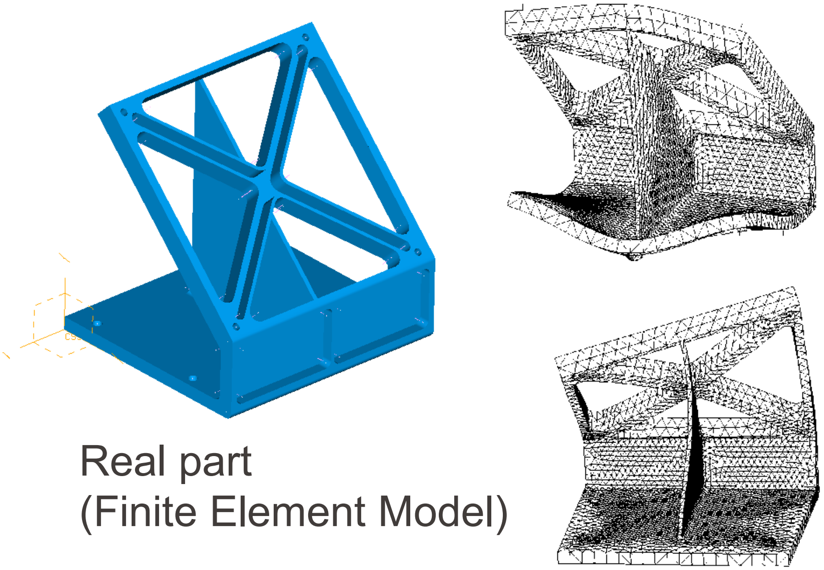
Coupled oscillator



Vibrating beam
Distributed mass



Model for calculation
Concentrated masses



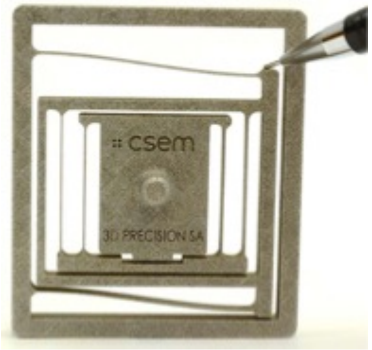
Real part
(Finite Element Model)

Compliant Mechanisms

Source: FreeFlex Pivot

Source: Ruland

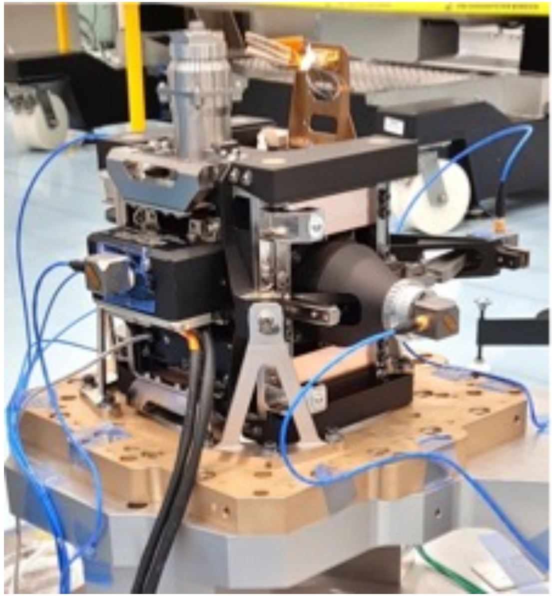
Source: Almatech



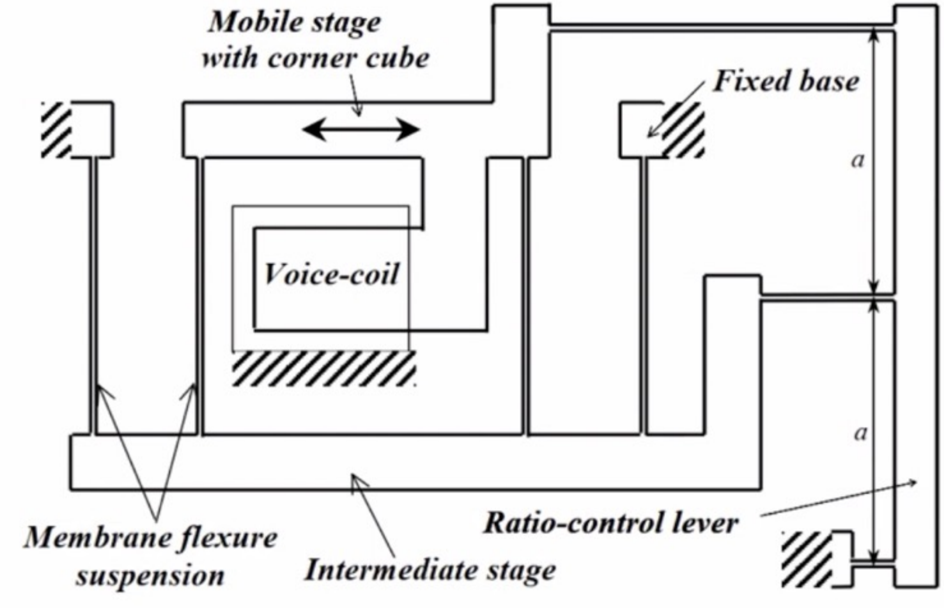
Source: CSEM

Source: ESA

Compliant Mechanisms

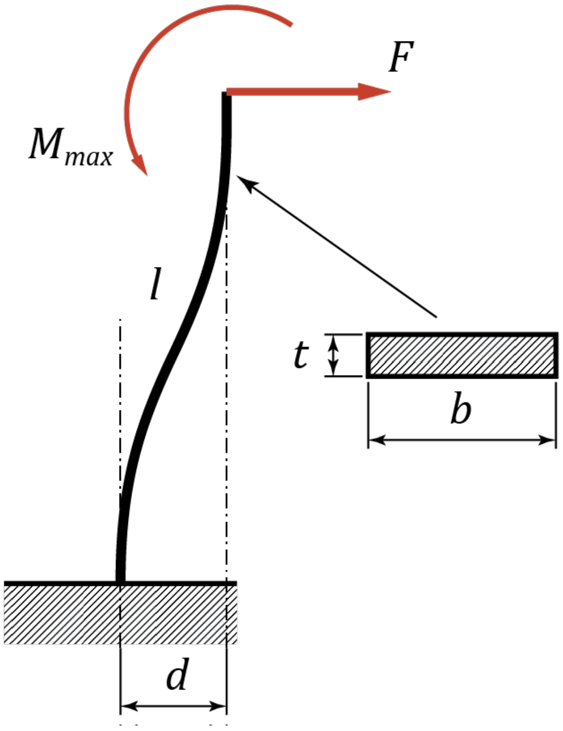


Source: CSEM



Source:
P. Spanoudakis et al. "*Design and Production of the METOP Satellite IASI Corner Cube Mechanisms*", European Space Mechanisms and Tribology Symposium, San Sebastian (2003)

Bending beam



Top end guided, bottom end fixed

- Maximum bending moment $M_{max} = \frac{F \cdot l}{2}$
- Elastic deformation $d = \frac{F \cdot l^3}{12EI}$
- Area moment of inertia $I = \frac{b \cdot t^3}{12}$
- Maximum bending stress

$$\boldsymbol{\sigma_{max}} = \frac{M_{max}}{I} \cdot \frac{t}{2} = \frac{3Fl}{b \cdot t^2} = \frac{\boldsymbol{3d \cdot t}}{\boldsymbol{l^2}} \cdot \boldsymbol{E}$$

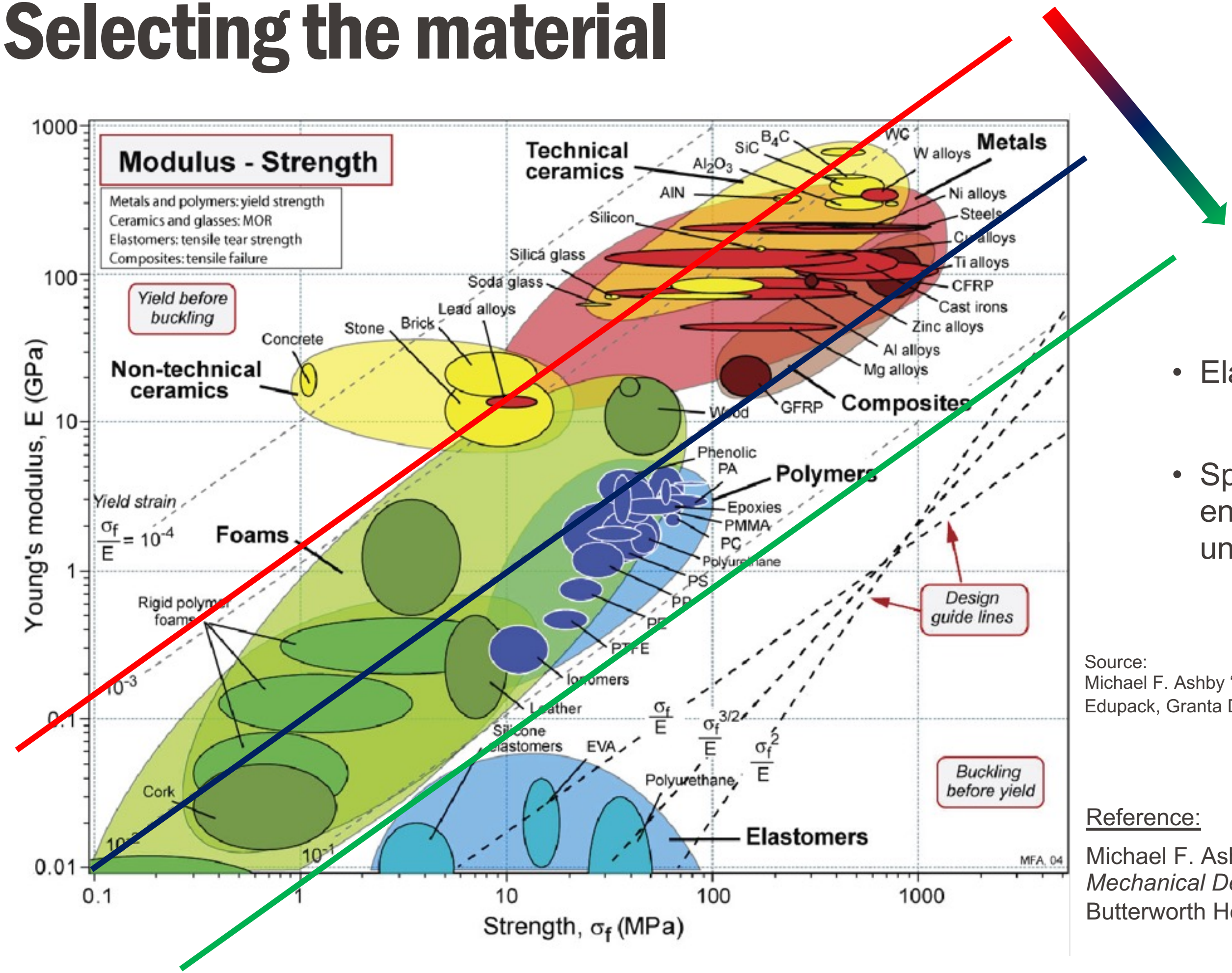
or $$\boldsymbol{d} = \frac{\boldsymbol{l^2}}{\boldsymbol{3t}} \cdot \frac{\boldsymbol{\sigma_f}}{\boldsymbol{E}}$$

Bending beam

$$\sigma_{max} = \frac{3d \cdot t}{l^2} \cdot E \qquad d = \frac{l^2}{3t} \cdot \frac{\sigma_f}{E}$$

- High allowable strength to Young's modulus ratio $\frac{\sigma_f}{E}$ ↗
- Reducing thickness ➡ lowering max stress
- Thin, long structure are best suited
- Max deformation and max stress are independent from beam width

Selecting the material



- Elastic hinges: $\frac{\sigma_f}{E}$
- Springs, elastic energy storage per unit volume: $\frac{\sigma_f^2}{E}$

Source:
Michael F. Ashby "*Material and Process Selection Charts*", CES Edupack, Granta Design (2009)

Reference:

Michael F. Ashby "*Materials Selection in Mechanical Design*" 3rd edition, Elsevier-Butterworth Heinemann, Oxford (2005)

Advantages of Compliant Mechanisms

- Friction free
  - No wear particle
  - No lubricant
- Fewer parts
- Potential saving on material and production costs
- Infinite life
- Low energy dissipation
- High precision movement

Drawbacks of Compliant Mechanisms

- Design complexity
- Analysis complexity
  - Large displacement
  - Non-linearity
  - Fatigue limit
  - Strength limitation
  - Vibrations (amplification factor)
- Limited movements
- Limited out of plane stiffness
- Energy Retention

Theme 5 Summary

- Roles of structures
- Assembly of structures
- Mechanical properties: stress, deformation, mass
- Vibrations
  - Harmonic oscillator (refresher)
  - Sine Vibrations
  - Dynamic amplification factor (overload)
  - Random Vibrations
  - Miles’s Formula

- Compliant Mechanisms

Next Week

- Theme 6 – Components: Introduction and the ball-bearings

Note:

- Mini Project part 2 Architecture: functions and components
  (cf. EE580_MP2_2025_v1 Architecture.pdf)
  Due date (next week): **April 10th, 11:00**