* **Zonal pricing: market splitting/coupling**

This method is compatible with the pool market mechanism only. When congestion occurs between 2 physical zones managed by a unique pool energy market, this market is split into 2 submarkets, one for each zone. The submarket prices are determined in order to relieve the congestion in the interconnection line.

*Example*:

Borduria and Syldavia power systems are interconnected and decided to be managed by a unique energy market. Basically the corresponding submarkets have the following characteristics:

*Borduria*: $\pi_B = S(q_B) = 10 + 0.01.\ q_B$ [\$/MWh], supply function

$D_B = 500$ MW, inelastic demand

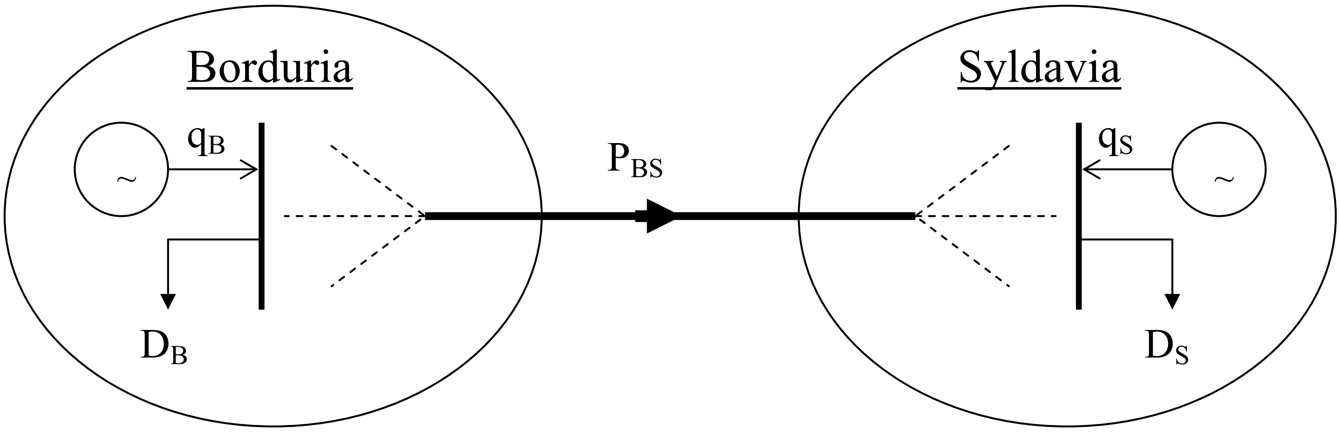
*Syldavia*: $\pi_S = S(q_S) = 13 + 0.02.\ q_S$ [\$/MWh], supply function

$D_S = 1500$ MW, inelastic demand

With no interconnection each submarket operates as follow:

$$q_B^* = D_B \ \&\ \pi_B^* = 15 \text{ \$/MWh};\ q_S^* = D_S \ \&\ \pi_S^* = 43 \text{ \$/MWh}$$

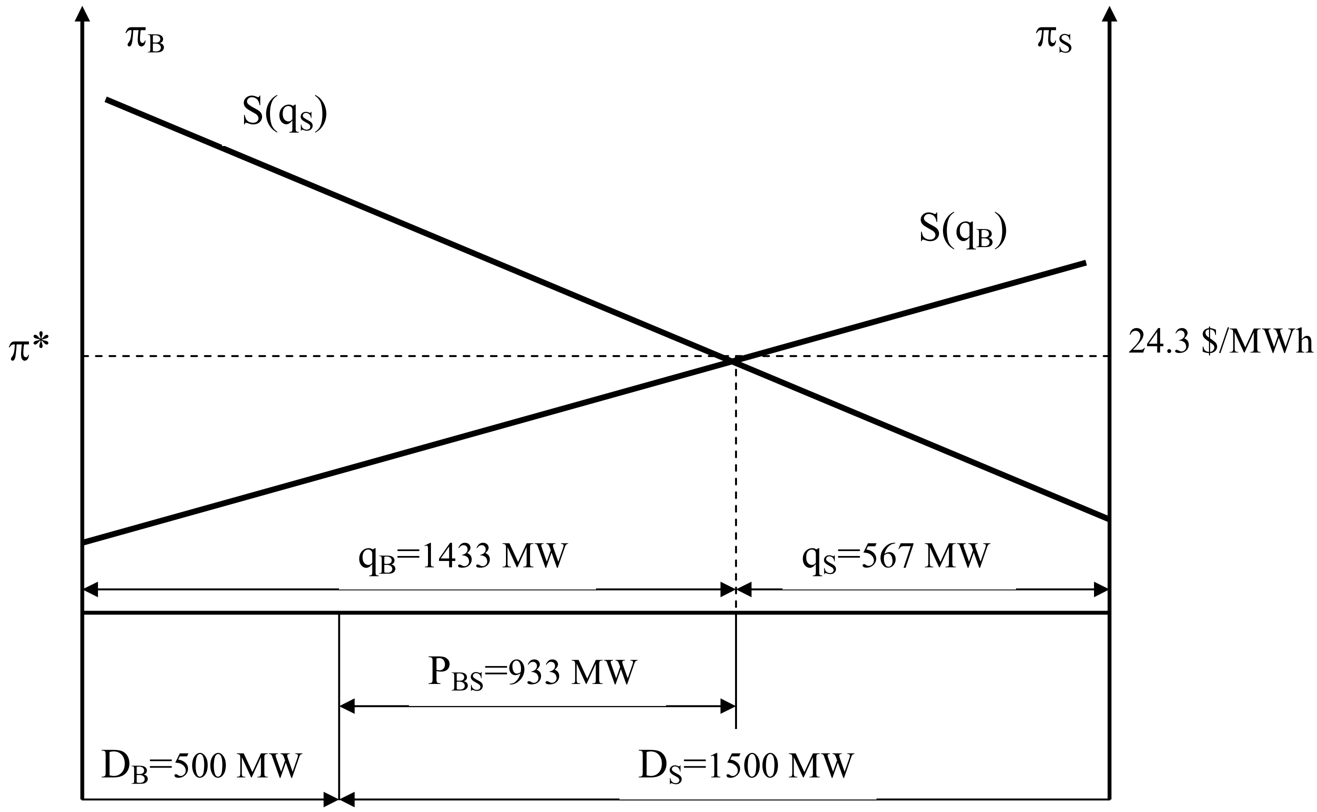
This shows clearly the interest of exchanging power between Borduria and Syldavia.



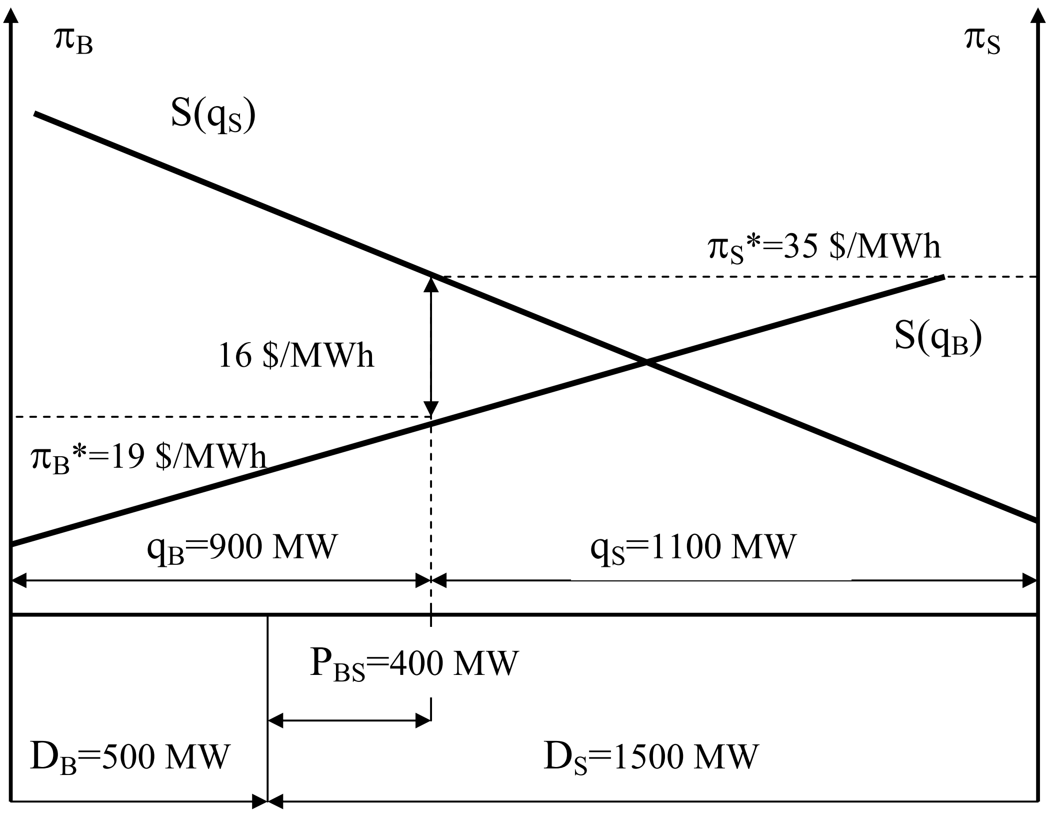

In the case of unique pool market, the MCP $\pi^*$ is determined as follow:

$$\pi^* = \pi_B = \pi_S \text{ and } q^* = q_B + q_S = D_B + D_S = 2000 \text{ MW}$$

It gives $q_B$ and $q_S$ , then $\pi^*$ and $P_{BS}$. Graphically it corresponds to:



If we limit the capacity of the interconnection line to 400 MW, we split this global market into submarket Borduria and submarket Syldavia determining for each a different price. These prices are calculated using the flow to be reduced in the interconnected line in order to remove the congestion. The problem could be solved graphically as follow.




We define the *congestion* (or *merchandizing*) *surplus* as $P_{BS} \cdot (\pi_S - \pi_B)$. This is the difference between total payments and total revenues.

In this example it is equal to $ 6400. It is not normally kept by ISO. It is distributed over the consumers holding *financial transmission rights* (FTR's).

* **Nodal pricing & Locational Marginal Prices (LMP)**

This is a generalization of the zonal pricing method. A market price, defined as *locational marginal pricing* (LMP), is determined at every node of the power system. It accounts implicitly of the possible congestion. Of course, if no congestion then all the prices are equal.

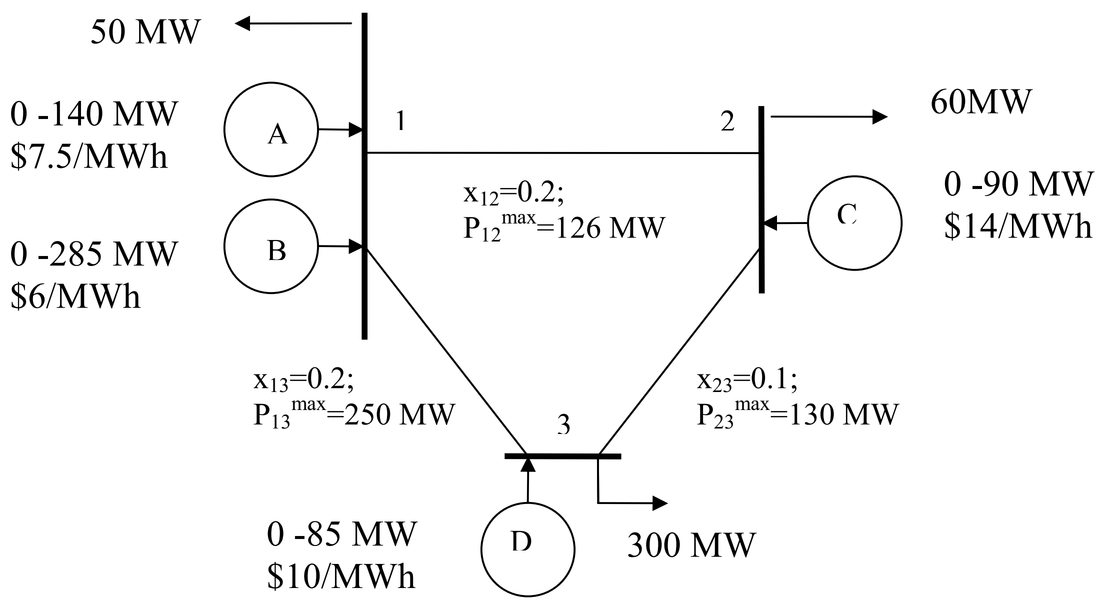
If we consider an inelastic load and a perfect competition (bid prices equal to marginal production costs) LMP's are derived from the *constrained economic dispatch solution*.

This is the minimization of the total generation costs (see pages 36 & 37) which is subject to the balancing equation (load = generation) and the constraints on the capacities of the transmission lines. The Lagrangian multipliers, related to the binding capacity constraints that are different from zero, serve to calculate the LMP's.

In the case of *constrained economic dispatch*, at least one capacity constraint is binding. The flow in the corresponding transmission line is equal to the maximum limit.

A LMP is the cost of supplying an additional megawatt of load at the node under consideration by the cheapest possible means.

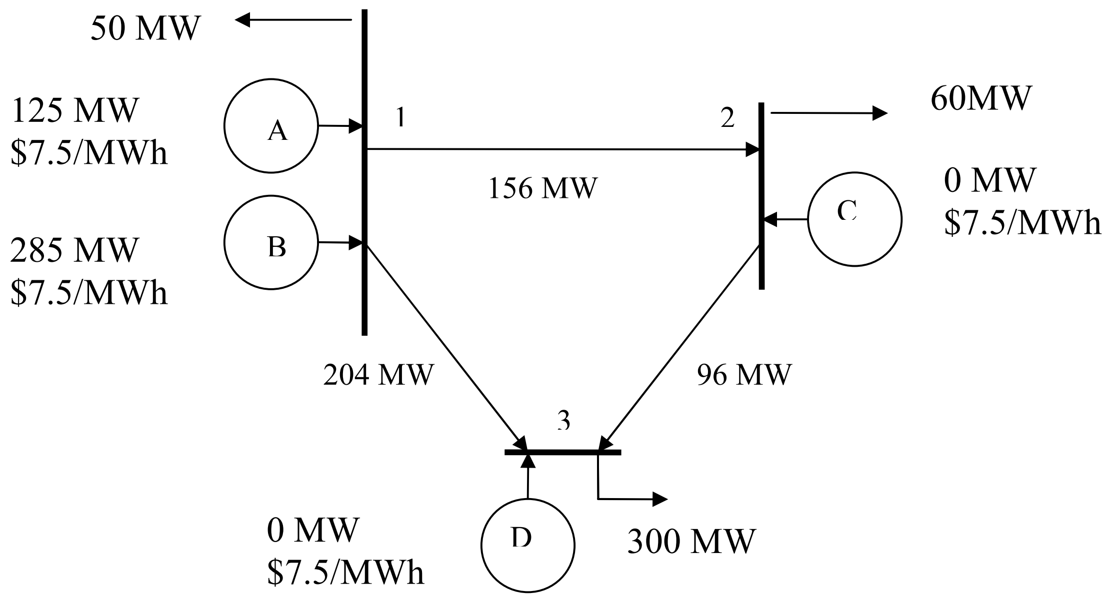
<u>*Example*</u>:




The *unconstrained economic dispatch* would give:

$q_A$= 125 MW; $q_B$= 285 MW, $q_C$= 0 MW, $q_D$= 0 MW.

The corresponding flows in the grid are:



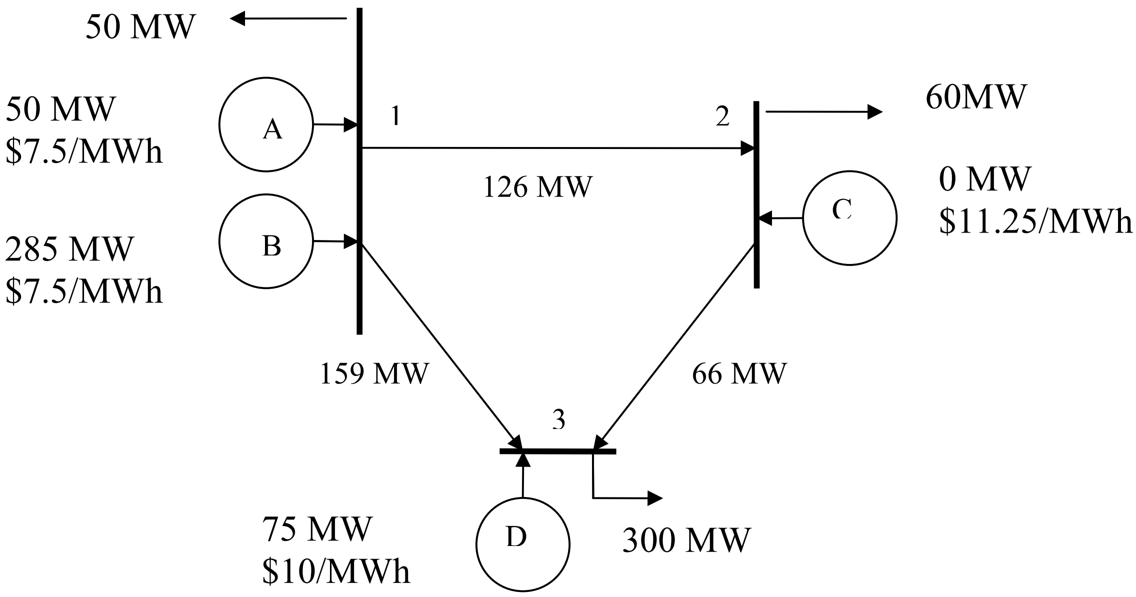

It could be calculated using the DC-Flow method or simply the PTDF's.

The line 1-2 is congested of 30 MW. To get the constrained economic dispatch solution, we must remove the congestion by decreasing the most expensive production at node 1, namely generator A, and increasing by the same quantity the least expensive generator elsewhere, namely generator D. This is the *counter-flow* rule.

Since $PTDF_{31}^{21} = 0.4 = x_{13} / (x_{23} + x_{12} + x_{13})$, the generator D must be increased of 75 MW (=30 MW / 0.4) and the generator A must be decreased of 75 MW.

The constrained economic dispatch solution is:
$q_A$= 50 MW; $q_B$= 285 MW, $q_C$= 0 MW, $q_D$= 75 MW.

The corresponding flows in the grid are:




The LMP's are determined as follow:

- for node 1, any increase of 1 MW of load will be covered at the cheapest cost by the generator A. Then $LMP_1$ = 7.5 $/MWh.

- for node 3, the load increase of 1 MW will be covered at the cheapest cost by the generator D. Then, the $LMP_3$ = 10 \$/MWh. In fact, in this respect any increase of generation at node 1 will overload the line 1-2 and the generator C at node 2 is too expensive.

- For node 2, since the generator 2 is too expensive, we can imagine a combination between generator A and D to cover cheaply the additional MW. However, we can notice that taking generator A or D alone in this respect will lead to overload the line 1-2. The possible combination is to increase one of them and to decrease the other while covering the additional MW and preventing the overload. It means that:

$$\Delta q_A + \Delta q_D = 1 \text{ MW}$$

and

$$PTDF_{12}^{12}\ \Delta q_A + PTDF_{32}^{12} . \Delta q_D = 0$$

with $PTDF_{12}^{12} = 0.6 = (x_{13} + x_{23}) / (x_{13} + x_{23} + x_{12})$ and $PTDF_{32}^{12} = 0.2 = x_{23} / (x_{23} + x_{13} + x_{12})$

We get $\Delta q_A$ = -0.5 MW and $\Delta q_D$ = 1.5 MW and we determine $LMP_2$ as:

$$LMP_2 = 1.5*10\$/MWh - 0.5*7.5\$/MWh = 11.25\ \$/MWh$$

We define the *congestion surplus or merchandizing surplus* as *consumer payments* minus *generator revenues*.

***Generator revenues*:**

$$LMP_1 \,.\, q_A + LMP_1 \,.\, q_B + LMP_2 \,.\, q_C + LMP_3 \,.\, q_D =$$

$$7.5 \,.\, 50 + 7.5 \,.\, 285 + 11.25 \,.\, 0 + 10 \,.\, 75 = 3262.50 \,\$$$

***Consumer payments*:**

$$LMP_1 \,.\, D_1 + LMP_2 \,.\, D_2 + LMP_3 \,.\, D_3 =$$

$$7.5 \,.\, 50 + 11.25 \,.\, 60 + 10 \,.\, 300 = 4050.00 \,\$$$

The ***congestion surplus*** is 787.50 \$. It is distributed over the consumers holding *financial transmission rights* (FTR's) as *hedging* mechanism.