Mathematical Formulation of Exercises 4.2/ 4.4/ 4.5

For this unit and time period $t \in \mathbf{T}$, let us consider three variables: $x_t \in \{0,1\}$ indicates if this unit is up at time t; $u_t \in \{0,1\}$ indicates whether the unit starts up at time t; and p_t is the quantity of power produced by the unit at time t. We also consider that $L, l \leq |\mathbf{T}|$ are the up and down times, respectively. To find the optimal dispatch, the following optimization problem should be solved:

$$\max \sum_{t=1}^{|\mathbf{T}|} (\lambda_t p_t - c_p(p_t) - c_0 u_t - c_{NL} x_t)$$

subject to:

$$\sum_{t'=t-L+1}^{t} u_{t'} \leq x_t \qquad \forall t \in \{L, ..., |\mathbf{T}|\} \tag{1a}$$

$$\sum_{t'=t-l+1}^{t} u_{t'} \leq 1 - x_{t-l} \qquad \forall t \in \{l, ..., |\mathbf{T}|\} \tag{1b}$$

$$u_t \geq x_t - x_{t-1} \qquad \forall t \in \{2, ..., |\mathbf{T}|\} \tag{1c}$$

$$P_{min} x_t \leq p_t \leq P_{max} x_t \qquad \forall t \in \mathbf{T} \tag{1d}$$

$$x_t, u_t \in \{0,1\} \qquad \forall t \in \mathbf{T} \tag{1e}$$

here, λ_t is the price of energy in $\$/MWh$, $c_p(p_t)$ is the cost function of generating unit in $\$/h$ ($c_p(0) = 0$), c_0 ($c_0 = 500$ \$) is the start-up cost of generating unit in \$, and c_{NL} ($c_{NL} = 144$ \$$/h$) is the no-load cost in $\$/h$.

The inequality (1a) is the minimum up-time constraint: it states that if the unit is down at time t, then it cannot have started up during the L previous periods. Inequality (1b) is the minimum down-time constraint, which is symmetric to the minimum up-time constraint. Inequality (1c) ensures that if the unit starts-up at time t ($x_t - x_{t-1} = 1$) then the start-up variable u_t must be 1. Inequality (1d) sets bounds to the quantity of power produced by the unit.

References:

[1] Rajan, D., & Takriti, S. (2005). Minimum up/down polytopes of the unit commitment problem with start-up costs. IBM Res. Rep, (RC23628), W0506-050.

[2] Bendotti, P., Fouilhoux, P., & Rottner, C. (2018). The min-up/min-down unit commitment polytope. Journal of Combinatorial Optimization, 36(3), 1024-1058.