EE-565 - W4
DC MACHINES CHARACTERISTICS

Prof. D. Dujic
École Polytechnique Fédérale de Lausanne
Power Electronics Laboratory
Switzerland

MATHEMATICAL MODEL

Dynamic model of DC machine is given as:

- voltage balance in the excitation winding:

$$u_f = R_f i_f + \frac{d\Psi_f}{dt} = R_f i_f + L_f \frac{di_f}{dt}$$

- voltage balance in the armature winding:

$$u_a = R_a i_a + L_a \frac{di_a}{dt} + E_a$$

- electromotive force:

$$E_a = k_e \Phi_f \Omega_m$$

- electromagnetic torque:

$$T_{em} = k_m \cdot \Phi_f \cdot i_a$$

- relation of excitation current to excitation flux:

$$\Phi_f = L_f^{'} i_f$$

- Newton equation:

$$J \frac{d\Omega_m}{dt} = T_{em} - T_m - k_F \Omega_m$$

- in case of DC machine with permanent magnets, the first equation is removed and excitation flux is constant

BLOCK DIAGRAM OF THE MODEL

With the help of Laplace Transform block diagram of model is obtained

- inputs into model are: excitation and armature voltage
- state variables are: armature current, excitation current and rotor speed
- outputs are: armature current, excitation flux, electromagnetic torque and the rotor speed

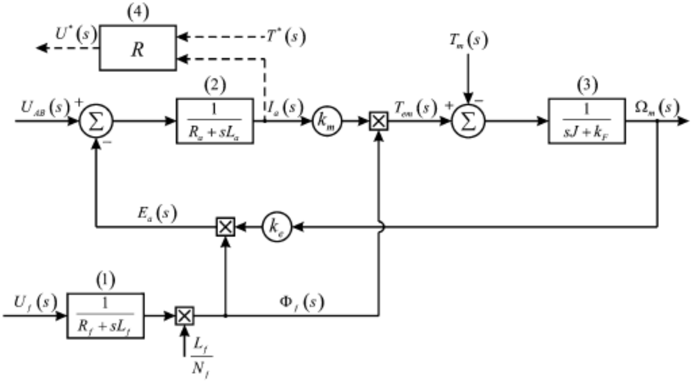

Figure 1 Model of DC machine presented as a block diagram (We can simplify stator part considering constant flux conditions).

TORQUE CONTROL

In the motion control applications, electrical machines are used to provide variable torque

- the torque of DC machine is proportional to the product of armature current and flux:

$$T_{em} = k_m \cdot \Phi_f \cdot i_a$$

Theoretically, torque could be controlled by changing either:

- the flux - too slow (large time constant; $L_f \gg L_a$) but also not possible with permanent magnet DC machines,
- the armature current - actually used in practice (small time constant)
- the speed of torque response is defined by the speed of response of the armature current:

$$\frac{di_a}{dt} = \frac{1}{L_a}(u_a - R_a i_a - E_a)$$

- variation of the armature current can be accomplished by varying the armature voltage
- regulator is acting on the current error (torque error) and generates required armature voltage command

STEADY-STATE EQUIVALENT CIRCUIT

In the steady-state there is no change in rotor speed or electrical currents

- steady state in excitation winding is defined by:

$$U_f = R_f I_f$$

- steady state in the armature winding:

$$U_a = R_a I_a + k_e \cdot \Phi_f \cdot \Omega_m$$

- armature current in the steady state is:

$$I_a = \frac{U_a - E_a}{R_a}$$

- external source internal resistance R_m is rather neglected...

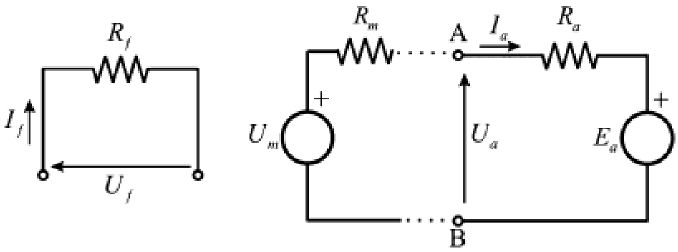

Figure 2 Steady-state equivalent circuits.

MECHANICAL CHARACTERISTIC (I)

Mechanical characteristic relates torque and speed in the steady-state:

- DC machine in steady-state has voltage balance equation:

$$U_m = U_a = R_a I_a + k_e \cdot \Phi_f \cdot \Omega_m$$

- if the EMF equals supply voltage, armature current is zero: $I_a = 0$
- electromagnetic torque is also zero!
- rotor angular speed (*no load speed*), under these conditions is:

$$\Omega_0 = \frac{U_m}{k_e \Phi_f}$$

- in the steady-state rotor speed is also constant:

$$J \frac{d\Omega_m}{dt} = 0$$

- neglecting friction torque, electromagnetic torque equals load torque!
- armature current is thus proportional to the load torque
- any increase of load torque increases armature current
- increase of armature current reduces the rotor speed

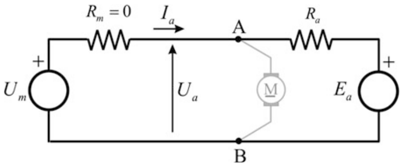

Figure 3 Steady-state equivalent circuits.

Mechanical characteristic relates torque and speed in the steady-state

- mechanical characteristic is expressed as: $T_{em}(\Omega_m), \Omega_m(T_{em})$
- armature current in the steady state is:

$$I_a = \frac{U_m - E_a}{R_a} = \frac{U_m - k_e \Phi_f \Omega_m}{R_a}$$

- electromagnetic torque is:

$$T_{em} = k_m \Phi_f \frac{U_m - k_e \Phi_f \Omega_m}{R_a} = k_m \Phi_f \frac{U_m}{R_a} - \frac{k_m k_e \Phi_f^2}{R_a} \Omega_m = T_0 - S\Omega_m$$

- at standstill, start-up torque is:

$$T_0 = k_m \Phi_f \frac{U_m}{R_a}$$

- at standstill, start-up current can be very large and has to be limited
- slope or the *stiffness* of mechanical characteristics is:

$$S = \frac{k_m k_e \Phi_f^2}{R_a}$$

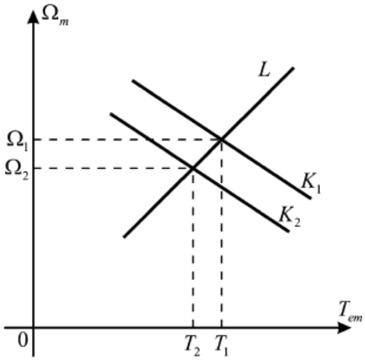

Figure 4 Mechanical characteristic.

- work machine has its own L
- steady-state operating point is at the intersection of two points

$$T_{em} = T_m$$

STABLE EQUILIBRIUM

Intersection of machine and load mechanical characteristic can be stable or unstable?

$$T_{em} = T_0 - S_{em}\Omega_m \qquad \Delta T_{em} = -S_{em}\Delta\Omega_m$$
$$T_m = T_{0m} - S_m\Omega_m \qquad \Delta T_m = -S_m\Delta\Omega_m$$

- assume that from steady-state, small change of speed occurs due to external disturbance:

$$J\frac{d(\Omega_1 + \Delta\Omega_m)}{dt} = J\frac{d\Delta\Omega_m}{dt} = T_{em} - T_m = (S_m - S_{em})\Delta\Omega_m$$

- **unstable** case:

$$S_m - S_{em} > 0 \quad \Delta\Omega_m > 0 \rightarrow \frac{\Delta\Omega_m}{dt} > 0 \quad \Delta\Omega_m < 0 \rightarrow \frac{\Delta\Omega_m}{dt} < 0$$

- small change of the speed (disturbance) results in even larger change of speed that progressively increase
- **stable** case:

$$S_m - S_{em} < 0 \quad \Delta\Omega_m > 0 \rightarrow \frac{\Delta\Omega_m}{dt} < 0 \quad \Delta\Omega_m < 0 \rightarrow \frac{\Delta\Omega_m}{dt} > 0$$

- small change of the speed (disturbance) gradually decrease and bring system to original operating point

PROPERTIES OF MECHANICAL CHARACTERISTIC

Basic properties of the electrical machine mechanical characteristic:

$$T_{em} = k_m \Phi_f \frac{U_m - k_e \Phi_f \Omega_m}{R_a} = k_m \Phi_f \frac{U_m}{R_a} - \frac{k_m k_e \Phi_f^2}{R_a} \Omega_m = T_0 - S\Omega_m$$

- *no load speed* – intersection of mechanical characteristic with the vertical axis

$$\Omega_0 = \frac{T_0}{S} = \frac{U_m}{k_e \Phi_f}$$

- *starting torque* – intersection of mechanical characteristic with the horizontal axis

$$T_0 = k_m \Phi_f \frac{U_m}{R_a}$$

- *stiffness* – considered positive if torque drops with increase of speed

$$S = \frac{k_m k_e \Phi_f^2}{R_a} \quad S = -\frac{\Delta T_{em}}{\Delta \Omega_m}$$

- alternative convention:

$$\Omega_m = \Omega_0 - \frac{1}{S} T_{em}$$

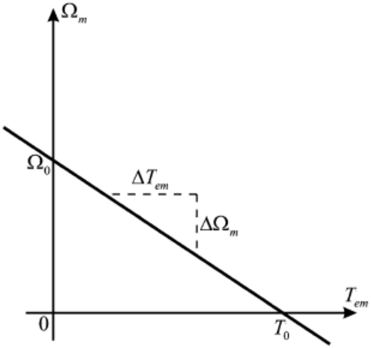

Figure 5 Mechanical characteristic properties.

Armature resistance is rather low and external resistances are often needed

- due to small R_a the slope or the stiffness is rather large: large torque change leads to small speed change

In case that supply voltage is always constant:

- speed could be changed by inserting external resistor to modify armature voltage / current
- inefficient approach: $U_a = U_m - R_{ext} I_a$

Use of a supply with variable voltage is a better approach

- change of armature supply voltage does not affect slope of mechanical characteristics
- *no load speed* is proportional to supply voltage
- family of mechanical characteristics is obtained
- Q1: motor, $T_{em} > 0, \Omega_m > 0$
- Q2: generator, $T_{em} < 0, \Omega_m > 0$
- Q3: motor, $T_{em} < 0, \Omega_m < 0$
- Q4: generator $T_{em} > 0, \Omega_m < 0$

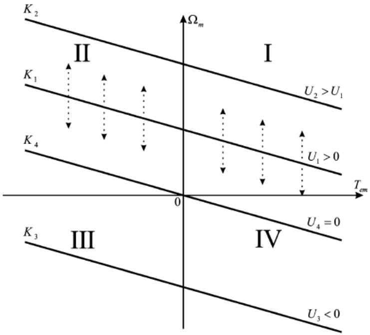

Figure 6 Speed - Torque operating quadrants.

DC GENERATOR

DC machines can operate in the generator mode

- mechanical power is supplied to the shaft and keeps the rotor in motion
- induced EMF supply the load connected between brushes (armature current changes direction)

$$U_G = E_a - R_a I_G$$

- current-voltage characteristics is characterized by slope determined by external load (voltage drop)
- to maintain output voltage constant, excitation voltage must be manipulated by control

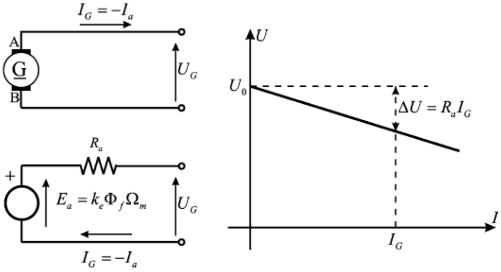

Figure 7 DC generator characteristics

RATED RATINGS OF DC MACHINES

Each machine is characterized with rated quantities

Safe operating areas need to be known, both for steady-state and transients

Rated voltage:

- the highest voltage that can be permanently applied without causing breakdown, or premature ageing
- limited by insulation design and dielectric breakdown

Rated current:

- maximum permissible current in continuous operation
- limited by Joule losses

Rated flux:

- the higher the better, since lower armature current is required (lower losses)
- limited by magnetic saturation of the ferromagnetic material

Rated speed:

- achieved with rated flux when EMF equals the rated voltage
- limited by mechanical constraints, but also electrical limits on the allowed EMF

THERMAL MODEL AND INTERMITTENT OPERATION

Losses in electrical machine lead to increase of temperature

- in steady-state heat generated in machine is equal to the heat transferred to the environment
- temperature does not change and *heating* is equal to *cooling*

Simplified model considers

- *thermal resistance:* determines heat emission from the machine to environment – cooling power
- *thermal capacity:* determines the heat accumulated within a machine - accumulation
- the change of machine temperature is described as:

$$P_\gamma = \frac{\Delta\theta}{R_T} + C_T \frac{d\Delta\theta}{d_t}$$

- thermal time constant

$$\tau = R_T C_T$$

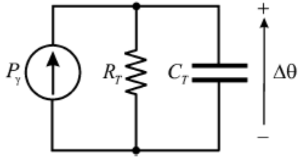

Figure 8 Simple thermal model

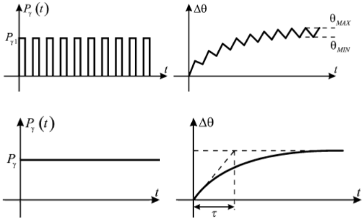

Figure 9 Temperature rise for different operating conditions

FIELD WEAKENING

DC machine can be operated above the rated speed

- rated speed is determined as:

$$\Omega_n = \frac{U_n}{k_e \Phi_n}$$

- to go above the rated speed, flux must be reduced
- care should be taken that EMF does not exceed rated voltage

$$E_a = k_e \Phi_n \Omega_m$$

- flux reduction at high speeds is called *field weakening*
- power available in the field weakening is *constant* and equal to rated power
- there are cases when flux reduction is desired even at low speeds
- e.g. machine operates with low torque (low armature current)
- if the flux is kept rated, magnetic losses are the dominant
- lowering the flux and increasing the armature current is more efficient

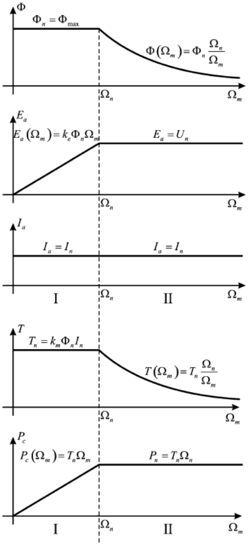

Figure 10 Typical operating range