EE-565 - W14
SM
MODELING

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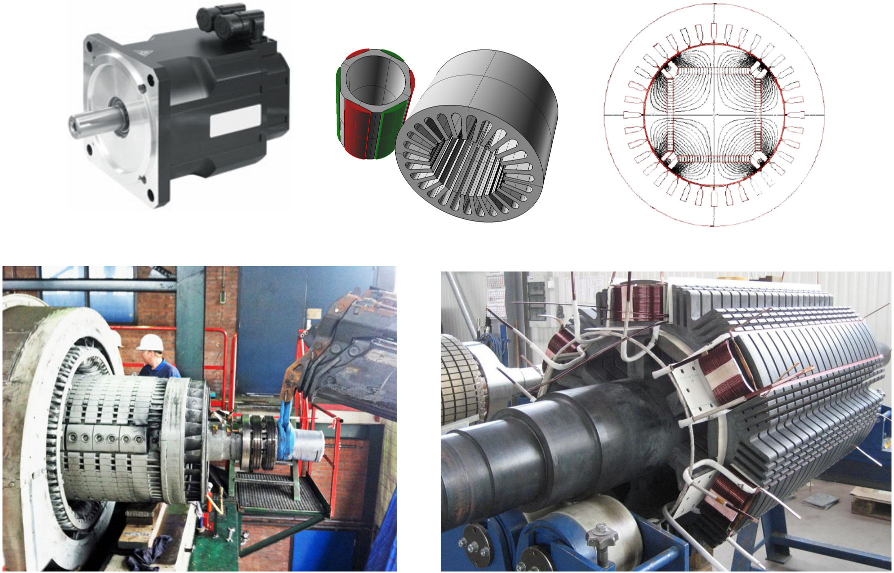

Figure 1 Synchronous machines - various parts.

The same modelling assumptions as for IM are followed:

- stator and rotor distributed windings are replaced with concentrated windings
- both stator and rotor have three-phase windings
- flux distribution along the air gap is assumed perfectly sinusoidal
- magnetic saturation is neglected
- core losses (hysteresis and eddy currents) are neglected
- skin effect is neglected
- parasitic capacitances are neglected
- resistances and inductances of the windings are constant

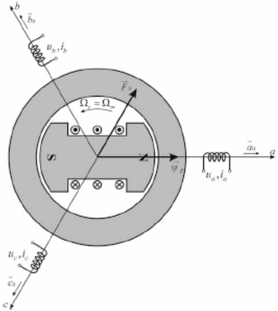

Figure 2 Rotor flux vector and stator magnetomotive force.

BASE VALUES - NORMALIZATION

Normalization is done with following base values:

- Phase voltage

$$U_b = \sqrt{2} U_n$$

- Phase current

$$I_b = \sqrt{2} I_n$$

- Angular frequency

$$\omega_n = 2\pi f_n$$

- Total flux

$$\Psi_b = U_b / \omega_n$$

- Apparent power

$$S_b = 3 U_n I_n = 3 U_b I_b / 2$$

- Mechanical angular speed

$$\Omega_n = \omega_n / p$$

(p is the number of pole pairs)

- Torque

$$T_n = S_n / \Omega_n$$

THREE-PHASE SYNCHRONOUS MACHINE MODEL

As with IM, we have to consider four basic parts:

- Differential equations of voltage balance:

$$\underline{u} = r\underline{i} + \frac{d\underline{\Psi}}{dt}$$

- Relation between the fluxes and currents, given by inductance matrix:

$$\underline{\Psi} = \underline{L}(\theta_m)\underline{i}$$

- Equation for the electromagnetic torque:

$$T_{em} = \frac{1}{2}\underline{i}^T \frac{d\underline{L}}{d\theta_m}\underline{i}$$

- Newton differential equation of motion:

$$J\frac{d\Omega_m}{dt} = T_{em} - T_m$$

Revolving magnetomotive force:

- stator of SM is supplied from a symmetrical power supply:

$$I_a = I_m cos(\omega_e t)$$
$$I_b = I_m cos(\omega_e t - 2\pi/3)$$
$$I_c = I_m cos(\omega_e t - 4\pi/3)$$

- the stator phase currents create the stator rotating magnetomotive force vector
- the speed of rotation depends on the number of pole pairs

$$\Omega_e = \frac{\omega_e}{p}$$

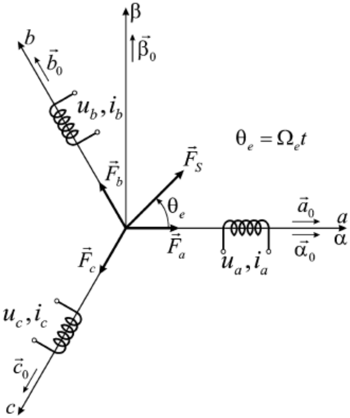

Figure 3 Revolving vector of the stator magnetomotive force.

CLARKE TRANSFORMATION

Using Clarke Transformation we can work with two phase equivalent machine:

$$\begin{bmatrix} I_\alpha \\ I_\beta \end{bmatrix} = K \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

- tranformation apply to: voltage, current, flux
- K = 2/3: peak values are the same before and after transformation (power is not)

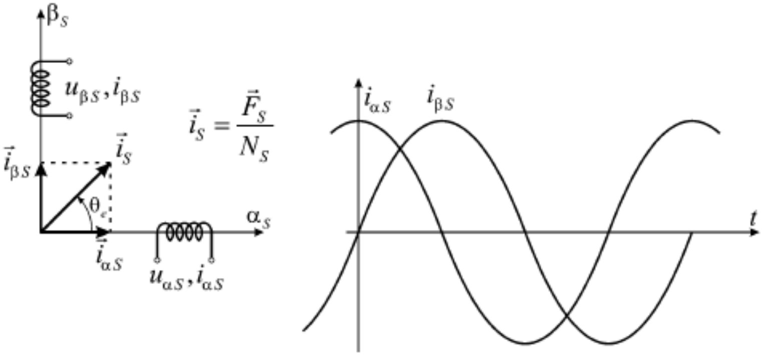

Figure 4 Two-phase representation of the stator winding.

VOLTAGE BALANCE

SM with two phase stator and wound rotor (generic case):

$$U_{\alpha s} = R_s I_{\alpha s} + \frac{d\Psi_{\alpha s}}{dt}$$
$$U_{\beta s} = R_s I_{\beta s} + \frac{d\Psi_{\beta s}}{dt}$$
$$U_r = R_r I_r + \frac{d\Psi_r}{dt}$$

Note that rotor voltage balance is described with single equation

U_r is externally provided

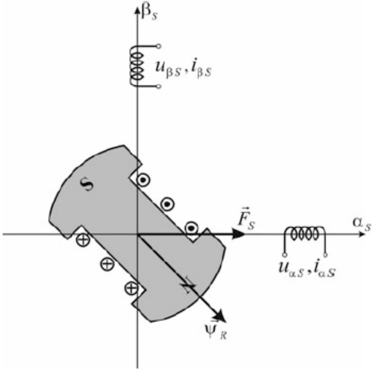

Figure 5 Synchronous machine with the two-phase stator winding and the excitation winding on the rotor.

FLUX LINKAGES

Relation between current and flux linkages is given by inductance matrix:

$$\begin{bmatrix} \Psi_{\alpha s} \\ \Psi_{\beta s} \\ \Psi_r \end{bmatrix} = \begin{bmatrix} L_s & 0 & L_m cos\theta_m \\ 0 & L_s & L_m sin\theta_m \\ L_m cos\theta_m & L_m sin\theta_m & L_r \end{bmatrix} \cdot \begin{bmatrix} I_{\alpha s} \\ I_{\beta s} \\ I_r \end{bmatrix}$$

- since the rotor is in motion, mutual inductances are variable and function of rotor position
- in addition, state variables are sinusoidal functions
- as with IM, there is a need for another transformation of the coordinate frame
- we need to apply Park Transformation

PARK TRANSFORMATION

d-q coordinate frame which rotates at the rotor speed:

- d-axis is aligned with magnetic axis of the excitation flux or PM flux
- rotor variables are already in the correct reference frame
- only stator variables must be transformed

$$\begin{bmatrix} I_d \\ I_q \end{bmatrix} = \begin{bmatrix} cos\theta_m & sin\theta_m \\ -sin\theta_m & cos\theta_m \end{bmatrix} \cdot \begin{bmatrix} I_{\alpha s} \\ I_{\beta s} \end{bmatrix}$$

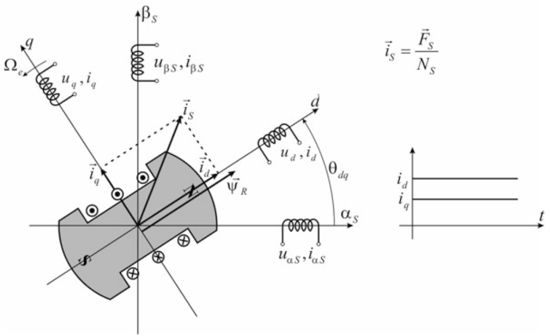

Figure 6 Transformation of stator variables to a synchronously rotating coordinate system. The angle θ_{dq} is equal to the rotor angle θ_m.

FLUX LINKAGES IN $d - q$ FRAME

After application of Park Transformation we have:

$$\begin{bmatrix} \Psi_{ds} \\ \Psi_{qs} \\ \Psi_r \end{bmatrix} = \begin{bmatrix} L_s & 0 & L_m \\ 0 & L_s & 0 \\ L_m & 0 & L_r \end{bmatrix} \cdot \begin{bmatrix} I_{ds} \\ I_{qs} \\ I_r \end{bmatrix}$$

- trigonometric dependencies are removed
- stator virtual d-winding coincide with magnetic axis of the excitation winding
- mutual inductance between q-winding of stator and excitation winding is zero
- there are only 5 non zero elements

$$\begin{aligned} \Psi_{ds} &= L_d I_{ds} + L_m I_r \\ \Psi_{qs} &= L_q I_{qs} \\ \Psi_r &= L_m I_{ds} + L_r I_r \end{aligned}$$

VECTORS AS COMPLEX NUMBERS

Current, voltage and flux vector can be represented using complex notation:

- taking currents as example:

$$\underline{I}_{\alpha\beta s} = I_{\alpha s} + jI_{\beta s}$$
$$\underline{I}_{dqs} = I_{ds} + jI_{qs}$$

- considering that:

$$e^{j\theta} = cos(\theta) + jsin(\theta)$$

- Park transformation is identical to:

$$\underline{I}_{dqs} = e^{-j\theta}\underline{I}_{\alpha\beta s}$$

- the same considerations are valid for voltage and flux

VOLTAGE BALANCE IN THE $d - q$ FRAME

Application of Park Transformation to stator voltage balance equation:

$$\underline{U}_{\alpha\beta s} = R_s \underline{I}_{\alpha\beta s} + \frac{d\underline{\Psi}_{\alpha\beta s}}{dt}$$

- results in complex voltage balance equation in d-q frame:

$$\underline{U}_{dqs} = R_s \underline{I}_{dqs} + \frac{d\underline{\Psi}_{dqs}}{dt} + j\omega_m \underline{\Psi}_{dqs}$$

- separation into real components yields:

$$U_{ds} = R_s I_{ds} + \frac{d\Psi_{ds}}{dt} - \omega_m \Psi_{qs}$$

$$U_{qs} = R_s I_{qs} + \frac{d\Psi_{qs}}{dt} + \omega_m \Psi_{ds}$$

- in addition we have rotor voltage balance equation as well:

$$U_r = R_r I_r + \frac{d\Psi_r}{dt}$$

SM MODEL IN ROTATING REFERENCE FRAME

Complete model in rotating reference (d-q) plane is:

$$U_{ds} = R_s I_{ds} + \frac{d\Psi_{ds}}{dt} - \omega_m \Psi_{qs}$$

$$U_{qs} = R_s I_{qs} + \frac{d\Psi_{qs}}{dt} + \omega_m \Psi_{ds}$$

$$U_r = R_r I_r + \frac{d\Psi_r}{dt}$$

$$\begin{bmatrix} \Psi_{ds} \\ \Psi_{qs} \\ \Psi_r \end{bmatrix} = \begin{bmatrix} L_s & 0 & L_m \\ 0 & L_s & 0 \\ L_m & 0 & L_r \end{bmatrix} \cdot \begin{bmatrix} I_{ds} \\ I_{qs} \\ I_r \end{bmatrix}$$

$$T_{em} = \frac{3}{2} p (\Psi_{ds} I_{qs} - \Psi_{qs} I_{ds})$$

ISOTROPIC / ANISOTROPIC PMSM

Isotropic: magnetic resistance is identical in all directions $\Rightarrow L_d = L_q$

Anisotropic: magnetic resistance in not identical in all directions $\Rightarrow L_d \neq L_q$

SM with surface mounted PM (cylindrical rotor)

- self-inductance of each winding depends on the ratio

$$L = N^2 / R_\mu$$

- with constant magnetic resistance, self-inductances are constant as well
- stator phase windings have the same number of turns and therefore:

$$L_d = L_q = L_s$$

SM with interior mounted PM

- magnetic resistance in not the same and therefore inductances are different
- voltage balance equations are the same in both cases, while Inductance matrix is not the same

ISOTROPIC / ANISOTROPIC PMSM – FLUX LINKAGES

For Isotropic PMSM we have

$$L_d = L_q = L_s$$

- flux linkage expressions are:

$$\begin{aligned}\Psi_{ds} &= L_s I_{ds} + L_m I_r \\ \Psi_{qs} &= L_s I_{qs} \\ \Psi_r &= L_m I_{ds} + L_r I_r\end{aligned}$$

For Anisotropic PMSM we have

$$L_d \neq L_q$$

- flux linkage expressions are:

$$\begin{aligned}\Psi_{ds} &= L_d I_{ds} + L_m I_r \\ \Psi_{qs} &= L_q I_{qs} \\ \Psi_r &= L_m I_{ds} + L_r I_r\end{aligned}$$

ISOTROPIC PMSM – TORQUE GENERATION (I)

Electrical power delivered to the machine is:

$$P_e = \frac{3}{2}(U_{ds}I_{ds} + U_{qs}I_{qs})$$

- combining with stator voltage balance equations:

$$U_{ds} = R_s I_{ds} + \frac{d\Psi_{ds}}{dt} - \omega_m \Psi_{qs}$$
$$U_{qs} = R_s I_{qs} + \frac{d\Psi_{qs}}{dt} + \omega_m \Psi_{ds}$$

- one obtains:

$$P_e = \frac{3}{2}R_s(I_{ds}^2 + I_{qs}^2) + \frac{3}{2}(I_{ds}\frac{d\Psi_{ds}}{dt} + I_{qs}\frac{d\Psi_{qs}}{dt}) + \frac{3}{2}\omega_m(\Psi_{ds}I_{qs} - \Psi_{qs}I_{ds})$$

$$P_e = P_{Cu} + \frac{dW_m}{dt} + P_{em}$$

- the 1^{st} part are stator copper losses
- the 2^{nd} part defines rate of change of accumulated energy in magnetic coupling field
- the 3^{rd} part defines rate of change of electrical energy into mechanical work - the power of electromechanical conversion

ISOTROPIC PMSM – TORQUE GENERATION (II)

The power of electromechanical energy conversion is:

$$P_{em} = \frac{3}{2}\omega_m(\Psi_{ds}I_{qs} - \Psi_{qs}I_{ds})$$

- rotor mechanical and electrical angular frequency (speed) are related as:

$$\omega_m = p\Omega_m$$

- the power P_{em} is passed to the shaft and converted into mechanical power
- in reality there are losses: friction, air resistance, excitation winding (if wound rotor)
- electromagnetic torque is per definition:

$$T_{em} = \frac{P_{em}}{\Omega_m}$$

- finally:

$$T_{em} = \frac{P_{em}}{\Omega_m} = p\frac{P_{em}}{\omega_m} = \frac{3}{2}p(\Psi_{ds}I_{qs} - \Psi_{qs}I_{ds})$$

ISOTROPIC PMSM – TORQUE GENERATION (III)

Combining flux linkage expressions:

$$\begin{aligned}\Psi_{ds} &= L_s I_{ds} + L_m I_r \\ \Psi_{qs} &= L_s I_{qs} \\ \Psi_r &= L_m I_{ds} + L_r I_r\end{aligned}$$

- into torque expression:

$$T_{em} = \frac{3}{2} p (\Psi_{ds} I_{qs} - \Psi_{qs} I_{ds})$$

- yields:

$$T_{em} = \frac{3}{2} p (L_s I_{ds} I_{qs} + L_m I_r I_{qs} - L_s I_{qs} I_{ds})$$

- finally:

$$T_{em} = \frac{3}{2} p L_m I_r I_{qs} = \frac{3}{2} p \Psi_{rm} I_{qs}$$

ANISOTROPIC PMSM

Anisotropic PMSM has different magnetic resistances along d- and q- axis

- self inductances are therefore also different as shown in Figure.
- a) Salient pole rotor – wound rotor
- b) Interior mounted (buried) PM

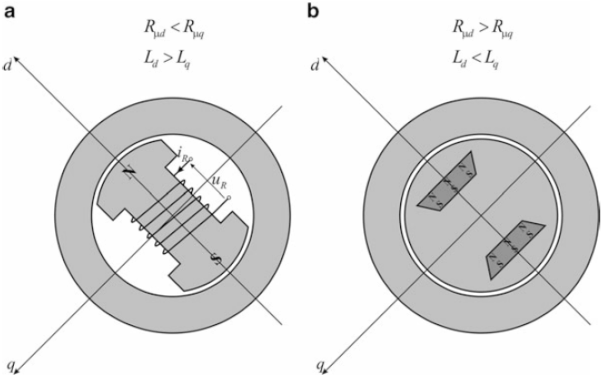

Figure 7 (a) Anisotropic rotor with excitation winding and with different magnetic resistances along $d-$ and $q-$ axes. (b) Anisotropic rotor with permanent magnets.

ANISOTROPIC PMSM – TORQUE GENERATION (I)

Starting from Torque expression:

$$T_{em} = \frac{3}{2}p(\Psi_{ds}I_{qs} - \Psi_{qs}I_{ds})$$

- and combining with flux linkages of anisotropic PMSM:

$$\begin{aligned} \Psi_{ds} &= L_d I_{ds} + L_m I_r \\ \Psi_{qs} &= L_q I_{qs} \\ \Psi_r &= L_m I_{ds} + L_r I_r \end{aligned}$$

- yields the expression for the Torque as:

$$T_{em} = \frac{3}{2}p(L_d I_{ds} I_{qs} + L_m I_r I_{qs} - L_q I_{qs} I_{ds})$$

- or in final form

$$T_{em} = \frac{3}{2}p\Psi_{rm}I_{qs} + \frac{3}{2}p(L_d - L_q)I_{ds}I_{qs}$$

ANISOTROPIC PMSM – TORQUE GENERATION (II)

Obtained Torque expression:

$$T_{em} = \frac{3}{2} p \Psi_{rm} I_{qs} + \frac{3}{2} p (L_d - L_q) I_{ds} I_{qs}$$

- has two parts:
- the 1^{st} part is the same as for Isotropic machine and is due to interaction of part of rotor flux that reaches stator winding and stator current q-component
- the 2^{nd} part is proportional to difference of self-inductance in d- and q- axis, and is called Reluctant Torque
- Reluctant Torque can exists in machine without any rotor windings or permanent magnets
- it tends to bring rotor in position with minimum magnetic resistance to the stator flux
- there are synchronous machines without excitation windings or permanent magnets, which rely purely on Reluctant Torque – Synchronous Reluctance Machines

SRM relies purely on Reluctant Torque:

- aimed at applications where size and weight are not so important
- SRM are robust, simple and low cost machines
- rotor has only magnetic circuit made to have different magnetic resistance in d- and q-axis
- this can be obtained by stacking iron sheet and leaving air gaps inside
- cylindrical rotor can be made with low air drag, allowing for high speed operation
- control is only from the stator side so Field Weakening is possible
- since there is no excitation on rotor, torque expression:

$$T_{em} = \frac{3}{2} p \Psi_{rm} I_{qs} + \frac{3}{2} p (L_d - L_q) I_{ds} I_{qs}$$

- is reduced to:

$$T_{em} = \frac{3}{2} p (L_d - L_q) I_{ds} I_{qs}$$

- shortcomings of SRM are low specific power and torque

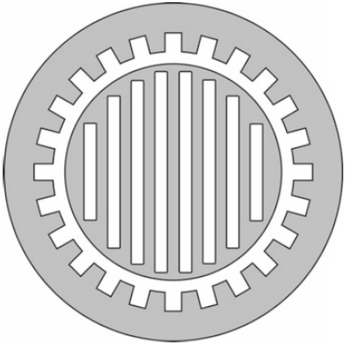

Figure 8 Rotor of a synchronous reluctance machine.