Chapter V-2 - Loudspeakers

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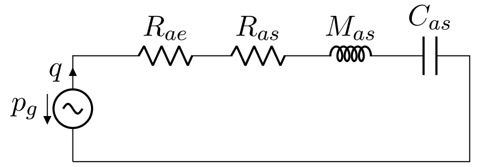
Exercise 1. Measurement of loudspeaker small-signal parameters (Thiele-Small)

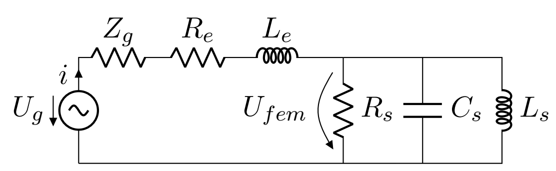
a. Loudspeaker on a screen

— $Z_{hp} = R_e + \dfrac{(B\ell)^2}{j\omega M_{ms} + R_{ms} + \dfrac{1}{j\omega C_{ms}}}$

— Equivalent acoustic scheme :
We introduce the acoustic equivalent R_{ae} of the dc electric resistance R_e $R_{ae} = \dfrac{(B\ell)^2}{S_d^2 R_e}$, and the acoustic equivalent (R_{as},M_{as},C_{as}) of the mechanical components (R_{ms},M_{ms},C_{ms}) :
$R_{as} = \dfrac{R_{ms}}{S_d^2}$, $M_{as} = \dfrac{M_{ms}}{S_d^2}$ and $C_{as} = C_{ms}S_d^2$.



Equivalent electric scheme :
We also introduce the electrical equivalent (R_s,C_s,L_s) of the mechanical components (R_{ms},M_{ms},C_{ms}) :
$R_s = \dfrac{(B\ell)^2}{R_{ms}}$, $C_s = \dfrac{M_{ms}}{(B\ell)^2}$ and $L_s = C_{ms}(B\ell)^2$



— If we introduce the equivalent electric impedance $R_s = \dfrac{(B\ell)^2}{R_{ms}}$:

$$Zhp = R_e + \frac{R_s}{Q_{ms}} \frac{\left(\dfrac{j\omega}{\omega_s}\right)}{\left(\dfrac{j\omega}{\omega_s}\right)^2 + \dfrac{1}{Q_{ms}}\left(\dfrac{j\omega}{\omega_s}\right) + 1}$$

— $R_e = 5.6\Omega$

— $\widehat{Z}_{hp}(f_s) = R_e + R_s = R_e + \dfrac{(B\ell)^2}{R_{ms}}$. We miss one other equation to retried $B\ell$ and R_{ms}.

— We chose to find the roots of the following equation $|Z_{hp}(f_{1,2})| = r_1 R_e = \sqrt{1+\dfrac{R_s}{R_e}}R_e$ which yields :
$|Z_{hp}(f_{1,2})|^2 = R_e^2\left(1+\dfrac{R_s}{R_e}\right)$ then
$\left(\dfrac{f_{1,2}}{f_s}\right)^4 - \left(2+\dfrac{1}{Q_{ms}^2}(1+\dfrac{R_s}{R_e})\right)\left(\dfrac{f_{1,2}}{f_s}\right)^2 + 1 = 0$
It yields :
$\left(\dfrac{f_{1,2}}{f_s}\right)^2 = \dfrac{1}{2}\left[\left(2+\dfrac{1}{Q_{ms}^2}(1+\dfrac{R_s}{R_e})\right) \pm \sqrt{\left(2+\dfrac{1}{Q_{ms}^2}(1+\dfrac{R_s}{R_e})\right)^2 - 4}\right]$
then :
$f_1^2 f_2^2 = \dfrac{1}{4}f_s^4\left[\left(2+\dfrac{1}{Q_{ms}^2}(1+\dfrac{R_s}{R_e})\right)^2 - \left(\left(2+\dfrac{1}{Q_{ms}^2}(1+\dfrac{R_s}{R_e})\right)^2 - 4\right)\right]$
and finally, after simplifications : $f_1 f_2 = f_s^2$

— $f_s = \sqrt{f_1 f_2} = 40.4$ Hz.

— We can notice that $\left(\dfrac{f_2-f_1}{f_s}\right)^2 = \left(\dfrac{f_2}{f_s}\right)^2 + \left(\dfrac{f_2}{f_s}\right)^2 - 2\dfrac{f_1 f_2}{f_s^2} = \left(\dfrac{f_2}{f_s}\right)^2 + \left(\dfrac{f_2}{f_s}\right)^2 - 2$
Since $\left(\dfrac{f_2}{f_s}\right)^2 + \left(\dfrac{f_2}{f_s}\right)^2 = 2 + \dfrac{1}{Q_{ms}^2}(1+\dfrac{R_s}{R_e}) = 2 + \dfrac{1}{Q_{ms}^2}r_0$
with $r_0 = 1 + \dfrac{R_s}{R_e}$
Then $Q_{ms}^2\left(\dfrac{f_2-f_1}{f_s}\right)^2 = r_0$ and finally $Q_{ms} = \dfrac{f_s}{f_2-f_1}\sqrt{r_0}$

— $Q_{es} = Q_{ms}\dfrac{R_e}{R_s} = \dfrac{Q_{ms}}{r_0-1}$

— r_0=65/5.6=11.6 $\rightarrow Q_{ms} = 4.12$, $Q_{es} = 0.38$ and $Q_{ts} = 0.35$

b. Loudspeaker with an additional mass
The measurement gives the two new frequencies $f_1' = 22$ Hz and $f_2' = 40.7$ Hz.

— $f_s' = \sqrt{f_1' f_2'} \approx 29.9$ Hz

— We remind that without the mass, $f_s = \dfrac{1}{2\pi\sqrt{C_{ms}M_{ms}}}$
With the added mass, the resonance frequency becomes : $f_s' = \dfrac{1}{2\pi\sqrt{C_{ms}(M_{ms}+M_{add})}}$
By processing $\dfrac{f_s}{f_s'} = \sqrt{\dfrac{M_{ms}+M_{add}}{M_{ms}}}$ we obtain $\dfrac{f_s^2}{f_s'^2} = 1 + \dfrac{M_{add}}{M_{ms}}$

— Thanks to the last relationship, we deduce $M_{ms} = \dfrac{M_{add}}{\dfrac{f_s^2}{f_s'^2} - 1} \approx 11g$

$C_{ms} = \dfrac{1}{4\pi^2 f_s^2 M_{ms}} \approx 1.4mm.N^{-1}$

— $Q_{ms} = \dfrac{1}{\omega_s R_{ms} C_{ms}} \rightarrow R_{ms} = \dfrac{1}{\omega_s Q_{ms} C_{ms}} = 0.67N.m^{-1}.s$
$R_s = \dfrac{(B\ell)^2}{R_{ms}} \rightarrow B\ell = \sqrt{R_{ms} R_s} = 6.9T.m$, where $R_s = 65 - 5.6 = 59.4\Omega$

c. Loudspeaker with a closed-box cabinet We now remove the masse M_{add} and close the rear face of the loudspeaker with a sealed cabinet of volume $V_b = 20$ L. We do once again the same measurement, which gives :
We introduce the compliance factor $\alpha = \dfrac{C_{as}}{C_{ab}}$, where $C_{ab} = \dfrac{V_b}{\rho c^2}$

— The new acoustic compliance of the closed-box loudspeaker is $C_{ac} = \dfrac{C_{ab} C_{as}}{C_{ab} + C_{as}}$, where $C_{as} = C_{ms} S_d^2$ and then
$f_c = \dfrac{1}{2\pi\sqrt{M_{ac} C_{ac}}} \approx \dfrac{1}{2\pi\sqrt{M_{as} C_{ac}}}$ if we consider the radiation acoustic mass in the cabinet M_{ab} is the same as the screen-mounted radiation acoustic mass M_{ar}.
We can also notice that $C_{ac} = \dfrac{C_{as}}{1+\alpha}$, then $f_c = \sqrt{1+\alpha} f_s$

— $Q_{ec} = \dfrac{1}{2\pi f_c C_{ac} R_{ae}}$, if we introduce $R_{ae} = \dfrac{(B\ell)^2}{S_d^2 R_e}$ the acoustical equivalent of the dc electrical resistance R_e.
Then, since $Q_{es} = \dfrac{1}{2\pi f_s C_{as} R_{ae}}$ and $C_{ac} = \dfrac{C_{as}}{1+\alpha}$, we deduce $Q_{ec} = \sqrt{1+\alpha} Q_{es}$

— After the 2 preceding expressions, we deduce $Q_{ec} f_c = (1+\alpha) Q_{es} f_s$, then $\alpha = \dfrac{Q_{ec} f_c}{Q_{es} f_s} - 1$
If we repeat the same procedure as in the 2 preceding questions, we first observe the values f_{1c} and f_{2c} of f for which $Z_{hp} = \sqrt{r_0} R_e$: $f_{1c} = 46.74Hz$ and $f_{2c} = 76.89Hz$ yields
$f_c = 59.95Hz$ and $Q_e c = 0.639$ (once again here, $r_0 = 65/5.6 = 11.6$).
Then $\alpha = 1.4413$, then $V_{as} = \alpha V_b = 28.8L$

— We can also deduce S_d : $C_{ms} = \dfrac{V_{as}}{\rho c^2 S_d^2} \rightarrow S_d = \sqrt{\dfrac{V_{as}}{\rho c^2 C_{ms}}} = 121cm^2$