Chapter V - 1. Microphones

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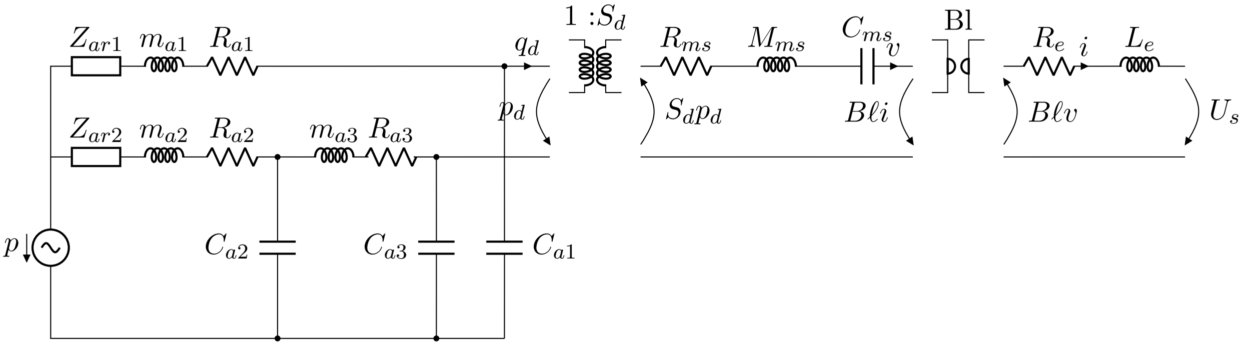
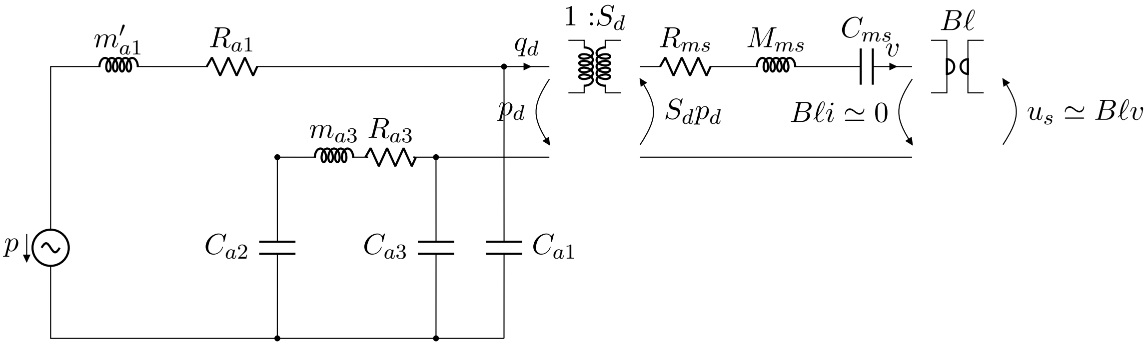
Exercice 1. Omnidirectional electrodynamic microphone

1. This microphone is a pressure microphone (only accessible from the front side), thus omnidirectional.
2. The role of the hole is to ensure the static pressure balance. If well designed it acts as a second-order high-pass filter (together with compliance C_{a2}).
3. We want an omnidirectional microphone (pressure action) with an electrodynamic transducer. It imposes to have the total mechanical impedance controlled by resistances, thus the grid and the internal fibrous layer.
4. 

5. Thorough derivation : We should assume that :
   — the output current i is null (due to the connection of the microphone to a high impedance)
   — the hole (m_{a2}, R_{a2}) is assimilated to an open circuit (the equalization only occurs in the DC range)
   — we can assimilate impedance Z_{ar1} to an acoustic mass m_{ar1} on the whole bandwidth. We denote $m'_{a1} = m_{a1} + m_{ar1}$ the new acoustic mass of the grid, and $Z_{a1} = R_{a1} + j\omega m_{a1}$ the acoustic impedance in front of the diaphragm.

   Therefore, the circuit should rather look like :



   Let's first derive the output voltage : $u_s \simeq B\ell v = \dfrac{B\ell S_d}{Z_{ms}} p_d$
   where $p_d = p_1 - p_3$ and $Z_{ms} = R_{ms} + j\omega M_{ms} + \frac{1}{j\omega C_{ms}}$.

Let's also denote Z_{a3} the equivalent acoustic impedance of compliance C_{a3} in parallel with (R_{a3}, m_{a3}, C_{a2}). We get :

$$Z_{a3} = \frac{(j\omega)^2 C_{a2} m_{a3} + (j\omega) C_{a2} R_{a3} + 1}{(j\omega)^3 C_{a2} C_{a3} m_{a3} + (j\omega)^2 C_{a2} C_{a3} R_{a3} + (j\omega)(C_{a2} + C_{a3})}$$

Then, $p_3 = Z_{a3} q_d$.

If we apply the nodes equation on node 1, we get :

$$\frac{p - p_1}{Z_{a1}} - j\omega C_{a1} p_1 - q_d = 0 \rightarrow p_1 = \frac{1}{j\omega C_{a1} Z_{a1} + 1} p - \frac{Z_{a1}}{j\omega C_{a1} Z_{a1} + 1} q_d \, .$$

Then, we just need to derive

$$p_1 - p_3 = \frac{Z_{ms}}{B\ell\, S_d} u_s = \frac{1}{j\omega C_{a1} Z_{a1} + 1} p - \left[\frac{j\omega C_{a1} Z_{a1} Z_{a3} + Z_{a1} + Z_{a3}}{j\omega C_{a1} Z_{a1} + 1} \right] \frac{S_d}{B\ell} u_s$$

And turning the equation :

$$\frac{u_s}{p} = \frac{S_d B\ell (j\omega C_{a1} Z_{a1} + 1)}{Z_{ms}(j\omega C_{a1} Z_{a1} + 1) + S_d^2 (Z_{a3}(j\omega C_{a1} Z_{a1} + 1) + Z_{a1})}$$

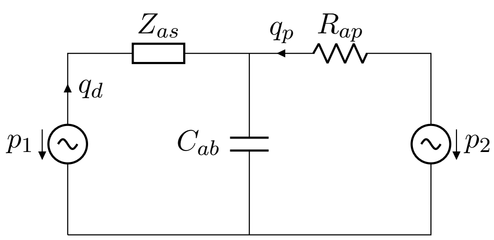
Finally, we obtain :

$$\frac{u_s}{p} = S_d B\ell C_{ms} \frac{(j\omega)^5 n_5 + (j\omega)^4 n_4 + (j\omega)^3 n_3 + (j\omega)^2 n_2 + (j\omega) n_1}{(j\omega)^6 d_6 + (j\omega)^5 d_5 + (j\omega)^4 d_4 + (j\omega)^3 d_3 + (j\omega)^2 d_2 + (j\omega) d_1 + d_0}$$

with :

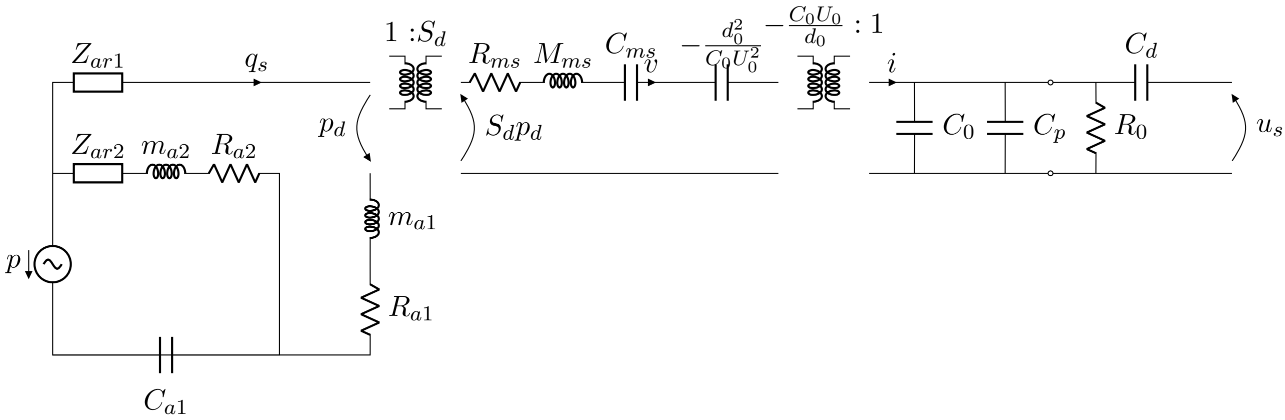
$$\begin{array}{llll}
 & & d_6 = & C_{ms} M_{ms} C_{a1} m'_{a1} C_{a2} C_{a3} m_{a3} \\
n_5 = & C_{a1} C_{a2} C_{a3} m'_{a1} m_{a3} & d_5 = & C_{ms} M_{ms} C_{a1} m'_{a1} C_{a2} C_{a3} R_{a3} \\
 & & & +(C_{ms} M_{ms} C_{a1} R_{a1} + C_{ms} R_{ms} C_{a1} m'_{a1}) C_{a2} C_{a3} m_{a3} \\
n_4 = & C_{a1} C_{a2} C_{a3} m'_{a1} R_{a3} & d_4 = & C_{ms} M_{ms} C_{a1} m'_{a1} (C_{a2} + C_{a3}) \\
 & +C_{a1} C_{a2} C_{a3} R_{a1} m_{a3} & & +(C_{ms} M_{ms} C_{a1} R_{a1} + C_{ms} R_{ms} C_{a1} m' a1) C_{a2} C_{a3} R_{a3} \\
 & & & +(C_{ms} M_{ms} + C_{a1} m'_{a1} + C_{ms} R_{ms} C_{a1} R_{a1} + S_d^2 C_{ms} m'_{a1}) C_{a2} C_{a3} m_{a3} \\
 & & & +S_d^2 C_{ms} C_{a1} m'_{a1} C_{a2} m_{a3} \\
n_3 = & C_{a1}(C_{a2} + C_{a3}) m'_{a1} & d_3 = & (C_{ms} M_{ms} C_{a1} R_{a1} + C_{ms} R_{ms} C_{a1} m' a1)(C_{a2} + C_{a3}) \\
 & +C_{a1} C_{a2} C_{a3} R_{a1} R_{a3} & & +(C_{ms} M_{ms} + C_{a1} m'_{a1} + C_{ms} R_{ms} C_{a1} R_{a1} + S_d^2 C_{ms} m'_{a1}) C_{a2} C_{a3} R_{a3} \\
 & & & +(C_{ms} R_{ms} + C_{a1} R_{a1} + S_d^2 C_{ms} R_{a1}) C_{a2} C_{a3} m_{a3} \\
 & & & +S_d^2 C_{ms} C_{a1} m'_{a1} C_{a2} R_{a3} + S_d^2 C_{ms} C_{a1} R_{a1} C_{a2} m_{a3} \\
n_2 = & C_{a1}(C_{a2} + C_{a3}) R_{a1} & d_2 = & (C_{ms} M_{ms} + C_{a1} m'_{a1} + C_{ms} R_{ms} C_{a1} R_{a1} + S_d^2 C_{ms} m'_{a1})(C_{a2} + C_{a3}) \\
 & +C_{a2} C_{a3} R_{a3} & & +(C_{ms} R_{ms} + C_{a1} R_{a1} + S_d^2 C_{ms} R_{a1}) C_{a2} C_{a3} R_{a3} \\
 & & & +S_d^2 C_{ms} C_{a1} m'_{a1} + S_d^2 C_{ms} C_{a1} R_{a1} C_{a2} R_{a3} \\
 & & & +S_d^2 C_{ms} C_{a2} m_{a3} \\
n_1 = & C_{a2} + C_{a3} & d_1 = & (C_{ms} R_{ms} + C_{a1} R_{a1} + S_d^2 C_{ms} R_{a1})(C_{a2} + C_{a3}) \\
 & & & +C_{a2} C_{a3} R_{a3} \\
 & & & +S_d^2 C_{ms}(C_{a1} R_{a1} + C_{a2} R_{a3}) \\
 & & d_0 = & C_{a2} + C_{a3} + S_d^2 C_{ms}
\end{array}$$

6. We get a unidirectional microphone : the back opening acts as a series resistance, together with the compliance in parallel which leads to the following scheme :



Exercice 2. Omnidirectional electrostatic (measurement) microphone

1. This is an omnidirectional microphone → closed cavity (pressure action) + small hole to equalize static pressure.
   Since we have an electrostatic transducer, working in the far field (not the proximity effect), the total mechanical impedance should be controlled by compliances, so the only important acoustic component here is the compliance C_{a1}. The holes in the back plate of the condenser are just designed to let the acoustic wave pass through the membrane, back to the cavity V_1. The size of the holes should be sufficiently large to not add too much mass (m_{a1}) or resistance (R_{a1}) in the back acoustic circuit.s



2.
3. If we consider the voltage at the output of the microphone (thus without the cable R_0, C_d), we can observe that $u_s \approx \dfrac{1}{j\omega(C_0 + C_p)}$ (here C_p is a parasitic capacitance due to the metallic casing of the microphone, that can be neglected in the following).
   With these simplifications, we observe
   $u = -\dfrac{C_0U_0}{d_0}\dfrac{1}{j\omega C_0}v = -\dfrac{U_0}{j\omega d_0}\dfrac{S_d}{j\omega M_{ms} + R_{ms} + \dfrac{1}{j\omega C'_{ms}}}p_d,$
   where $C'_{ms} = \dfrac{C_{ms}d_0^2}{d0^2 - C_{ms}C_0U_0^2}$.
   On the acoustic side, we can consider the equalization hole (m_{a2}, R_{as}) is only effective for the DC component, and then this mesh may be discarded. Finally :
   $p_d = \dfrac{Z'_{ms}}{Z'_{ms} + Z_{a1}S_d^2}p$ where $Z'_{ms} = j\omega M_{ms} + R_{ms} + \dfrac{1}{j\omega C'_{ms}}$ and $Z_{a1} = j\omega M_{a1} + R_{a1} + \dfrac{1}{j\omega C_{a1}}$.
   Then $\dfrac{u_s}{p} = -\dfrac{U_0}{j\omega d_0}\dfrac{S_d}{j\omega(M_{ms} + S_d^2M_{a1}) + (R_{ms} + S_d^2R_{a1}) + \dfrac{1}{j\omega C'_{ms}} + \dfrac{S_d^2}{j\omega C_{a1}}}$
4. We get a unidirectional microphone.

Exercise 3. Microphone sensitivity

1. Full-scale $U_{max} = U_0 10^{\frac{L_{V,max}}{20}} = 100$ mV.
2. — $L_p = 20log_{10}\left(\dfrac{\tilde{p}}{p_0}\right)$ (in dB re. 20 μPa)
   — $L_U = 20log_{10}\left(\dfrac{\tilde{U}}{U_0}\right)$ (in dB re. 1 μV)

— $L_p = 20log_{10}\left(\frac{\tilde{M}}{M_0}\right)$ (in dB re. 1 V/Pa)

3. $M = \frac{\tilde{U}}{\tilde{p}} = \frac{\tilde{U}}{U_0}\frac{p_0}{\tilde{p}}\frac{U_0}{M_0\,p_0}$

   Then $L_p = L_U - L_M - 20log_{10}\left(\frac{U_0}{M_0\,p_0}\right)$

   Below the VU-meter, we can read that "full-scale" corresponds to 100 mV (red light), and on the scale we see it corresponds to 20 dB. Also, 100 mV corresponds to $L_{\text{full-scale}} = 20\log_{10}(\frac{100\text{mV}}{1\mu\text{V}}) = 100$ dB (re. 1 μV).
   The VU-meter indicates the value "14 dB" (out of 20) which is -6 dB from the max 20 dB. So it means $L_U = 94$ dB (re. 1 μV).
   Knowing the microphone sensitivity is $L_M = -26.2$ dB (re. 1 V/Pa), it means M=50 mV/Pa. We can then deduce $L_p \approx 94$ dB (re 20 μPa), then $\tilde{p} = 1$ Pa.