Chapter IV - Electroacoustic systems and radiation

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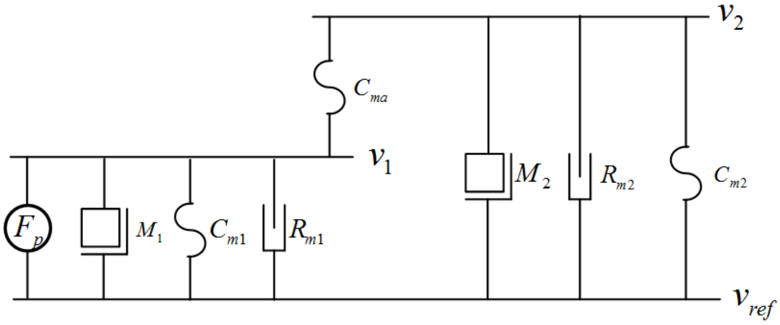
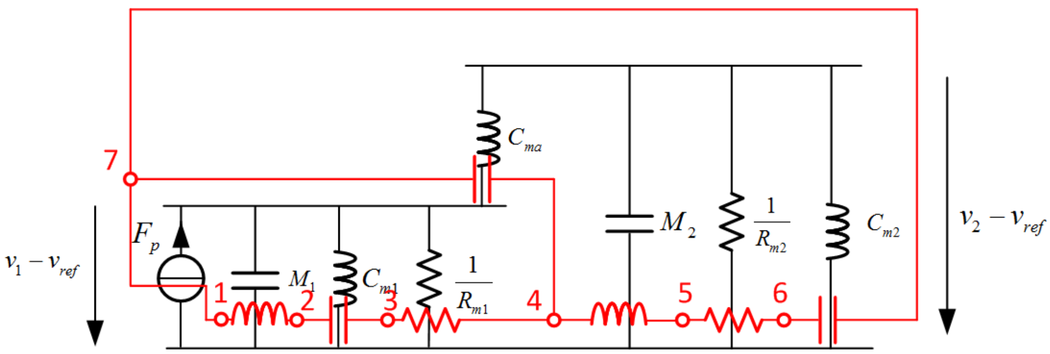
Exercise 1. Double panel partition

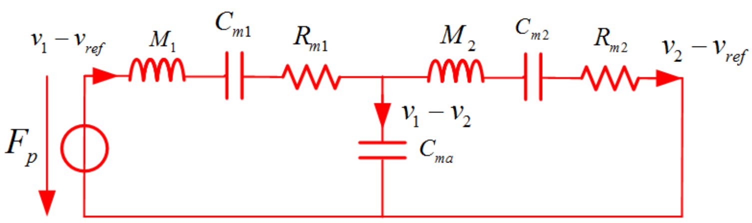
1. We see two velocities v_1 and v_2 corresponding to the two individual panels (masses M_1 and M_2), and we also add a line for the reference velocity $v_{ref} = 0$. We obtain the following symbolic scheme (velocities v_i corresponding to "potentials") :



2. We deduce the inverse scheme (in black) :



And finally the direct scheme (see the red scheme overlaying the black inverse scheme) :



3. $H = \frac{v_2}{F_p}$, since $Z_{meq} = C_{ma} \parallel (R_{m2}, C_{m2}, M_2) = \frac{j\omega M_2 + R_{m2} + \frac{1}{j\omega C_{m2}}}{1 + j\omega C_{ma}(j\omega M_2 + R_{m2} + \frac{1}{j\omega C_{m2}})}$,

we get : $\frac{Z_{meq}}{Z_{meq} + (j\omega M_1 + R_{m1} + \frac{1}{j\omega C_{m1}})} F_p = (j\omega M_2 + R_{m2} + \frac{1}{j\omega C_{m2}}) v_2$.

Then, $H = \dfrac{1}{j\omega M_2 + R_{m2} + \frac{1}{j\omega C_{m2}}} \dfrac{Z_{meq}}{Z_{meq} + (j\omega M_1 + R_{m1} + \frac{1}{j\omega C_{m1})}}$

Finally : $H = \dfrac{1}{j\omega M_2 + R_{m2} + \frac{1}{j\omega C_{m2}}(1 + j\omega C_a(j\omega M_1 + R_{m1} + \frac{1}{j\omega C_{m1}}) + j\omega M_1 + R_{m1} + \frac{1}{j\omega C_{m1}}}$.

Exercise 2. Helmholtz resonators

$f_s = \dfrac{1}{2\pi\sqrt{m_a C_a}} = \dfrac{c}{2\pi}\sqrt{\dfrac{S}{VL}}$ The resonance frequencies can be arranged in the following order : $f_b < f_d < f_a < f_c$

Exercise 3. Silencer

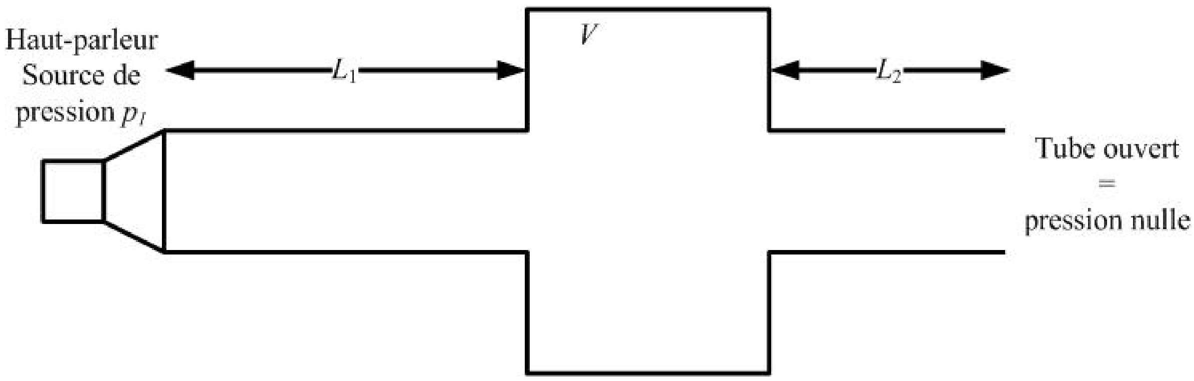


Figure 1 – Schematic representation of a silencer.

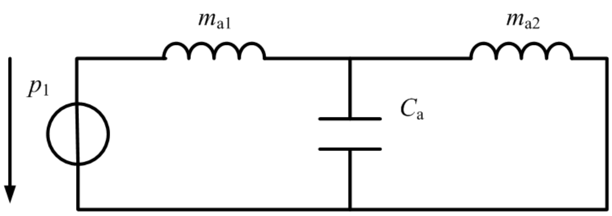


Figure 2 – Equivalent circuit of the silencer.

The acoustic mass in the duct of length L_1 is given by $m_{a1} = \rho L_1/(\pi r_d^2)$, and that the duct of length L_2 by $m_{a2} = \rho L_2/(\pi r_d^2)$. The effective acoustic compliance in the coupling cavity is given by $C_a = V/(\rho c^2)$. The analog acoustic scheme is illustrated in Fig. 7.

The input impedance of the silencer can be derived as

$$Z_a = \frac{p_1}{q_1} = \frac{(j\omega^2)m_{a1}C_a + \frac{m_{a1}}{m_{a2}} + 1}{j\omega C_a + \frac{1}{j\omega m_{a2}}}$$

Exercise 4. Boomwhacker

1. $\begin{cases} \dfrac{\partial p}{\partial x} &= -\dfrac{\rho_0}{S}\dfrac{\partial q}{\partial t} \\ \dfrac{\partial q}{\partial x} &= -\chi_s S \dfrac{\partial p}{\partial t} \end{cases}$

   Then $\dfrac{\partial^2 p}{\partial x^2} - \dfrac{1}{c_0^2}\dfrac{\partial^2 p}{\partial t^2} = 0$, where $c_0 = \dfrac{1}{\sqrt{\rho_0 \chi_s}}$

2. If we derive $p(x,t) = P(x)e^{j\omega t}$ in the wave equation, we obtain the Helmholtz equation $\dfrac{d^2 P}{dx^2} + k^2 P = 0$, where $k = \dfrac{\omega}{c_0}$

   The solution of the wave equations are then of the form :

   $P(x) = P_{0+}e^{-jkx} + P_{0-}e^{+jkx}$.

   Taking the first equation (Euler's generalized equation), and since $\dfrac{\partial q}{\partial t} = j\omega Q(x)e^{j\omega t}$, we deduce :

   $Q(x) = \dfrac{1}{Z_{ac}}\left(P_{0+}e^{-jkx} - P_{0-}e^{+jkx}\right)$.

3. $$\forall x, \begin{cases} P(x) = & P_{0+}e^{-jkx} + P_{0-}e^{jkx} \\ Q(x) = & \dfrac{S}{\rho_0 c_0}\left(e^{-jkx} - P_{-}e^{jkx}\right) \end{cases}$$

   If we denote $P_+(x) = P_{0+}e^{-jkx}$ and $P_-(x) = P_{0-}e^{jkx}$, we can write the diffusion relationship :

   $$\begin{pmatrix} P(x) \\ Q(x) \end{pmatrix} = \begin{bmatrix} 1 & 1 \\ \dfrac{1}{Z_{ac}} & -\dfrac{1}{Z_{ac}} \end{bmatrix} \begin{pmatrix} P_+(x) \\ P_-(x) \end{pmatrix}$$

   Then : $\begin{pmatrix} P(x-L) \\ Q(x-L) \end{pmatrix} = \begin{bmatrix} e^{jkL} & e^{-jkL} \\ \dfrac{e^{jkL}}{Z_{ac}} & -\dfrac{e^{-jkL}}{Z_{ac}} \end{bmatrix} \begin{pmatrix} P_+(x) \\ P_-(x) \end{pmatrix}$

   We can also invert the diffusion matrix and it yields :

   $$\begin{pmatrix} P_+(x) \\ P_-(x) \end{pmatrix} = \dfrac{Z_{ac}}{2} \begin{bmatrix} -\dfrac{1}{Z_{ac}} & -1 \\ -\dfrac{1}{Z_{ac}} & 1 \end{bmatrix} \begin{pmatrix} P(x) \\ Q(x) \end{pmatrix}$$

   And finally :

   $$\begin{pmatrix} P(x-L) \\ Q(x-L) \end{pmatrix} = \begin{bmatrix} e^{jkL} & e^{-jkL} \\ \dfrac{e^{jkL}}{Z_{ac}} & -\dfrac{e^{-jkL}}{Z_{ac}} \end{bmatrix} . \dfrac{Z_{ac}}{2} \begin{bmatrix} -\dfrac{1}{Z_{ac}} & -1 \\ -\dfrac{1}{Z_{ac}} & 1 \end{bmatrix} \begin{pmatrix} P(x) \\ Q(x) \end{pmatrix} = \begin{bmatrix} \cos kL & jZ_{ac}\sin kL \\ \dfrac{j}{Z_{ac}}\sin kL & \cos kL \end{bmatrix} \begin{pmatrix} P(x) \\ Q(x) \end{pmatrix}$$

   which also holds for $x = L$.

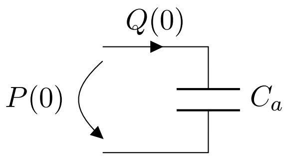
4. The duct is closed at the right termination, then $Q(L) = 0$.

   Then $Z_a(0) = \dfrac{P(0)}{Q(0)} = -jZ_{ac}\cot kL$

5. If the left termination is open ($P(0) = 0$), then we should have $Z_a(0) = 0$. The resonance frequencies correspond then to $\cot kL = 0$, then $f_n = \dfrac{(2n+1)c_0}{4L}$

For a length of 19 cm, the first resonance frequency occurs at $f_1 = 447$ Hz (almost A_{440}).

6. at low frequencies ($kL << 1$), $Z_a(0) \approx -j\dfrac{Z_{ac}}{kL} = \dfrac{1}{j\omega\dfrac{V}{\rho_0 c_0^2}}$. The low-frequency behavior is an acoustic compliance $C_a = \dfrac{V}{\rho_0 c_0^2}$, and the acoustical scheme is the following :



Exercise 5. Bi-directional and cardioid sources

The directivity factor is computed as : $\Delta = \dfrac{4\pi}{\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} D^2(\theta,\phi)\sin\theta d\theta d\phi}$ (referred to an omnidirectional source with solid angle 4π).

1. Here $D(\theta,\phi) = \cos\theta$, then $\Delta = \dfrac{4\pi}{2\pi\int_{\theta=0}^{\pi}\cos^2(\theta)\sin\theta d\theta}$.

   Integrating by parts, $u = \cos(\theta)$ and $du = -\sin(\theta)d\theta$, one gets : $\Delta = 3$ or $L_\Delta = 4.8$ dB. Half-power beamwidth : $D^2(\theta) = \frac{1}{2} \to \theta_{-3\text{dB}} = 45°$, the total aperture (beamwidth) is then 90° (symmetry over the axis)
2. Same as before : $\Delta = 3$ or $L_\Delta = 4.8$ dB, and $\theta_{-3\text{dB}} = 45°$.

Exercise 6. Directivity of a loudspeaker

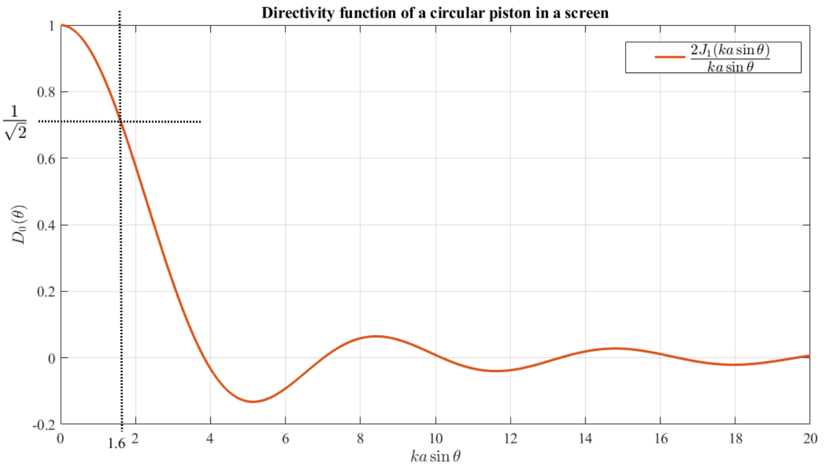


Figure 3 – Directivity function of a piston on screen

We assume that a source remains "omnidirectional" (with a tolerance of 3 dB) if its directivity function remains higher than $D_0(\theta) \geq \dfrac{1}{\sqrt{2}}, \forall\theta \in [-\frac{\pi}{2}\frac{\pi}{2}]$ (since $20.\log_{10}\dfrac{1}{\sqrt{2}} = -3$ dB(re. $D_0(0) = 1$)). According to the figure above, that means that $ka\sin\theta_{min} = 1.6$ (where θ_{min} designates the angle at which the directivity reaches $\frac{1}{\sqrt{2}}$).

1. For $f = 500$ Hz and $\theta_{min} = 90°$, $a \leq \dfrac{c}{f \sin\theta_{min}} = 21{,}8$ cm (here we consider $\theta_{min} = \pi/2$ so that the half-bandwidth angle is ± 90°, ie. omnidirectional)
2. For $f = 2000$ Hz and $\theta_{min} = 90°$, $a \leq 5{,}46$ cm (same assumption)
3. For $f = 1000$ Hz and $\theta_{min} = 30°$, $a \leq 21{,}8$ cm (here the half-bandwidth should be ± 30°).

Exercise 7. Radiation of a 2-way loudspeaker - monopole hypothesis

Reminder :
Field radiated by a monopole :

$$\underline{p}_M(r) = j\frac{\rho c}{4\pi r}k\underline{q}e^{-jkr}$$

Field radiated by a monopole on a closed box (semi-monopole) :

$$\underline{p}_E(r) = j\frac{\rho c}{2\pi r}k\underline{q}e^{-jkr}$$

1. Monopole on a closed box with flow velocity q_1 or q_2 :

$$q_1 = \frac{2\pi r_1}{\rho c k}p_{M_1}(r_1) = \frac{2\pi r_1}{\rho c 2\pi f/c} \cdot (20 \cdot 10^{-6} \cdot 10^{\frac{L_{p1}}{20}}) = 4,9 \cdot 10^{-3} \text{ m.s}^{-3}$$
$$q_2 = 3,9 \text{ m.s}^{-3}$$

If synchronous source, microphone ∼ in the loudspeaker's main axis, with wavelength $\lambda \sim$ 1m
$\Longrightarrow$ the pressure waves are in phase and therefore :

$$p_{1+2} = p_1 + p_2 \Longrightarrow L_{p_{1+2}} = 20 \cdot \log\left[10^{\frac{L_{p1}}{20}} + 10^{\frac{L_{p2}}{20}}\right] = 93 \text{ dB}$$

2. At 350 Hz, the boomer becomes slightly directive $\Longrightarrow$ attenuation of the off-axis measurements. For the medium however, no changes.
If measurement points are off-axis, the phase difference varies between the two signals $\Longrightarrow$ a quadratic summation must be done (of the energy) and the expression before changes to be :

$$L_{p_{1+2}} = 10 \cdot \log\left[10^{\frac{L_{p1}}{10}} + 10^{\frac{L_{p2}}{10}}\right] = 89,6 \text{ dB}$$

Exercise 8. Radiation of a small speaker

1. At low-frequencies, $R_{ar} \approx \dfrac{\rho c}{\pi a^2}\dfrac{(ka)^2}{2}$. It is valid until $ka \approx 1$ then for $f < \dfrac{c}{2\pi a} = 909$ Hz.
2. On axis, the sound pressure reads $p(r, \theta = 0) = j\dfrac{\rho c}{2\pi r}kqe^{-jkr}$, where q is the volume velocity, linked to the excursion ξ as $q = j\omega(\pi a^2)\xi$.
Then (if we denote $\tilde{\nu}$ the rms value of quantity ν) : $\tilde{p}(r, 0) = \dfrac{\rho\omega^2(\pi a^2)}{2\pi r}\tilde{\xi} = \dfrac{\sqrt{2}\rho\omega^2 a^2}{4r}\xi_{\max}$ (here the peak-to-peak elongation is given).
   - at 250 Hz, $\tilde{p}(r = 10\text{m}) = 0.158$ Pa or $L_p \approx 78$ dB (re. 20 μPa)
   - at 500 Hz, $\tilde{p}(r = 10\text{m}) = 0.633$ Pa or $L_p \approx 90$ dB (re. 20 μPa)
   - at 1000 Hz, we should (theoretically) not consider the low-frequency approximation. However, the actual limitation corresponds to $ka = \sqrt{2}$ (instead of $ka = 1$), then the frequency bound is $f_{\max} \approx 1287$Hz. In this cas we can still consider the low-frequency approximation.
     $\tilde{p}(r = 10\text{m}) = 2.5$ Pa or $L_p \approx 102$ dB (re. 20 μPa)