# Chapter IV - Electroacoustic systems and radiation

Laboratory of Electromagnetism and Acoustics

Fall semester 2017

## Exercise 1. Double pane partition

Let's consider the double pane partition system, made of 2 panels of wood attached to a rigid structure (null velocity), separated by a plenum (small volume) of air as shown on figure 1. We assume the left side panel (1) is subject to a (pressure) force $F_p$, and the right-side panel (2) presents a vibratory velocity $v_2$ (the left-side panel can be characterized by the velocity $v_1$).
We assume that each wooden panel behaves as a mass-spring-losses system ($M_i$, $R_{mi}$, $C_{mi}$), and the air plenum can be modeled as a pure compliance (additional spring) $C_{ma}$ between the two panels.

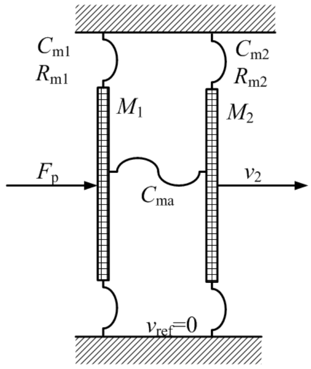



**Figure 1** – double pane partition

1. Identify all the velocities, and draw the symbolic scheme of the mechanical system.
2. Draw the inverse, and then the direct mechanical scheme of the double pane partition.
3. Write the transfer function
$$Y_m = \frac{\underline{v}_2}{\underline{F}_p}$$

## Exercise 2. Helmholtz resonators

4 Helmholtz resonators with different dimensions are presented on figure 2. Arrange the resonators by increasing order of resonance frequency.

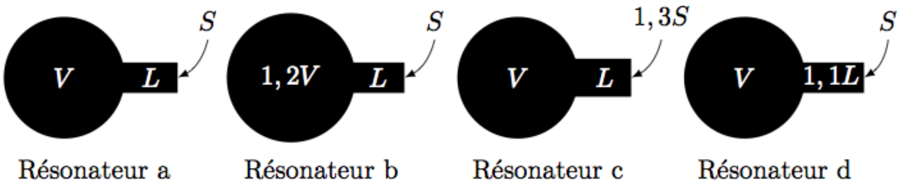



**Figure 2** – Helmholtz resonators

## Exercise 3. Silencer

Draw the analogue acoustic scheme of the silencer, assuming :

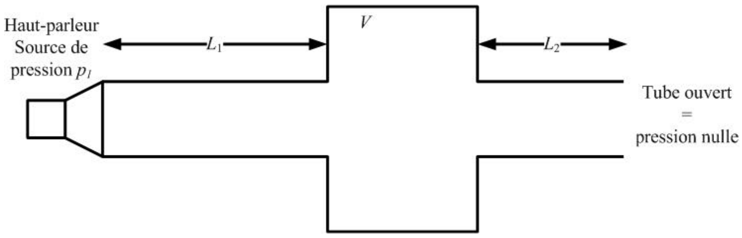
— the loudspeaker is an ideal source of sound pressure $p_1$

— the termination at the right side is open ($p = 0$)

— the radiation of all ducts is negligible

— all ducts of constant section are considered as acoustic masses $m_a$ (remind the expressions of $m_{a1}$ and $m_{a2}$, for ducts of radius $r_d$ and lengths $L_1$ and $L_2$).

— all volumes are considered as acoustic compliances $C_a$ (remind the expression of $C_a$)




Express the input impedance $Z_a = \dfrac{p_g}{q_1}$, where $q_1$ is the flow velocity in the duct at the left of the silencer.

## Exercise 4. Boomwhacker

Let's consider a musical toy called "boomwhacker", consisting of a tubes of different lengths made of plastic (see `http://en.wikipedia.org/wiki/Boomwhacker`). The considered system is then made of a tube of section $S$ and length $L$, open at one extremity and closed at the other. We propose to derive the boomwhacker resonance frequencies and calculate the equivalent acoustic impedance at the input of the boomwhacker ($x = 0$) as illustrated on figure 3.

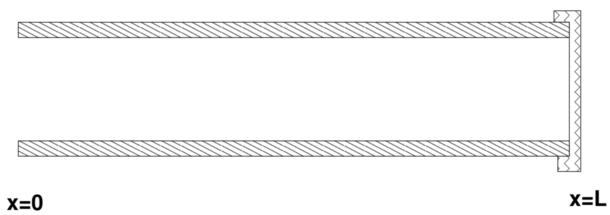



**Figure 3** – Boomwhacker principle

We note $\rho_0$ the mass density of the air, and $c_0$ the sound celerity in the air.

1. Remind the 2 equations of the wave propagation linking pressure $p(x,t)$ and volume velocity $q(x,t)$ applied to a short portion of tube $dx$, and then derive the d'Alembert equation applied to the pressure $p(x,t)$.

We can introduce the characteristic acoustic impedance of the duct $Z_{ac} = \dfrac{\rho_0 c_0}{S}$.

2. If we assume the pressure can be written as $p(x,t) = P(x)e^{j\omega t}$ (where $\omega$ is the pulsation of the wave), and the volume velocity can then be written as $q(x,t) = Q(x).e^{j\omega t}$, derive the general solutions of the sound pressure $P(x)$ and of the volume velocity $Q(x)$.

3. Show that the input values of pressure and volume velocity $P(0)$ and $Q(0)$, and the values $P(L)$ and $Q(L)$ at the termination $x = L$ can be read :

$$\begin{pmatrix} P(0) \\ Q(0) \end{pmatrix} = \begin{pmatrix} coskL & jZ_{ac}sinkL \\ \frac{j}{Z_{ac}}sinkL & coskL \end{pmatrix} \begin{pmatrix} P(L) \\ Q(L) \end{pmatrix}$$

4. What is the boundary condition at $x = L$ ? Deduce the input acoustic impedance $Z_a(0) = \frac{P(0)}{Q(0)}$
5. Now, let's assume the termination at $x = 0$ is open. What are the expression of the resonance frequencies of the boomwhacker ?
   Numerical application : what is the fundamental frequency of a boomwhacker of length $L = 19$ cm ? (we assume here $c_0 = 340$ m.s$^{-1}$)
6. What is the low-frequency behavior ($kL << 1$) of the input acoustic impedance ? Draw the analogue scheme representing this input impedance at low frequencies ?

## Exercise 5. Bi-directional and cardioid sources

1. What is the directivity factor and the directivity index of a bi-directional source $D_0(\theta) = \cos\theta$.

   Compute the half-power beamwidth of the source.
2. We remind that the directivity of a cardioid source is : $D_0(\theta) = \frac{1}{2}(1 + \cos\theta)$

   What are the directivity factor and the directivity index of a cardioid source ?

## Exercise 6. Directivity of a loudspeaker

1. Calculate the radius of a loudspeaker for it to be omnidirectional (with a tolerance of 3 dB) up to 500 Hz. (Use the plane piston model and charts for the Bessel functions).
2. Same question up to 2000 Hz.
3. Calculate the diameter of a loudspeaker for it to have a half-power beamwidth at 1000 Hz which is inferior to 30°.

## Exercise 7. Radiation of a 2-way loudspeaker - monopole hypothesis

A 2-way loudspeaker(boomer and medium) is placed in an anechoic chamber and the sound pressure level is measured at 4 m in the main axis of the loudspeaker for a 350 Hz sinusoidal input signal.
Three configurations are studied :

1. only the boomer is active,
2. only the medium loudspeaker is active,
3. both loudspeakers are active.

The measured pressure levels are :

1) $L_{p1} = 88$ dB 2) $L_{p2}$= 86 dB $L_{p3} = 93$ dB

1. From $L_{p1}$ and $L_{p2}$, calculate the effective flow velocities $\underline{q}_1$ and $\underline{q}_2$ of the two monopoles on a closed box which represent the boomer and the medium loudspeaker.
   Use the monopole model to predict the sound pressure level when both loudspeakers are radiating and compare with $L_{p3}$.

The same measurement (at 350 Hz) is repeated but at 10 points outside of the main axis. The averaged measured sound pressure levels are :
1) $L'_{p1} = 87.2$ dB 2) $L'_{p2}$= 86 dB $L'_{p3} = 90$ dB

2. Explain (without caluculating) the differences between on-axis and off-axis values and propose a new relationships which allows to predict $L'_{p1+2}$ from $L'_{p1}$ and $L'_{p2}$. Demonstrate that a monopole model is insufficient when describing the behavior of the loudspeaker.

## Exercise 8. Radiation of a small loudspeaker

We consider a loudspeaker of 12 cm diameter mounted on an infinite screen.

1. Remind the low-frequency approximation of the radiation resistance of a piston of radius $a$. Up to what frequency is this approximation valid ?
2. What is the sound pressure level, at 10 m on-axis, if the loudspeaker excursion is 0.28 mm (peak-peak), at $f = 250$ Hz, 500 Hz, 1000 Hz ?