3.3 Statistical acoustics

Introduction

- 1885: Sabine observations:
  - in closed space, sound is sustained after source stops
  - If the prolongation is too long, intelligibility is deteriorated
  - There exist limit values of duration with respect to the intelligibility deterioration
  - This phenomenon depends on walls nature and their covering area

Introduction

- Statistical acoustics: considering statistical variations of acoustic quantities rather than an accurate description of sound fields (wave acoustics)

➔assume theoretical **diffuse fields**: *it is composed of many rays with the average properties of* ***equal intensity*** *and* ***equal spatial distribution***

  - Constant sound energy density over the room
  - No dominant sound incident direction: $\vec{I} = \vec{0}$ over all directions

  ➔ All rays, on average, have been reflected the same number of times and traveled the same distance

  ➔ **Same mean free-path length d_m between 2 reflections for all rays**

In practice:

  - Only little overall absorption
  - Homogenous distribution of absorption

Reverberation

It is the prolongation of sound in a closed space

explanation:

- extinction of free state (cf. wave acoustics)
- late reflections constitute a *continuum* in geometrical acoustics

quantification: *reverberation time,*

corresponding to a **decrease of 60 dB** of sound level

Sabine absorption factor

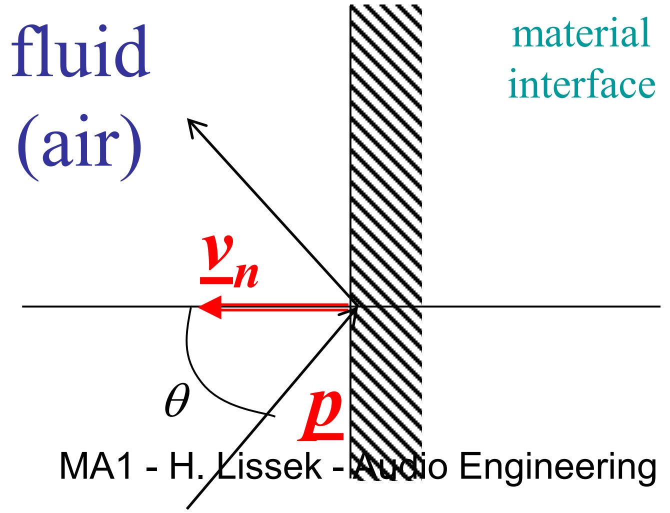
quantification of the absorbing nature of walls: *absorption coefficient* α

Energy absorbed by a wall:

$$\alpha(\theta) = 1 - \rho(\theta)$$

or in terms of volumic energy

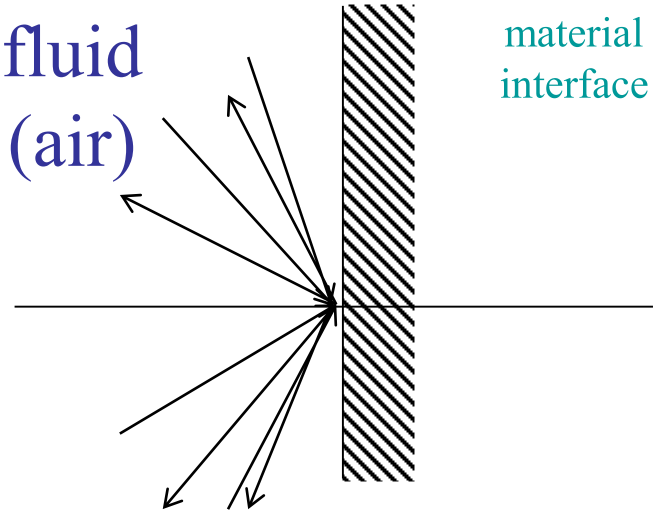
$$w_r = (1 - \alpha) w_i$$



Sabine absorption factor

Absorption factor α depends on incidence angle

In diffuse fields, it is possible to define a mean absorption factor α_S for equiprobable incidences



Sabine's law

see Kutruff, pp 127-160

Relationship between:

- Reverberation time T_s of a room
- absorption factors of walls α_i

Hypothesis:

- geometrical acoustics description
- ideally *diffuse* field, and remaining diffuse after the source stops

Sabine's law

see Kutruff, pp 127-160

At each reflection, sound rays lose the ratio α_s of their energy:

$$\delta\overline{w} = -\alpha_s \overline{w}_i$$

Mean free path d_m between 2 reflections induces mean free time

$$t_m = d_m / c$$

➔statistically the energy loss δw occurs every t_m

Diffuse field ➔

$$\frac{\delta\overline{w}}{\overline{w}} = -\frac{\alpha_s}{t_m} dt$$

Then:

$$\overline{w}(t) = \overline{w}_0 \exp(-\alpha_s t / t_m)$$

Sabine's law

see Kutruff, pp 127-160

Reverberation time T_s is defined as the time for the energy to decrease of 60 dB (corresponding to a factor 1/1000000)

In diffuse field, estimated mean free time can be derived as $t_m = \frac{4V}{cS}$

Then the *Sabine's law* becomes:

$$T_s \cong 55,4 \frac{V}{c\alpha_s S} \cong 0,16 \frac{V}{\alpha_s S}$$

equivalent sound absorption area – Sabine formula

Sound *absorption area* A (m^2) =denominator of Sabine's law

Generalization to heterogeneous walls:

$$A = \sum_i A_i = \sum_i \alpha_{si} . S_i$$

Sabine's law ➔ Sabine's formula

$$T_s \cong 0,16 \frac{V}{A}$$

Sabine’s law

alternative derivation

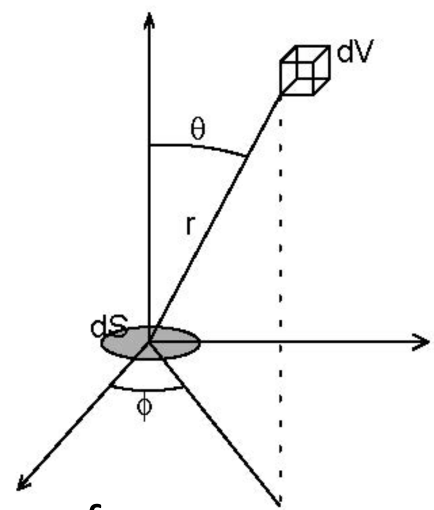
We want to estimate the sound intensity *I* hiting a surface d*S* of walls inside the room.

We consider a diffuse field (ie random incidence plane waves, uniform energy density w)

Let’s consider a volume dV with energy wdV.

The portion of energy stemming from dV and hiting dS is:

$$W = \frac{dS\cos\theta}{4\,\pi r^2} wdV$$



The power that hits dS in 1 second =energy within a half-sphere of radius $R = c \times 1\text{s}$

$$P = \frac{wdS}{4\pi}\int_{r=0}^{c}\int_{\theta=0}^{\pi/2}\int_{\phi=0}^{2\pi}\frac{\cos\theta}{r^2}dr.rd\theta.r\sin\theta d\phi = \frac{wc}{4}dS = I.dS$$

Since the total intensity impinging the wall is $I = \frac{wc}{4}$, the whole absorbed intensity is $I_a = A\frac{wc}{4}$

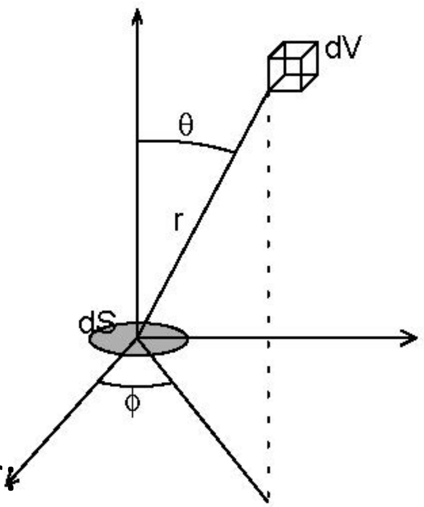
Sabine's law

alternative derivation

Since all the power delivered by a source is absorbed,

$P_{source} = P_{absorbed} = A\frac{wc}{4}$ then $w = \frac{4P_{source}}{Ac}$

A diffuse field = superposition of plane waves with arbitrary directions ➔ $I = wc = \frac{p^2}{\rho c}$



Reverberation time:

The total power in the room, at any time, is the difference between the power of the source and the absorbed power:

$$\frac{dW}{dt} = V\frac{dw}{dt} = P_{source} - P_{absorbed}$$

When the source stops:

$$V\frac{dw}{dt} = -A\frac{wc}{4} \quad ➔ \quad \frac{dw}{w} = -\frac{cA}{4V}dt$$

Leads to the Sabine 's law

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Properties

Diffuse field

➔ at cut-off, sound levels (L_I and L_p) linearly decrease

➔ do not depend on repartition of absorbent

➔ T_s can be modified by changing the surface and nature of materials

➔ inversely, measuring T_s leads to the assessment of materials' absorption factors

Comments

Sabine valid only in diffuse field

➔ still valid with diffuse reflections (Lambert's law, see Kutruff book)

This condition is often verified in reality

Estimation of t_m with rigorous methods lead to the same result

Sabine is generally verified (~orders of magnitudes)

Particular case : Sabine applied on ideally absorbing walls (α=1)

➔ T_s finite (not null)!

Important caution: for $\frac{A}{S} > 0.3$, Sabine is overestimated!

Eyring and Millington laws

see Kutruff, pp 127-160

Absorbing walls ➔ Sabine is insufficient

Eyring's law: same concept than Sabine, but with a different law for energy decrease:

- At each reflection, energy drop by (1-α): $W_i = W_{i-1}(1-\alpha)$
- After N reflections, energy $W_N = W_0(1-\alpha)^N$
- For a decrease of 10^6 (as for Sabine), it yields: $\frac{W_N}{W_0} = 10^{-6} = (1-\alpha)^N$
- Then : $N = \frac{\ln(10^{-6})}{1-\alpha} \simeq \frac{-13.8}{1-\alpha}$
- Considering mean free time t_m, the total time for the N reflections is:

$$T_E = N t_m = N \frac{4V}{cS} = 0.16 \frac{V}{\underbrace{-\ln(1-\alpha)\mathrm{S}}_{A_E}}$$

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Eyring and Millington laws

see Kutruff, pp 127-160

Absorbing walls ➔ Sabine is insufficient

Eyring's law: same concept than Sabine, but with a different law for energy decrease

$$T_e = 0{,}16 \frac{V}{\underbrace{-S \ln(1-\alpha_e)}_{A_E}}$$

Millington's law: consider each material independently, and a mean free path for each pair of walls (no more averaged on the whole room):

$$T_m = 0{,}16 \frac{V}{\underbrace{\sum_{i=1}^{n} S_i \ln(1-\alpha_{mi})}_{A_m}}$$

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Effects of losses in air

For wide rooms, we shall consider the absorption coefficient of the air: a in Np/m

$$\bar{w} = \bar{w}_0 \exp\left\{-(2ac + \frac{\alpha_s}{t_m})t\right\}$$

Increase of the equivalent absorption area of about $0{,}9 a_{\mathrm{dB}} V$

Schroeder frequency

Characteristic frequency above which the field is diffuse enough:

$$f_c = 2000(T_s/V)^{1/2}$$

Determines the limit between modal and diffuse sound fields

Useful relationship for determining the minimum volume of a room with respect to the diffuse field conditions

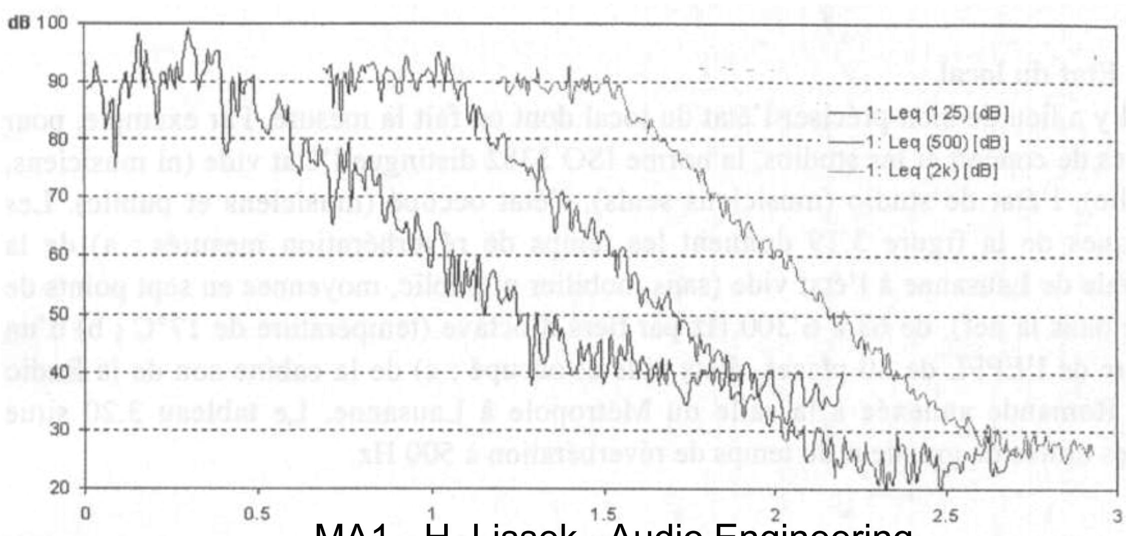
$$V_{min} = (2000/f_c)^2 T_s$$

For example: for a opera house requiring T_s=2 s, and instruments down to 60 Hz ➔ it yields a minimum volume of 2'222 m^3 !

T_s measurement

A pink noise is emitted in a room, with a loudspeaker, so that a high number of modes are excited

At source cut-off, the decrease of sound pressure level is sensed with a microphone



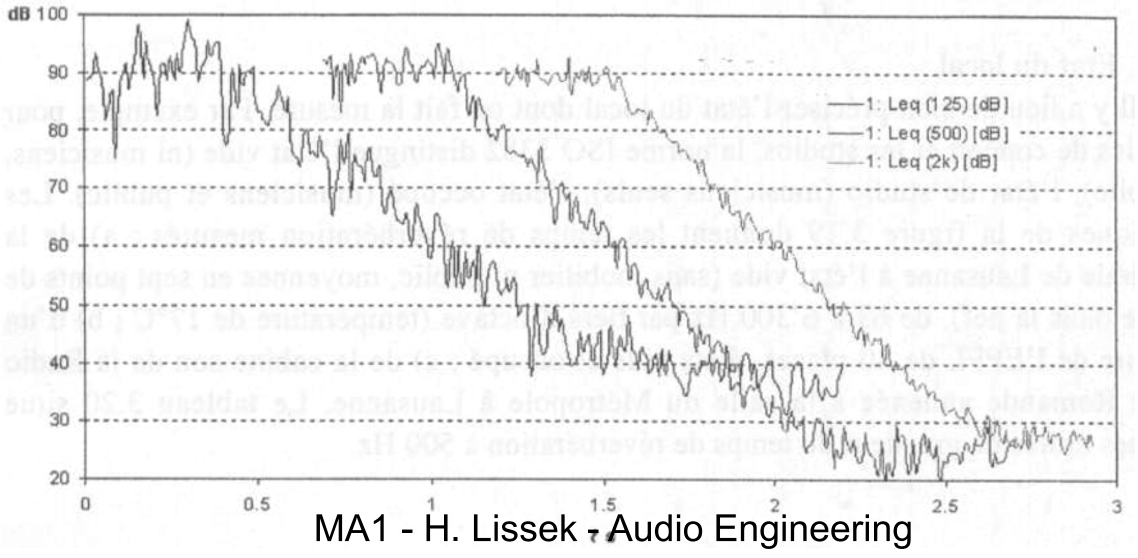
T_s measurement

In practice, background noise may disturb the measurements

Decrease of 60 dB not always possible

➔ Measurement of T_{20} or T_{30}

Other measurable metrics: *initial reverberation time* (10 first ms of decrease)



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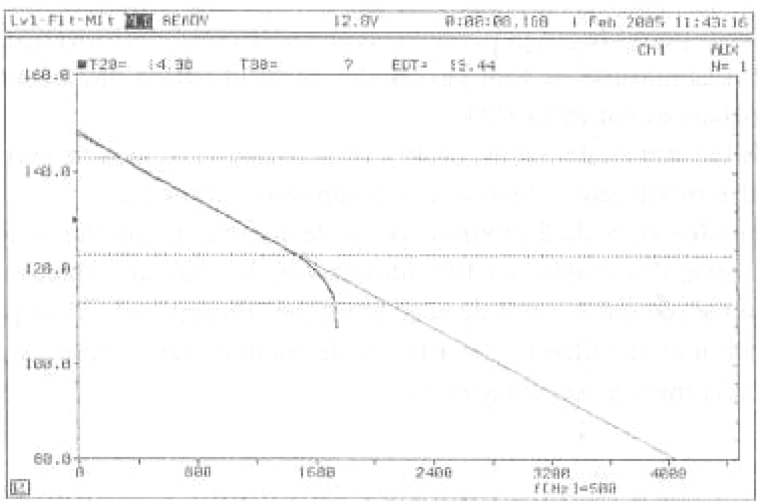
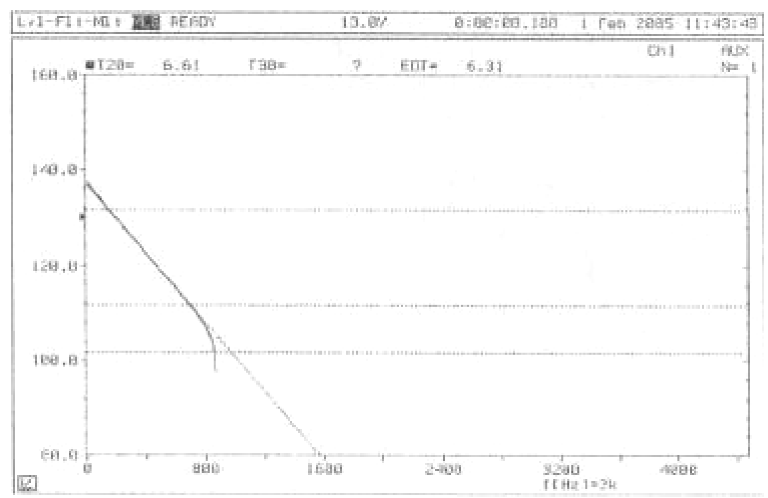
Schroeder method

Schroeder demonstrate that the mean decrease can be deduced from the impulse response $p(\tau)$

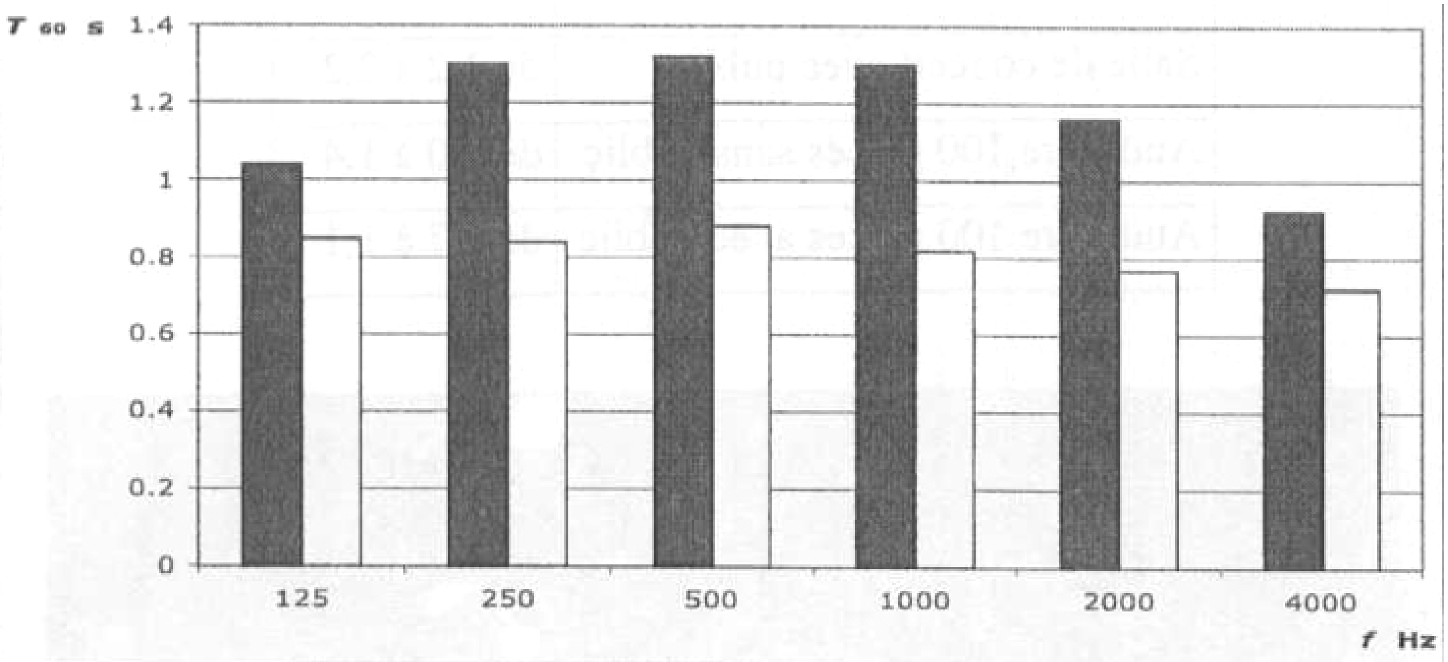
$$\tilde{p}^2(t) = \int_t^\infty p^2(\tau)d\tau = \int_0^\infty p^2(\tau)d\tau - \int_0^t p^2(\tau)d\tau$$

Pseudo-random excitation: multi-length sequences(MLS) ➔ impulse response $p(\tau)$





State of the room



Reverberation times in the ELA2 auditorium without and with audience

Absorption factors measurements

By comparing T_s in a reverberant room **without** and **with** absorbing material, one can compute absorption factor

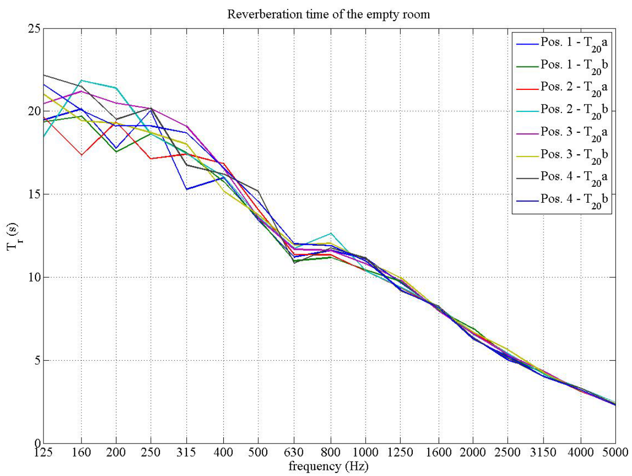
Sabine's formula allows to find the desired α_s

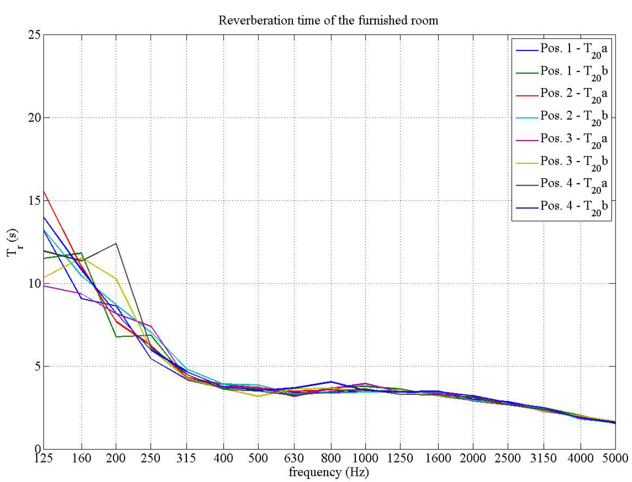
$$\begin{cases} T_{empty} = 0,16 \dfrac{V}{\alpha_{s0}.S_{room}} \\ T_{material} = 0,16 \dfrac{V}{\alpha_{s0}.(S_{room} - S_{material}) + \alpha_s.S_{material}} \end{cases}$$

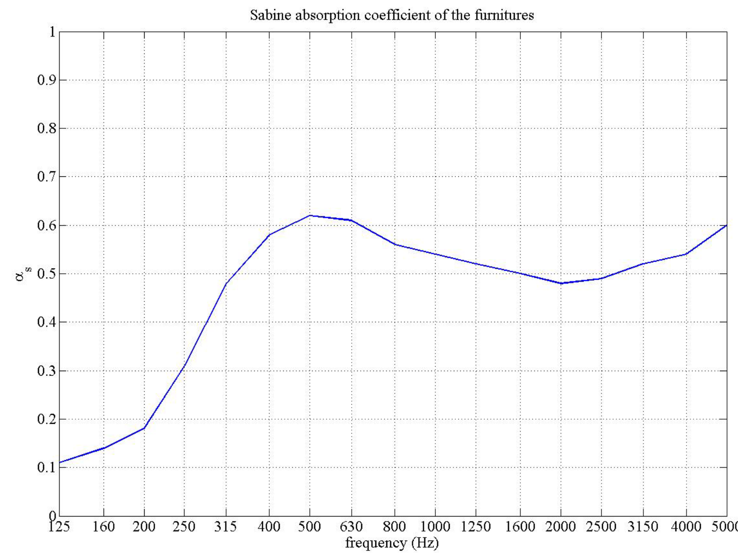
We can then derive the values of α_s

Note: it is possible that α_s >1

Absorption factors measurements



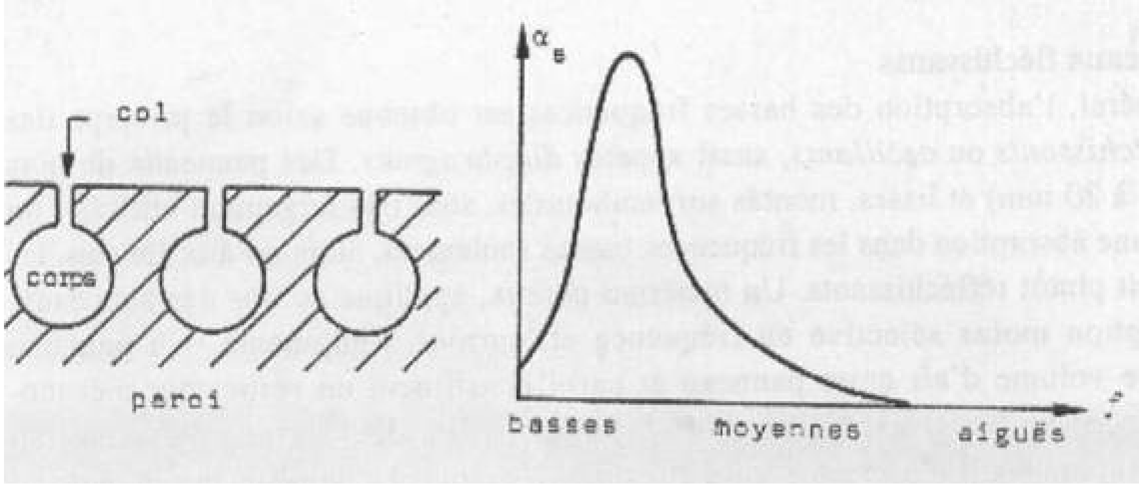




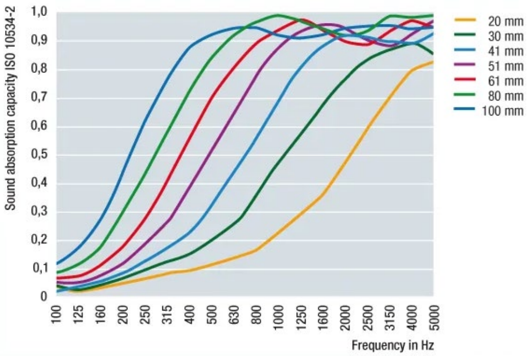
Absorption principles

3 fundamental principles, corresponding to different frequency bands:

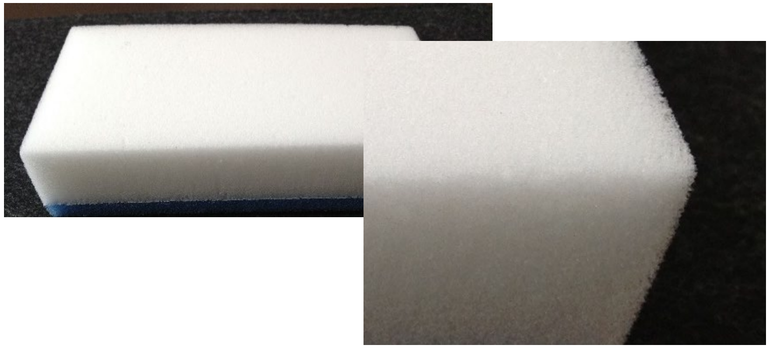
- porous materials (mineral fibers)
- panel absorbers (wooden panels)
- acoustic resonators (see perforated ceilings)

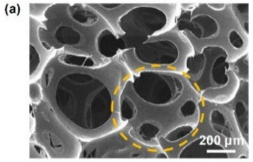


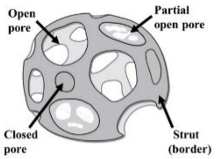
Porous materials

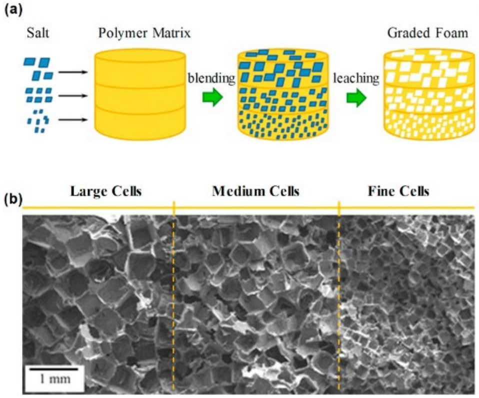


Foams

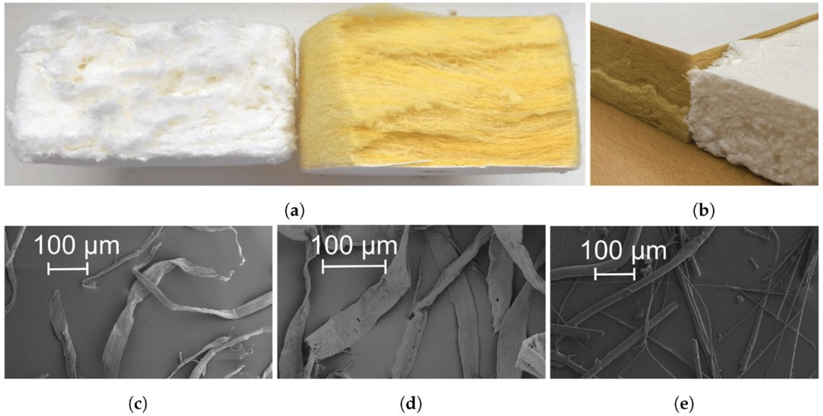


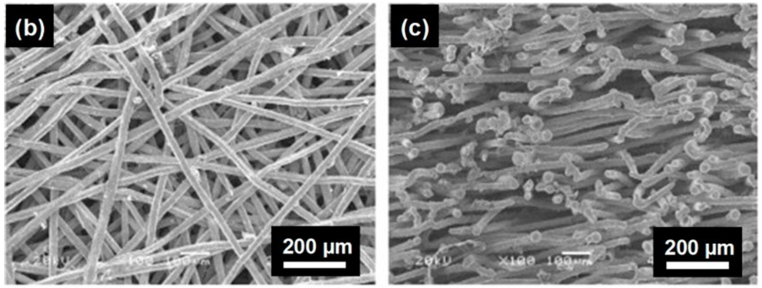






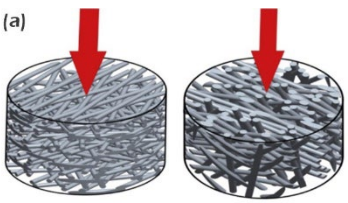
Fibrous materials



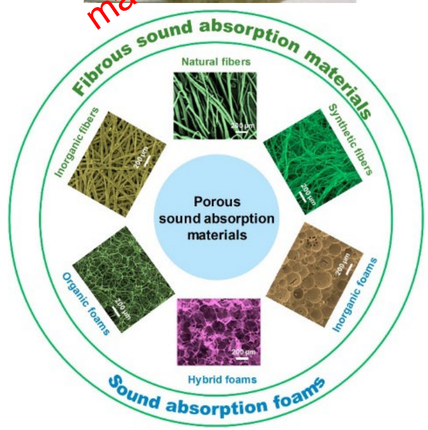


Porous materials

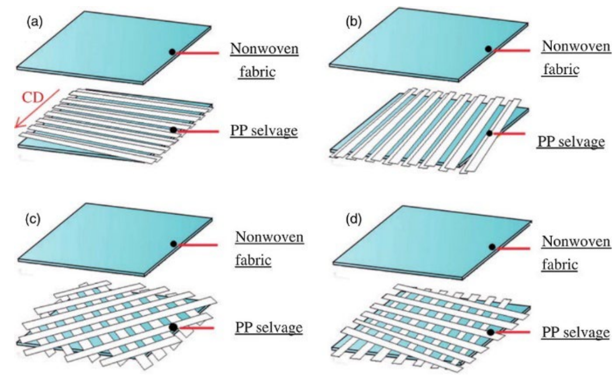
inorganic fibers (glasswool, carbon, metal, etc.)



incl. Sustainable materials

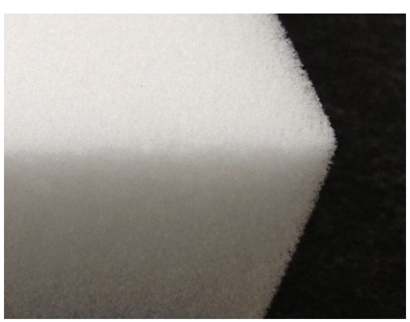


processed polymeric fibers (nylon, fabrics, etc.)

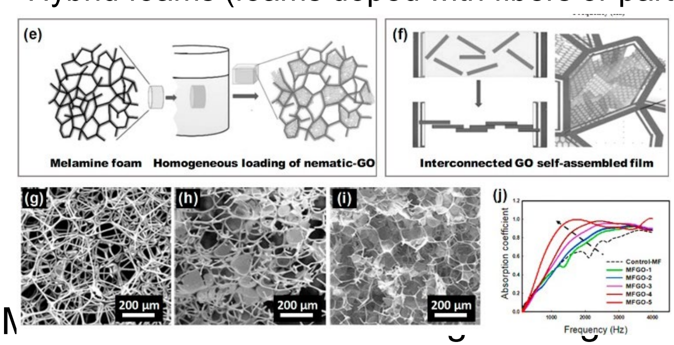


Inorganic foams (metallic foams, etc.)

Organic foams (eg PU, melamin, etc.)



Hybrid foams (foams doped with fibers or particles)



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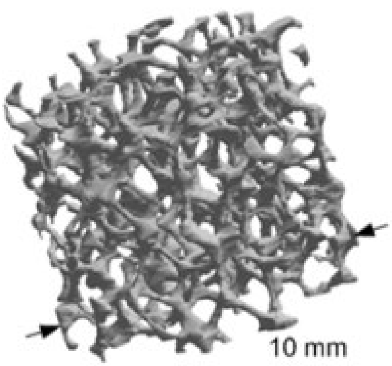
Porous layers

Mechanisms

- Viscous damping
- Thermal dissipation
- Mechanical vibrations of skeleton



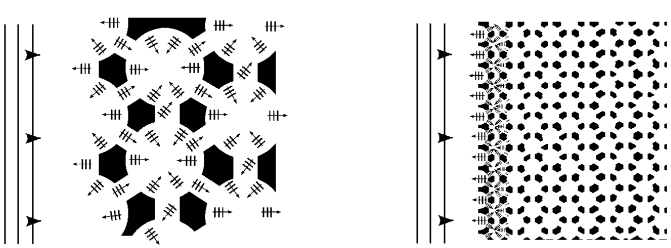
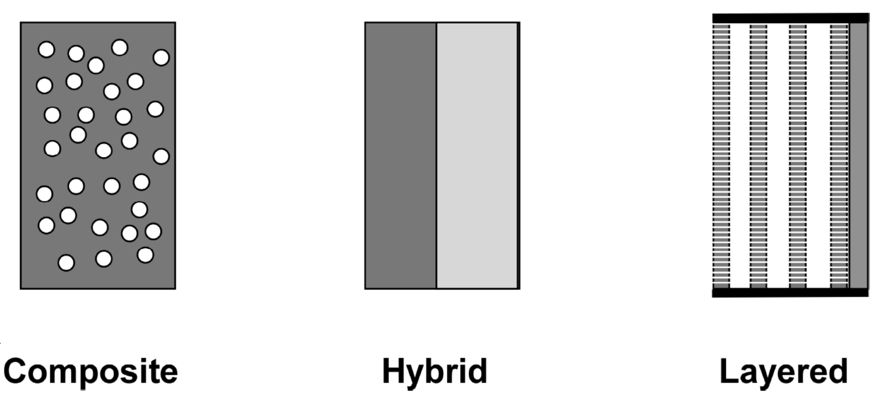


Figure 1.—Decrease in energy transfer potential as pore size decreases from large (left) to small (right).

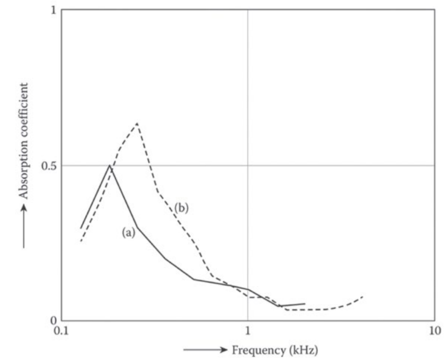
Absorption factors

- high values above 500 Hz,
- quite null below 500 Hz
- increase with thickness and density,
- optimal value of density,
- higher thickness increase absorption at low frequencies
- mounting conditions (plenum) modify the absorption at low frequencies



Panel absorbers

Panel absorbers



(a) wooden panel, 8 mm thick, m' = 5 kg/m2, 30 mm away from rigid wall, with d=20 mm Rockwool in the air gap; (b) panels, 9.5 mm thick, perforated at 1.6% (diameter of holes 6 mm), 50 mm distant from rigid wall, air space filled with glass wool.

Absorption at low frequencies

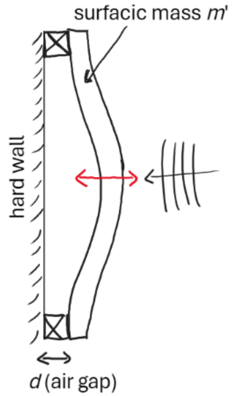
Thin wooden panels (4 to 20 mm) with smooth surface, mounted on beams

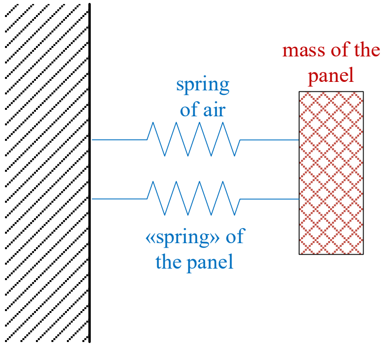
Possible adjunction of porous materials

Mass-spring-losses system

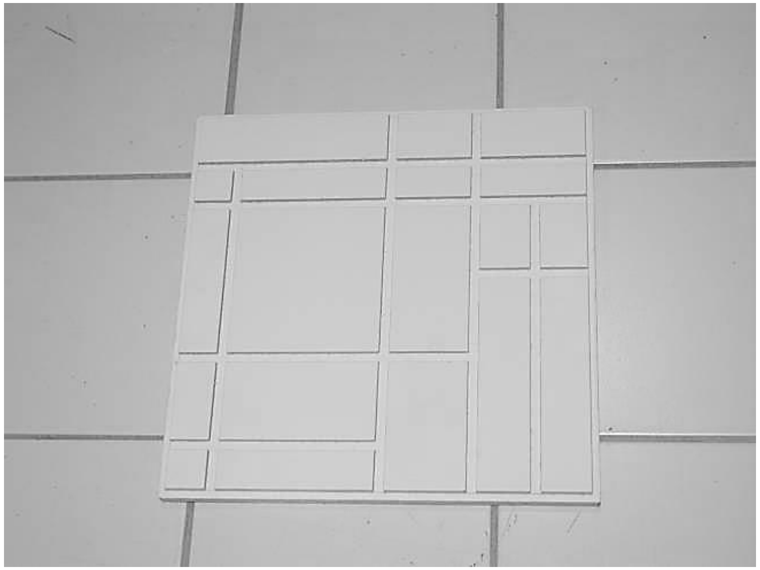
Resonance frequency determined by:

$$f_r = \frac{600}{\sqrt{m'd}}$$





Panel absorbers

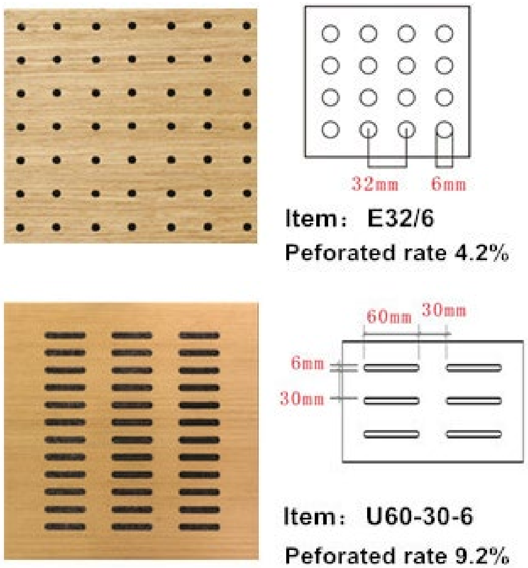


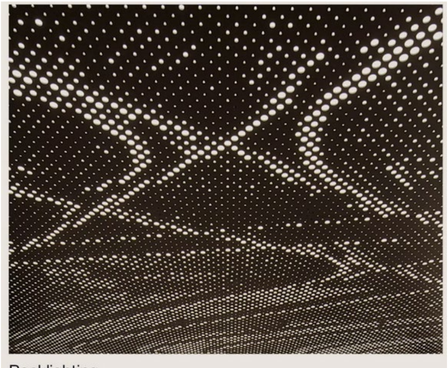
Acoustic panels:

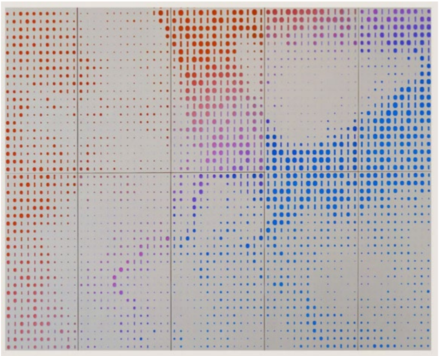
- with holes,
- with gaps,
- plaster or wood derivates,
- combined or not with porous layers

Absorption at low and medium frequencies, depending on the type and density of surface irregularities

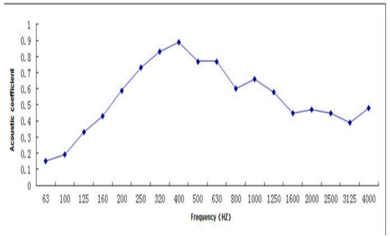
Acoustic resonators




Backlighting


RGB Lighting

Acoustic resonators

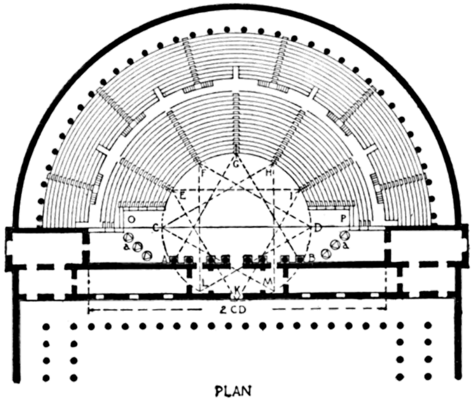


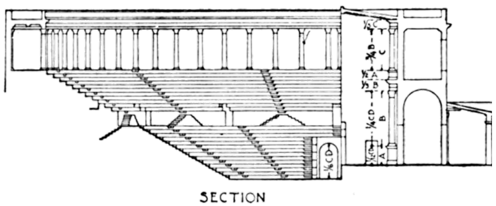
Their use relates back to Antiquity.

Principle: hollow body, communicating with outside by way of a neck or a hole

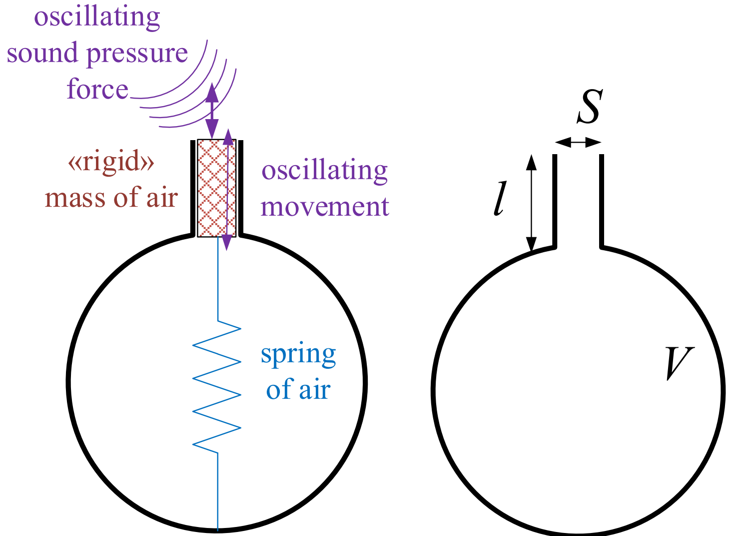
Absorption in the medium frequencies range

With quite small dimensions with respect to wavelength at the resonance, the resonator can be computed with an equivalent Kirchhoff model: acoustical mass-spring-losses system





THE ROMAN THEATRE ACCORDING TO VITRUVIUS, 1914



$$f_H = \frac{c_0}{2\pi}\sqrt{\frac{S}{lV}}$$

Examples

	Facteur d'absorption α_s à la fréquence indiquée en Hz					
Matériau et mise en oeuvre	125	250	500	1000	2000	4000
Crépi lisse sur murs de briques ou de béton	0,01	0,01	0,02	0,02	0,03	0,04
Plafond plâtre lisse 2 cm avec plénum 20 cm	0,25	0,20	0,10	0,05	0,05	0,10
Parois revêtues de bois ou panneaux de fibres de bois sur lambourdes	0,40	0,30	0,20	0,10	0,10	0,20
Revêtement de sol collé (bois, linoléum, etc.)	0,02	0,03	0,04	0,05	0,05	0,10
Parquets, etc., sur lattes	0,20	0,15	0,10	0,10	0,05	0,10
Tapis, épaisseur moyenne	0,05	0,08	0,20	0,30	0,35	0,40
Rideaux, épaisseur moyenne	0,10	0,15	0,30	0,40	0,50	0,60
Panneaux acoustiques sans plénum	0,10	0,15	0,40	0,60	0,70	0,70
Panneaux acoustiques avec plénum de 15 cm	0,20	0,30	0,60	0,70	0,70	0,70
Fenêtre fermée	0,10	0,04	0,03	0,02	0,02	0,02
1 m^2 de public assis et dense	0,60	0,75	0,90	0,95	0,95	0,85
1 m^2 de sièges rembourrés épais	0,45	0,55	0,60	0,60	0,60	0,50

Audience, furniture

The audience modifies the absorption in the room at medium frequencies – idem for furnitures

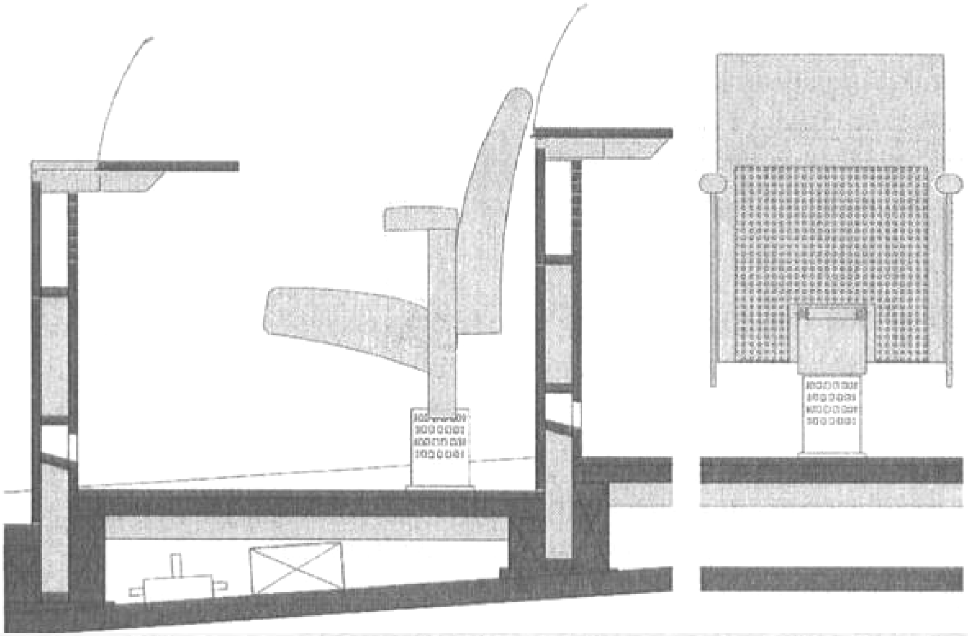
2 possibilities for computing:

- equivalent sound absorption area per person
- absorption factor α_s per square meter

Fundamental issue: realize a T_s that is the less sensitive to the presence of audience

➔ "absorbing disposals" in the furniture to compensate the absence of audience

Audience, furnitures



Désignation	Aire d'absorption A_1 en m^2 / unité à la fréquence indiquée en Hz					
	125	250	500	1000	2000	4000
Public debout ou sur sièges en bois	0,15	0,30	0,50	0,55	0,60	0,50
Idem, sur sièges rembourrés	0,20	0,40	0,55	0,60	0,60	0,50
Musiciens avec instruments sur podium	0,40	0,80	1,0	1,4	1,3	1,2
Mobilier en bois	0,01	0,01	0,02	0,03	0,05	0,05
Mobilier rembourré en tissu	0,10	0,30	0,35	0,45	0,50	0,40
Mobilier en cuir	0,10	0,25	0,35	0,35	0,20	0,10

Energy conservation

see Kutruff, pp 127-160

Energy variation = source power – absorbed power

If the source power variates slowly:

➔ we consider $\overline{w}$ and $\tilde{p}$ (averages rather than instantaneous values)

Diffuse field=sum of plane waves with equal-probability directions

Sound intensity within an unitary face (of a wall) is: $I = \frac{\tilde{p}^2}{4Z_c} = \frac{c\overline{w}}{4}$

➔ Absorbed power: $P_{aa}(t) = I.A = \overline{w}\frac{Ac}{4}$

➔ Energy conservation equation:

$$P_a(t) = V\dot{\overline{w}} + \overline{w}\frac{Ac}{4}$$

EPFL

Steady state and reverberated level

In case of a constant source (ie. mean variation of energy per unit volume is null):

$$P_a = IA_s = \overline{w}\frac{Ac}{4}$$

The reverberated level is defined as sound pressure level in diffuse field $L_r \cong L_{Pa} - 10\log A + 6$

➔ Sound pressure level computation takes into account the contribution of reflections in the room, assuming the field is diffuse

Steady state and reverberated level

Beranek observed that the reverberated energy is the one of the first reflection, then A is replaced by $R=A/(1-\alpha_{mean})$

The reverberated level L_r is the same in the whole room, whatever

- the distribution of absorbant,
- position of the source
- position of the listener
- their relative distances

Direct sound level, for a known source (directivity δ, power P_a) is:

$$L_d \cong L_{Pa} + L_\delta - 20\log d - 11$$

where

$$L_\delta = 10\log_{10}\delta$$

Critical distance, critical radius

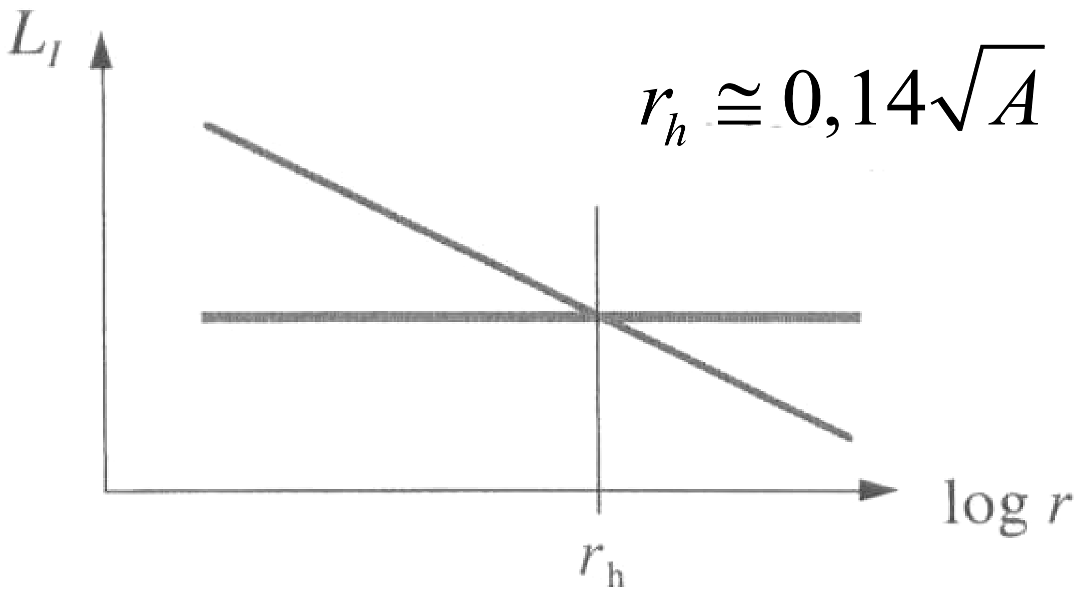
L_d decreases with d whereas L_r remains constant

➔ *Critical distance* for which $L_r = L_d$

$$d_c \cong 0{,}14\sqrt{\delta . A}$$

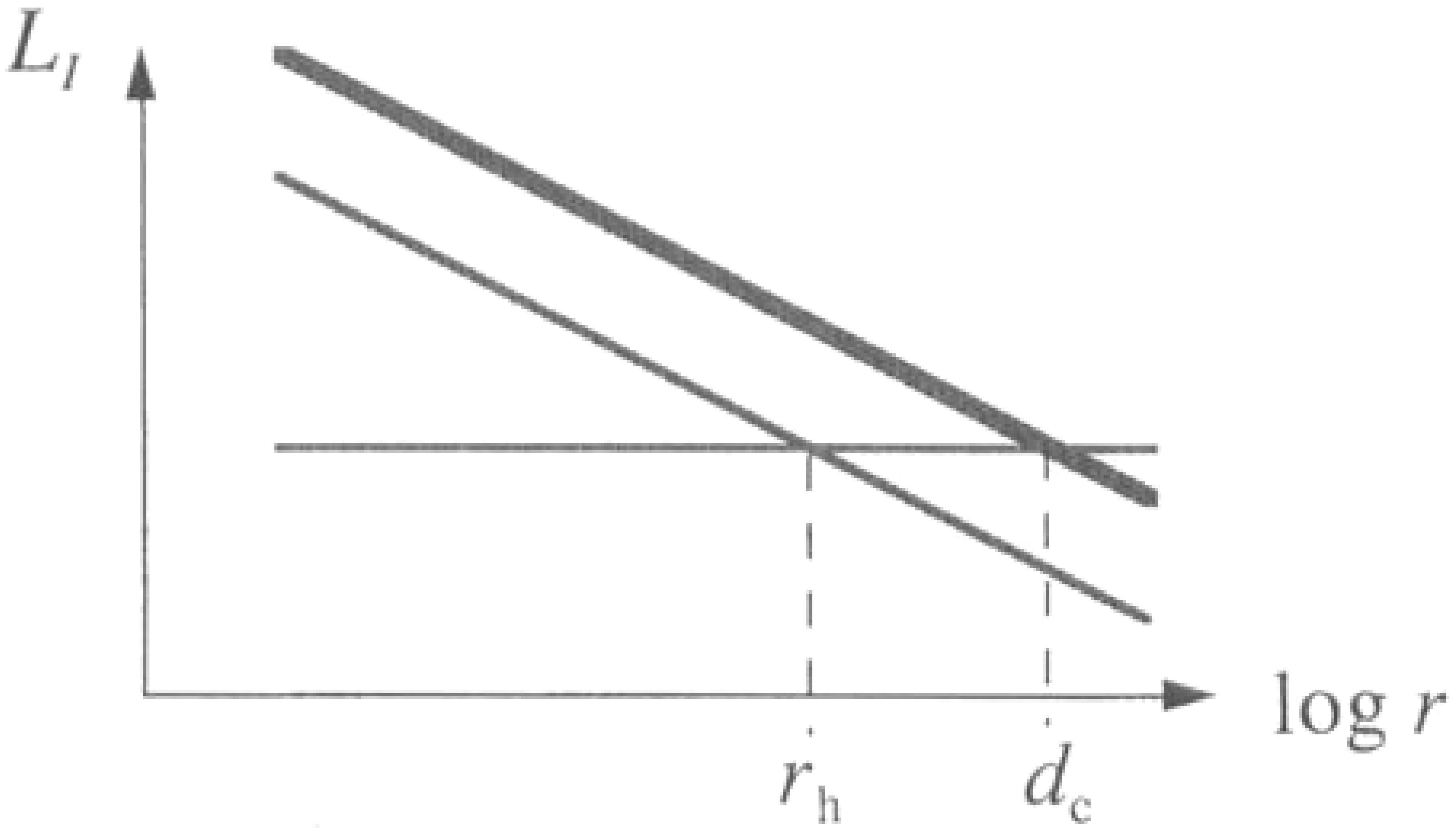
If omnidirectional source (δ=1), *critical radius*

$$r_h \cong 0{,}14\sqrt{A}$$

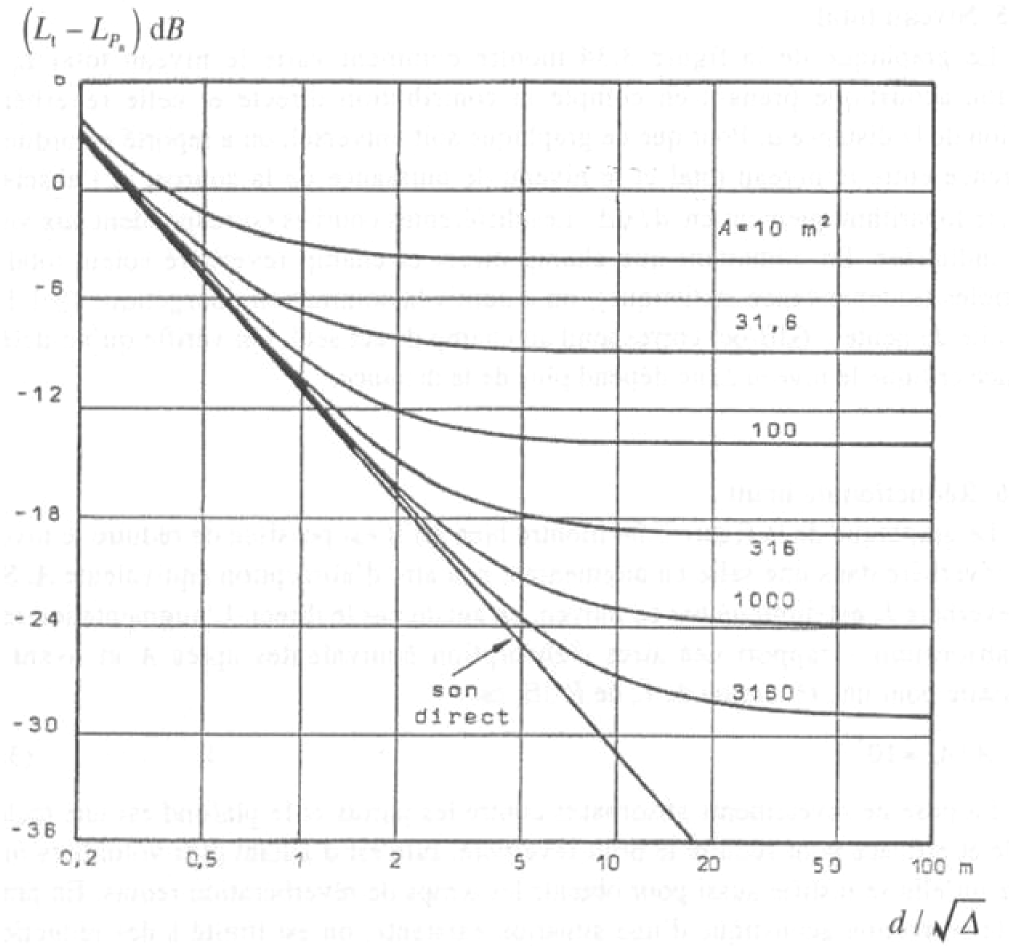


EPFL

Properties



Total level



$(L_t - L_{P_s})$ dB
6
0
-6
-12
-18
-24
-30
-36
A = 10 m²
31,6
100
316
1000
3160
son direct
0,2
0,5
1
2
5
10
20
50
100 m
$d / \sqrt{\Delta}$

Applications

- **Noise reduction**

Possibility to reduce the reverberated level (noise) by enhancing (increasing) A

➔ relative increase of absorption leads to a decrease of R dB of the reverberated level Lr:

$$A/A_0 = 10^{(R/10)}$$

- **Acoustic power measurement**

In a reverberant room, the measure of the sound pressure level $Lp=Lr$ (dominating reveberant field) gives L_{Pa}

Unsteady state

When a source power varies with time (see speech or music):

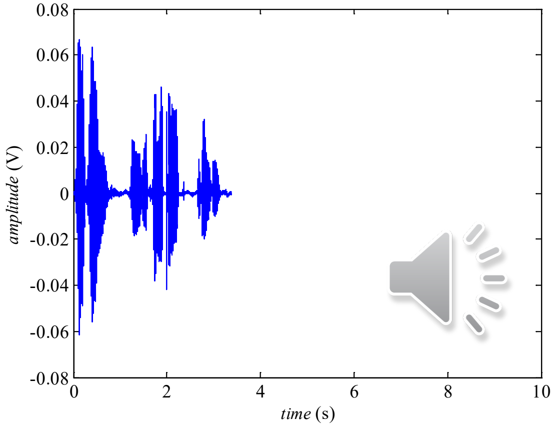
➔ evolution/modulation of the acoustic fields in a room

Conservation equation ➔solutions after

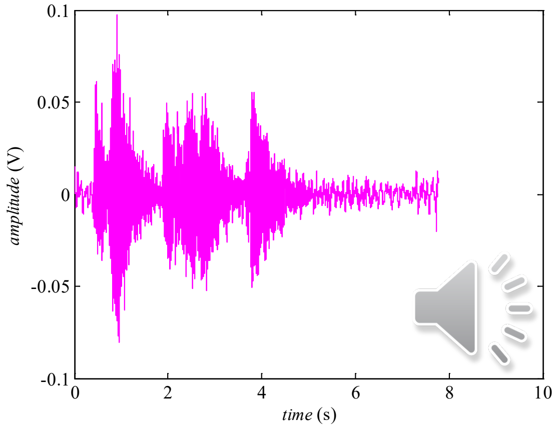
$$V\overline{w}(t) = P_a(t) * \exp(-t/\tau)$$

where $\tau = 4V/cA$ or $T_s/13{,}8$

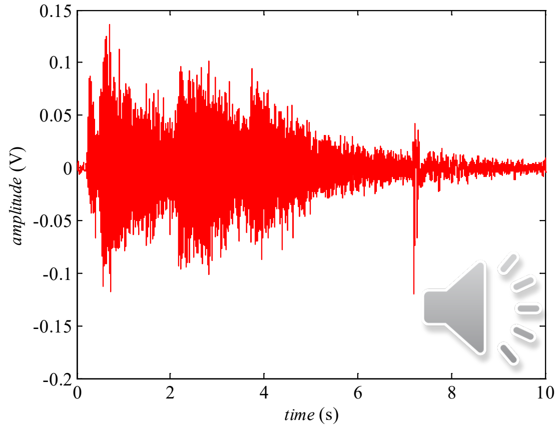
Anechoic recording



Semi-reverberant recording



Reverberant recording



Modulation transfer function

For a harmonic acoustic power of a source, modulated at 100 %:

$$P_a(t) = P_{a0}\left[1+\cos\Omega t\right]$$

we obtain: $\overline{w}(t) = \overline{w}_0\left[1+m\cos\Omega(t-\tau_0)\right]$

where $\overline{w}_0 = \tau P_{a0/V}$

and delay τ_0

Modulation transfer function $$m(\Omega) = \frac{1}{\sqrt{1+\Omega^2\tau^2}}$$

In complex form: $$\underline{m}(\Omega) = \frac{1}{1+j\Omega\tau}$$

Modulation transfer function

Schroeder showed that m is the Fourier transform of the square of the impulse response of the room $h(t)$

$$\underline{m}(\Omega) = \frac{\int_0^\infty h^2(t)\exp(-j\Omega t)dt}{\int_0^\infty h^2(t)dt}$$

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