III. ROOM ACOUSTICS

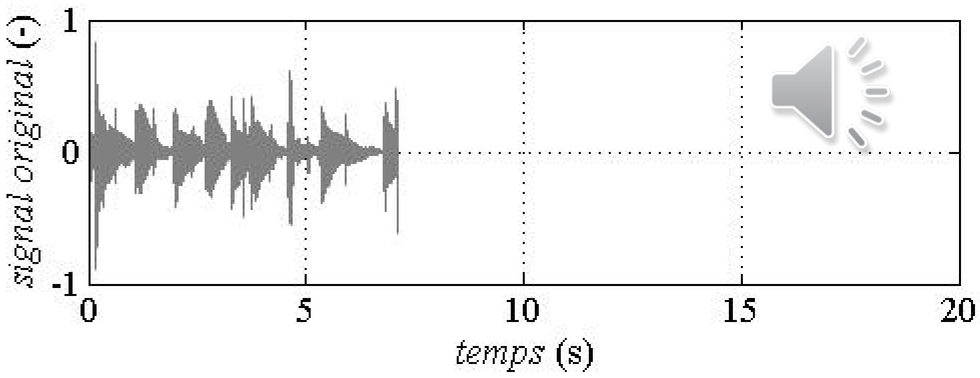
MA1 – Audio

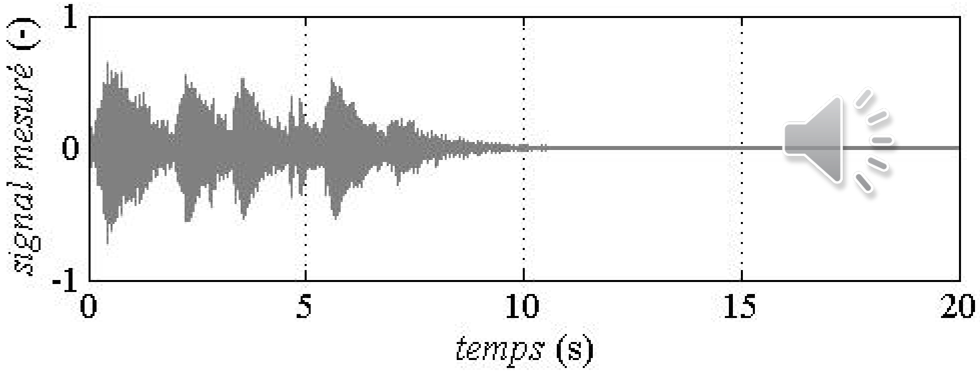
Hervé Lissek, Etienne Rivet, Vu Thach Pham

EPFL

CONTEXT

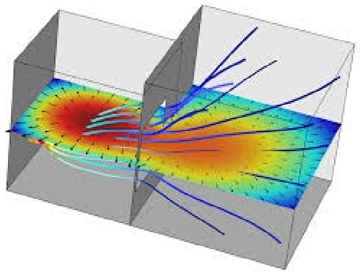
- Music rendering

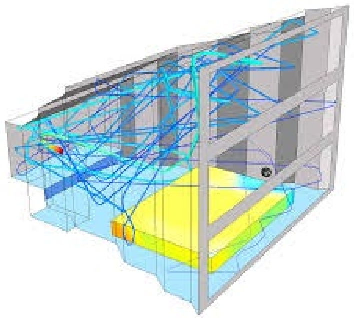


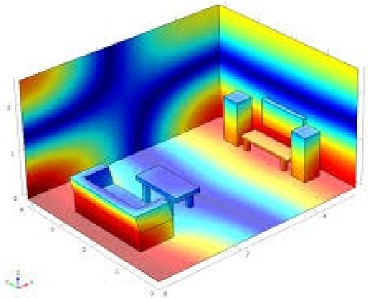


SOUND FIELD CHARACTERISATION

- Wave-based methods: boundary element method, finite element method, finite-difference time domain
- Modal methods: expression of the sound field as a linear combination of the modes
- Geometric methods: ray tracing method, image source method
- Statistical methods: hypothesis of a diffuse sound field, uniform acoustic energy traveling in all the directions with the same probability
- Diffusion-equation models (extension of the statistical theory to spatially varying reverberation times): mainly used for coupled rooms







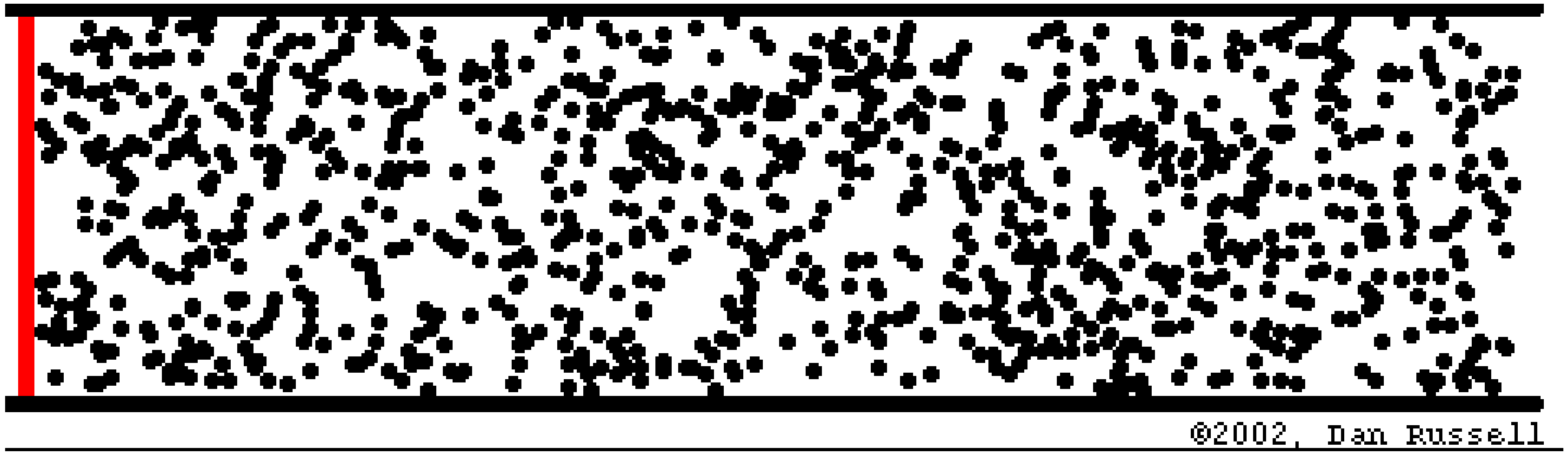
III.1 WAVE THEORY OF ROOM ACOUSTICS

MA1 – Audio

EPFL

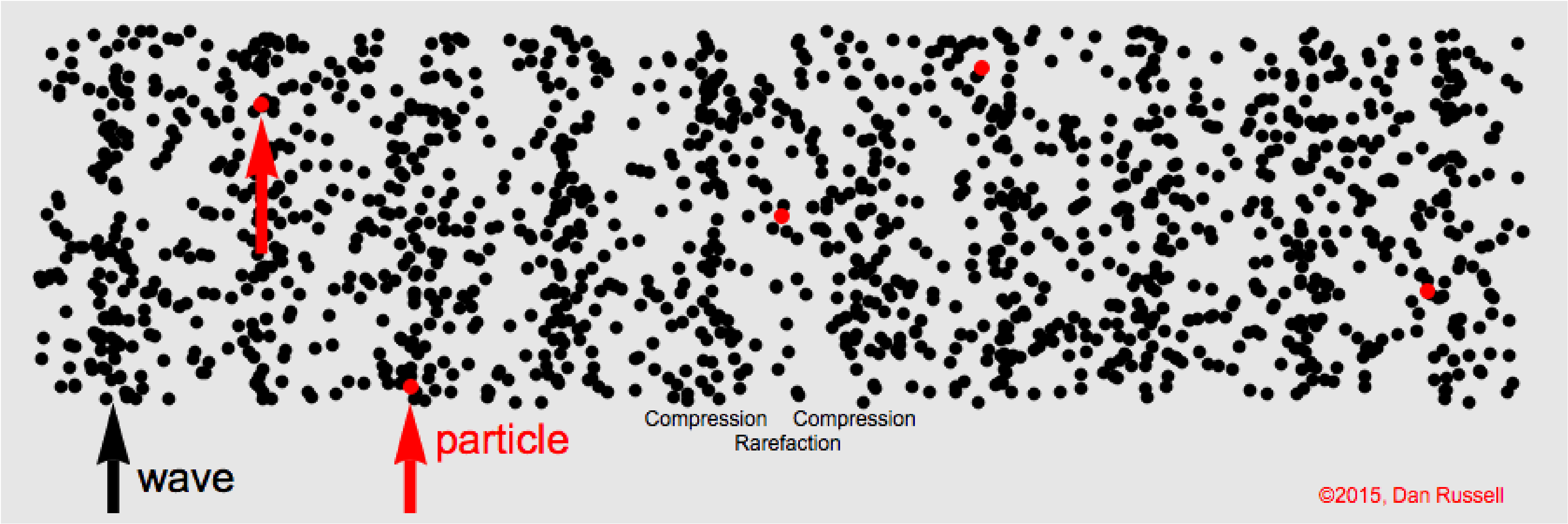
SOUND WAVES IN AIR

- Air particles oscillate back and forth about their equilibrium positions, while the wave disturbance propagates through the medium



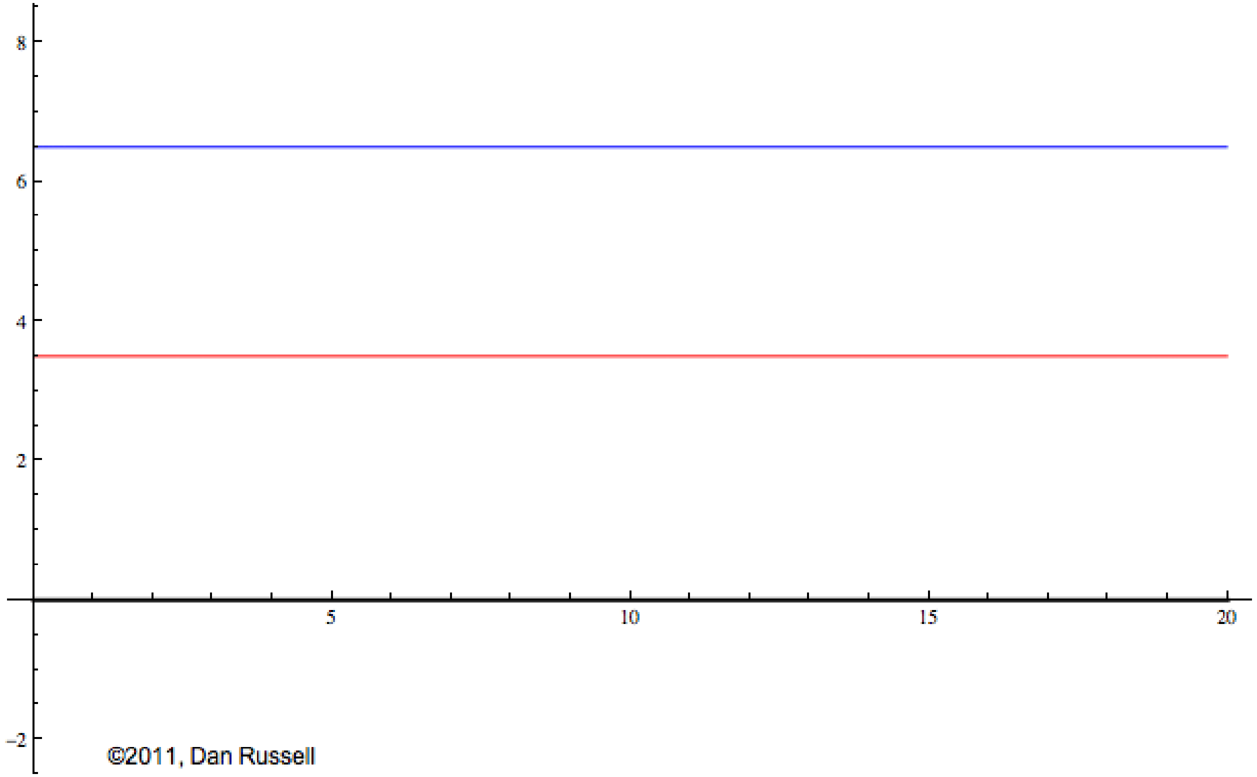
WHAT IS A WAVE?

- Longitudinal waves

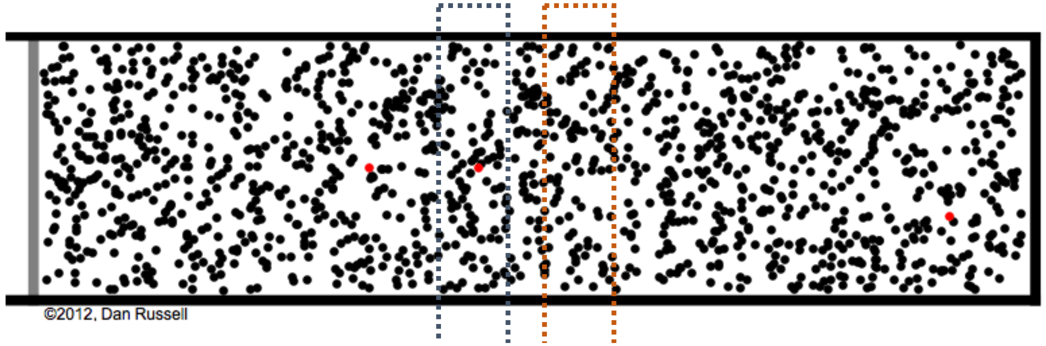


SUPERPOSITION OF WAVES

- Waves travelling in opposite directions



STANDING WAVES



PLANE WAVES IN WAVEGUIDES

- Steady-state solution of the wave equation $p(x,t) = \sqrt{2}\,\mathrm{Re}\left(p_x e^{j\omega t}\right)$

$$\frac{d^2 p_x}{dx^2} + k^2 p_x = 0$$

→ $p_x = Ae^{-jkx} + Be^{jkx}$

- Dynamic law

$$\frac{dp_x}{dx} = -j\rho\omega v_x$$

→ $v_x = \frac{1}{\rho c} Ae^{-jkx} - \frac{1}{\rho c} Be^{jkx}$

ρ : mass density of the medium

c : speed of sound in the medium

REFLECTION COEFFICIENT AND WALL IMPEDANCE

- Wall impedance

$$Z_L = \frac{p_L}{v_L}$$

- Reflection coefficient

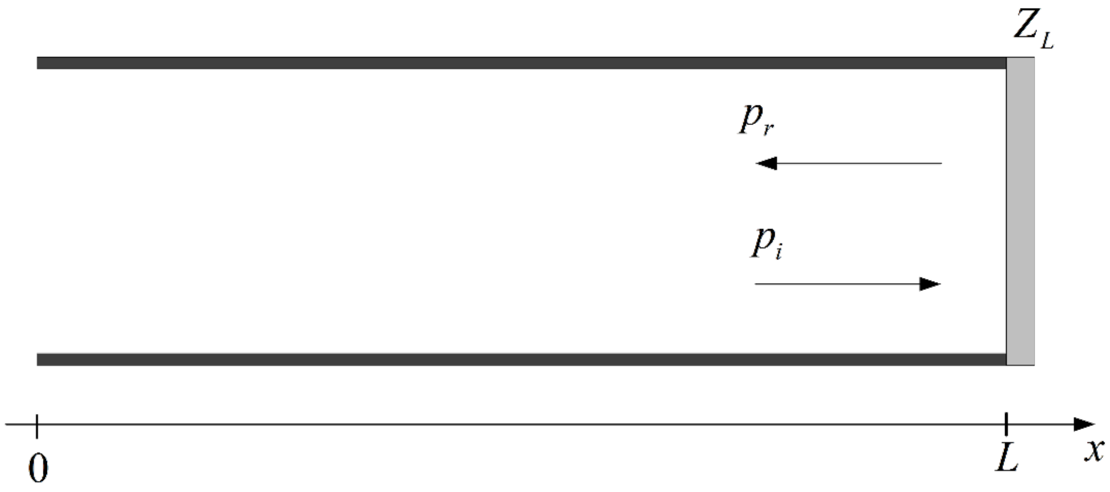
$$R_L = \frac{p_r}{p_i} = \frac{Z_L - \rho c}{Z_L + \rho c}$$

→ $$Z_L = \rho c \frac{1 + R_L}{1 - R_L}$$

→ $$p_x = A\left(e^{-jkx} + R_L e^{jk(x-2L)}\right)$$

- Absorption coefficient

$$\alpha = 1 - |R_L|^2$$

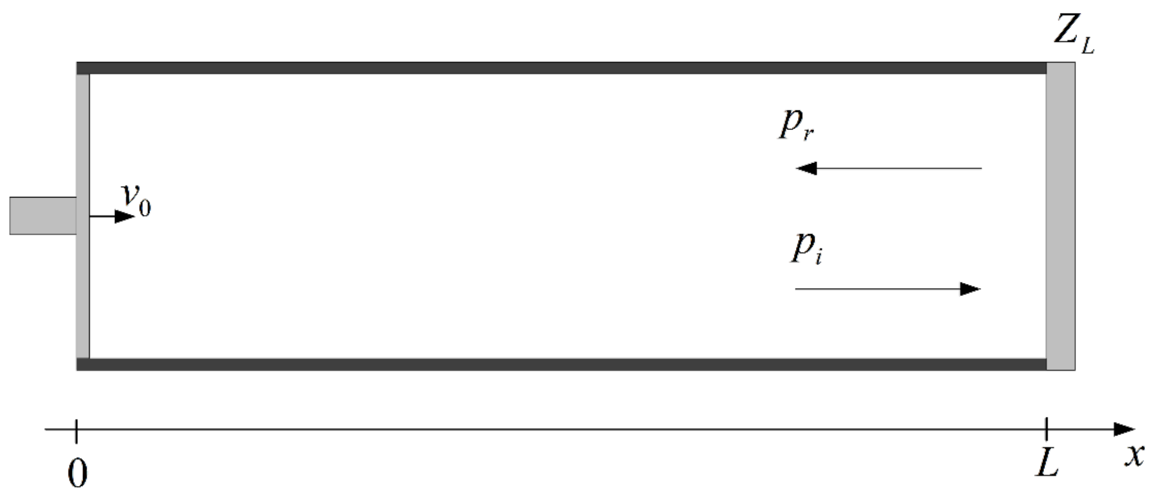


$$\frac{dp_L}{dx} + jk\beta_L p_L = 0$$

with $\beta_L = \frac{\rho c}{Z_L}$ (reduced admittance)

SOLUTION OF THE WAVE EQUATION IN WAVEGUIDES

$$p_x = \rho c \frac{e^{jk(L-x)} + R_L e^{-jk(L-x)}}{e^{jkL} - R_L e^{-jkL}} v_0$$



- Input impedance

$$Z_{x_0} = \rho c \frac{Z_L + j\rho c \tan(kL)}{\rho c + jZ_L \tan(kL)}$$

  - For a rigid wall

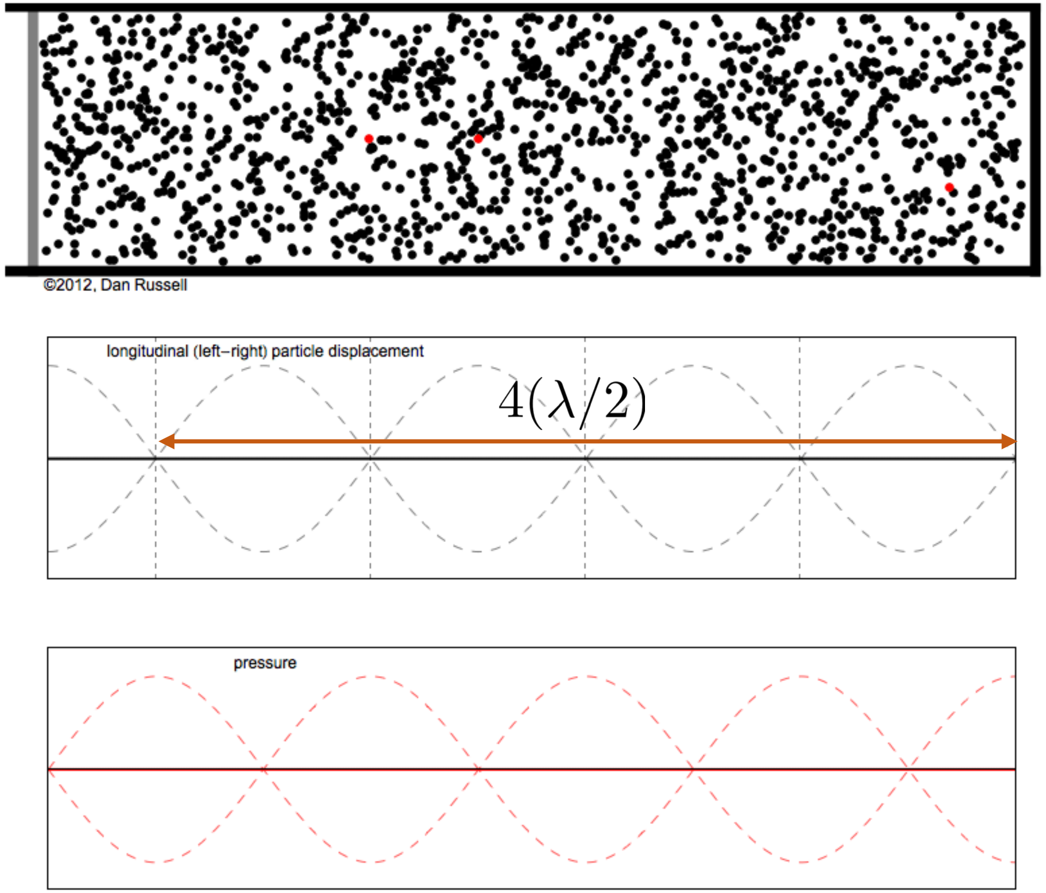
$$Z_0 = -j\rho c \cot(kL)$$

$$v_x = \frac{sin(k(L-x))}{sin(kL)} v_0 \qquad p_x = -j\rho c \frac{cos(k(L-x))}{sin(kL)} v_0$$

    - Resonances for sin(kL) = 0, i.e. f = nc/(2L)

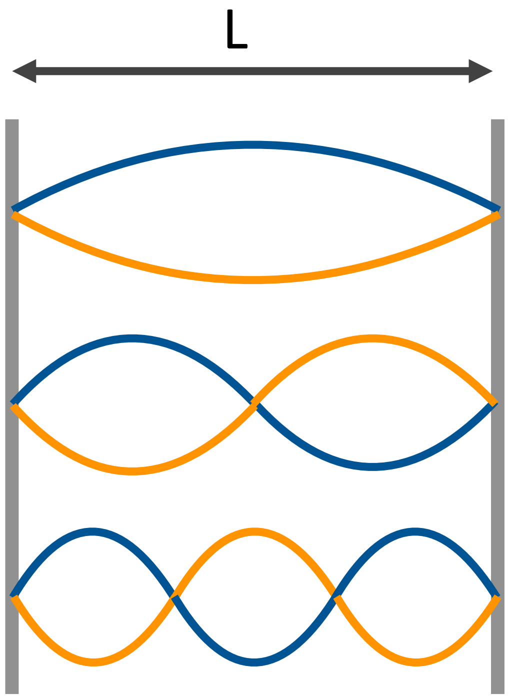
(be careful: in books $Z_0 = \rho c$)

SOLUTION OF THE WAVE EQUATION IN WAVEGUIDES



STANDING WAVES

Particle displacement



$$\lambda = 2L, f = \frac{c}{2L}$$

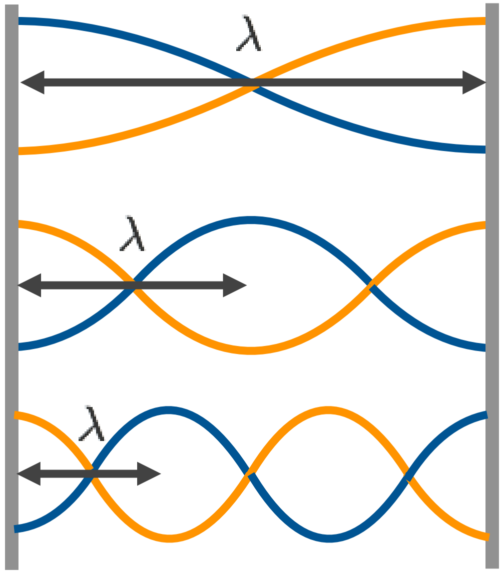
$$\lambda = L, f = \frac{c}{L}$$

$$\lambda = \frac{2L}{3}, f = \frac{3c}{2L}$$

⋮

$$\lambda = \frac{2L}{n}, f = \frac{nc}{2L}$$

Sound pressure



Mode 1

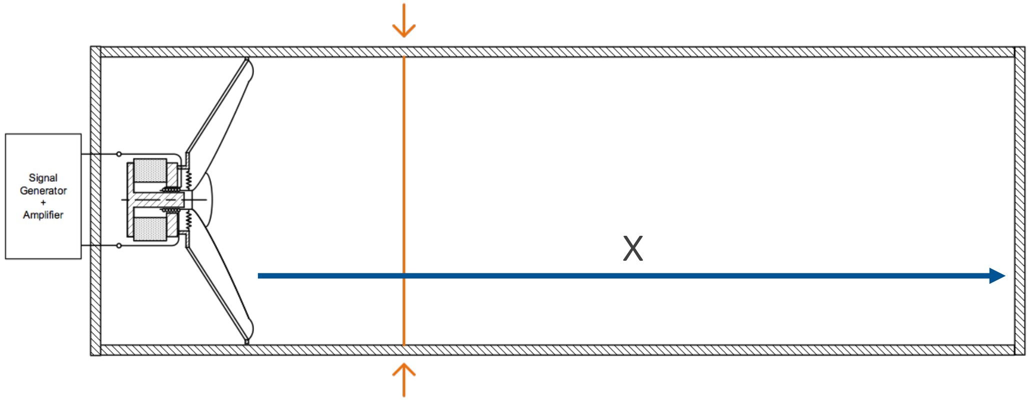
Mode 2

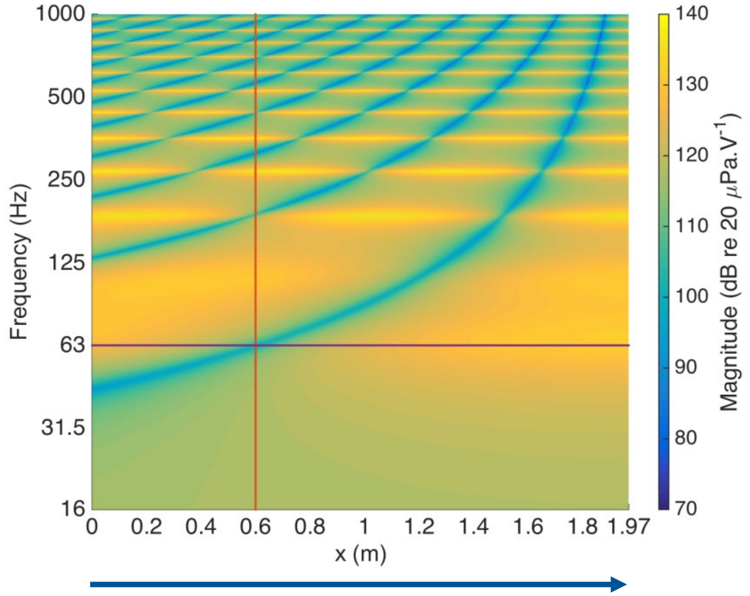
Mode 3

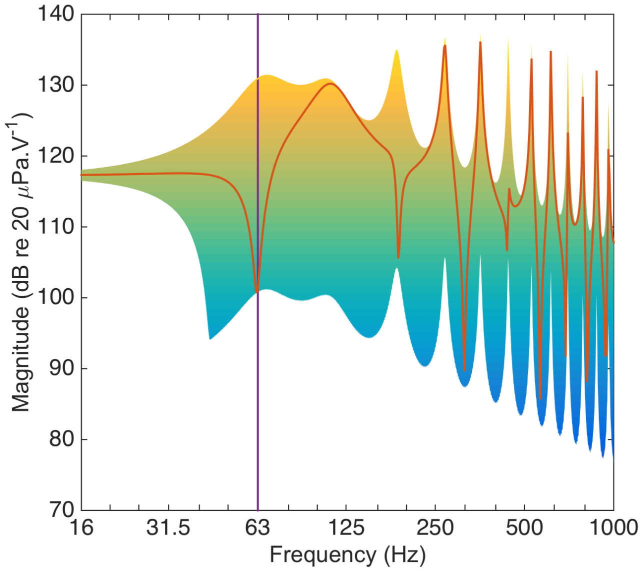
⋮

Mode n

SOUND SOURCE IN DUCT WITH RIGID TERMINATION

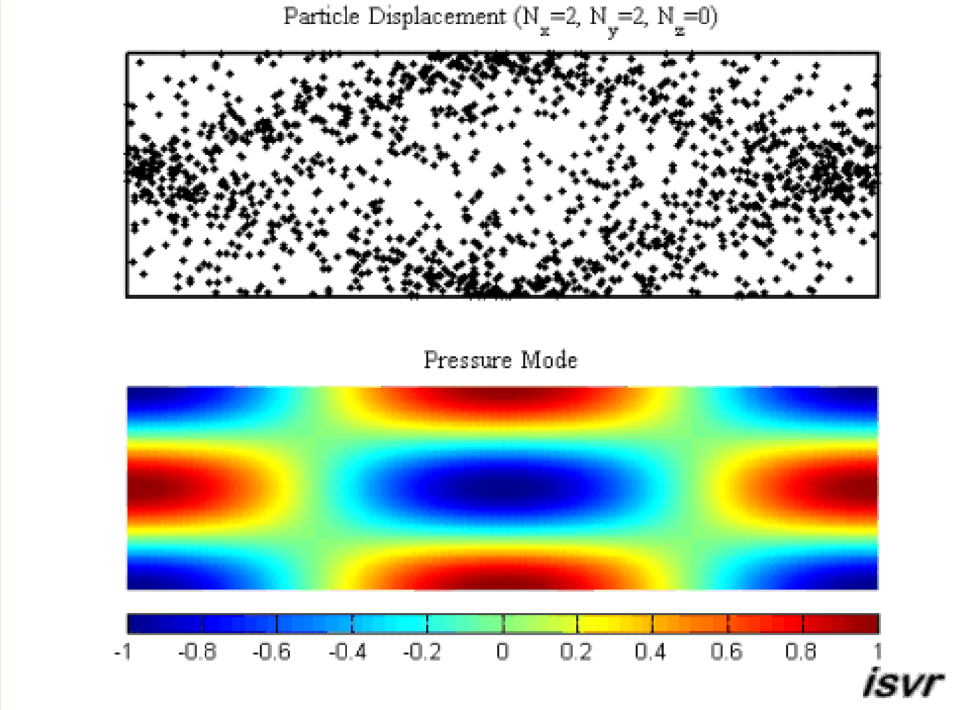




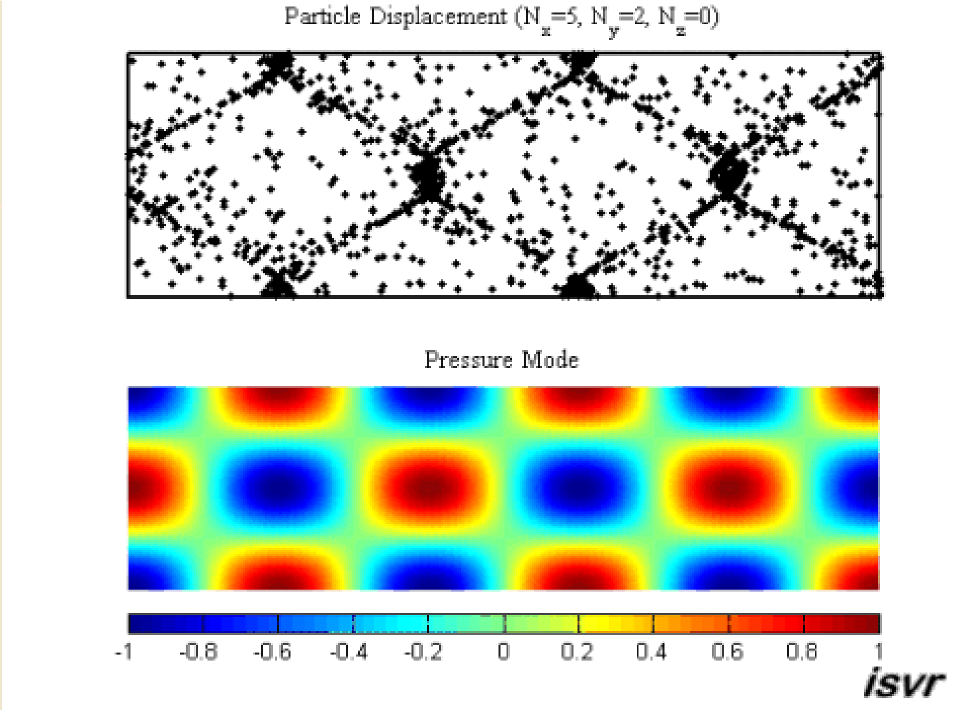


ROOM MODES

Mode (2,2,0)



Mode (5,2,0)



MODAL REPRESENTATION OF THE SOUND PRESSURE IN ROOMS

- Helmholtz equation associated to a source function

$$\Delta p(\mathbf{r}) + k^2 p(\mathbf{r}) = -j\omega\rho q(\mathbf{r})$$

- Boundary condition assuming locally reacting boundaries

$$\frac{Z}{\rho c}\nabla p(\mathbf{r}) \cdot \mathbf{n} = -jkp(\mathbf{r})$$

where $\mathbf{n}$ is a unit vector normal to the boundary surface area (outward direction)

- Source function expanded in a series of eigenfunctions

$$q(\mathbf{r}) = \sum_n Q_n \Phi_n(\mathbf{r})$$

where

$$Q_n = \frac{1}{K_n}\int\int\int_V \Phi_n(\mathbf{r})q(\mathbf{r})\mathrm{d}v$$

with K_n is a constant expressed in $\mathrm{Pa}^2\mathrm{m}^3$

MODAL REPRESENTATION OF THE SOUND PRESSURE IN ROOMS

- Eigenfunctions satisfy the orthornormal property

$$\int\int\int_V \Phi_m(\mathbf{r})\Phi_n(\mathbf{r})\mathrm{d}v = K_n\delta_{mn}$$

- Solution of the wave equation

$$p_\omega(\mathbf{r}) = \sum_n P_n\Phi_n(\mathbf{r})$$

- Every term is related to the corresponding eigenvalue $\lambda_n = -{k_n}^2$ through $\Delta\Phi_n = \lambda_n\Phi_n$
- Assuming a point source located at r_0 $\quad q(\mathbf{r}) = q_0\delta(\mathbf{r}-\mathbf{r}_0)$
- Finally

$$p_\omega(\mathbf{r}) = j\omega\rho q_0 \sum_n \frac{\Phi_n(\mathbf{r})\Phi_n(\mathbf{r}_0)}{K_n\left({k_n}^2 - k^2\right)}$$

- Eigenvalues in complex quantities, i.e. $\quad k_n = \frac{\boldsymbol{\omega_n}}{c} = \frac{1}{c}(\omega_n + j\delta_n)$

$$p_\omega(\mathbf{r}) = \rho c^2\omega q_0 \sum_n \frac{\Phi_n(\mathbf{r})\Phi_n(\mathbf{r}_0)}{K_n\left[2\delta_n\omega_n + j\left(\omega^2 - {\omega_n}^2\right)\right]}$$

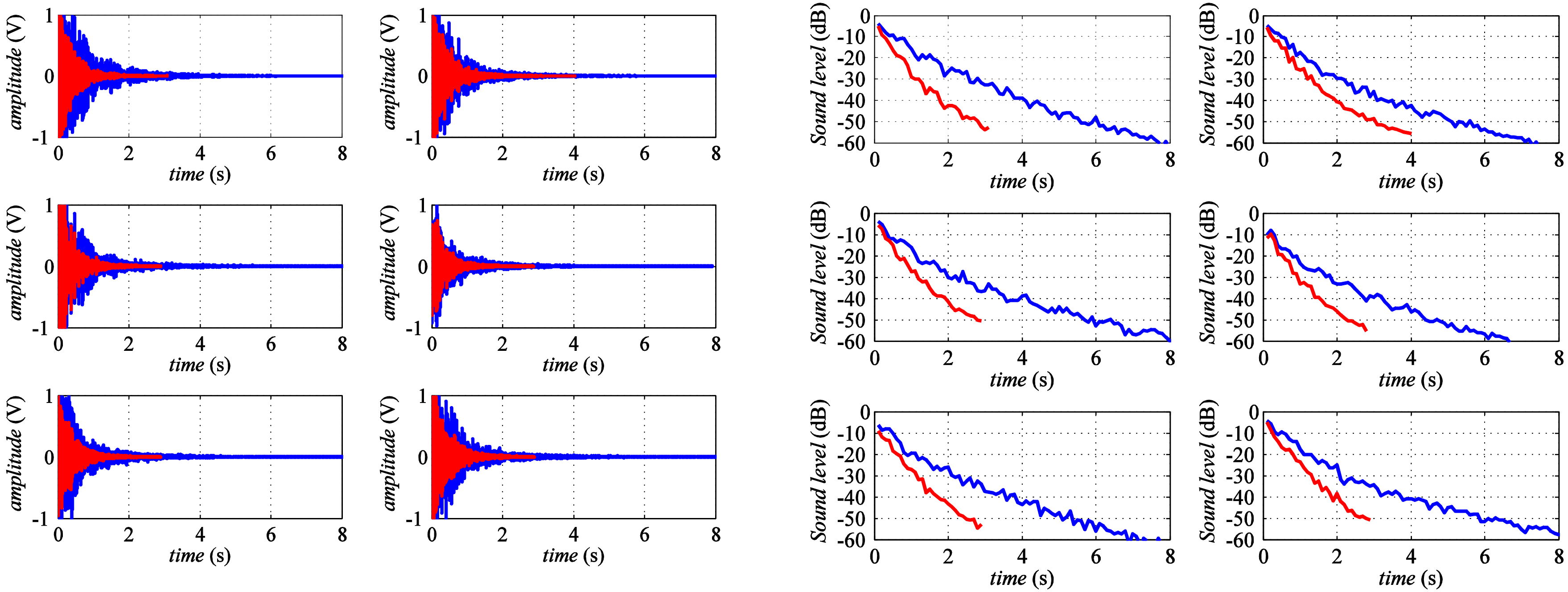
MODAL DECAY

$$\boldsymbol{\omega_n} = ck_n = \omega_n + j\delta_n = \omega_n + j\tau_n^{-1}$$

$$Re[\exp(j\boldsymbol{\omega_n}t)] = \cos(\omega_n t)\exp\left(-t/\tau_n\right)$$

- δ_n is the exponential decay of mode n
- τ_n is the relaxation time of mode n

→ exponential decay of mode n with the rate depends on δ_n



SOLUTION OF THE WAVE EQUATION IN RECTANGULAR ROOMS

- In the steady-state, for Cartesian cordinates, and assuming rigid walls

$$(\nabla^2 + k^2)p(x,y,z) = 0 \qquad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \qquad k = \omega/c$$

- Solution expressed as $p(x,y,z) = p_0 \Phi_x(x)\Phi_y(y)\Phi_z(z)$

- For instance, $\Phi_x$must satisfy $\dfrac{d^2\Phi_x}{dx^2} + {k_x}^2\Phi_x$

- Boundary condition $\dfrac{d\Phi_x(0)}{dx} = 0$ and $\dfrac{d\Phi_x(l_x)}{dx} = 0$

  → $k_x = n_x\pi/l_x$

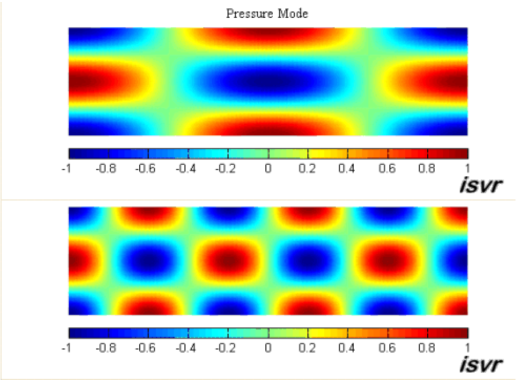
- ${k_x}^2 + {k_y}^2 + {k_z}^2 = k^2$

NORMAL MODES IN RECTANGULAR ROOMS WITH RIGID BOUNDARIES

- Eigenfunctions approximated by the mode shape functions

$$\Phi_{n_x n_y n_z}(x, y, z) = \cos\left(\frac{n_x \pi x}{l_x}\right)\cos\left(\frac{n_y \pi y}{l_y}\right)\cos\left(\frac{n_z \pi z}{l_z}\right)$$

where $(n_x,\, n_y,\, n_z) \in \mathbb{N}^3$ are non-simultaneously equal to zero



(2,2,0)

(5,2,0)

- Eigenfrequencies

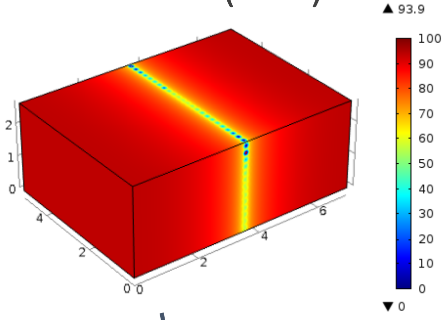
$$\omega_{n_x n_y n_z} = \pi c \sqrt{\left(\frac{n_x}{l_x}\right)^2 + \left(\frac{n_y}{l_y}\right)^2 + \left(\frac{n_z}{l_z}\right)^2}$$

- $K_n = V/(\epsilon_{n_x}\epsilon_{n_y}\epsilon_{n_z})$ where $\epsilon_{n_s} = 1$ if $n_s = 0$ and $\epsilon_{n_s} = 2$ if $n_s > 0$

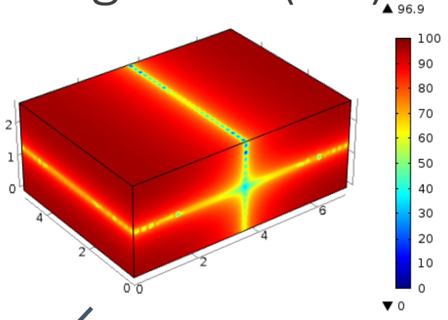
- Only for LIGHTLY DAMPED enclosures

DISTRIBUTIONS IN SPACE AND FREQUENCY OF SOUND PRESSURE

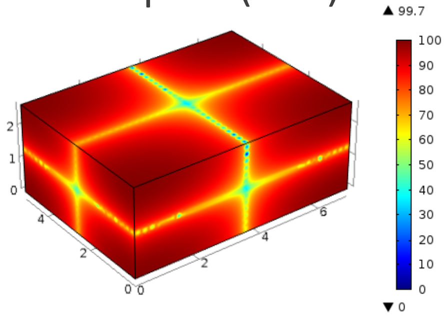
Axial (1D) mode

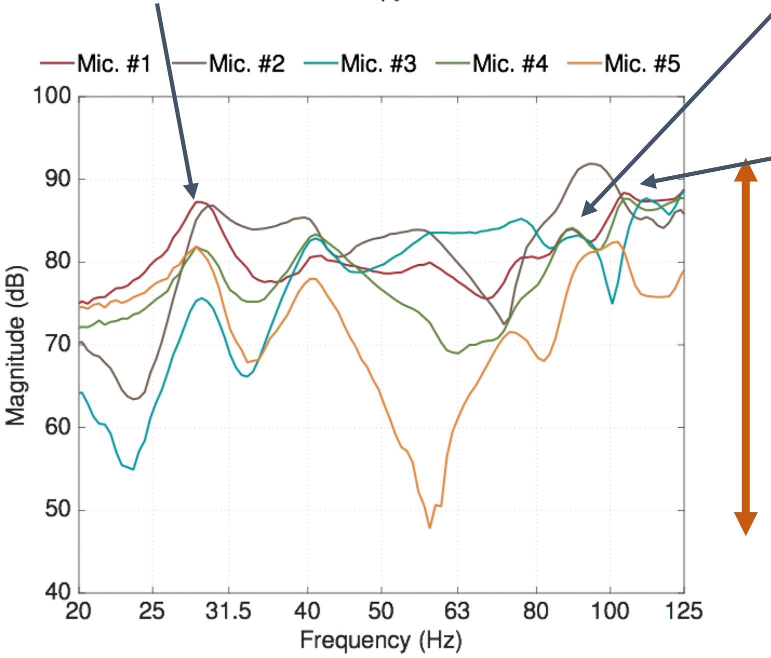


Tangential (2D) mode

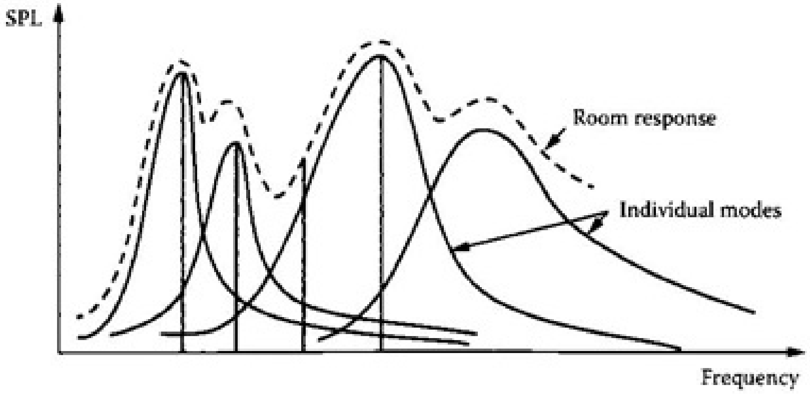


Oblique (3D) mode





$$p_\omega(\mathbf{r}) = \rho c^2 \omega q_0 \sum_n \frac{\Phi_n(\mathbf{r})\Phi_n(\mathbf{r}_0)}{K_n\left[2\delta_n\omega_n + j\left(\omega^2 - {\omega_n}^2\right)\right]}$$



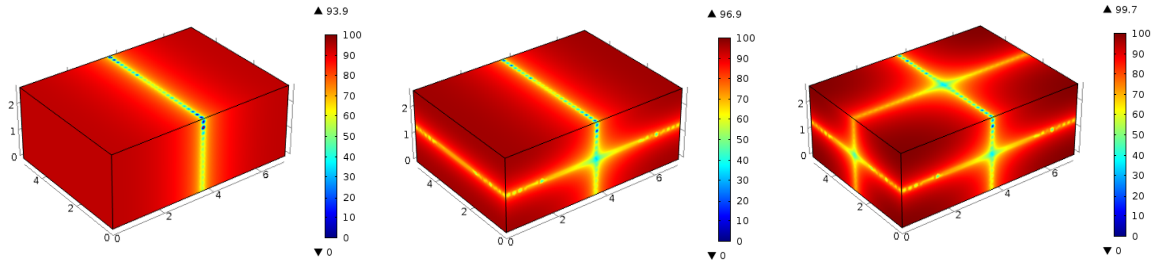
ROOM MODES CALCULATION: EXAMPLE

Room modes calculation for a shoe-box room with:

Length = 8m , Width = 6m , Height = 4.5m

$$f_{\{n_x,n_y,n_z\}} = \frac{c}{2}\sqrt{\left(\frac{n_x}{L_x}\right)^2 + \left(\frac{n_y}{L_y}\right)^2 + \left(\frac{n_z}{L_z}\right)^2}$$

$$L_x = 8\,m\,;\ L_y = 6\,m\ ;L_z = 4.5\,m$$



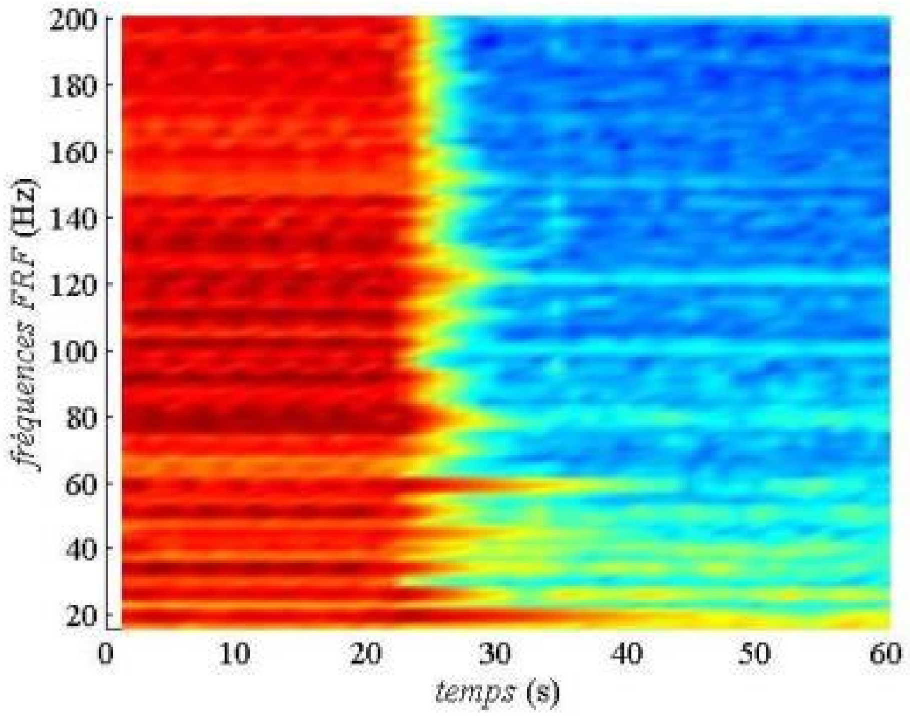
Frequency (Hz)	Nx	Ny	Nz	Type
21.44	1	0	0	Axial
28.58	0	1	0	Axial
35.73	1	1	0	Tangential
38.11	0	0	1	Axial
42.88	2	0	0	Axial
43.73	1	0	1	Tangential
47.64	0	1	1	Tangential
51.53	2	1	0	Tangential
52.24	1	1	1	Oblique
57.17	0	2	0	Axial
57.36	2	0	1	Tangential
61.05	1	2	0	Tangential
64.09	2	1	1	Oblique
64.31	3	0	0	Axial
68.71	0	2	1	Tangential
70.38	3	1	0	Tangential
71.46	2	2	0	Tangential
71.97	1	2	1	Oblique
74.76	3	0	1	Tangential
76.22	0	0	2	Axial
79.18	1	0	2	Tangential
80.03	3	1	1	Oblique

DISTRIBUTIONS IN SPACE OF SOUND FIELD: EXAMPLE

- Reverberation chamber (non shoe-box)
- Frequency increasing from 18Hz-60Hz
- Excitation from one corner of the room

TEMPORAL BEHAVIOUR

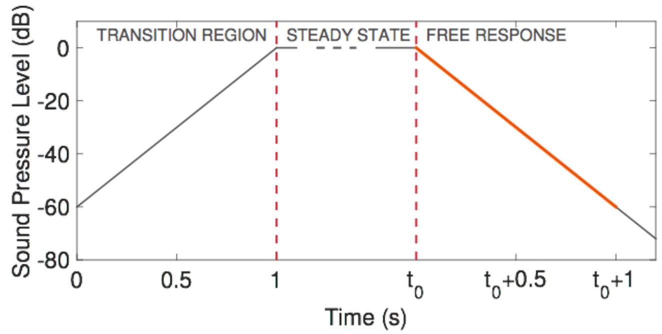
- Too long sustain, lack of precision, masking effect at higher frequencies

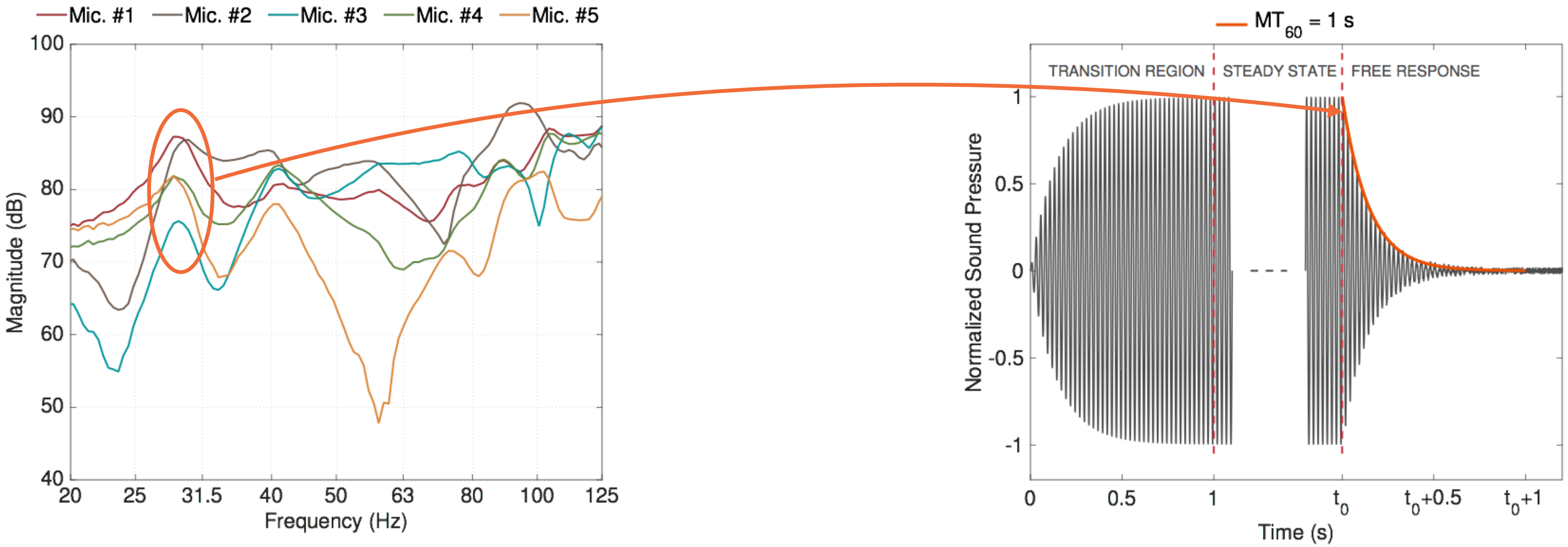


MODAL DECAY TIME

- Time interval corresponding to a decrease of 60 dB of the sound pressure level during the free response

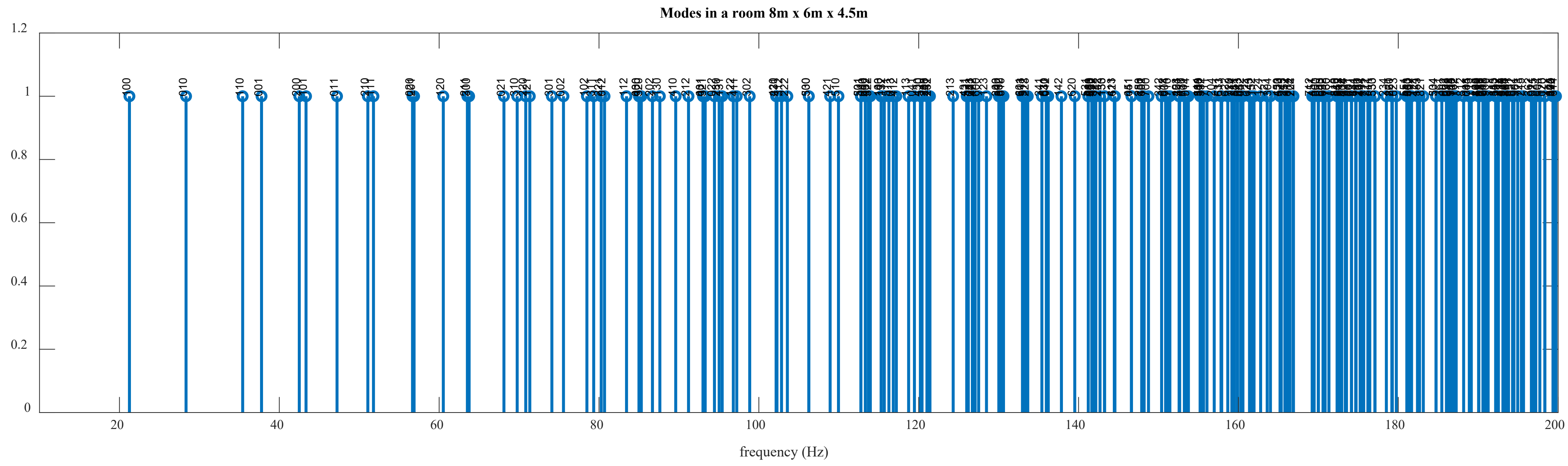
$$MT_{60_n} = \frac{3\ln(10)}{\delta_n} = \frac{3\ln(10)\,Q_n}{\pi f_n}$$





Conclusions

- Room modes occur due to the occurrence of standing waves between rigid boundaries
- They room mode density increases with frequency, up to a limit where it could be considered a «continuum» (see «Schroeder frequency»)



Conclusions

- Room modes occur due to the occurrence of standing waves between rigid boundaries
- They room mode density increases with frequency, up to a limit where it could be considered a «continuum» (see «Schroeder frequency»)
- The room modes have detrimental effects to music/sound reproduction
  - Affect the spatial rendering of sound
  - Affect the frequency linearity of sound diffusion
  - Affect the time response («sustain» «ringing», etc.)
- Unfortunately not much hardware available to solve this issue :
  - bass traps with bulk embodiment and narrow frequency performance,
  - signal processing affecting the phase response of the rendering)

➔ Electroacoustic absorbers developed in the lab are a recent breakthrough in the field (see last lecture of the semester)