Integrated Neurotechnologies Laboratory

EPFL

Advanced analog integrated circuit design (EE-523), Lecture 5

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Recap: Switched-Capacitor Circuits

- **Continuous-time circuits**: Input and output are continuous signals
- **Discrete-time circuits:** Process input at periodic instants of time

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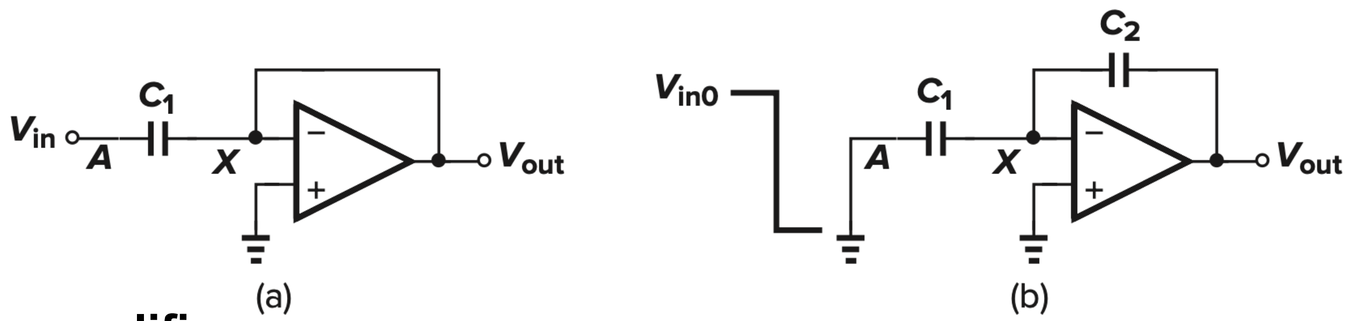
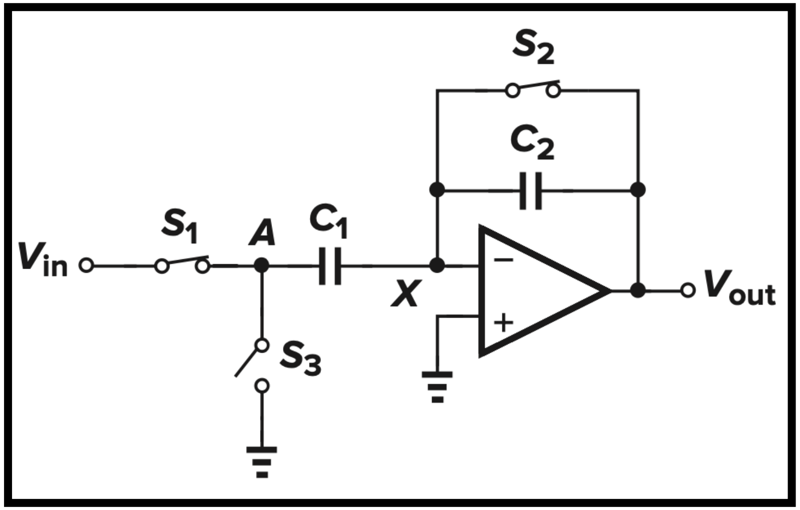
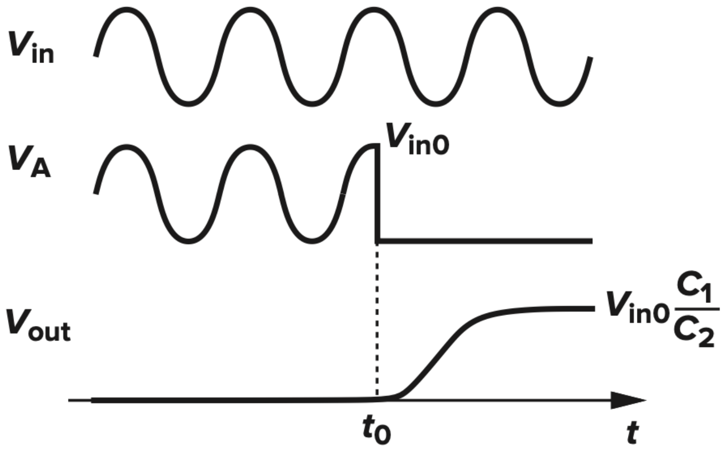
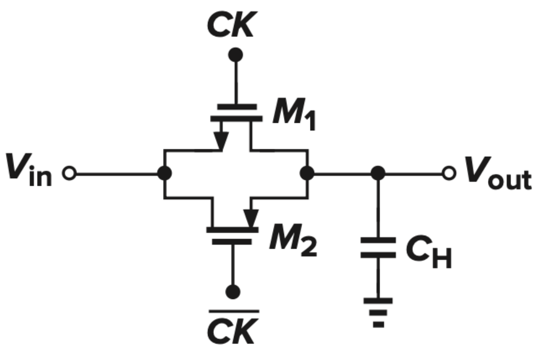


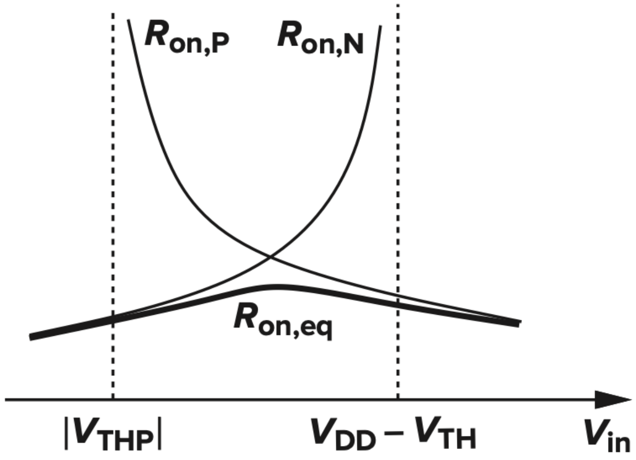
Switched-capacitor amplifier





Recap: SC Circuits – Speed and Precision





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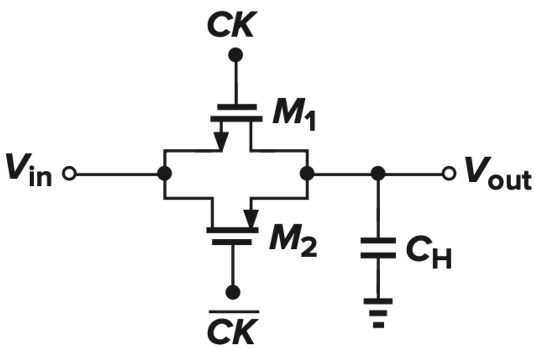
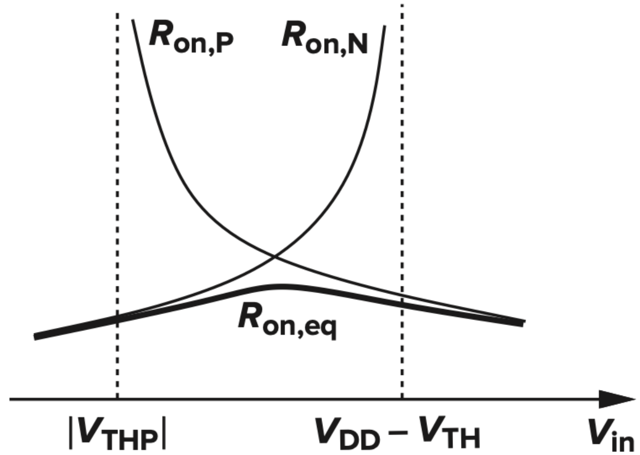
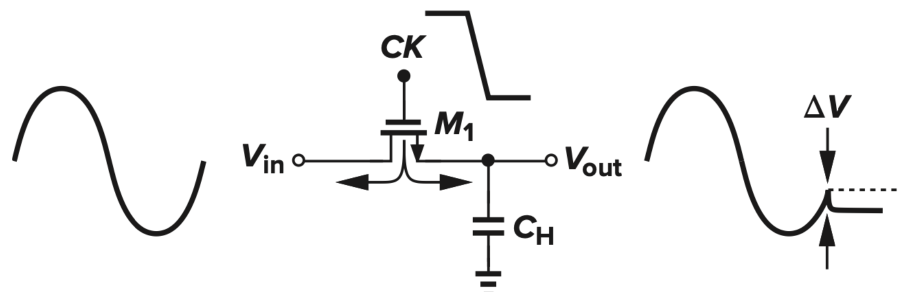




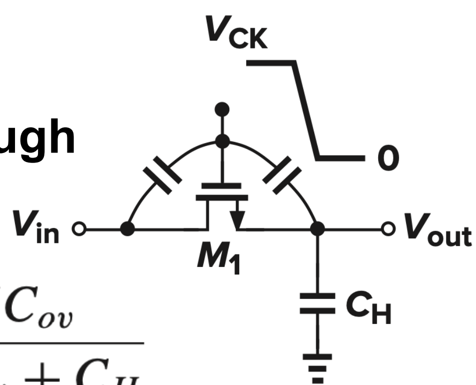
- **Three mechanisms introduce error at the instant the MOS switch turns off:**
  - **Channel Charge Injection**
  - **Clock Feedthrough**
  - ***kT*/*C* Noise**

Channel Charge Injection



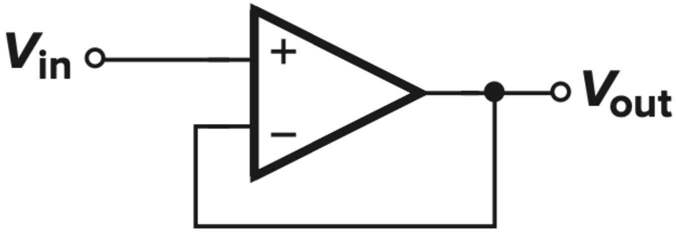
$$\Delta V = \frac{WLC_{ox}(V_{DD} - V_{in} - V_{TH})}{2C_H}$$

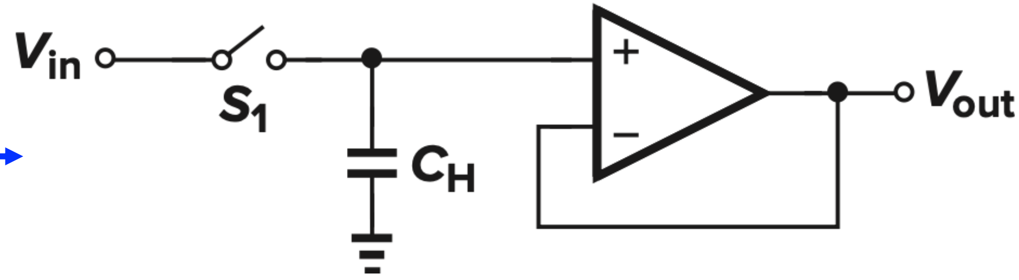
Clock Feedthrough



$$\Delta V = V_{CK}\frac{WC_{ov}}{WC_{ov} + C_H}$$

Switched-Capacitor Amplifiers





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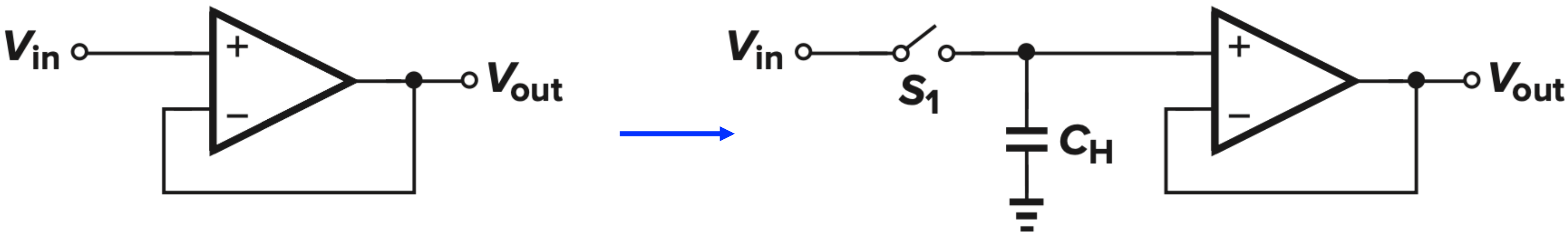
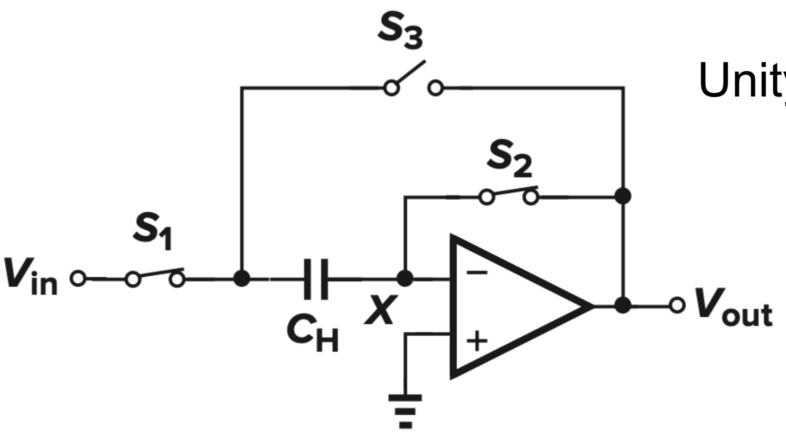
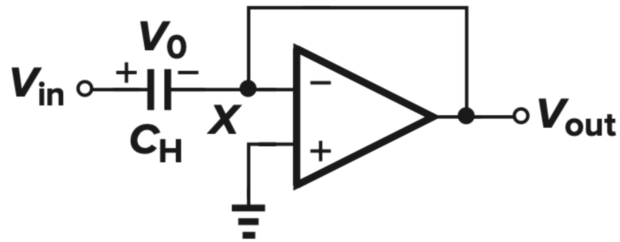
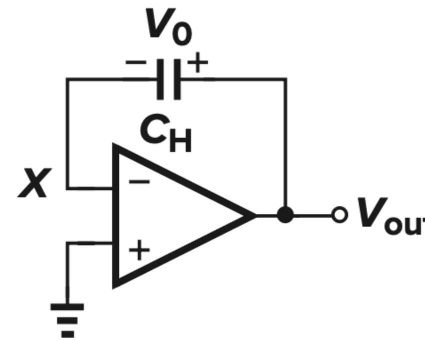




Unity-gain sampler





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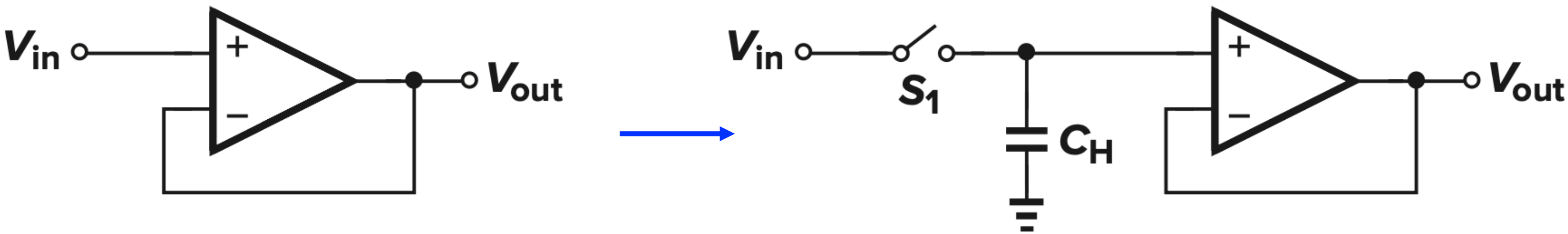

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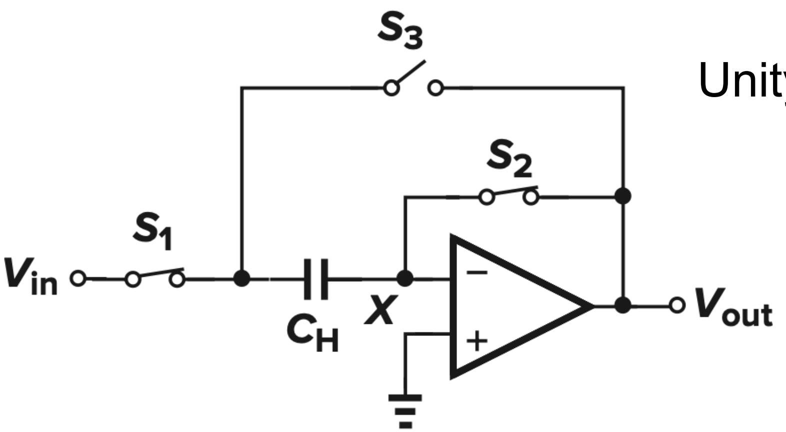
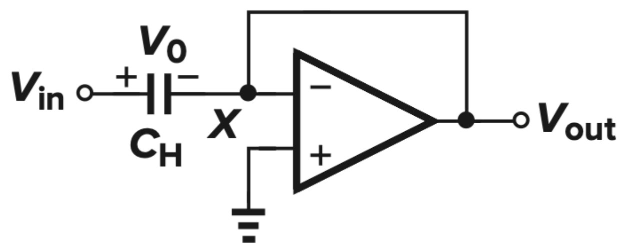
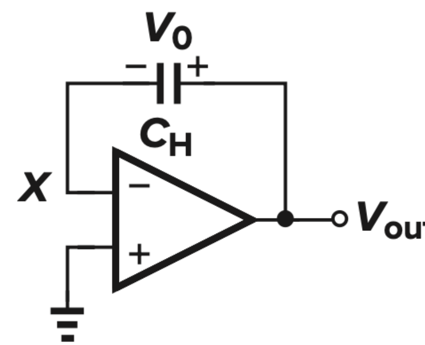
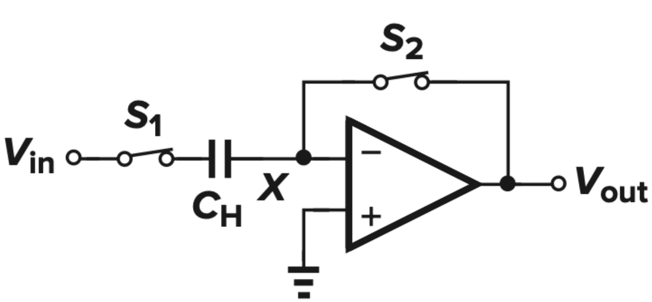
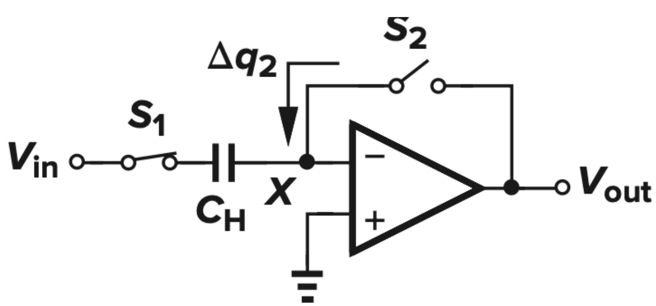
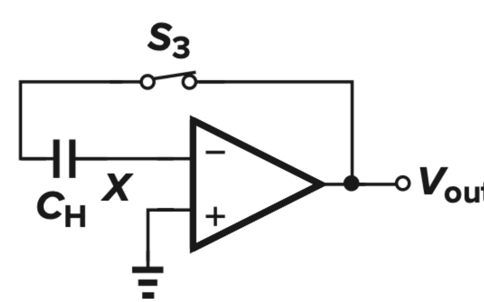






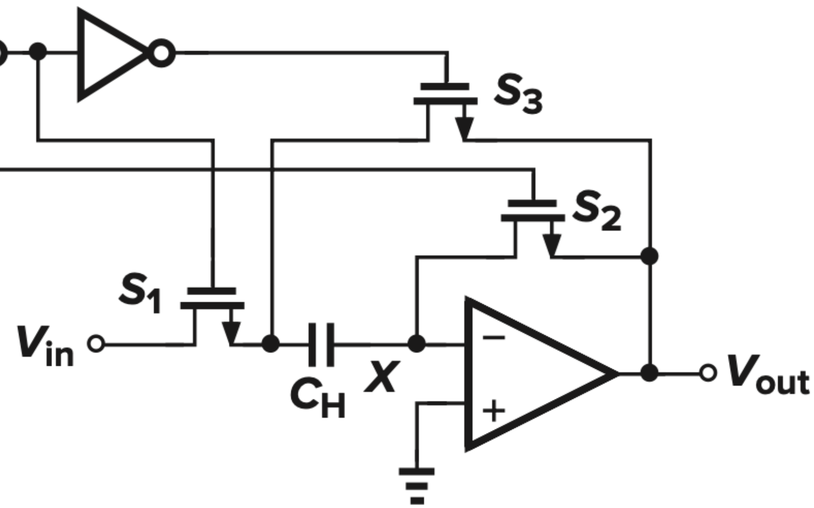


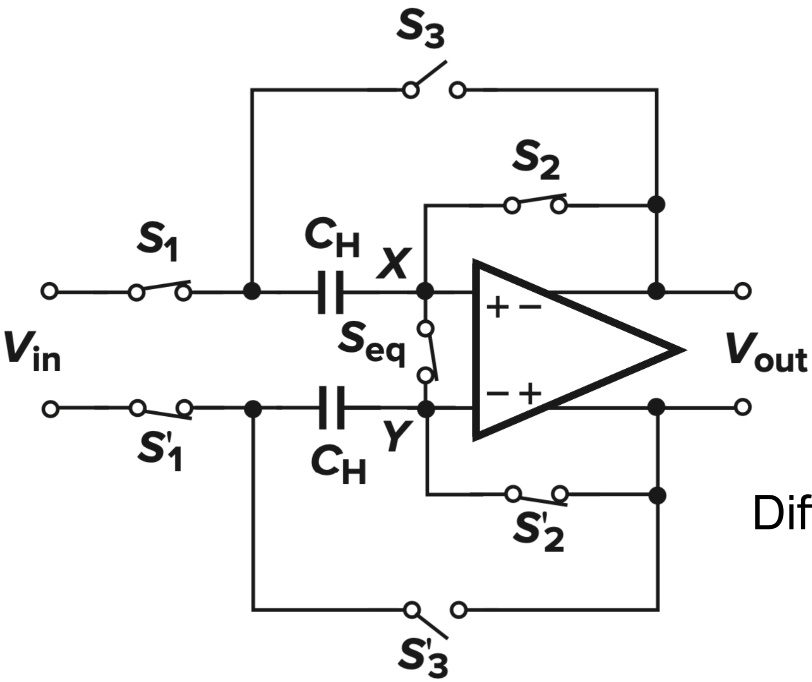




Unity-gain sampler

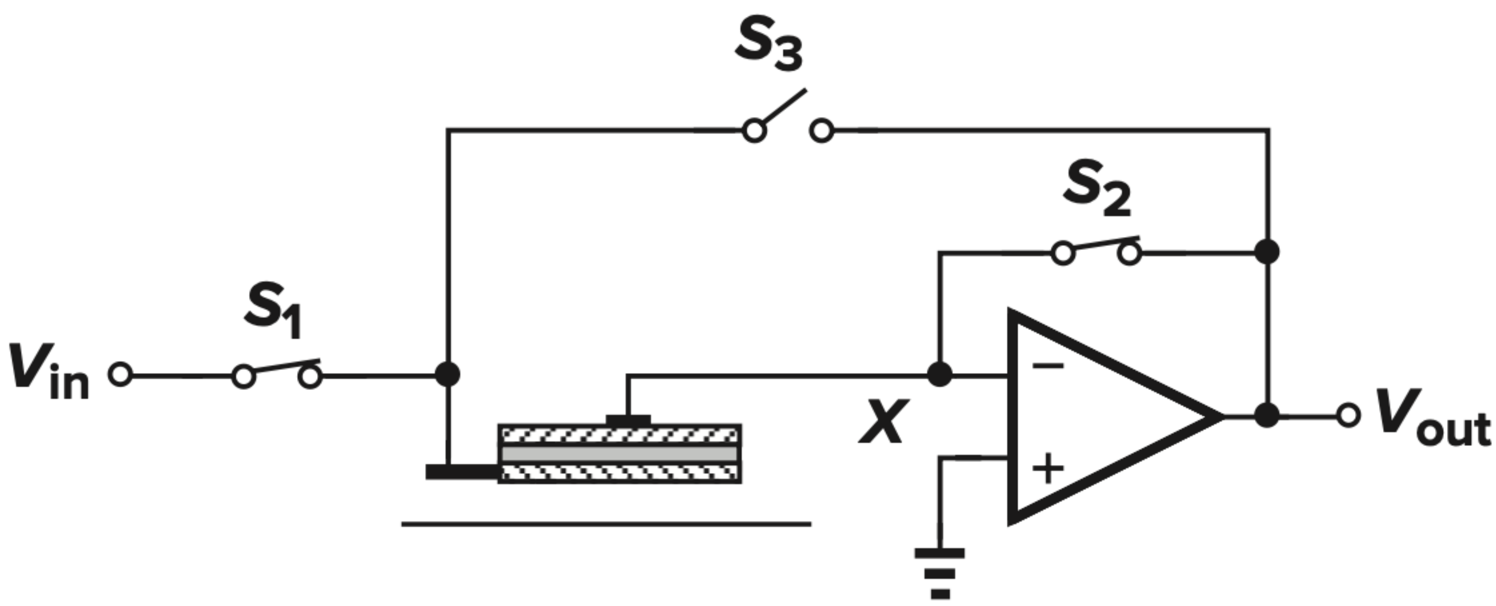
Generation of proper clock edges for unity-gain sampler

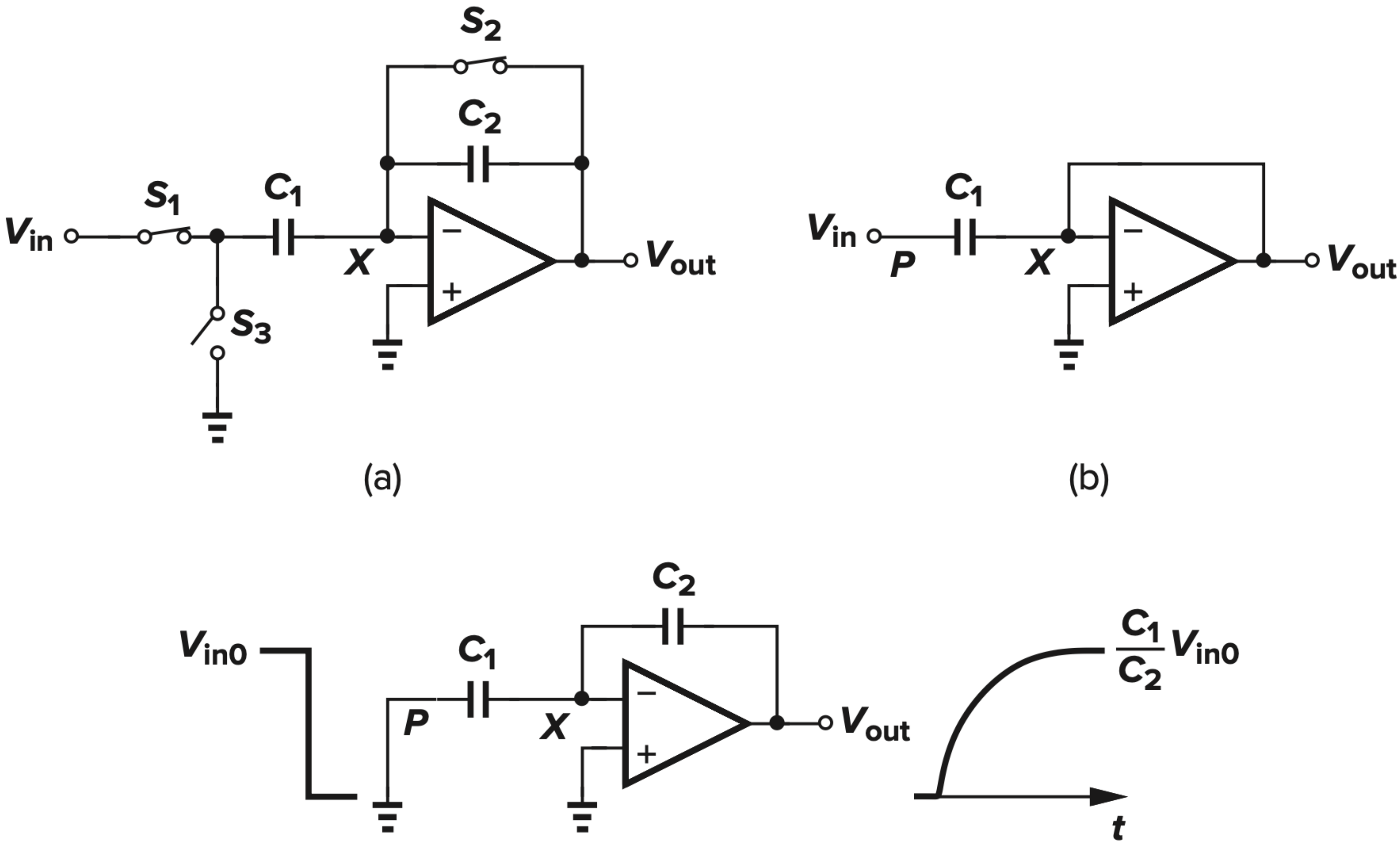




Differential realization of unity-gain sampler

Bottom-plate sampling

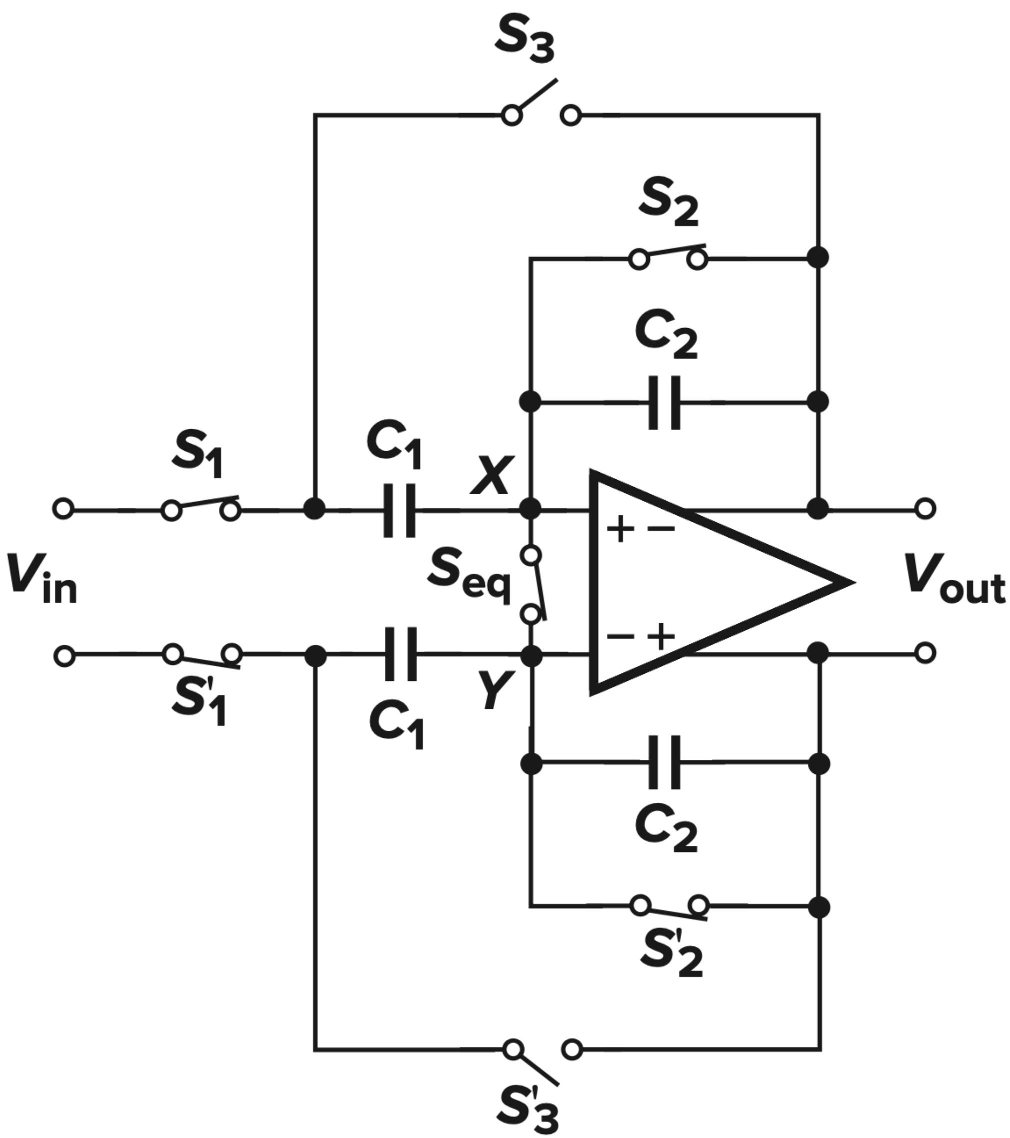




S2
C2
S1
C1
Vin
Vout
X
S3
C1
Vin
P
X
Vout
C2
Vin0
C1
P
X
Vout
C1
C2
Vin0
t

(a) (b) (c)

Differential realization of noninverting amplifier



Nonlinearity

- Two types of nonidealities: **nonlinearity and mismatch**

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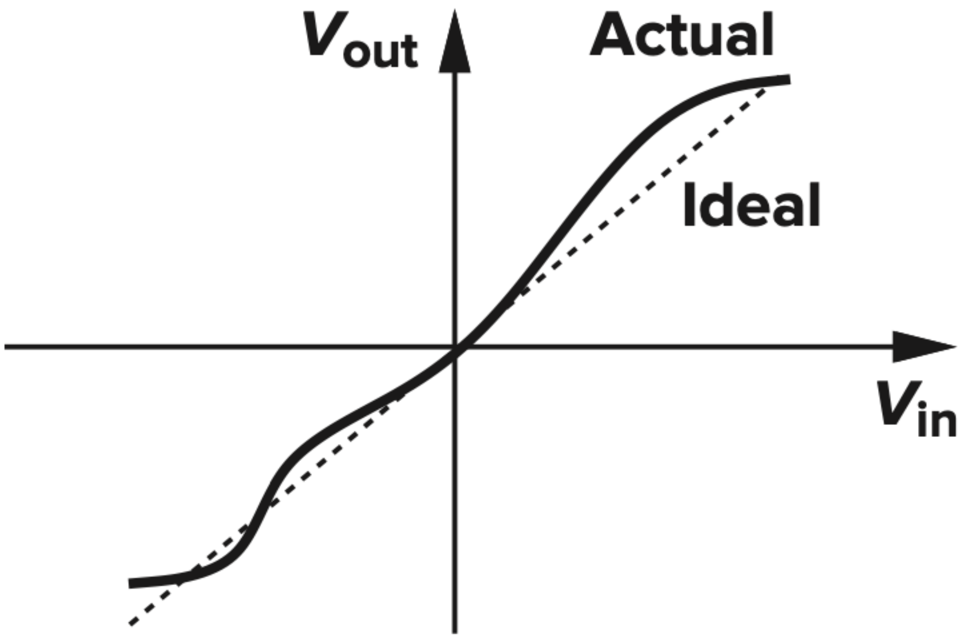
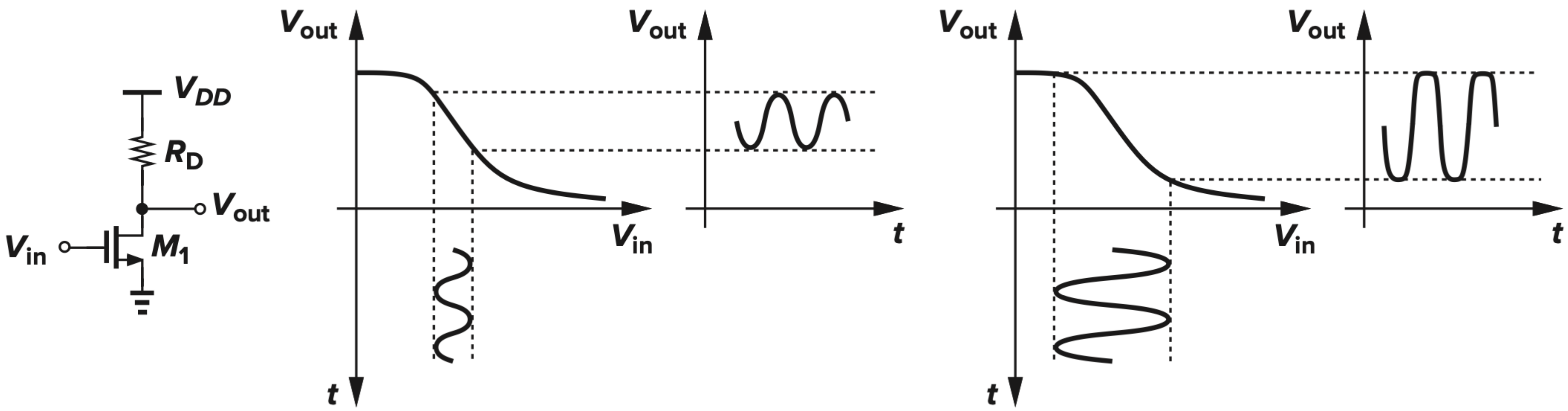
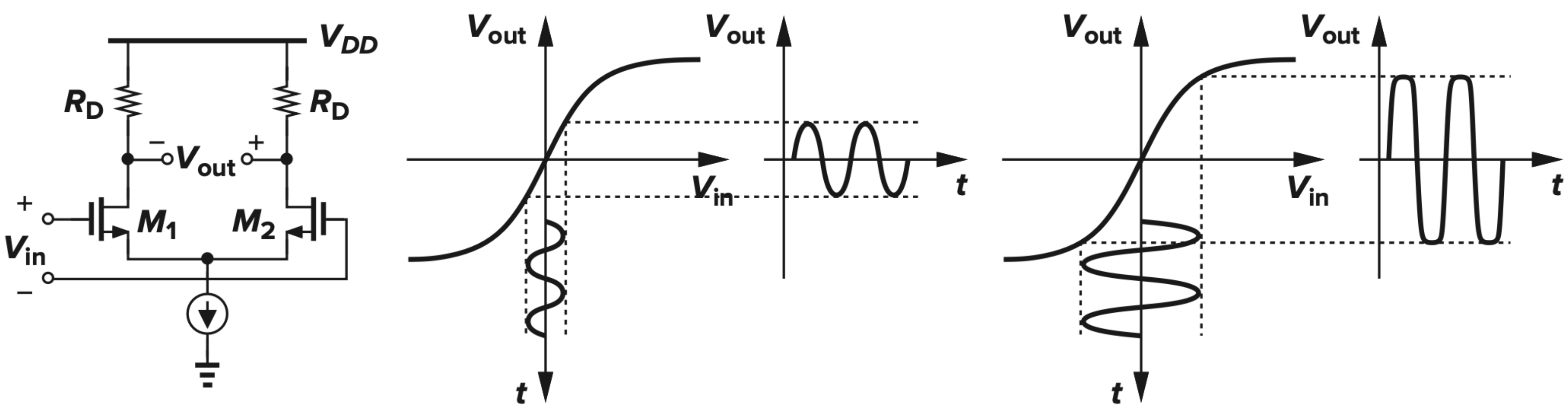


Nonlinearity Examples

Distortion in (a) a common-source stage and (b) a differential pair



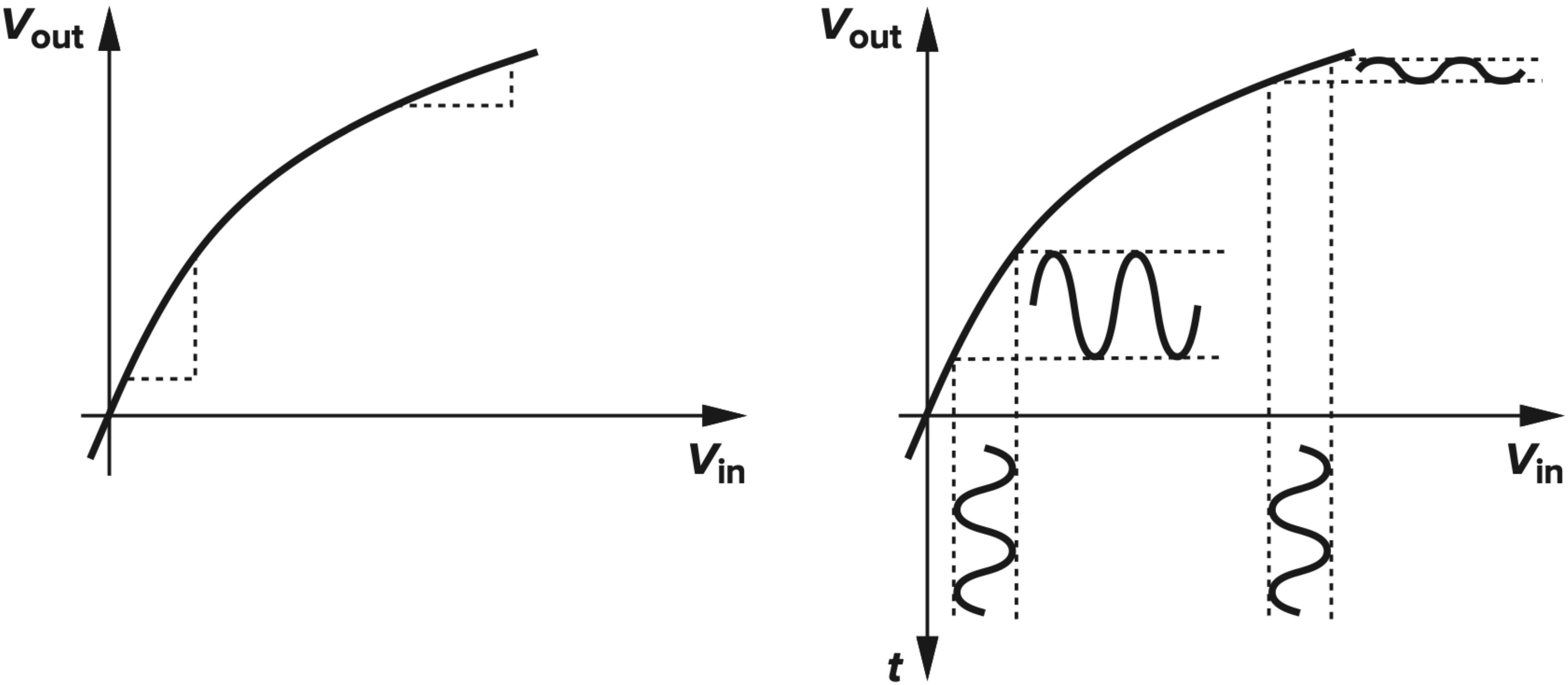
(a)



(b)

Nonlinearity Behavior

Variation of small-signal gain in a nonlinear amplifier



(a) (b)

Nonlinearity Definition

$$y(t) = \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t) + \cdots$$

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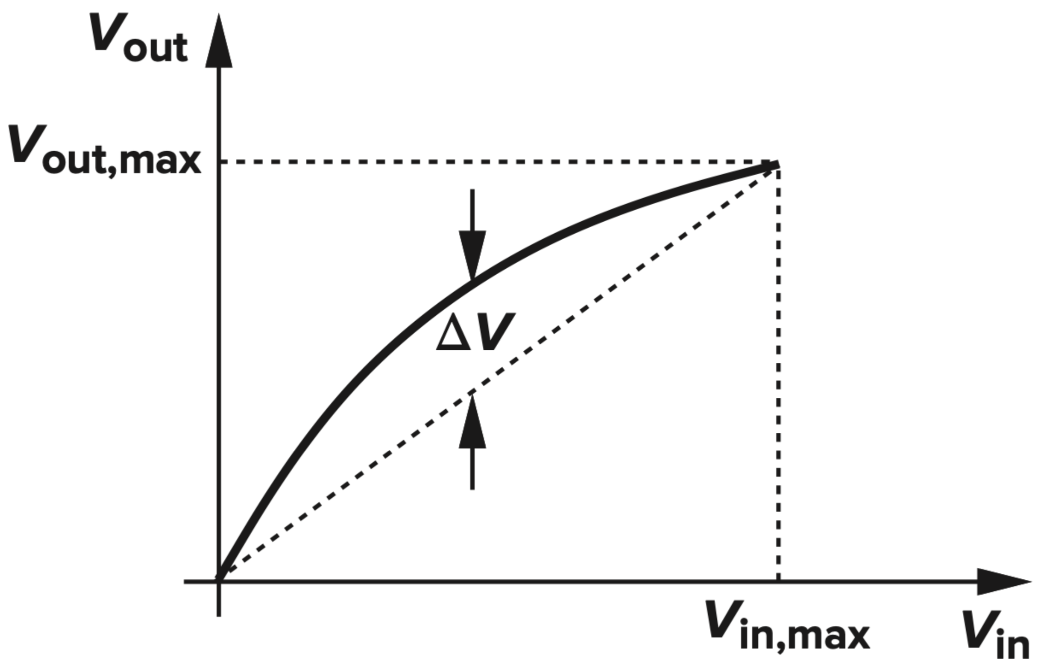

$$1\% \text{ nonlinearity } (\Delta V / V_{out,max} = 0.01)$$

Nonlinearity Definition

$$
\begin{aligned}
y(t) &= \alpha_1 A \cos \omega t + \alpha_2 A^2 \cos^2 \omega t + \alpha_3 \cos^3 \omega t + \cdots \\
&= \alpha_1 A \cos \omega t + \frac{\alpha_2 A^2}{2}[1 + \cos(2\omega t)] + \frac{\alpha_3 A^3}{4}[3 \cos \omega t + \cos(3\omega t)] + \cdots
\end{aligned}
$$

Total Harmonic Distortion

$$
\mathrm{THD} = \frac{(\alpha_2 A^2/2)^2 + (\alpha_3 A^3/4)^2}{(\alpha_1 A + 3\alpha_3 A^3/4)^2}
$$

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Total Harmonic Distortion

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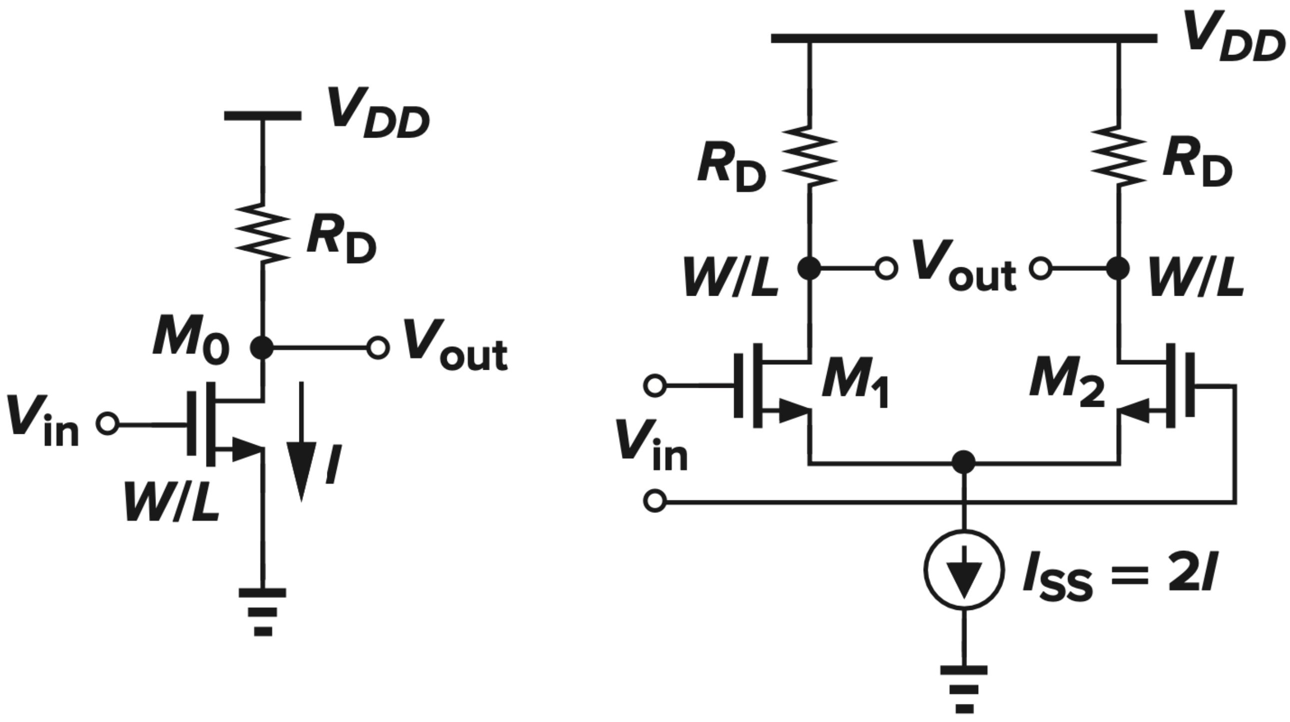
Nonlinearity of Differential Circuits:

$$y(t) = \alpha_1 x(t) + \alpha_3 x^3(t) + \alpha_5 x^5(t) + \cdots$$

Nonlinearity: Single-ended and differential amplifiers



$$|A_v| \approx g_m R_D$$

$$= \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH}) R_D$$

Nonlinearity: Single-ended amplifiers

input signal

$$I_{D0} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH} + V_m\ cos\omega t)^2$$

$$= \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})^2 + \mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})V_m \cos\omega t$$

$$+ \frac{1}{2}\mu_n C_{ox}\frac{W}{L}V_m^2 \cos^2\omega t$$

$$= I + \mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})V_m \cos\omega t + \frac{1}{4}\mu_n C_{ox}\frac{W}{L}V_m^2[1 + \cos(2\omega t)]$$

$$\boxed{\frac{A_{HD2}}{A_F} = \frac{V_m}{4(V_{GS} - V_{TH})}}$$

Nonlinearity: Differential amplifiers

$$I_{D1} - I_{D2} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}V_{in}\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - V_{in}^2}$$

$$= \frac{1}{2}\mu_n C_{ox}\frac{W}{L}V_{in}\sqrt{4(V_{GS} - V_{TH})^2 - V_{in}^2}$$

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$$= \frac{1}{2}\mu_n C_{ox}\frac{W}{L}V_{in}\sqrt{4(V_{GS} - V_{TH})^2 - V_{in}^2}$$

$$|V_{in}| \ll V_{GS} - V_{TH}$$

$$I_{D1} - I_{D2} = \mu_n C_{ox}\frac{W}{L}V_{in}(V_{GS} - V_{TH})\sqrt{1 - \frac{V_{in}^2}{4(V_{GS} - V_{TH})^2}}$$

$$\approx \mu_n C_{ox}\frac{W}{L}V_{in}(V_{GS} - V_{TH})\left[1 - \frac{V_{in}^2}{8(V_{GS} - V_{TH})^2}\right]$$

$$= \mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})\left[V_m \cos\omega t - \frac{V_m^3 \cos^3\omega t}{8(V_{GS} - V_{TH})^2}\right]$$

Nonlinearity: Differential amplifiers

$$I_{D1} - I_{D2} = \mu_n C_{ox} \frac{W}{L}(V_{GS} - V_{TH}) \left[V_m \cos \omega t - \frac{V_m^3 \cos^3 \omega t}{8(V_{GS} - V_{TH})^2} \right]$$

$$\cos^3 \omega t = [3 \cos \omega t + \cos(3\omega t)]/4$$

$$I_{D1} - I_{D2} = g_m \left[V_m - \frac{3V_m^3}{32(V_{GS} - V_{TH})^2} \right] \cos \omega t - g_m \frac{V_m^3 \cos(3\omega t)}{32(V_{GS} - V_{TH})^2}$$

$$\frac{A_{HD3}}{A_F} \approx \frac{V_m^2}{32(V_{GS} - V_{TH})^2}$$

Single-ended (for comparison)

$$\frac{A_{HD2}}{A_F} = \frac{V_m}{4(V_{GS} - V_{TH})}$$

Capacitor Nonlinearity

- In switched-capacitor circuits, the voltage dependence of capacitors may introduce substantial distortion
- While for a linear capacitor we have Q = C.V, for **a voltage-dependent capacitor dQ = C.dV**. Thus, the total charge on a capacitor sustaining a voltage V1 is:

$$Q(V_1) = \int_0^{V_1} C\, dV$$

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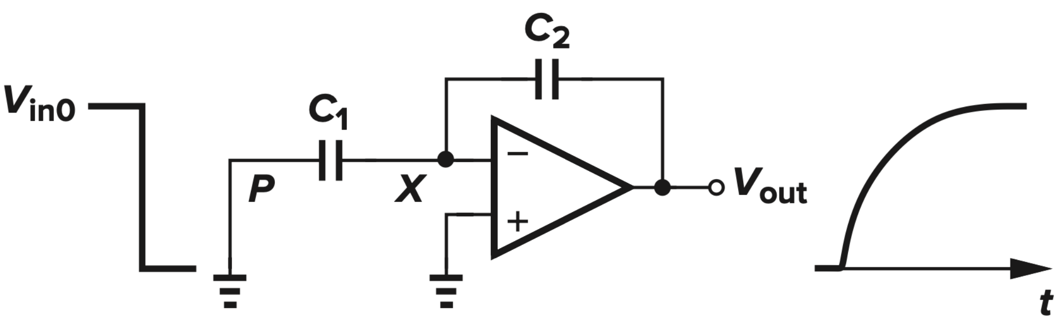
$$\boxed{C = C_0(1 + \alpha_1 V + \alpha_2 V^2 + \cdots)}$$



$$Q_1 = \int_0^{V_{in0}} C_1\,dV$$

$$= \int_0^{V_{in0}} MC_0(1 + \alpha_1 V)\,dV$$

$$= MC_0 V_{in0} + MC_0 \frac{\alpha_1}{2} V_{in0}^2$$

$$Q_2 = \int_0^{V_{out}} C_2\,dV$$

$$= C_0 V_{out} + C_0 \frac{\alpha_1}{2} V_{out}^2$$

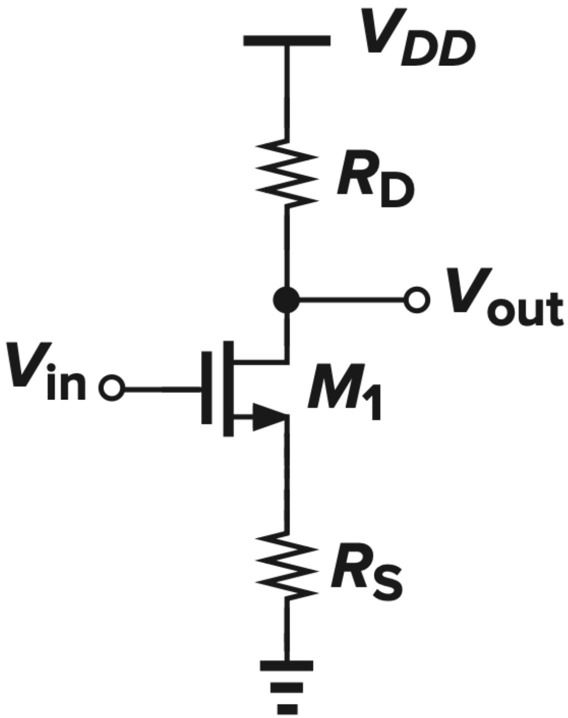
Capacitor Nonlinearity

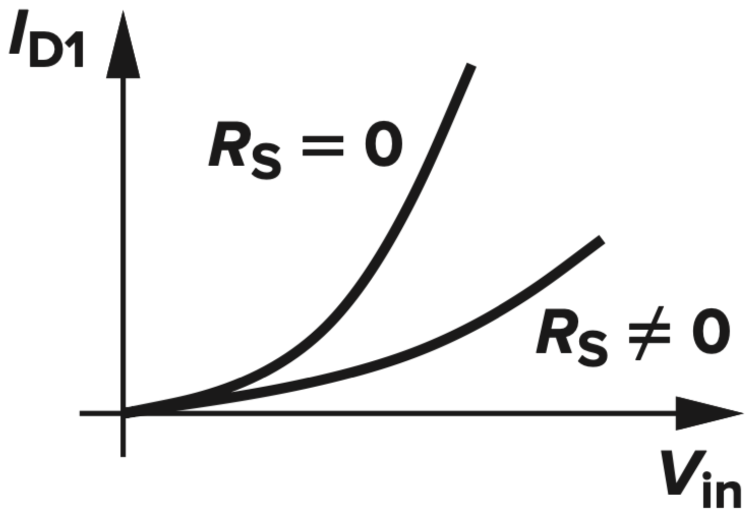
$$V_{out} = \frac{1}{\alpha_1}\left(-1 + \sqrt{1 + M\alpha_1^2 V_{in0}^2 + 2M\alpha_1 V_{in0}} \right)$$

$$\epsilon \ll 1, \sqrt{1+\epsilon} \approx 1 + \epsilon/2 - \epsilon^2/8$$

$$V_{out} \approx MV_{in0} + (1-M)\,\frac{M\alpha_1}{2} V_{in0}^2$$

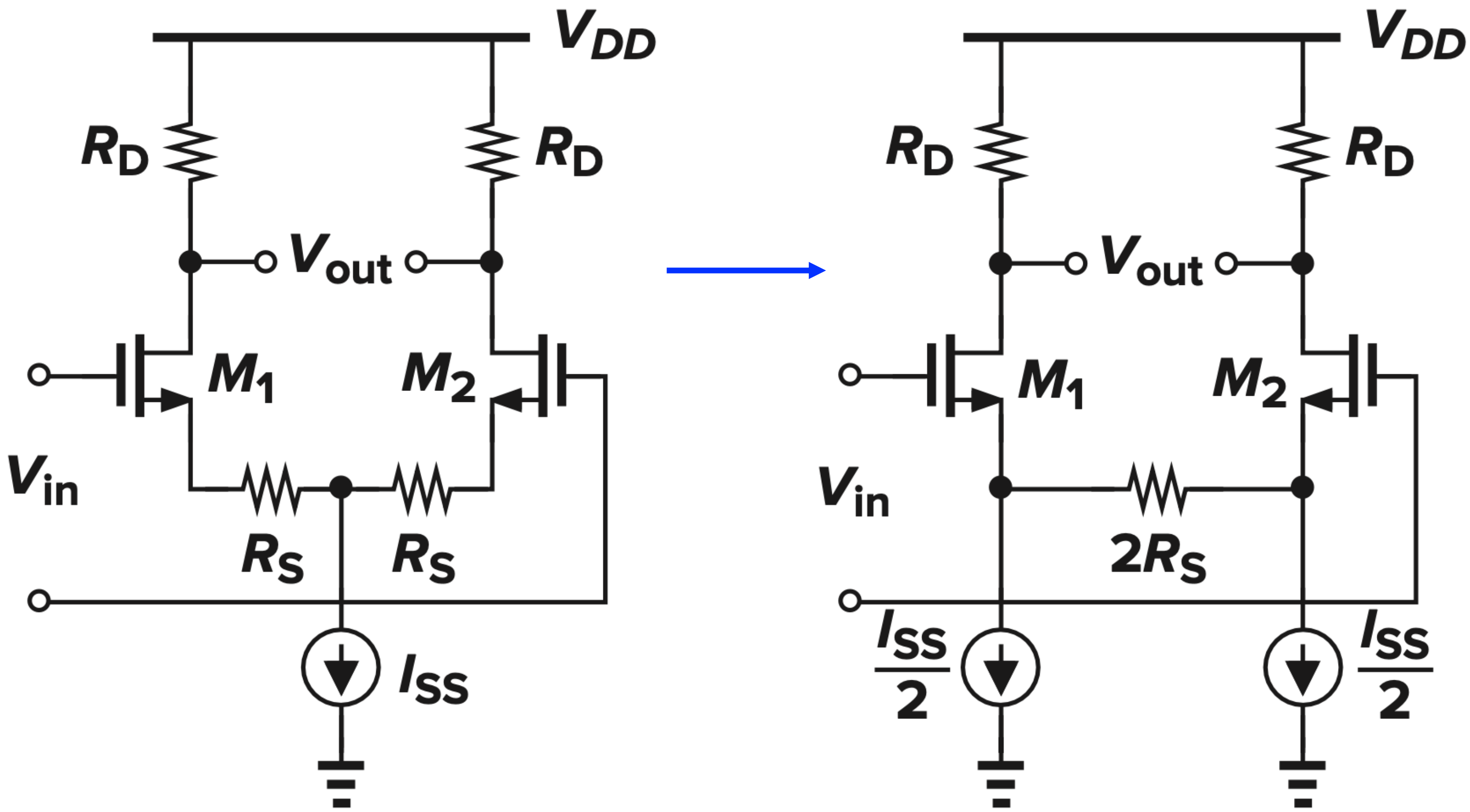
Linearization Techniques



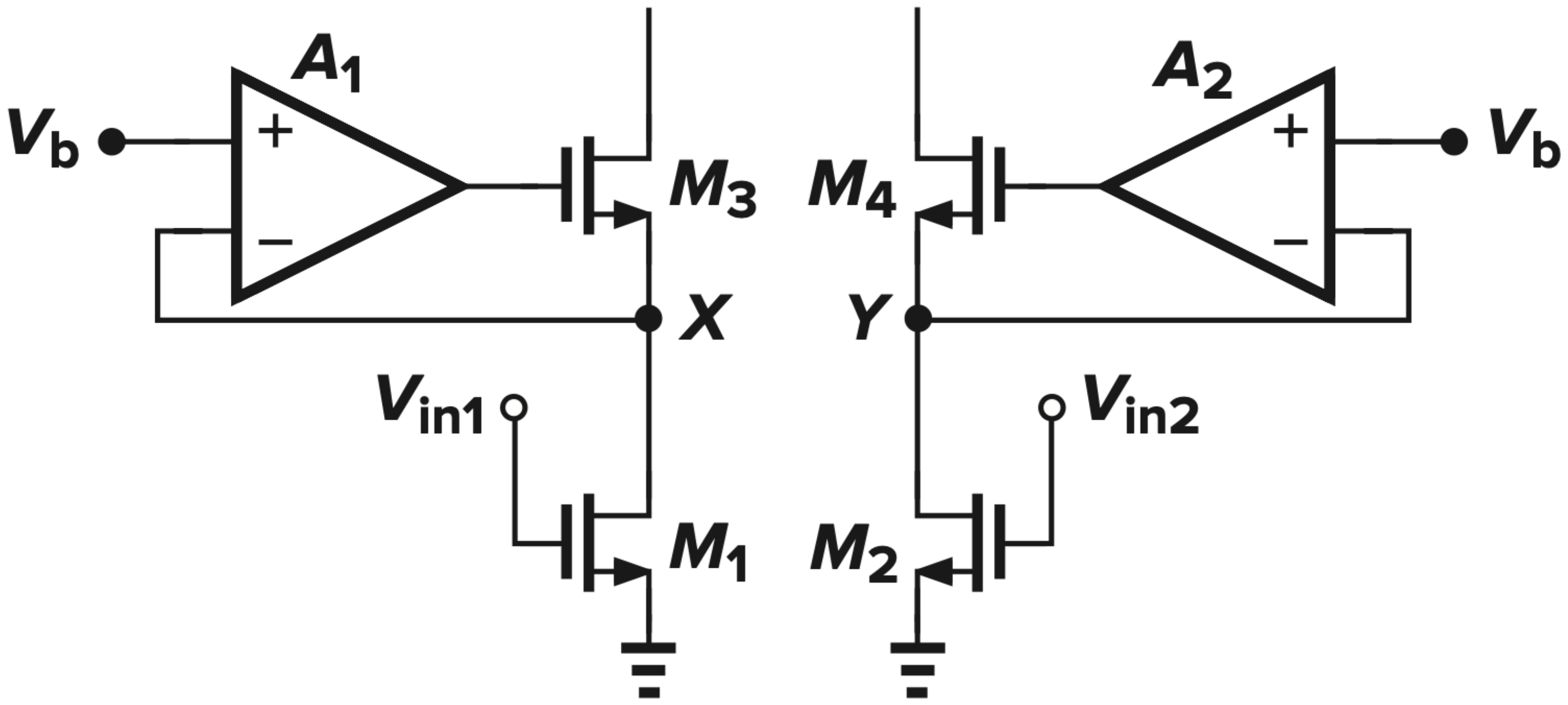


$$G_m = \frac{g_m}{1 + g_m R_S}$$

Source degeneration applied to a differential pair



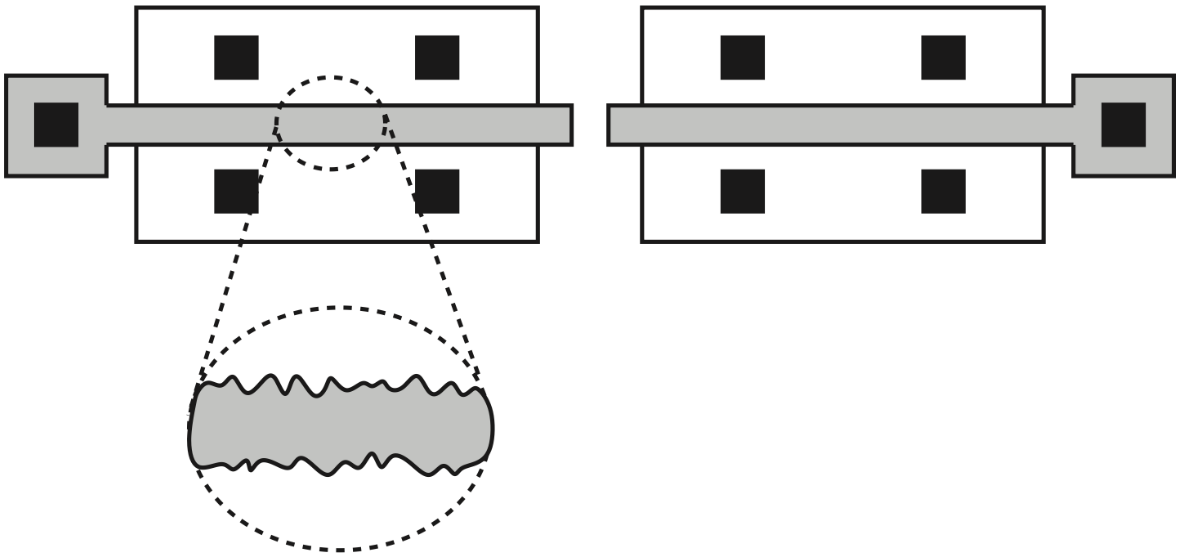
Linearization: Differential pair with input devices in triode



$$I_D = (1/2)\mu C_{ox}(W/L)[2(V_{GS} - V_{TH})V_{DS} - V_{DS}^2]$$

Mismatch

Random mismatches due to microscopic variations in device dimensions



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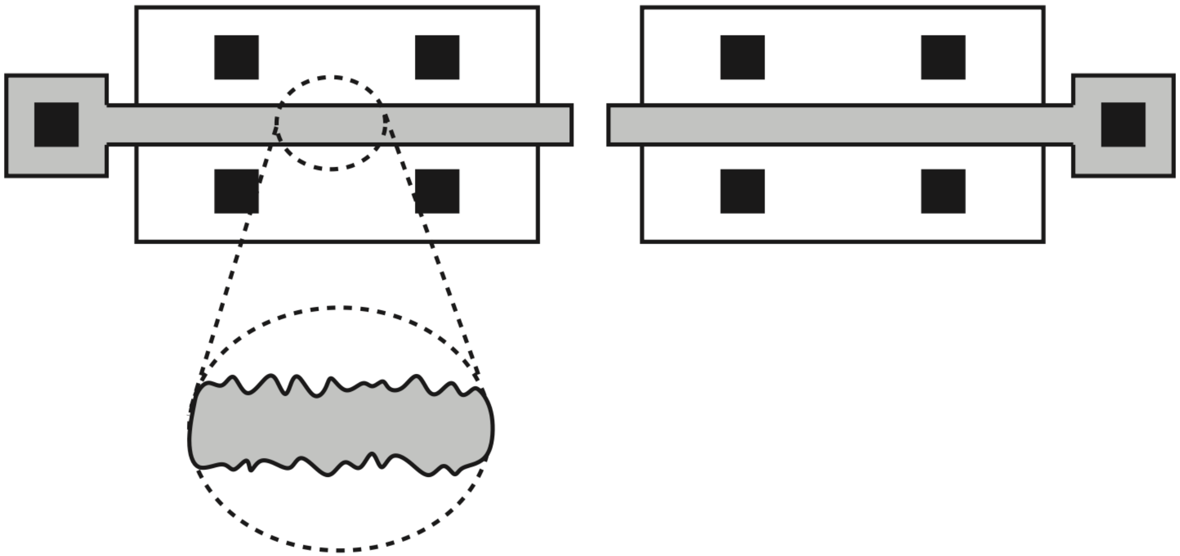
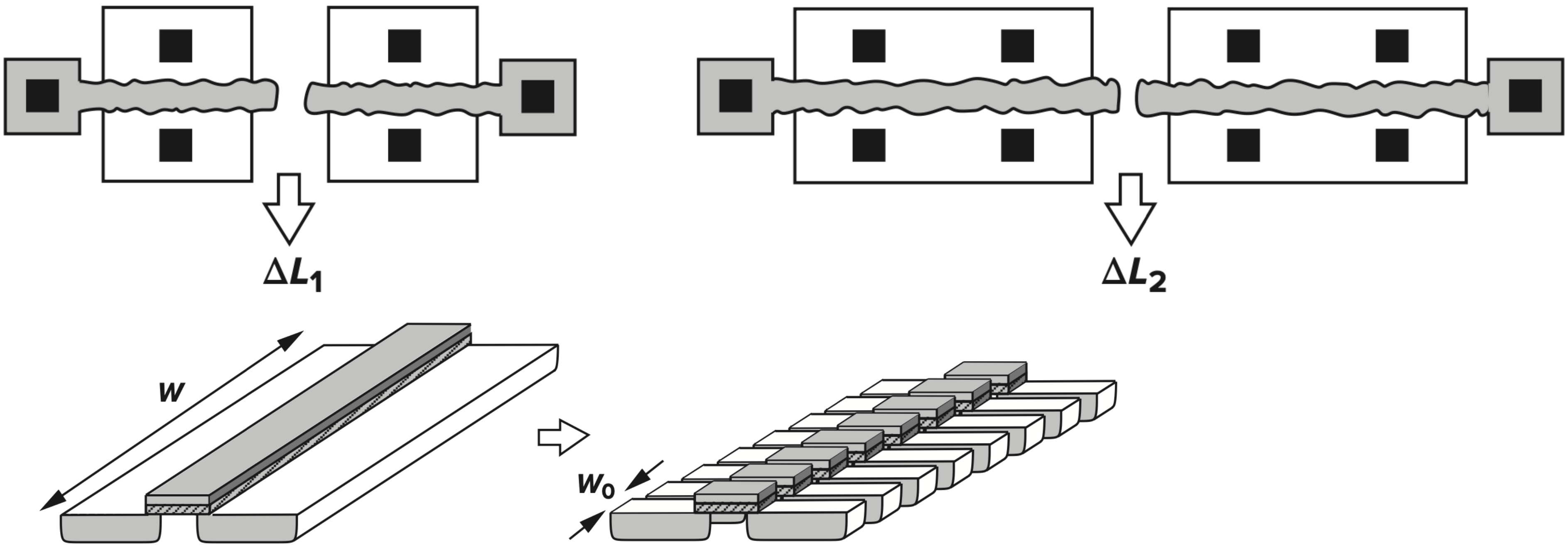


(1) Identify and formulate the mechanisms that lead to mismatch between devices
(2) Analyze the effect of device mismatches on the performance of circuits

$$I_D = (1/2)\mu C_{ox}(W/L)(V_{GS} - V_{TH})^2$$

Mismatch

Reduction of length mismatch as a result of increasing the width



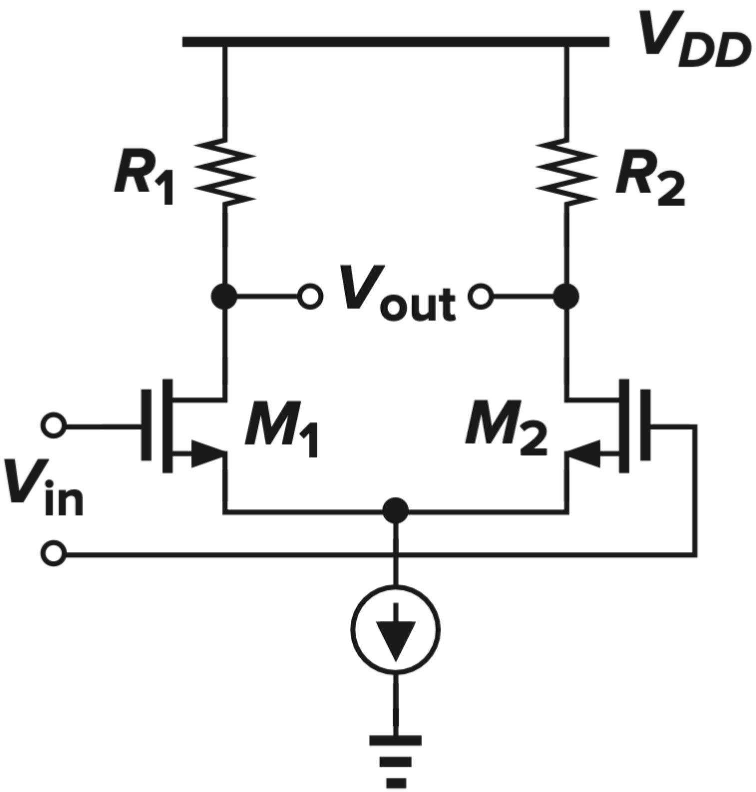
$$L_{eq} \approx (L_1 + L_2 + \cdots + L_n)/n$$

$$\Delta L_{eq} \approx \left(\Delta L_1^2 + \Delta L_2^2 + \cdots + \Delta L_n^2\right)^{1/2}/n$$

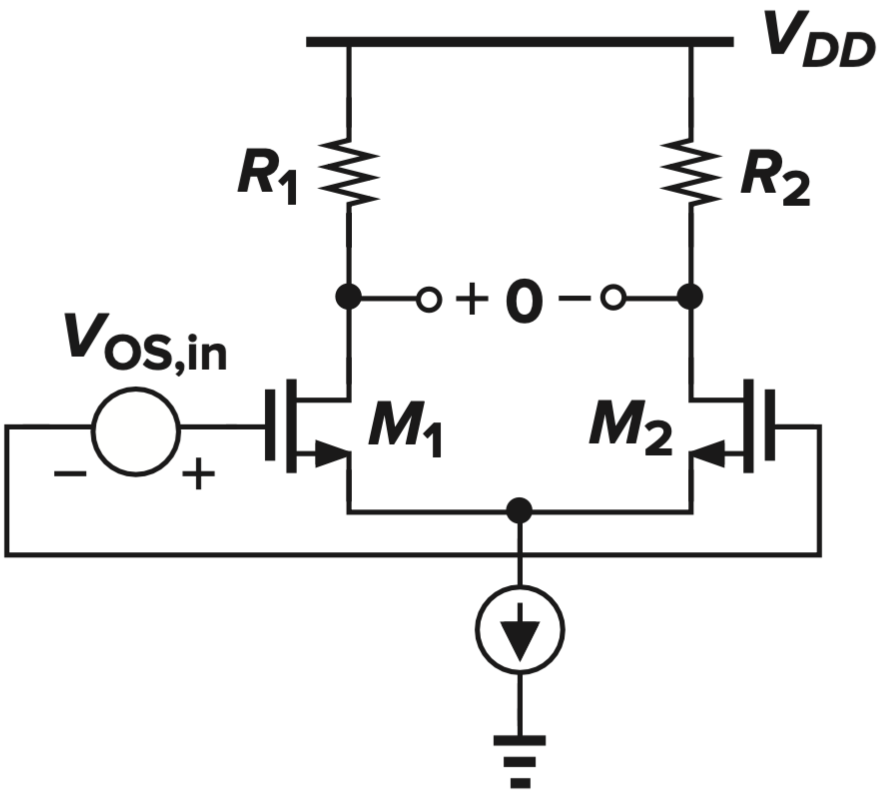
$$= \frac{\left(n\Delta L_0^2\right)^{1/2}}{n}$$

$$= \frac{\Delta L_0}{\sqrt{n}}$$

Effect of Mismatch: Offset

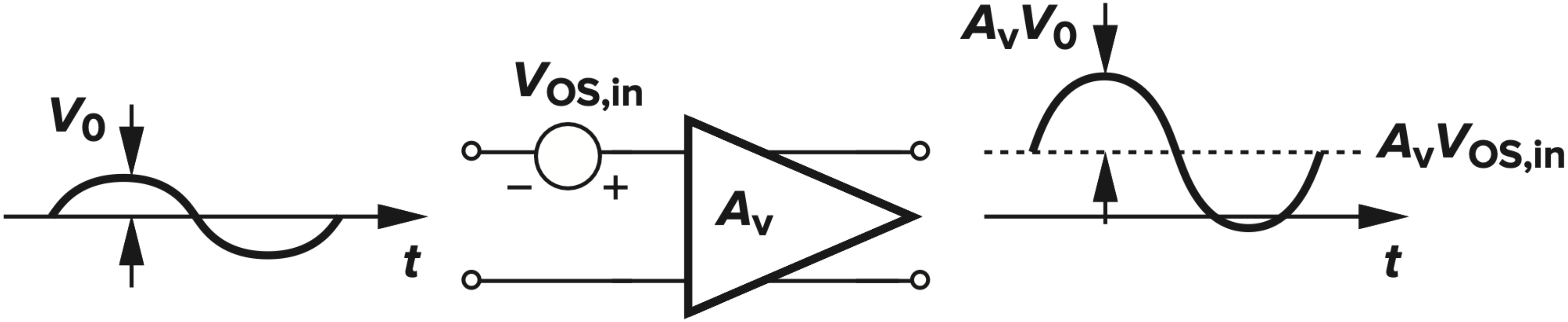


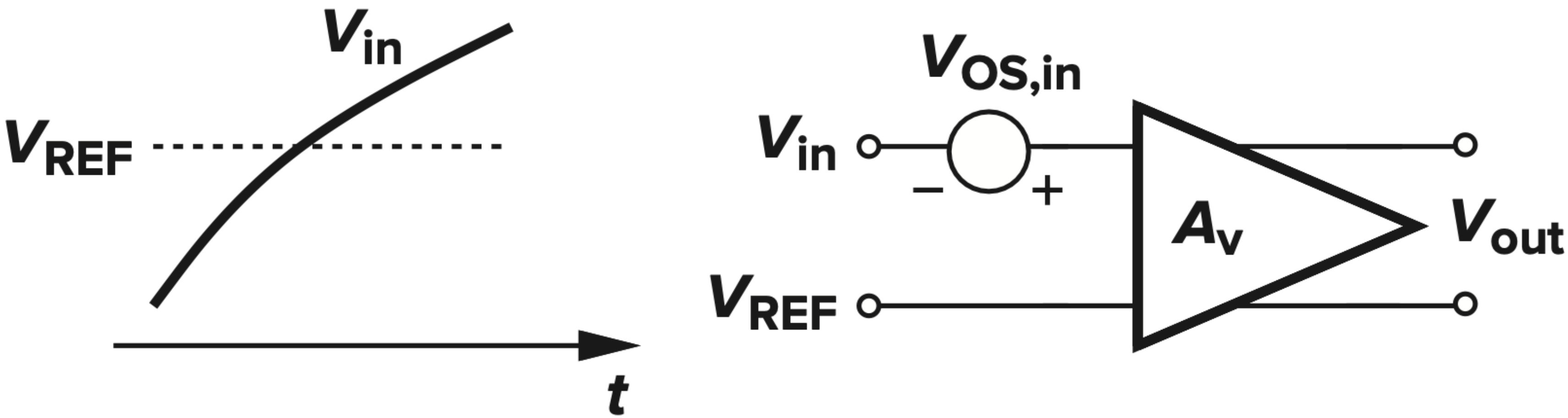
Offset voltage referred to the input



$$|V_{OS,in}| = |V_{OS,out}|/A_v$$

Effect of offset in an amplifier.



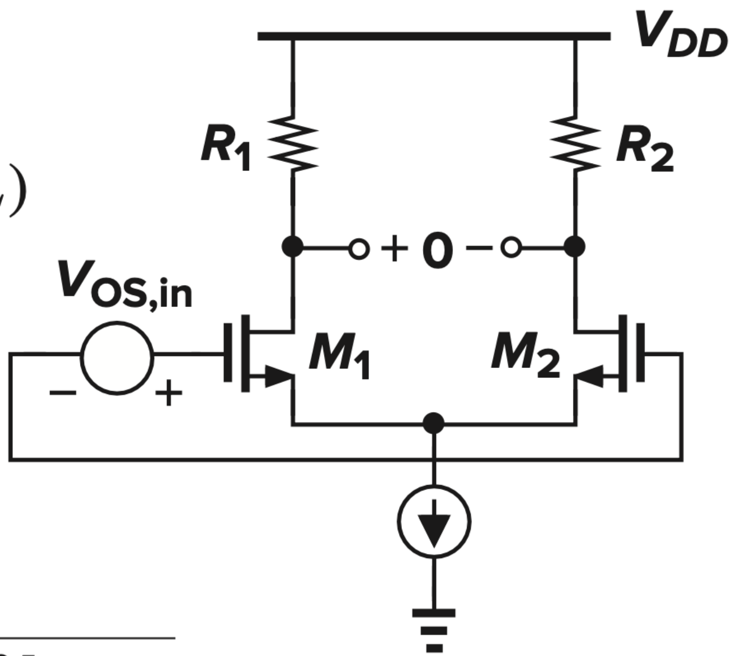


Input offset of a differential pair

$$V_{TH1} = V_{TH},\, V_{TH2} = V_{TH} + \Delta V_{TH}$$

$$(W/L)_1 = W/L,\, (W/L)_2 = W/L + \Delta(W/L)$$

$$R_1 = R_D,\, R_2 = R_D + \Delta R$$



$$I_{D1} = I_D,\, I_{D2} = I_D + \Delta I_D$$

$$V_{OS,in} = V_{GS1} - V_{GS2}$$

$$V_{OS,in} = \sqrt{\frac{2I_{D1}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1}} + V_{TH1} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_2}} - V_{TH2}$$

$$= \sqrt{\frac{2}{\mu_n C_{ox}}}\left[\sqrt{\frac{I_D}{\frac{W}{L}}} - \sqrt{\frac{I_D + \Delta I_D}{\frac{W}{L} + \Delta\left(\frac{W}{L}\right)}}\right] - \Delta V_{TH}$$

Input offset of a differential pair

$$= \sqrt{\frac{2}{\mu_n C_{ox}}}\sqrt{\frac{I_D}{W/L}}\left[1 - \sqrt{\frac{1+\frac{\Delta I_D}{I_D}}{1+\Delta\left(\frac{W}{L}\right)/\left(\frac{W}{L}\right)}}\right] - \Delta V_{TH}$$

$$\sqrt{1+\epsilon} \approx 1+\epsilon/2 \text{ and } (\sqrt{1+\epsilon})^{-1} \approx 1-\epsilon/2$$

$$V_{OS,in} = \sqrt{\frac{2I_D}{\mu_n C_{ox}\left(\frac{W}{L}\right)}}\left\{1-\left(1+\frac{\Delta I_D}{2I_D}\right)\left[1-\frac{\Delta(W/L)}{2(W/L)}\right]\right\} - \Delta V_{TH}$$

$$= \sqrt{\frac{2I_D}{\mu_n C_{ox}\left(\frac{W}{L}\right)}}\left[\frac{-\Delta I_D}{2I_D}+\frac{\Delta(W/L)}{2(W/L)}\right] - \Delta V_{TH}$$

Input offset of a differential pair

$$I_{D1}R_1 = I_{D2}R_2$$

$$I_D R_D = (I_D + \Delta I_D)(R_D + \Delta R_D) \approx I_D R_D + R_D \Delta I_D + I_D \Delta R_D$$

$$\Delta I_D / I_D \approx -\Delta R_D / R_D$$

$$V_{OS,in} = \frac{1}{2}\sqrt{\frac{2I_D}{\mu_n C_{ox}\left(\frac{W}{L}\right)}}\left[\frac{\Delta R_D}{R_D} + \frac{\Delta(W/L)}{(W/L)}\right] - \Delta V_{TH}$$

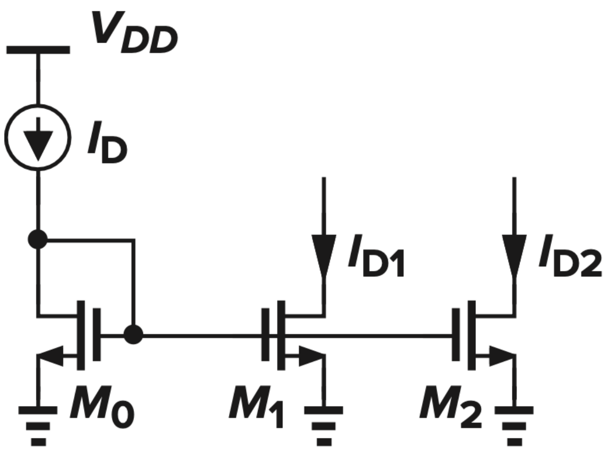
$$V_{OS,in} = \frac{V_{GS} - V_{TH}}{2}\left[\frac{\Delta R_D}{R_D} + \frac{\Delta(W/L)}{(W/L)}\right] - \Delta V_{TH}$$

Mismatch between current sources

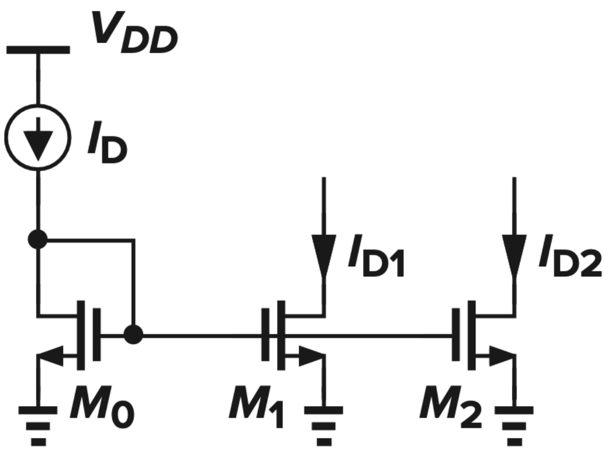
$$y = f(x_1, x_2, \ldots)$$

$$\Delta y = \frac{\partial f}{\partial x_1}\Delta x_1 + \frac{\partial f}{\partial x_2}\Delta x_2 + \cdots$$

$$I_D = (1/2)\mu_n C_{ox}(W/L)(V_{GS} - V_{TH})^2$$



Mismatch between current sources



$$y = f(x_1, x_2, \ldots)$$

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$$I_D = (1/2)\mu_n C_{ox}(W/L)(V_{GS} - V_{TH})^2$$

$$\Delta I_D = \frac{\partial I_D}{\partial (W/L)}\Delta\left(\frac{W}{L}\right) + \frac{\partial I_D}{\partial (V_{GS} - V_{TH})}\Delta(V_{GS} - V_{TH})$$

$$\Delta I_D = \frac{1}{2}\mu_n C_{ox}(V_{GS} - V_{TH})^2\Delta\left(\frac{W}{L}\right) - \mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})\Delta V_{TH}$$

$$\frac{\Delta I_D}{I_D} = \frac{\Delta(W/L)}{W/L} - 2\frac{\Delta V_{TH}}{V_{GS} - V_{TH}}$$

Even-Order Distortion

$$y_1 \approx \alpha_1 x_1 + \alpha_2 x_1^2 + \alpha_3 x_1^3 \text{ and } y_2 \approx \beta_1 x_2 + \beta_2 x_2^2 + \beta_3 x_2^3$$

$$y_1 - y_2 = (\alpha_1 x_1 - \beta_2 x_2) + \left(\alpha_2 x_1^2 - \beta_2 x_2^2\right) + \left(\alpha_3 x_1^3 - \beta_3 x_2^3\right)$$

$$y_1 - y_2 = (\alpha_1 + \beta_1)x_1 + (\alpha_2 - \beta_2)x_1^2 + (\alpha_3 + \beta_3)x_1^3$$