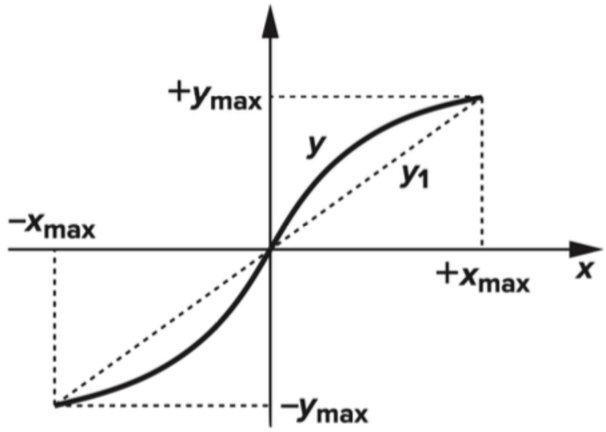
EE-523 – Exercise 4 Solutions

1. $y(t) = \alpha_1 x(t) + \alpha_3 x^3(t)$.

$$\text{The input range } x = -x_{max} \text{ to } x = +x_{max}$$



The line passing through the end points:

$$y_1 = (\alpha_1 x_{max} + \alpha_3 x_{max}^3)\frac{x}{x_{max}}$$

$$y_1 = (\alpha_1 + \alpha_3 x_{max}^2)x$$

$$\Delta y = y - y_1 = \alpha_1 x + \alpha_3 x^3 - (\alpha_1 + \alpha_3 x_{max}^2)x$$

$$\frac{\partial \Delta y}{\partial x} = 0 \rightarrow 3\alpha_3 x^2 - \alpha_3 x_{max}^2 = 0$$

$$x = x_{max}/\sqrt{3}$$

$$\Delta y_{max} = 2\alpha_3 x_{max}^3/3\sqrt{3}$$

$$\Delta y_{max}/y_{max} = \frac{2\alpha_3 x_{max}^3}{3\sqrt{3} \times 2(\alpha_1 x_{max} + \alpha_3 x_{max}^3)}$$

$$\text{the maximum peak} - \text{to} - \text{peak output swing}: 2(\alpha_1 x_{max} + \alpha_3 x_{max}^3)$$

$$\Delta y/y_{max} \approx \frac{\alpha_3}{(3\sqrt{3}\alpha_1)} x_{max}^2$$

We can ignore the $\alpha_3 x_{max}^3$ term with respect to $\alpha_1 x_{max}$.

2. Assuming square-low behavior, we have $g_m \propto \sqrt{I_D}$ in saturation.
   - For the case of no degeneration:

$$\frac{g_{m,high}}{g_{m,low}} = \sqrt{\frac{1.25}{0.75}}$$

   - With $g_m R_S = 2$

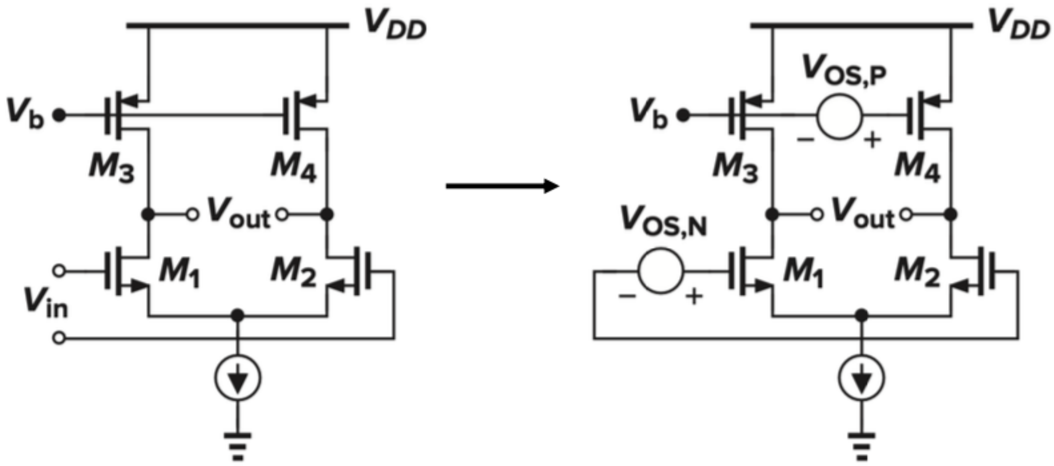
$$\text{with generation: } G_m = \frac{g_m}{1 + g_m R_S}$$

$$\frac{G_{m,high}}{G_{m,low}} = \frac{\dfrac{\sqrt{1.25}g_m}{1+\sqrt{1.25}g_m R_S}}{\dfrac{\sqrt{0.75}g_m}{1+\sqrt{0.75}g_m R_S}} = \sqrt{\frac{1.25}{0.75}} \times \frac{1+2\sqrt{0.75}}{1+2\sqrt{1.25}}$$

$$= 0.84\sqrt{\frac{1.25}{0.75}}$$

Degeneration reduces the gain variation by $\sim 16\%$.

3. Assume all the transistors operate in saturation.



We insert the offsets of the NMOS and PMOS pairs.
To have $I_{D1} = I_{D2}$ and $I_{D3} = I_{D4}$

$$V_{OS,N} = \frac{(V_{GS}-V_{TH})_N}{2}\left[\frac{\Delta\left(\frac{W}{L}\right)}{\frac{W}{L}}\right]_N + \Delta V_{TH,N}$$

$$V_{OS,P} = \frac{|V_{GS}-V_{TH}|_P}{2}\left[\frac{\Delta\left(\frac{W}{L}\right)}{\frac{W}{L}}\right]_P + \Delta V_{TH,P}$$

From the noise analysis basics, we know that $V_{OS,P}$ is amplified by a gain of $g_{m,P}\left(r_{O_N}\|r_{O_P}\right)$ and divided by $g_{m,N}\left(r_{O_N}\|r_{O_P}\right)$ when referred to the input.

$$V_{OS,in} = \frac{(V_{GS}-V_{TH})_N}{2}\left[\frac{\Delta\left(\frac{W}{L}\right)}{\frac{W}{L}}\right]_N + \Delta V_{TH,N} + \left[\frac{|V_{GS}-V_{TH}|_P}{2}\left[\frac{\Delta\left(\frac{W}{L}\right)}{\frac{W}{L}}\right]_P + \Delta V_{TH,P}\right](g_{m,P}/g_{m,N})$$