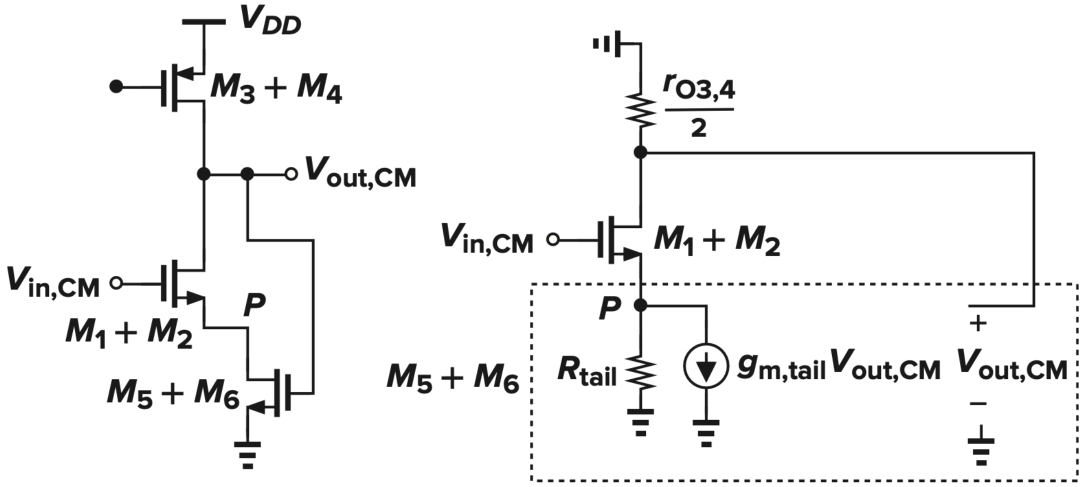
EE-523 – Exercise 2 Solutions

1. **For CM input and assuming symmetry, the output nodes can be shorted.**



$$R_{tail} = \frac{R_{on}}{2}$$

$$g_{m_{5,6}}(\text{triode}) = \mu_n C_{ox}\left(\frac{W}{L}\right) V_{DS}$$

$$2g_{m_{5,6}} = 2\mu_n C_{ox}\left(\frac{W}{L}\right)_{5,6} V_P$$

$$\frac{R_{on}}{2} = \frac{1}{2\mu_n C_{ox}\left(\frac{W}{L}\right)_{5,6}(V_{out,CM} - V_{TH_{5,6}})} \text{ where } V_{out,CM} = V_{GS}$$

If $\boldsymbol{\lambda = \gamma = 0}$ for $\mathbf{M_1, M_2}$:

$$-\frac{V_{out,CM}}{r_{o3,4}/2} = 2g_{m1,2}(V_{in,CM} - V_P)$$

$$V_P = V_{in,CM} + \frac{V_{out,CM}}{g_{m1,2} r_{o3,4}}$$

$$\frac{-V_{out,CM}}{r_{o3,4}/2} = \frac{V_{in,CM} + \frac{V_{out,CM}}{g_{m1,2} r_{o3,4}}}{\left(\frac{1}{2\mu_n C_{ox}\left(\frac{W}{L}\right)_{5,6}(V_{out,CM} - V_{TH5,6})}\right)} + 2g_{m5,6} V_{out,CM}$$

$$\frac{V_{out,CM}}{V_{in,CM}} = \frac{-1}{\frac{2 \times \frac{R_{on}}{2}}{r_{03,4}} + \frac{1}{g_{m1,2} r_{o3,4}} + 2g_{m5,6}(R_{on}/2)}$$

It can be further simplified by replacing $g_{m5,6}$ based on V_P.

2.

$$\mathbf{A_{V,diff} = A_{V1} \times A_{V2}}$$

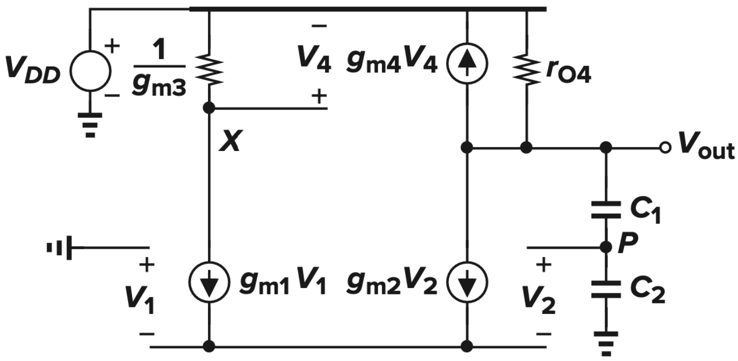
$$|A_{V1}| = g_{m1,2}(r_{o1,2} \| r_{03,4} \| R_{1,2})$$

$$|A_{V2}| = g_{m5,6}(r_{o5,6} \| r_{o7,8} \| R_{3,4})$$

Assuming symmetry:

$$|A_V| = g_{m1}(r_{o1} \| r_{03} \| R_1) \times g_{m5}(r_{o5} \| r_{o7} \| R_3)$$

3. **Small signal model:**



$$V_2 = \frac{C_1}{C_1 + C_2} V_{out} - (-V_1) \quad \& \quad g_{m1} V_1 + g_{m2} V_2 = 0$$

$$V_2 = \frac{C_1}{(C_1 + C_2)} V_{out} - \frac{g_{m2}}{gm_1} V_2$$

$$V_2 = \frac{\frac{C_1}{C_1 + C_2} V_{out}}{1 + \frac{g_{m2}}{g_{m1}}}$$

$$\text{also } V_4 = -g_{m1} V_1 / g_{m3}$$

$$\textbf{KCL at } \mathbf{V_{out}}: -\frac{g_{m1}V_1}{gm_3}g_{m4} + (V_{out} - V_{DD})/r_{o4} + g_{m2}V_2 = 0$$

Assuming symmetry: $g_{m1} = g_{m2}$ & $g_{m3} = g_{m4}$

$$\frac{V_{out}}{V_{DD}} = \frac{1}{g_{m2}r_{o4}\left(\frac{C_1}{C_1 + C_2}\right) + 1}$$

$$PSRR = \frac{A_V}{\left(\frac{V_{out}}{V_{DD}}\right)} \text{ where } A_V = \frac{C_1 + C_2}{C_1}$$

$$PSRR \approx \frac{C_1 + C_2}{C_1} \times (g_{m2}r_{o4}\left(\frac{C_1}{C_1 + C_2}\right) + 1) \approx g_{m2}r_{o4}$$

4.

$$|A_{V1}| = \frac{g_{m1}}{g_{m3}} = \sqrt{\frac{\mu_n}{\mu_p} \times \frac{\left(\frac{W}{L}\right)_1}{\left(\frac{W}{L}\right)_3}} = \sqrt{\frac{75 \times 50}{30 \times 10}} = 3.54$$

For (5,7) & (6,8) at the gate of M5:

$$\overline{V^2_{n,M5,M7,M6,M8}} = 2 \times \frac{4KT\gamma(g_{m5} + g_{m7})}{g^2_{m5}} \text{ where } \gamma = 2/3$$

Referred to input:

$$\overline{V^2_{n,in,M5,M7,M6,M8}} = 2 \times \frac{4KT\gamma(g_{m5} + g_{m7})}{g^2_{m5} \times (3.54)^2} \text{ where } \gamma = \frac{2}{3} \qquad (1)$$

$$\overline{V^2_{n,in,M1,M3,M4,M2}} = 2 \times \frac{4KT\gamma(g_{m1} + g_{m3})}{g^2_{m1}} \text{ where } \gamma = \frac{2}{3} \qquad (2)$$

$$\overline{V^2_{n,in}} = (1) + (2)$$