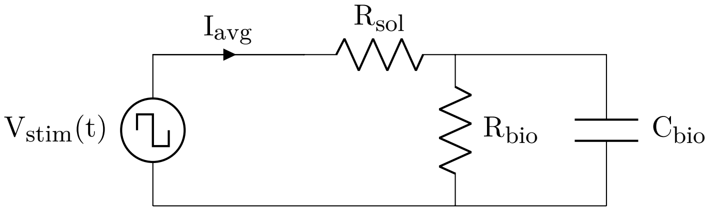
EE-517: Bio-Nano-Chip Design

Solutions 13: Bio-CMOS circuits for DNA detection

December 8, 2020

Problem 1

With *Charge-Based Capacitance Measurement* (CBCM) method, the bio/nano interface is assumed to behave as an ideal capacitor C, independent of frequency. The CBCM front-end circuit is displayed hereunder,



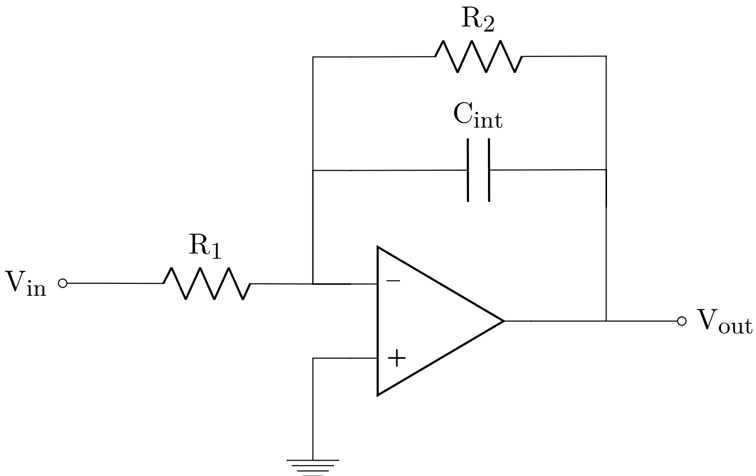
where the voltage step generator (period T and step voltage V_{step}) polarizes the bio/nano layer modeled as a resistance R_{bio} and a capacitor C_{bio}. The voltage drop across the solution resistance is neglected so that the average current through the bio/nano layer is

$$\begin{aligned} I_{avg} &= \frac{1}{T}\int_0^T \frac{V_{stim}(t)}{R_{bio}}\,dt + \frac{Q_{tot}}{T} \\ &= \frac{V_{step}}{2R_{bio}} + \frac{C_{bio}V_{step}}{T} \end{aligned} \tag{1}$$

Therefore, $C_{bio} = \frac{1}{fV_{step}}\left(I_{avg} - \frac{V_{step}}{2R_{bio}}\right)$. The upper and lower bounds are $\mathbf{C_{bio,min} = 3.3\,pF}$ and $\mathbf{C_{bio,max} = 3.3\,mF}$, respectively.

Problem 2

The integration stage of the CBCM is implemented with the following circuit.



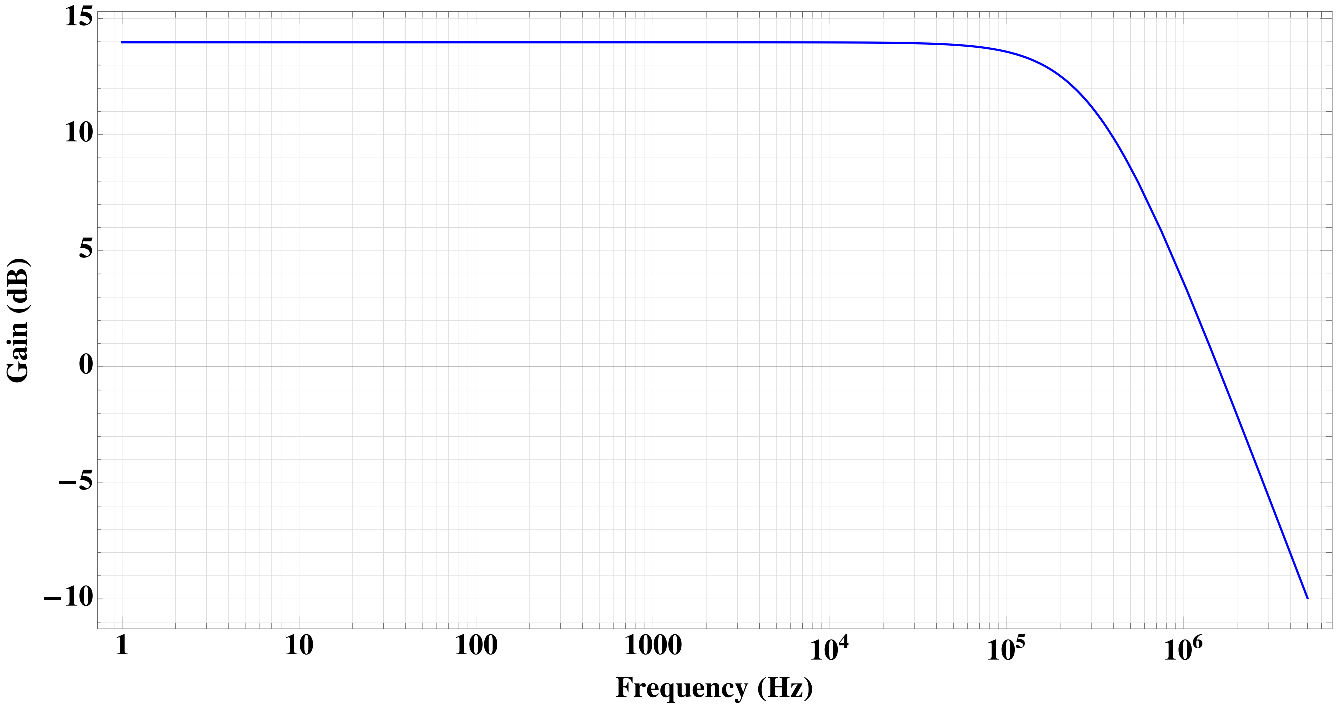
By applying nodal voltage analysis at the virtual ground of the opamp,

$$\frac{V_{out}(s)}{V_{in}(s)} = -\frac{R_2}{R_1}\frac{1}{1 + sR_2C_{int}}$$

The integrator acts as a low-pass filter and the DC gain is set by the ratio of resistors. The integrator is sized such that the signal is not attenuated, and by applying Nyquist criterion,

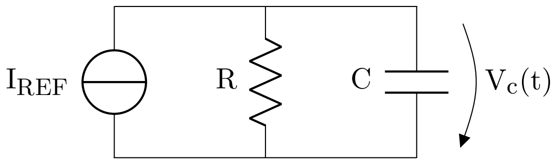
$$2f_{max} \leq \frac{1}{2\pi R_2 C_{int}} \iff R_2 C_{int} \leq \frac{1}{4\pi f_{max}}$$
$$\iff R_2 C_{int} \leq 0.796\,\mu s.$$

Therefore, $\mathbf{R_1 = 100\,k\Omega}$, $\mathbf{R_2 = 500\,k\Omega}$, and $\mathbf{C_{int} = 1\,pF}$ is a possible design choice. The corresponding Bode diagram is plotted below.



Problem 3

(a) By neglecting the solution resistance and the geometrical two-plates capacitance formed by the two electrodes, the Randles circuit of the bio/CMOS interface is simply



where R and C represent the contributions of resistive and capacitive elements variations at the electrode/solution interface. By applying Kirchhoff current law,

$$I_{REF} = \frac{V_C(t)}{R} + C\frac{dV_C(t)}{dt}$$

The differential equation is solved with $V_C(t=0) = 0$, yielding

$$V_C(t) = R\,I_{REF}(1 - e^{-\frac{t}{RC}})$$

The square signal V_{OUT} at the output of the comparator flips its state when a voltage V_{REF} is built across the electrodes. Thus,

$$V_C(t = \frac{T}{2}) = V_{REF} \iff R\,I_{REF}(1 - e^{-\frac{T}{2RC}}) = V_{REF}$$

$$\iff \mathbf{T = 2RC\ln\left(\frac{1}{1 - \frac{V_{REF}}{R\,I_{REF}}}\right)}$$

(b) The upper and lower bounds of the period of the output signal are $\mathbf{T_{min} = 2.68\,ms}$ and $\mathbf{T_{max} = 5.34\,ms}$, respectively. In terms of frequency, $\mathbf{f_{min} = 187\,Hz}$ and $\mathbf{f_{max} = 373\,Hz}$.

Problem 4

If the assumption $\frac{V_{ref}}{R\,I_{ref}} \longrightarrow 0$ is made, the period of the output signal is simplified to $T = 2C\frac{V_{ref}}{I_{ref}}$, by applying the following approximations:

- $\frac{1}{1-\frac{1}{x}} \overset{x\to 0}{\sim} 1 + x.$
- $\ln(1+x) \overset{x\to 0}{\sim} x.$

The upper bounds of the period of the output signal are $\mathbf{T_{min} = 2.67\,ms}$ and $\mathbf{T_{max} = 5.33\,ms}$, respectively.

The relative error made by doing the assumption is

$$\epsilon = \max\frac{|T_{approx} - T_{theo}|}{T_{theo}} = \mathbf{0.4\%}$$