EE512 – Applied Biomedical Signal Processing

Basics

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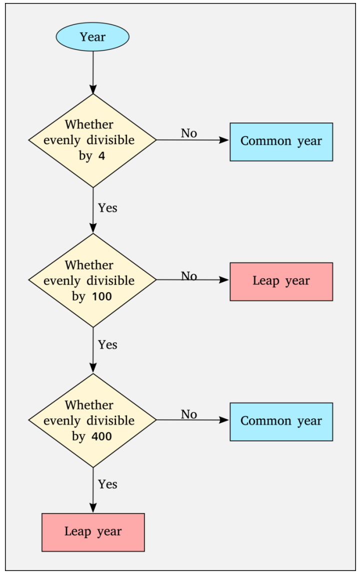
Outline

- Digital vs analog processing
- Signal sampling
- Fourier transform
- Digital filters
  - Finite impulse response filters (FIR)
  - Infinite impulse response filters (IIR)
  - Z transform

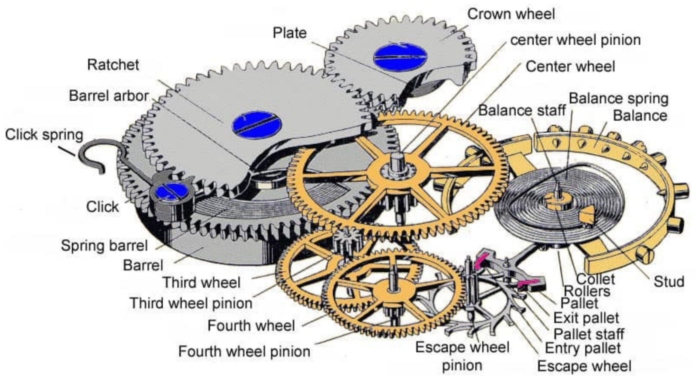
Digital vs analog signal processing

- **Digital processing**
- N = sum of quartz oscillations @ f_q
- $S = mod(N/f_q ,60)$
- $M = mod(floor(N/f_q/60), 60)$
- $H = mod(floor(N/f_q/3600), 24)$
- …
  - *if (month == February)*
    - *if (leap year)*
      - *if (day == 30)*
        - *month = March*
        - *day = 1*
    - …



Analog processing

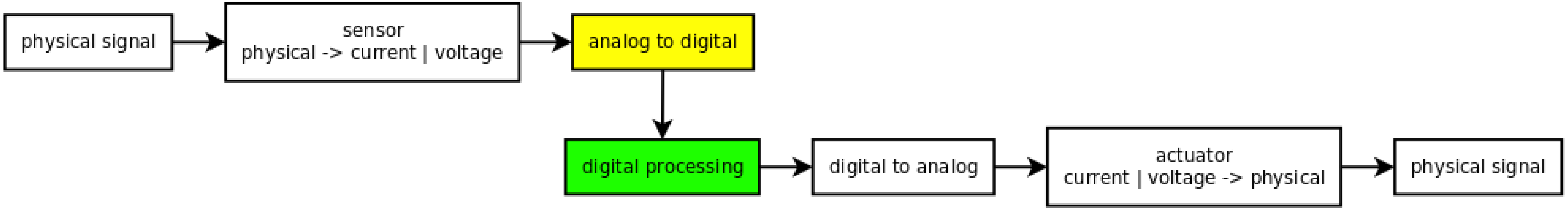
hours,
minutes,
seconds



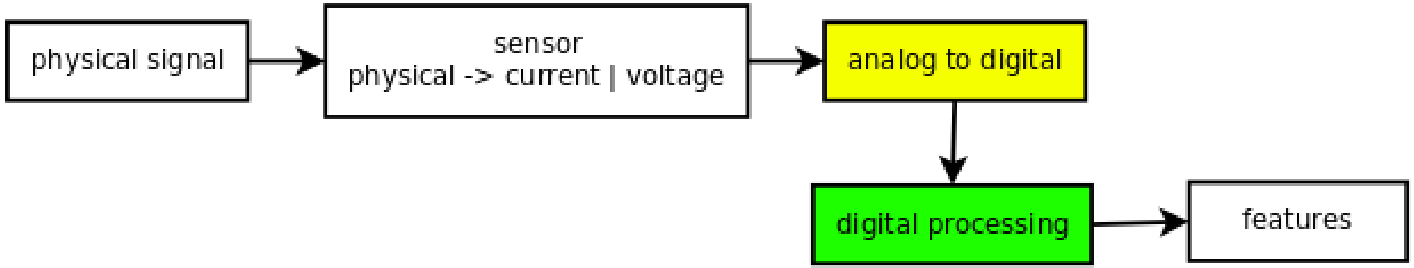
perpetual
calendar

Signal processing schemes

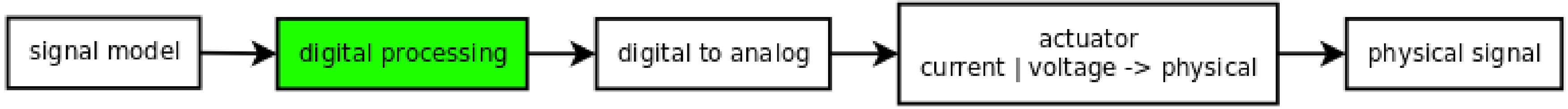
Filter



Analysis

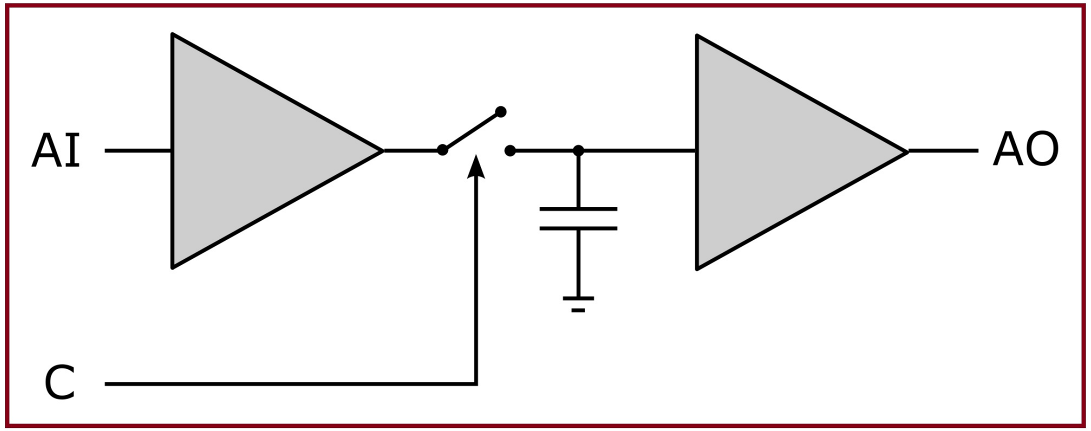


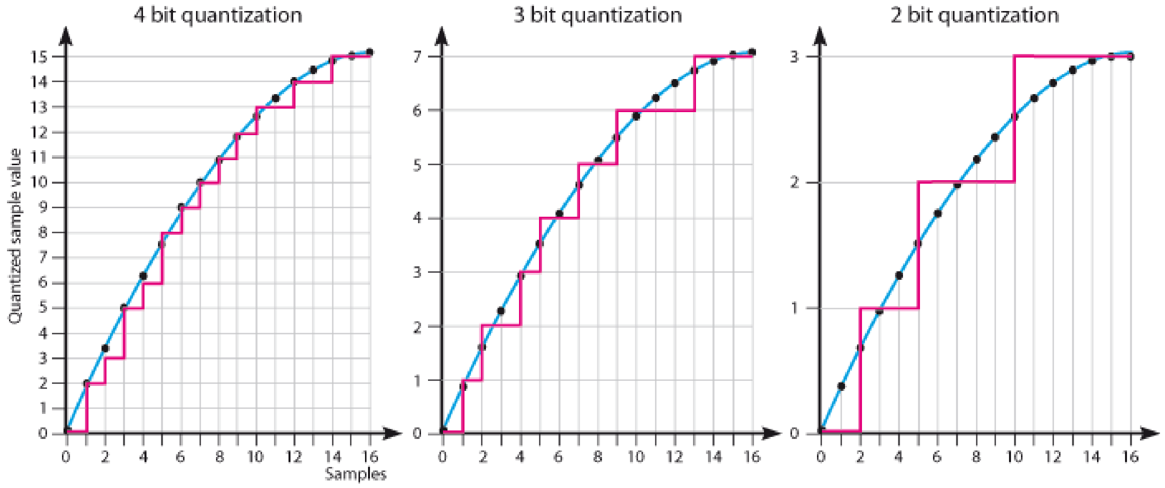
Synthesis



Analog to digital conversion (ADC)

- "Pick" the value of the signal at time intervals (**sampling**)
  - Electronic circuits that performs sample-and-hold
  - Capacitor + switch
- Convert the analog value into a digital number (**quantization**)
  - Integer values (binary values)
  - Resolution of the ADC in bits
  - Produce quantization noise





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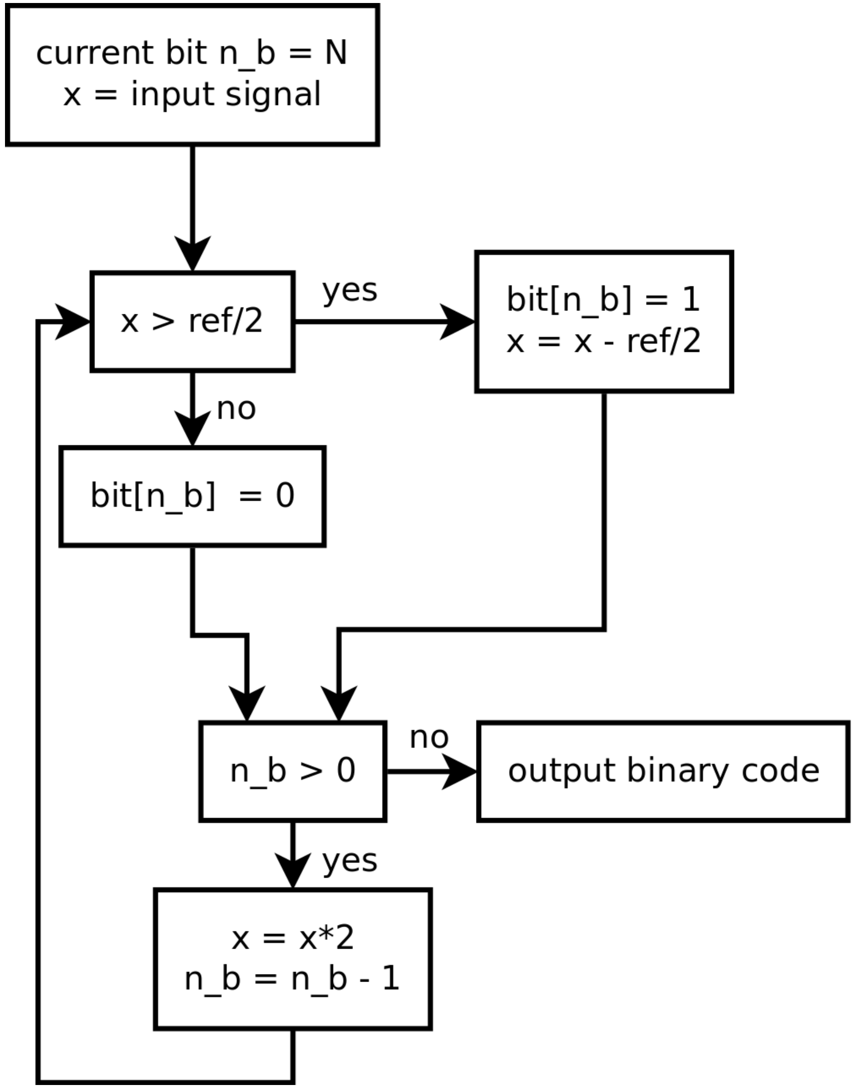
Quantization

- Quantization is obtained by iteratively comparing the analog value with reference
- Each comparison results in one bit that is either 0 or 1
- The result consists in a binary value x_b with relationship

$$x_a = x_b \cdot \frac{input\ voltage\ range}{2^N}$$

- Quantization produces and error of ± ½ LSB that can be neglected if N is large enough



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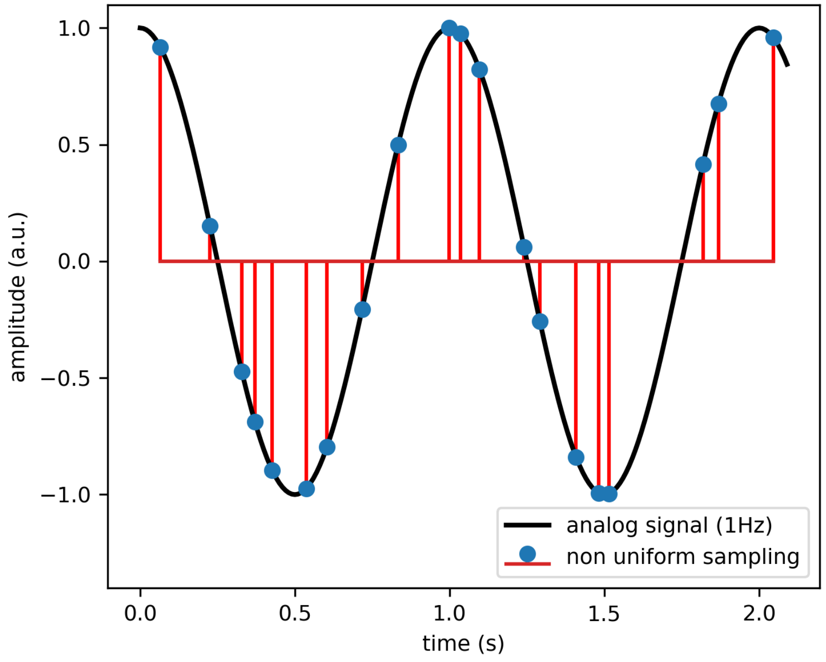
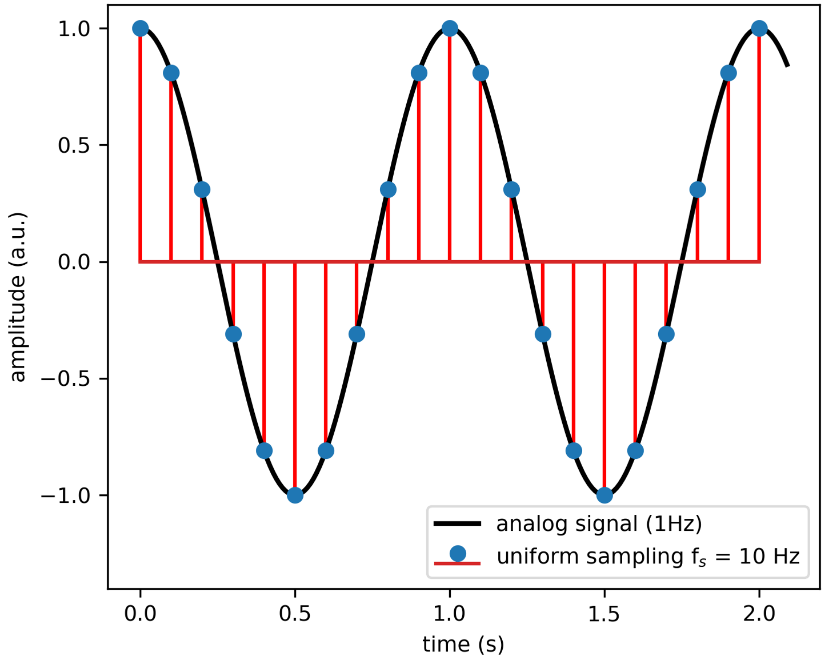
Sampling

- Sampling consist in "picking" the signal value at different **sampling times**
- Generally, the time intervals are constant (uniform sampling)
  - Non uniform sampling can / must be used in some specific cases, but it makes calculation more complex
  - Non uniform sampled signal can be transformed into uniformly sampled signal by interpolation
- When sampling is uniform it is defined by **sampling period** T_s
- The dual of the sampling period is the **sampling frequency** or **sampling rate**:

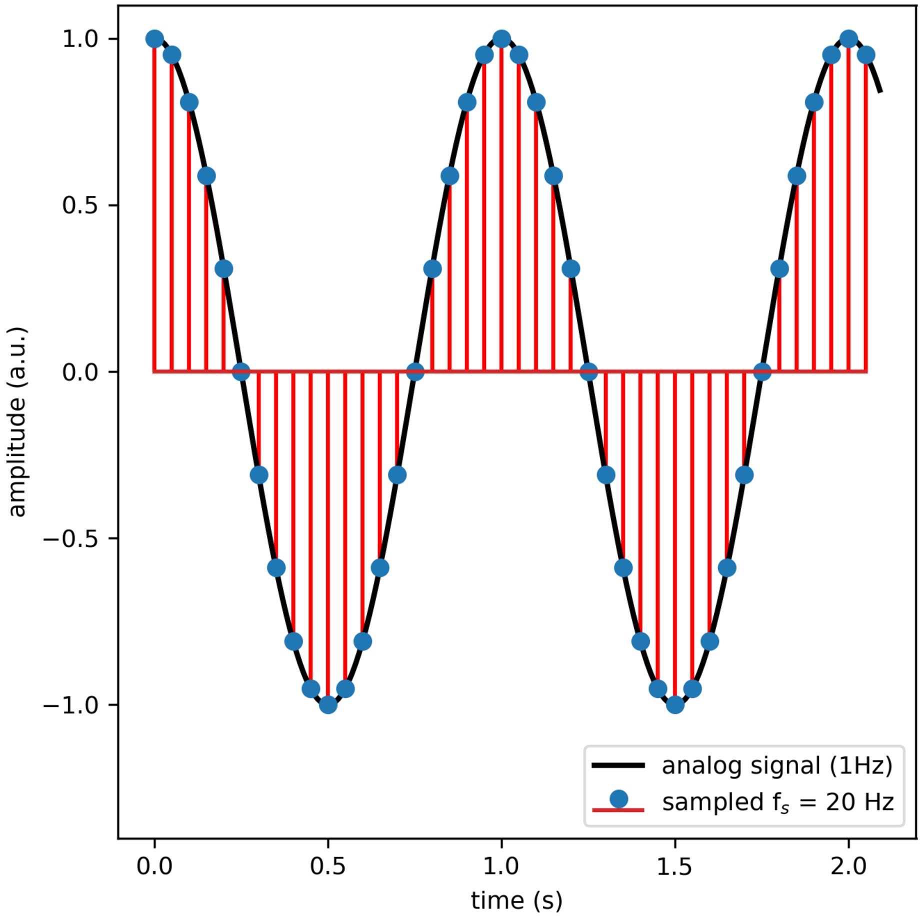
$$f_s = \frac{1}{T_s}$$





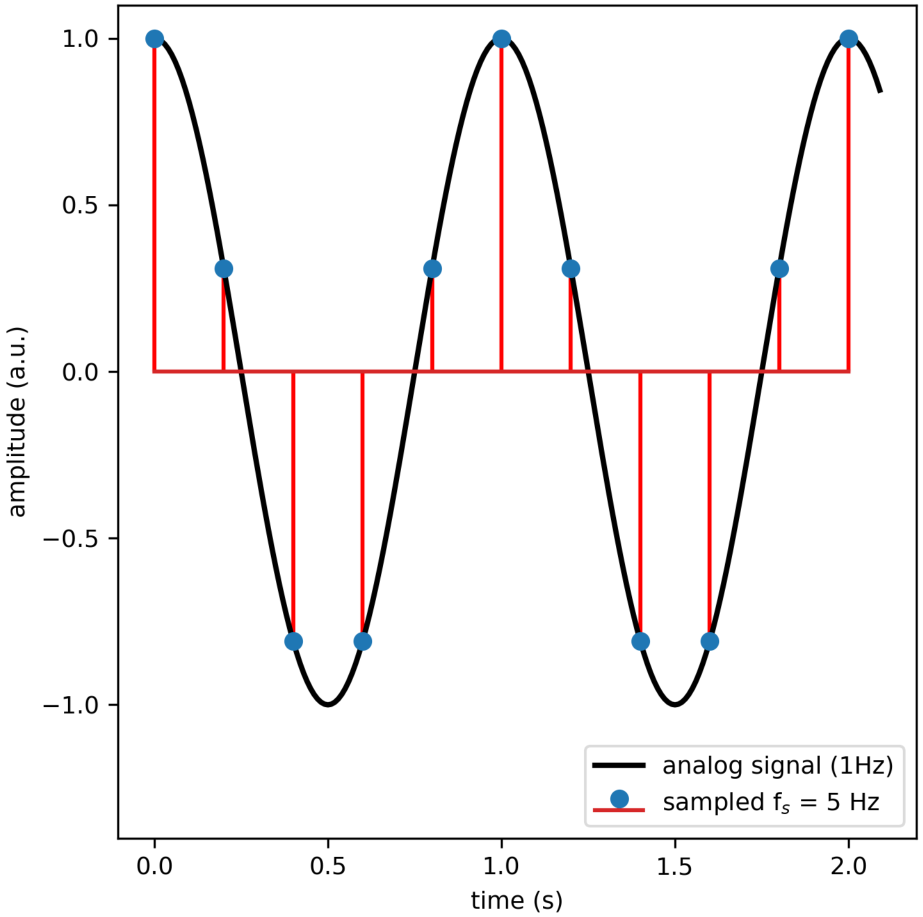
Sampling: fast sampling

- Cos signal with f_a = 1 Hz
- f_s = 20 Hz
- $f_a \ll \frac{f_s}{2}$
- High sampling frequency
  - High number of values in the numerical series
  - Increase required memory and computational power
- Oversampling can be used to improve accuracy and reduce noise



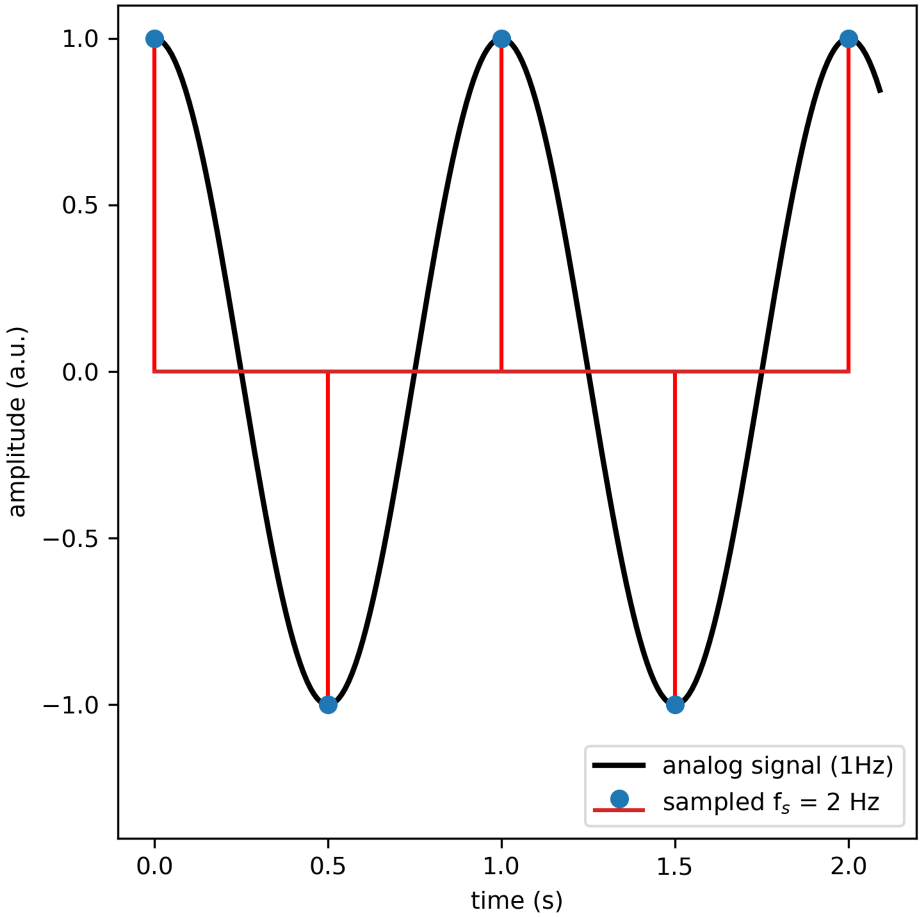
Sampling: "normal" sampling

- Cos signal with f_a = 1 Hz
- f_s = 5 Hz
- $f_a < \frac{f_s}{2}$
- "Normal" sampling
- The samples in the digital series represents unambiguously the analog signal



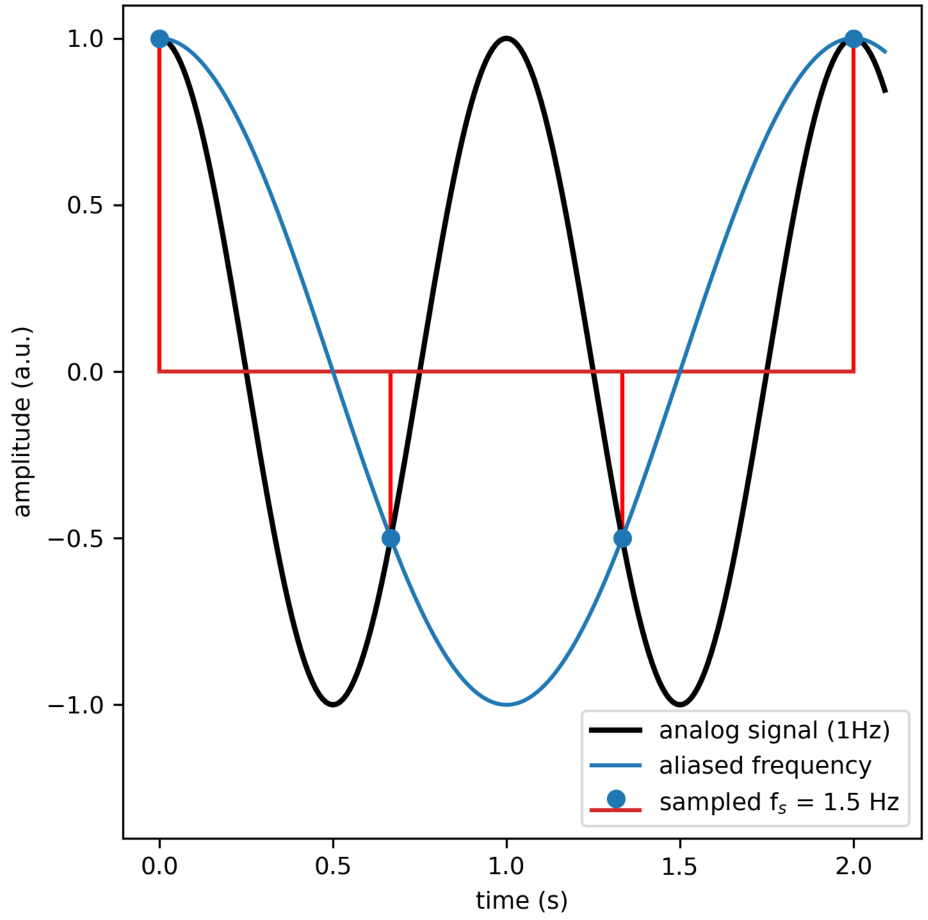
Sampling: critical sampling

- Cos signal with f_a = 1 Hz
- f_s = 2 Hz
- $f_a = \frac{f_s}{2}$
- Limit sampling frequency
- Analog signal is unambiguously represented by the digital series
- Limit case for perfect reconstruction of the signal

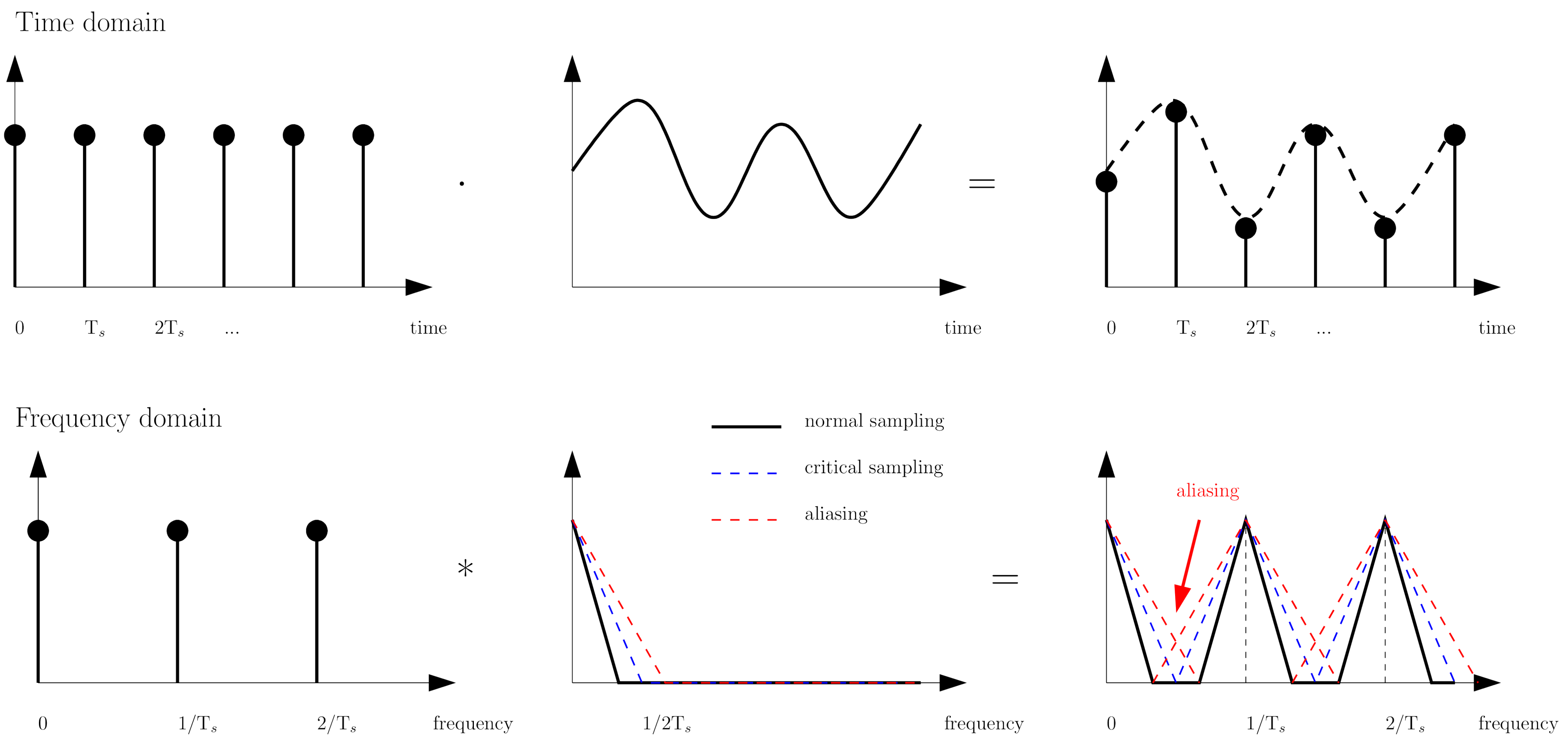


Sampling: aliasing

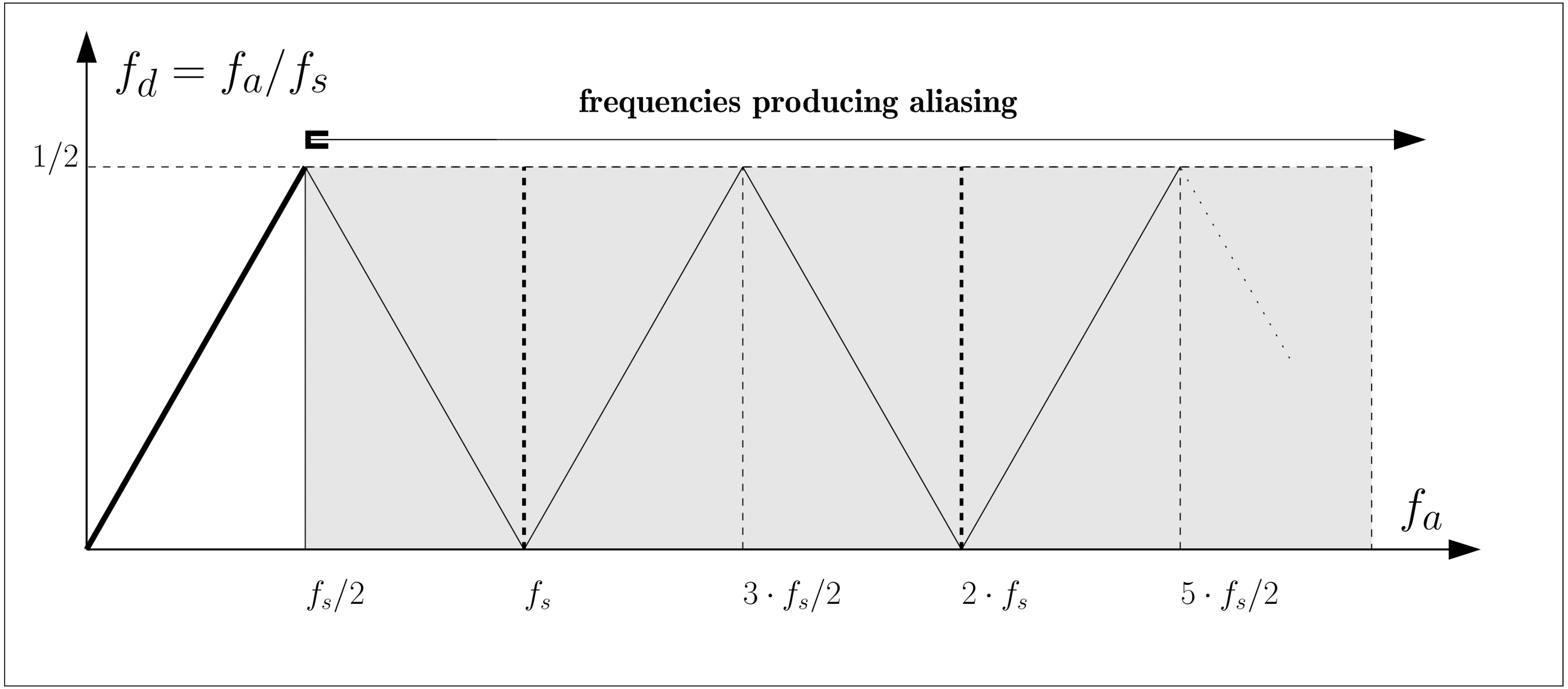
- Cos signal with f_a = 1 Hz
- f_s = 1.5 Hz
- $f_a > \frac{f_s}{2}$
- Sampling frequency is too low, and signal cannot be reconstructed unambiguously
- Alternative solution if called an *alias* and the phenomenon *aliasing*



Spectral representation of aliasing



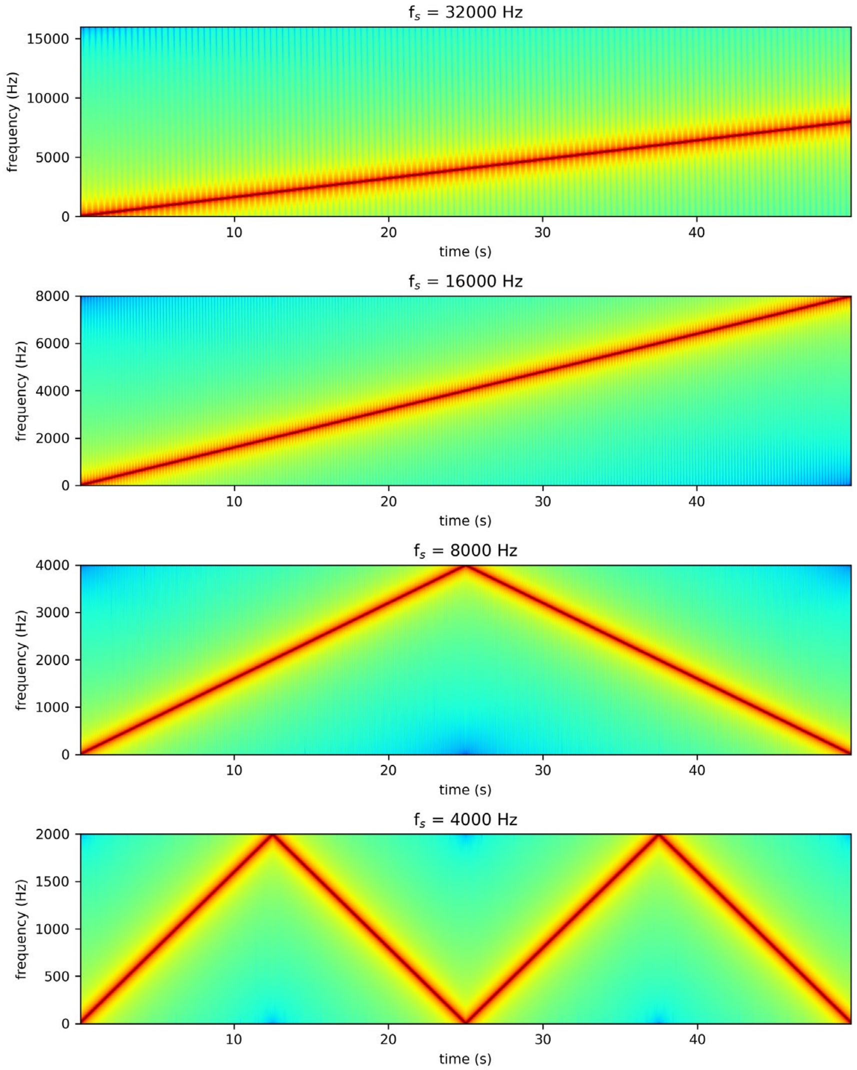
Frequency mapping



- **All analogic frequencies (f_a) > $f_s/2$ are aliased after sampling**
- **f is an alias frequency if** $\mathbf{mod}\left(|f - k \cdot f_s|, {}^{f_s}/_{2}\right) = |f - k \cdot f_s|, k \in \mathbb{N}^*$

Aliasing example (chirp)

- $x_a(t) = \cos(2\pi(t/50) \cdot 8000 \cdot t)$
- $f_s = 8\ \text{kHz}$:
  - $f_a > 4\ \text{kHz} \rightarrow f_d \cdot f_s = f_s - f_a$
- $f_s = 4\ \text{kHz}$:
  - $2\ \text{kHz} < f_a < 4\ \text{kHz} \rightarrow f_d \cdot f_s = f_s - f_a$
  - $4\ \text{kHz} < f_a < 6\ \text{kHz} \rightarrow f_d \cdot f_s = f_a - f_s$
  - $6\ \text{kHz} < f_a < 8\ \text{kHz} \rightarrow f_d \cdot f_s = 2 \cdot f_s - f_a$



Aliasing example (spatial frequency)

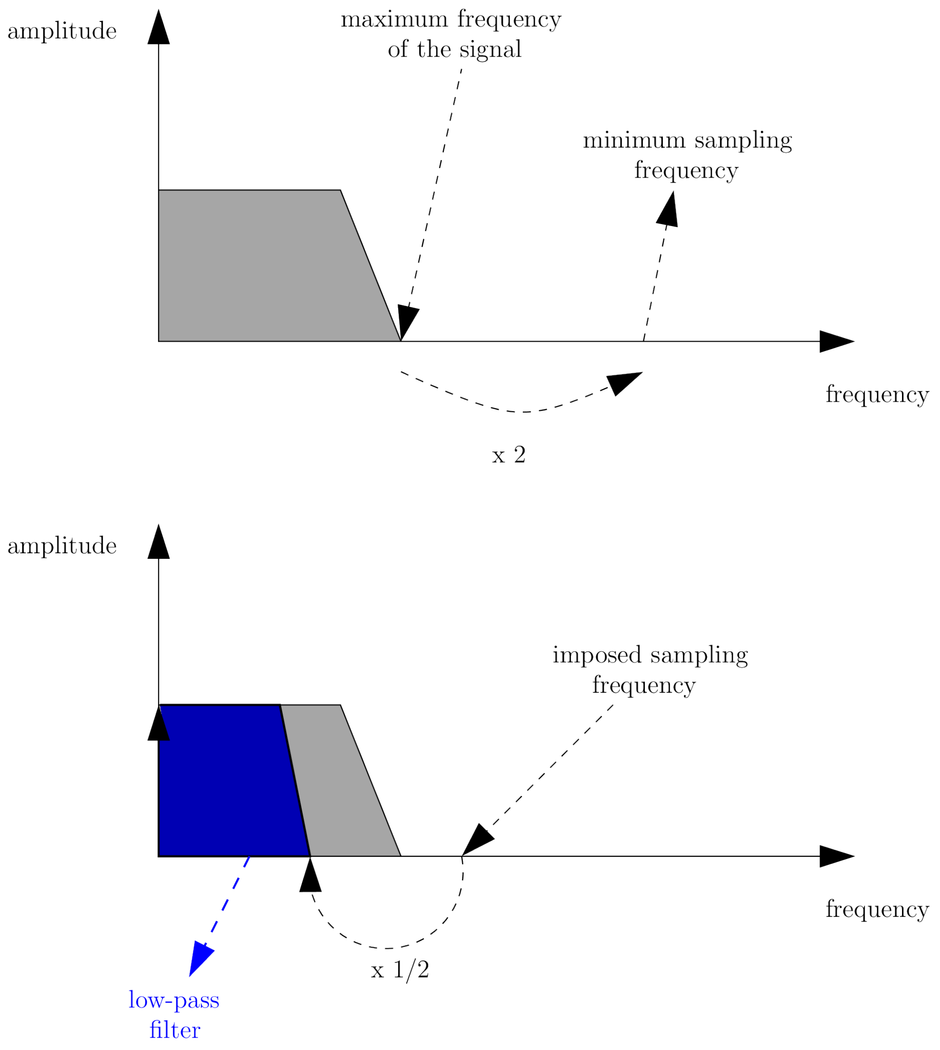
- In sampling the independent variable is time but it can also be space
- In the example the spatial frequency of the lines on the door is higher than the sensor resolution
- Production of aliasing (called Moiré for pictures)

Sampling: summary

- $f_a < \frac{1}{2} f_s$

- **$\max(f_a)$ is known**
  - select $f_s > 2 \cdot \max(f_a)$

- **f_s is imposed**
  - analog low-pass filter signal with a cut-off frequency below $\frac{1}{2} f_s$



Analog ↔ digital

- Analog domain
  - f_s → sampling frequency
  - T_s → sampling period
  - f_a → analog frequency
  - t_a → analog time
- Digital domain
  - f_d → digital frequency $\left[-\frac{1}{2}; \frac{1}{2}\right]$
  - ω_d → digital angular frequency $[-\pi; \pi]$
  - n → digital time
  - f_N → Nyquist frequency $[-1; 1]$ (Matlab, Python : filter design)

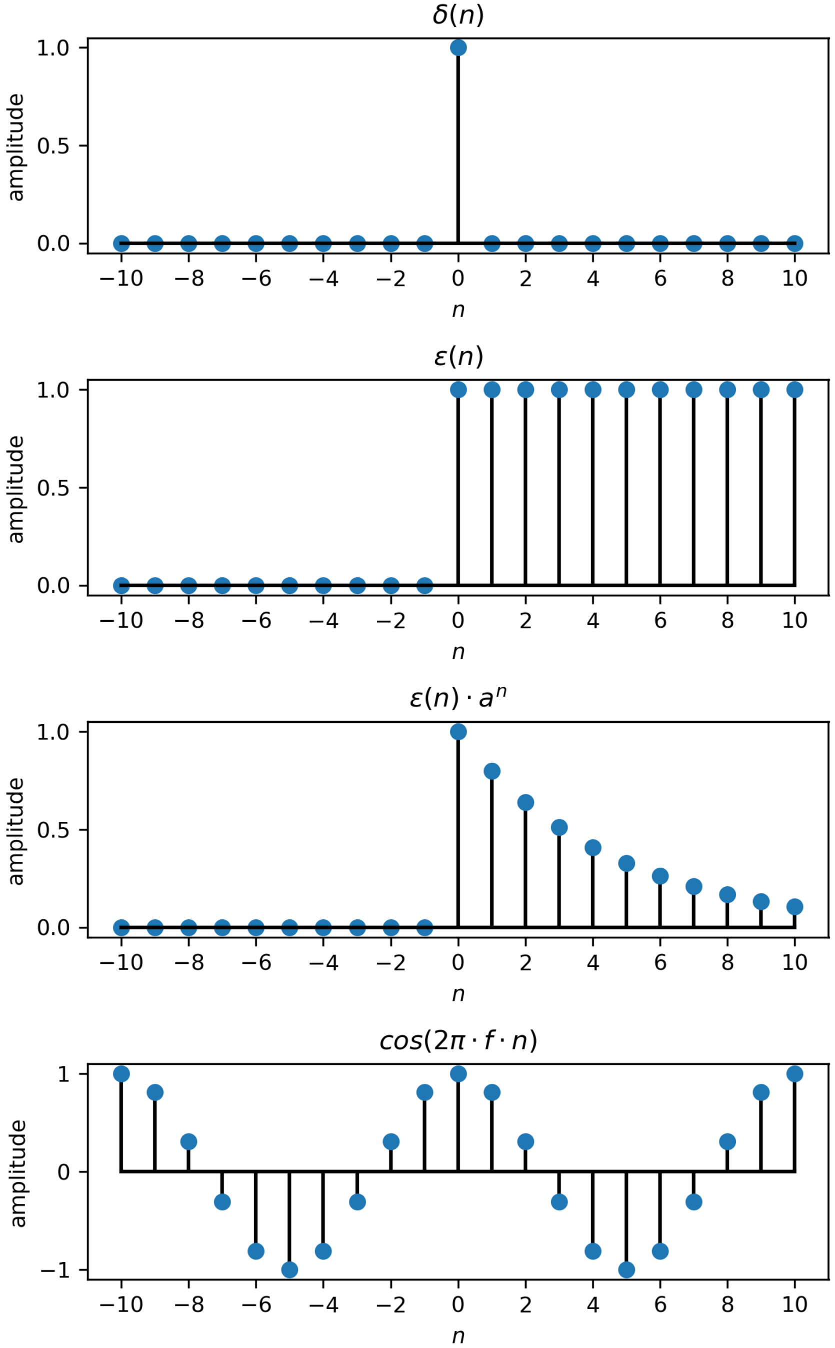
- $f_a = f_d \cdot f_s = f_N \cdot \frac{f_s}{2}$
- $t_a = n \cdot T_s = \frac{n}{f_s}$
- $\omega_d = f_d \cdot 2\pi = f_N \cdot \pi$
- $f_N = 2 \cdot f_d$

Typical exam questions

- An unknown signal is sampled at 6 Hz and its digital frequency corresponds to 1Hz. What are the four frequencies that can be aliased to this frequency?
- In a movie, with 25 frame per second, the wheel of a car is observed as rotating in reverse-to-normal direction at 1rps? What are the two first real rotation speeds that can produce this effect?

Basics signals

- Impulse (dirac)
  - $\delta(n) = \begin{cases} 1, n = 0 \\ 0, n \neq 0 \end{cases}$
- Step
  - $\varepsilon(n) = \begin{cases} 1, n \geq 0 \\ 0, n < 0 \end{cases}$
- Exponential
  - $\varepsilon(n) \cdot a^n$
- Sine and cosine
  - $\cos(2\pi \cdot f \cdot n)$



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Complex exponentials (Euler's notation)

- Processing of signals in spectral domain requires a projection on *sine* and *cosine* function
- Euler notation permits a more compact formulation of equations
- Euler notation permits a separation of amplitude and phase

$$\omega = 2\pi \cdot f$$

$$e^{j\cdot\omega\cdot k} = \cos(\omega \cdot k) + j \cdot \sin(\omega \cdot k)$$

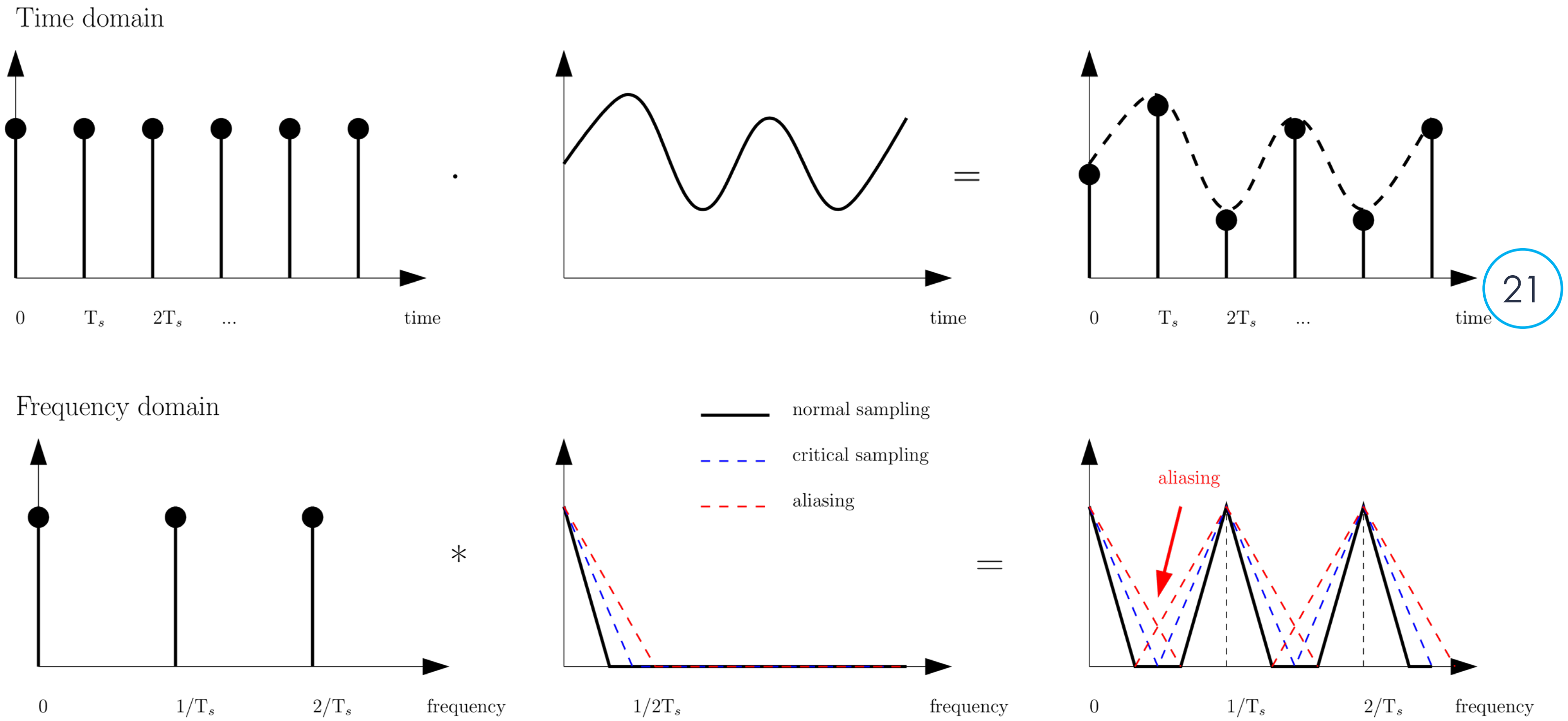
$$\cos(\omega \cdot k) = \frac{e^{j\cdot\omega\cdot k} + e^{-j\cdot\omega\cdot k}}{2}$$

$$\sin(\omega \cdot k) = \frac{e^{j\cdot\omega\cdot k} - e^{-j\cdot\omega\cdot k}}{2 \cdot j}$$

Fourier transform

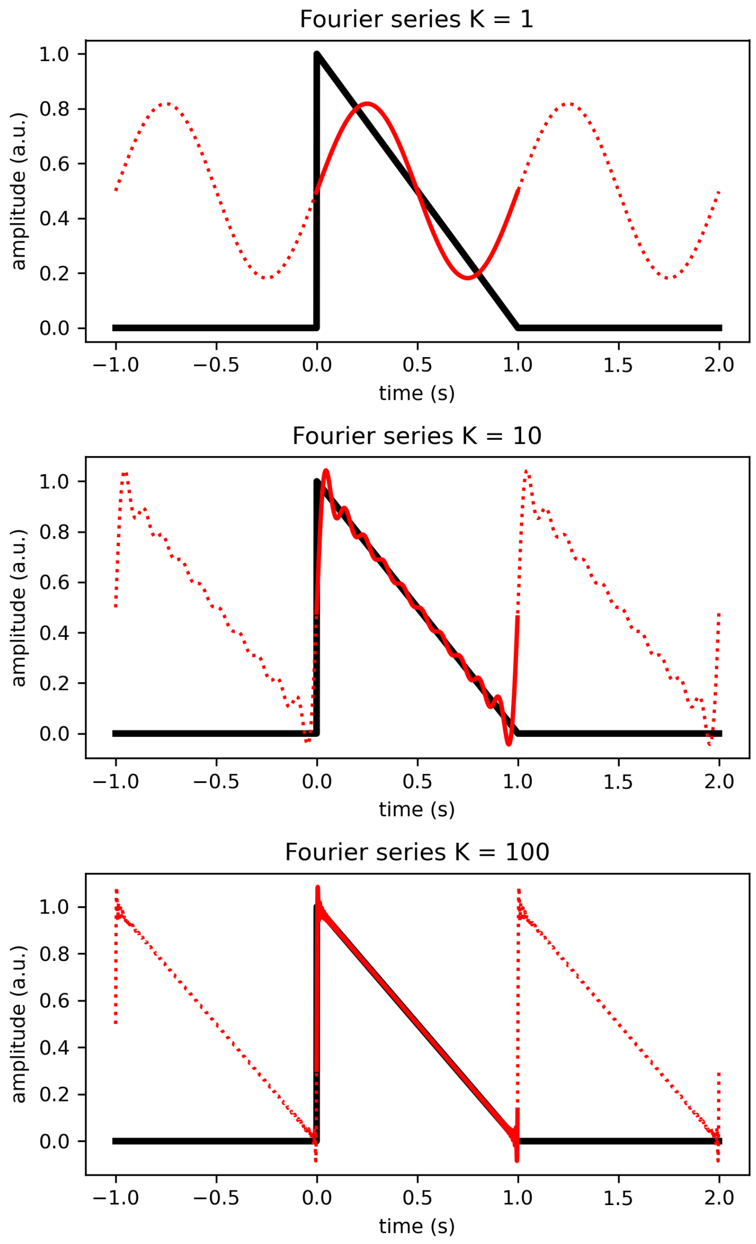
- $X(f) = \int_{-\infty}^{+\infty} x(t) \cdot e^{-j \cdot 2\pi \cdot f \cdot t} \mathrm{d}t$
  - time: continuous
  - frequency: continuous
- Convergence condition:
  - $\int_{-\infty}^{+\infty} (x(t))^2 \mathrm{d}t < \infty$
- Properties
  - $h(t) * x(t) \rightarrow H(f) \cdot X(f)$
  - $h(t) \cdot x(t) \rightarrow H(f) * X(f)$

Sampling in time domain and frequency domain



Fourier series

- $X(k) = \frac{1}{T}\int_0^T x(t) \cdot e^{-j\cdot\frac{2\pi}{T}\cdot k\cdot t}\mathrm{dt}$
- $\hat{x}(t) = \sum_{k=-K}^{K} X(k) \cdot e^{j\cdot\frac{2\pi}{T}\cdot k\cdot t}$
  - time: continuous
  - frequency: discrete
  - $k \in \mathbb{N}$
- Signal is assumed to be periodical
  - $x(t) = x(t \pm p \cdot T)$



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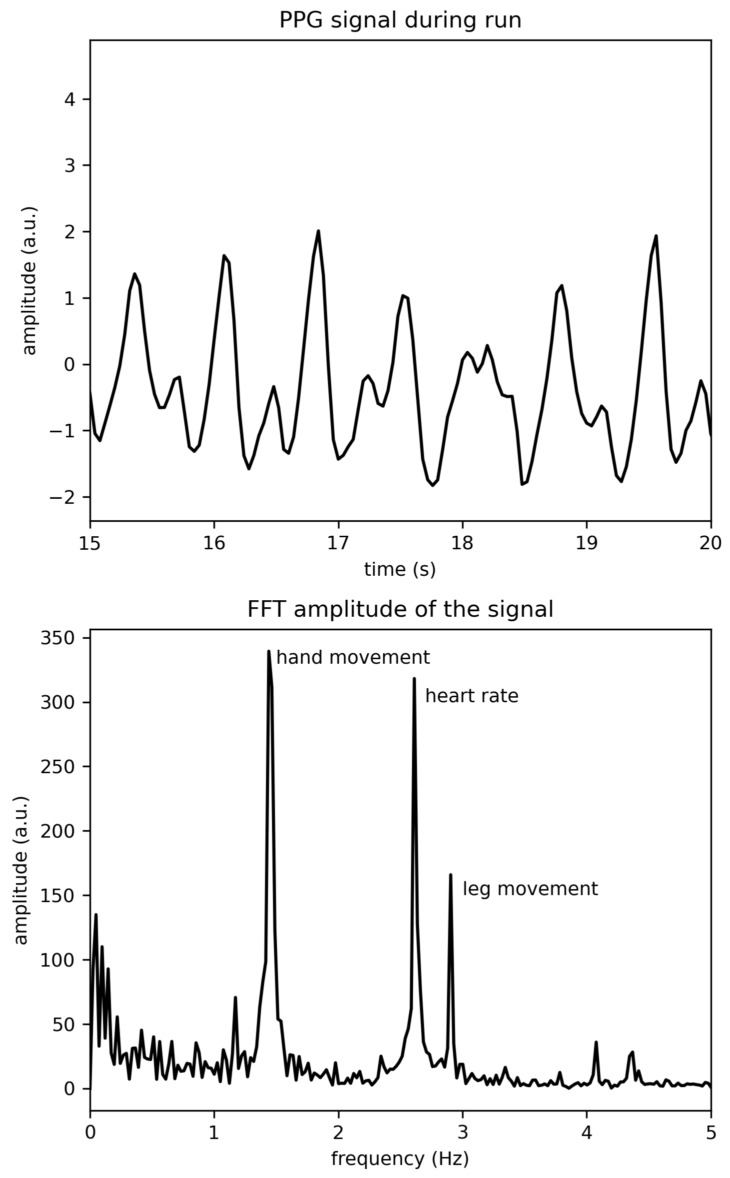
Discrete Fourier transform (DFT)

- $X(k) = \sum_{n=0}^{N-1} x(n) \cdot e^{-j \cdot \frac{2\pi}{N} \cdot k \cdot n}$
  - time: discrete
  - frequency: discrete
  - $k \in [0, N-1] \cap k \in \mathbb{N}_+$
  - $f_d = \frac{k}{N}$
    $f_a = \frac{k}{N} \cdot f_s$
- $x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) \cdot e^{j \cdot \frac{2\pi}{N} \cdot k \cdot n}$
- Complexity:
  - $N \neq 2^F \rightarrow DFT \rightarrow O(N^2)$
  - $N = 2^F \rightarrow FFT \rightarrow O(N \cdot \log(N))$
    (Fast Fourier Transform)



Discrete Fourier transform: properties

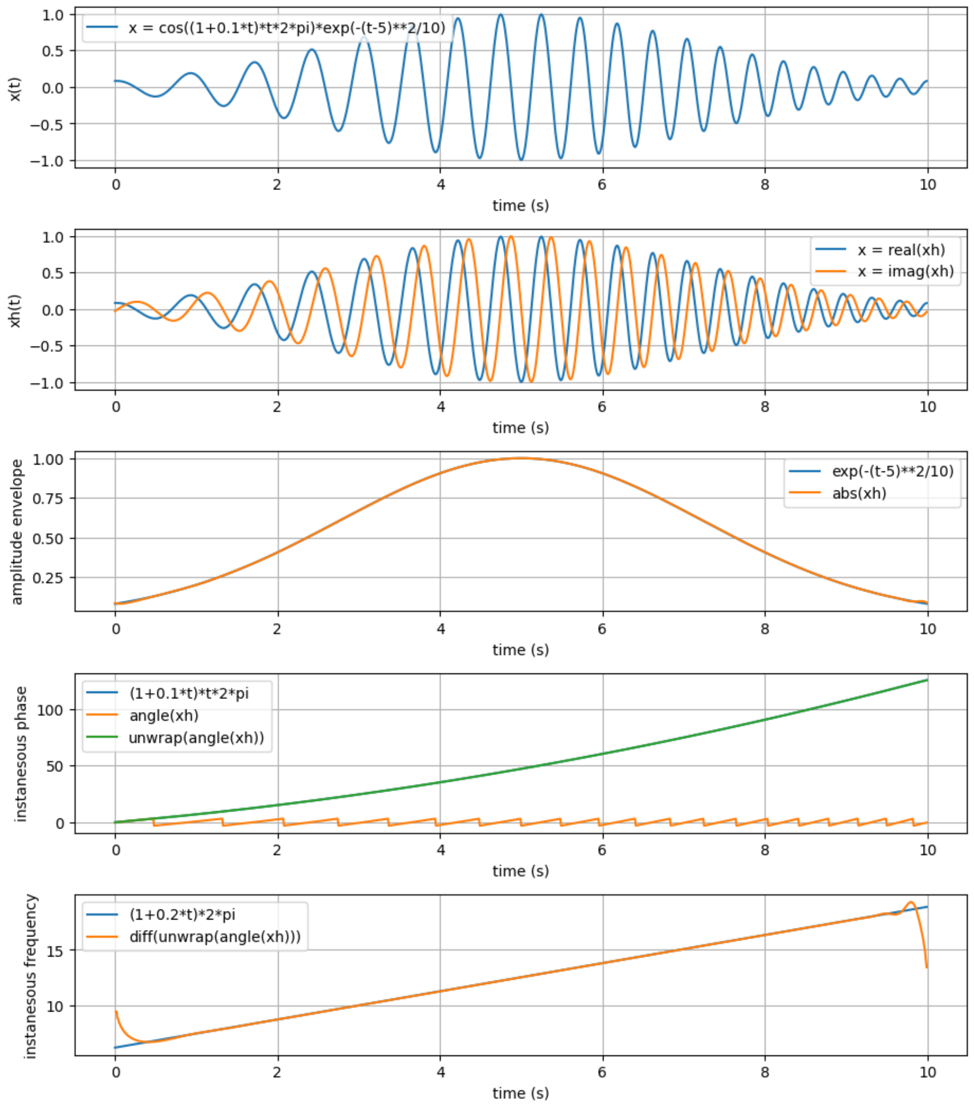
- Convolution
  - $h(n) * x(n) \rightarrow H(k) \cdot X(k)$
  - $h(n) \cdot x(n) \rightarrow H(k) * X(k)$
- Time shift
  - $\mathrm{DFT}\big(x(n-n_0)\big) = e^{-j\cdot\frac{2\pi}{N}\cdot k\cdot n_0} \cdot X(k)$
- $x(n) \in \mathbb{R}$
  - $X(-k) = X^*(k)$
  - $|X(-k)| = |X(k)|$
- Linearity
  - $\mathrm{DFT}\big(a\cdot x(n) + b\cdot y(n)\big) = a\cdot X(k) + b\cdot Y(k)$
- Amplitude spectrum
  - $|X(k)| = \sqrt[2]{X(k)\cdot X^*(k)}$
- Phase spectrum
  - $\varphi(X(k)) = \mathrm{angle}(X(k))$

Hilbert Transform

- **Fourier transform:**
- $x(t) \rightarrow \ X(f) = \int_{-\infty}^{+\infty} x(t) \cdot e^{-j\cdot 2\pi\cdot f\cdot t}\mathrm{d}t \rightarrow x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(\omega) \cdot e^{j\cdot 2\pi\cdot f\cdot t}\mathrm{d}t$
- $\cos(\omega \cdot t) \rightarrow \frac{1}{2}\big(\delta(-\omega) + \delta(\omega)\big) \rightarrow \cos(\omega \cdot t)$
- $\sin(\omega \cdot t) \rightarrow \frac{1}{2}j\big(\delta(-\omega) - \delta(\omega)\big) \rightarrow \sin(\omega \cdot t)$
- **Hilbert transform:**
- $x(t) \rightarrow \ X(f) = \int_{-\infty}^{+\infty} x(t) \cdot e^{-j\cdot 2\pi\cdot f\cdot t}\mathrm{d}t \rightarrow x_H(t) = \frac{1}{\pi}\int_{0}^{+\infty} X(\omega) \cdot e^{j\cdot 2\pi\cdot f\cdot t}\mathrm{d}t$
- $\cos(\omega \cdot t) \rightarrow \frac{1}{2}\big(\delta(-\omega) + \delta(\omega)\big) \rightarrow \cos(\omega \cdot t) + \mathrm{j} \cdot \sin(\omega \cdot t)$
- $\sin(\omega \cdot t) \rightarrow \frac{1}{2}j\big(\delta(-\omega) - \delta(\omega)\big) \rightarrow \sin(\omega \cdot t) - \mathrm{j} \cdot \cos(\omega \cdot t)$

Hilbert Transform Properties

- $x(t) = a \cdot \cos(\omega \cdot t)$
- $x_H(t) = a \cdot (\cos(\omega \cdot t) + \mathrm{j} \cdot \sin(\omega \cdot t)\ )$
- $|x_H(t)| = a \cdot (\cos^2(\omega \cdot t) + \sin^2(\omega \cdot t)) = a$
- $\measuredangle(x_H(t)) = \tan^{-1}\frac{\sin(\omega \cdot t)}{\cos(\omega \cdot t)} = \omega \cdot t$
- $\frac{d}{dt}\measuredangle(x_H(t)) = \frac{d}{dt}(\omega \cdot t) = \omega$
- **HILBERT transform is useful only for narrow band signals**



time ↔ frequency: summary

- Digital signal can be transformed from/to frequency domain using DFT or FFT
- **Convolution** in one domain is **multiplication** in the other domain
- The **spectral resolution** depends on the length of the DFT/FFT
- The DFT/FFT permits to analyze the **amplitude** and the **phase** of the signal

Typical exam question

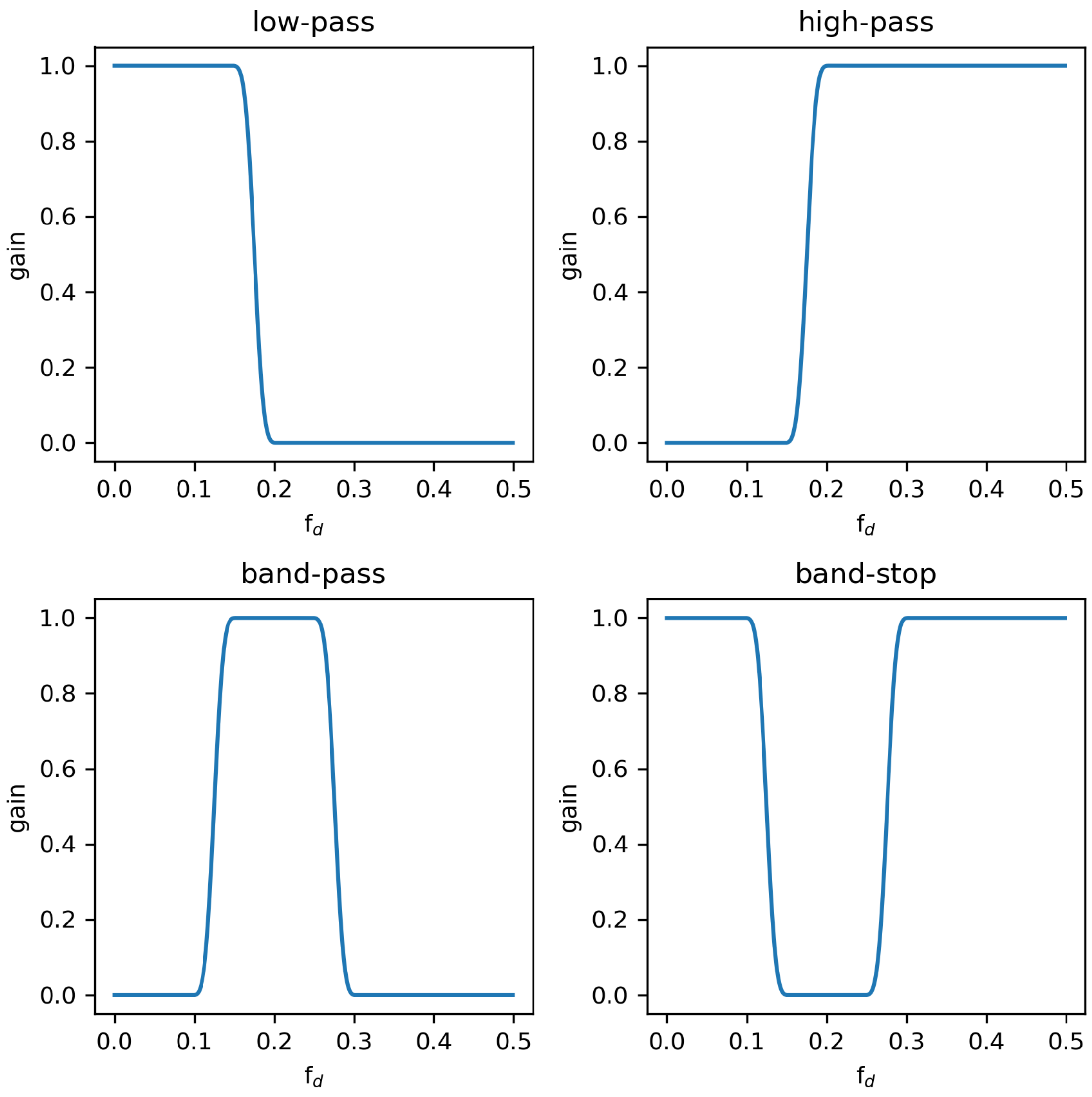
- In an ECG signal, sampled at 100 Hz, you want to estimate the heart rate with a resolution of ½ bpm using a DFT. What is the minimum length of the DFT that ensures this resolution?
- A jogger is using a smart watch to measure his heart rate that is 126 bpm. He is running at a cadence of 2 Hz. The optical signal is sampled at 10 Hz. With a DFT computed on 80 samples are you able to differentiate the peaks of the heart rate and the peak of the motion?

Digital filters

- Digital filtering is a linear operation that processes a numerical time series in order to attenuate some frequency intervals
- Digital filters are divided into two classes
  - Finite Impulse response (FIR)
    - Impulse response of the filter is finite
    - Filter is always stable
    - Larger number of coefficients to match IIR
  - Infinite Impulse response (IIR)
    - Impulse response of the filter is infinite
    - Coefficients must fulfill requirements to ensure stability
    - Smaller number of coefficients than FIR

Filter kinds

- There are four basic types of filters
  - Low-pass: high frequencies are attenuated
  - High-pass: low frequencies are attenuated
  - Band-pass: frequencies outside a frequency range are attenuated
  - Band-stop: frequencies within a frequency range are attenuated

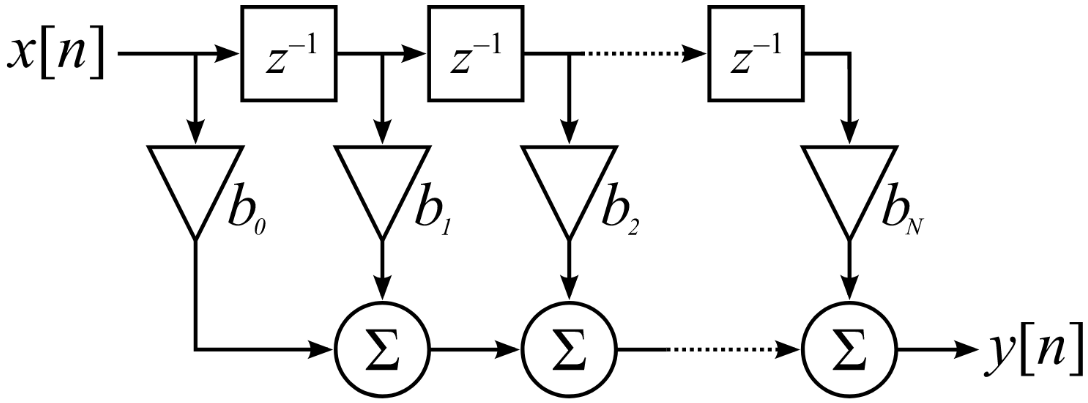


FIR and IIR filters

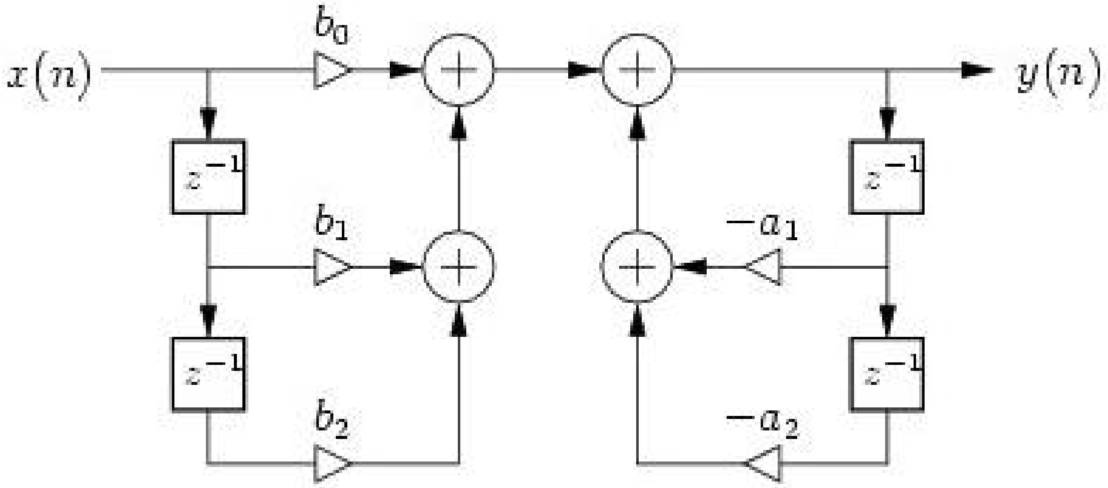
- FIR filter
  - $y(n) = \sum_{i=0}^{N} b_i \cdot x(n-i)$
  - output $y(n)$ depends only on the current and past values of the input $x(n)$
- IIR filter
  - $y(n) = \sum_{i=0}^{N_b} b_i \cdot x(n-i) - \sum_{i=1}^{N_a} a_i \cdot y(n-i)$
  - output $y(n)$ depends on the current and past values of the input $x(n)$ and on the past values of the output (**auto-regressive**)
- Cut-off frequency
  - The cut-off frequency is the frequency where the gain is -3dB or $1/\sqrt[2]{2}$

Z transform

- z^{-1} is defined as a delay of one sample
- FIR
  - $y(n) = \sum_{i=0}^{N} b_i \cdot x(n-i)$
  - $Y(z) = \left(\sum_{i=0}^{N} b_i \cdot z^{-i}\right) \cdot X(z)$
  - $\boldsymbol{Y(z) = B(z) \cdot X(z)}$



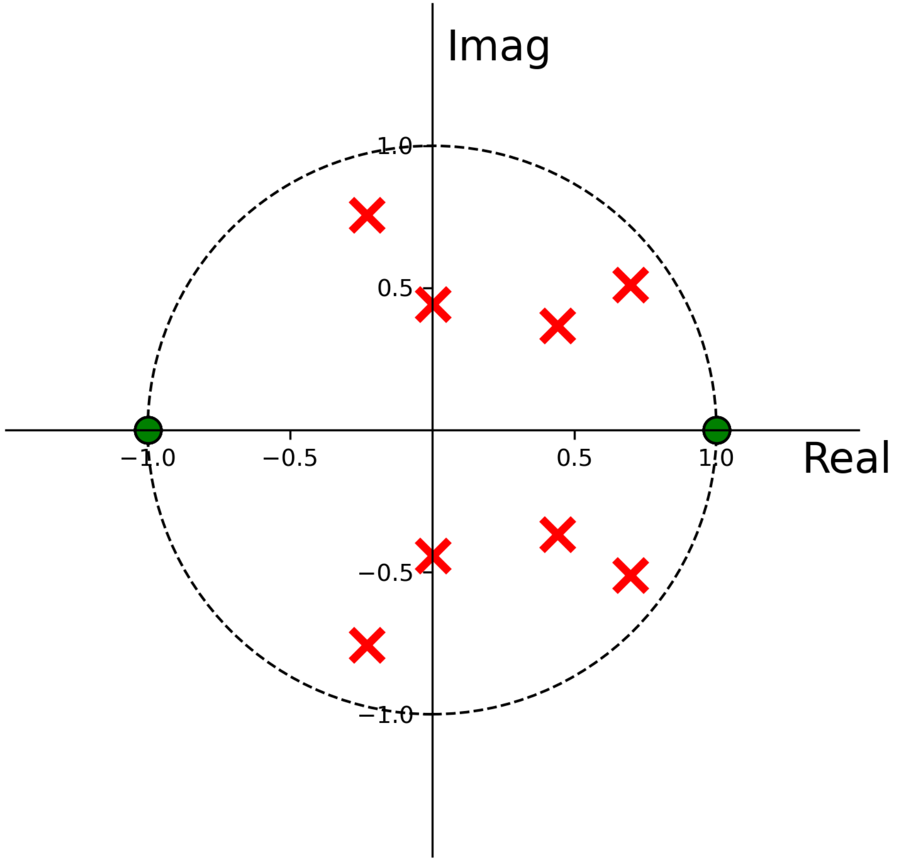
- IIR
  - $y(n) + \sum_{i=1}^{N_a} a_i \cdot y(n-i) = \sum_{i=0}^{N_b} b_i \cdot x(n-i)$
  - $A(z) \cdot Y(Z) = B(z) \cdot X(Z)$
  - $\boldsymbol{Y(Z) = \frac{B(z)}{A(z)} \cdot X(Z)}$



Z transform

- Filter transfer function
  - $H(z) = \frac{B(z)}{A(z)} = \frac{b_0 + b_1 \cdot z^{-1} + b_2 \cdot z^{-2} + \cdots}{1 + a_1 \cdot z^{-1} + a_2 \cdot z^{-2} + \cdots}$
- Numerator and denominator are polynomials of z. The polynomials can be factorized.
  - $H(z) = \frac{k_b \cdot \prod_{i=1}^{N_b} 1 - z_i \cdot z^{-1}}{\prod_{i=1}^{N_a} 1 - p_i \cdot z^{-1}}$
  - z_i are the zeros of transfer function
  - p_i are the poles of transfer function
  - k_b is the gain of the numerator
  - if a pole (zero) has complex value its complex conjugate is also a pole (zero)
- **The filter is stable if $|p_i| < 1 \; \forall \, i$**

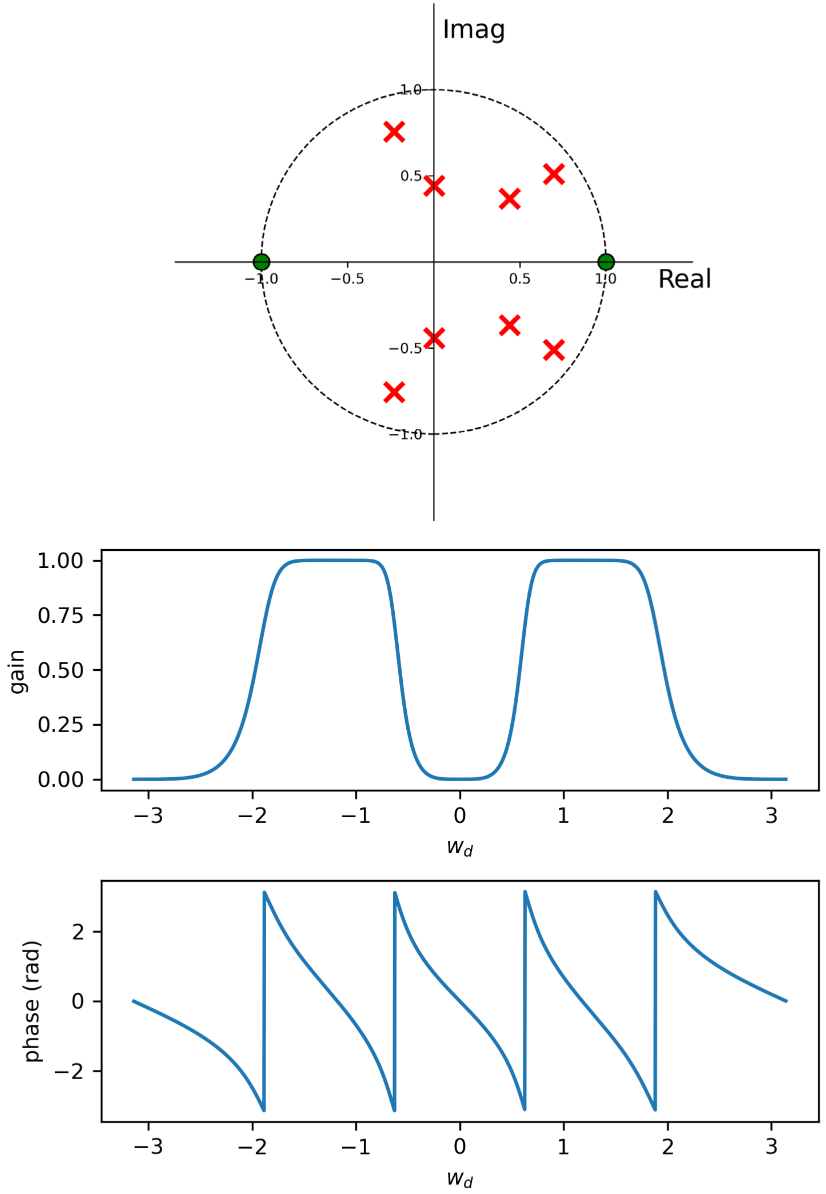
Band-pass filter [0.1, 0.3], $N_b = 8, N_a = 8$



Z transform (frequency response)

- $H(z) = \frac{B(z)}{A(z)}$
  - z^{-1} is a delay
  - $\mathrm{DFT}\big(x(n-n_0)\big) = e^{-j\cdot\frac{2\pi}{N}\cdot k\cdot n_0} \cdot X(k)$
  - $z = e^{j\omega}$ for frequency response
- $H(e^{j\omega_d}) \rightarrow$ frequency response
  - $|H(e^{j\omega_d})| \rightarrow$ amplitude response
  - $\mathrm{angle}\big(H(e^{j\omega_d})\big) \rightarrow$ phase response
  - $\omega_d \in [-\pi, \pi]$

Band-pass filter [0.1, 0.3], $N_b = 8, N_a = 8$



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Z transform (impulse response)

- FIR impulse response
  - $y(n) = \sum_{i=0}^{N} b_i \cdot \delta(n-i)$
  - $y(n) = [b_0, b_1, \cdots, b_N, 0, 0 \cdots]$
  - The impulse response of a FIR filter is given by the coefficients of the filters
- IIR impulse response
  - $y(n) = \sum_{i=0}^{N_b} b_i \cdot \delta(n-i) - \sum_{i=1}^{N_a} a_i \cdot y(n-i)$
  - The computation of the impulse response implies to solve the difference equation
  - The impulse response can be directly computed from the Z transform by implementing the division
  - $H(z) = \frac{B(z)}{A(z)} = \frac{b_0 + b_1 \cdot z^{-1} + b_2 \cdot z^{-2} + \cdots}{1 + a_1 \cdot z^{-1} + a_2 \cdot z^{-2} + \cdots}$

Z transform IIR impulse response (example)

- $H(z) = \frac{B(z)}{A(z)} = \frac{1}{1-a\cdot z^{-1}}$
- Implementing the division of the fraction permits to get the impulse response of IIR filters
- IIR filter can be approximated by a FIR by truncating the impulse response
- $a < 1 \rightarrow$ decaying exponential
  - **stable**
- $a = 1 \rightarrow$ step function
  - **unstable**
- $a > 1 \rightarrow$ increasing exponential
  - **unstable**

$$1 \;\Big|\; 1 - a\cdot z^{-1}$$

$$-1 + a\cdot z^{-1}$$

$$a\cdot z^{-1} \qquad\qquad 1 + a\cdot z^{-1} + a^2\cdot z^{-2} + a^3\cdot z^{-3} + \ldots$$

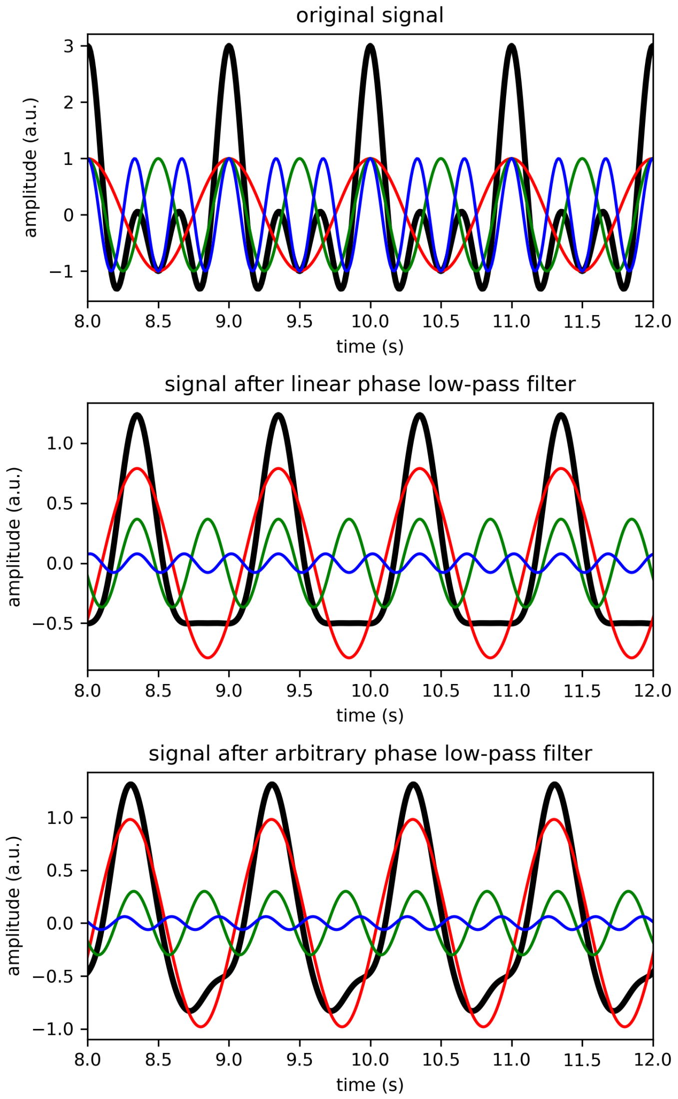
$$-a\cdot z^{-1} + a^2\cdot z^{-2}$$

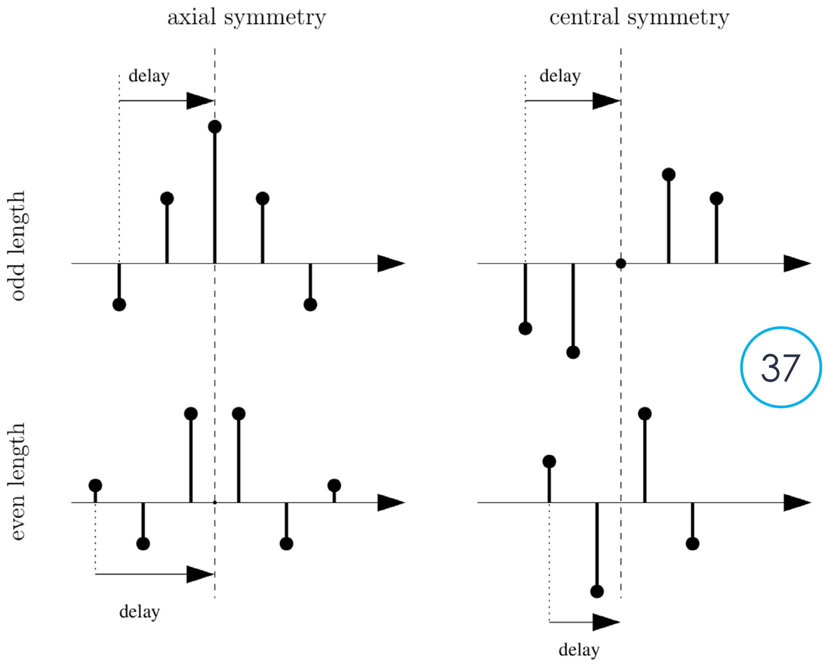
$$a^2\cdot z^{-2}$$

$$\ldots$$

Linear phase filters

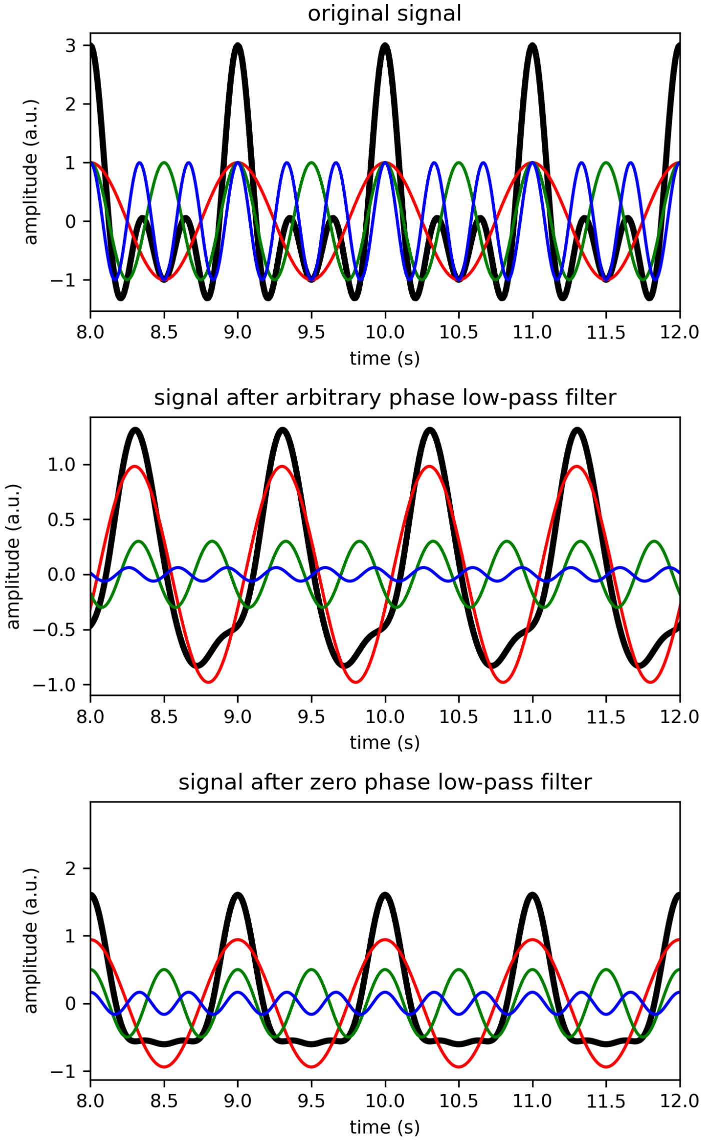
- **Linear phase filters** permit a **constant delay** of all the components of a signal
- Linear phase filter **can only be obtained with FIR filters**
- FIR filters with linear phase have to exhibit either an **axial symmetry** or a **central symmetry** in their coefficients
- Axial symmetry -> low-pass
- Central symmetry -> high or band pass





Zero-phase filters

- Zero-phase filter can be obtained with any filter
- Zero-phase filtering consist in two pass filtering with inversion of the time
  - $z(n) = h(n) * x(-n)$
  - $y(n) = h(n) * z(-n)$
- After zero phase filtering all the components of the signal have a delay of 0
- Zero phase filtering **can only be applied for analysis** because the **full time-range** of the signal **has to be available**
  - **(not for real time processing)**



Filters: summary

- Two categories of linear filters exists:
  - FIR filters:
    - Always stable
    - Linear phase filters (central or axial symmetry)
    - Larger number of coefficients to match IIR filters
  - IIR filters:
    - Stable only when the radius of all poles is smaller than 1
    - Smaller number of coefficients
    - No linear phase
  - Zero-phase filter:
    - Any linear filter can be used as a zero-phase filter by applying a two-step filtering
    - Only for post analysis of signals but not for real-time

Typical exam question

- A digital filter is given by

$$H(z) = \frac{1-z^{-2}}{1+1/4 \cdot z^{-2}}$$

- What are the poles and zeros of the filter?
- What kind of filter is it?
- What are the 4 first term of the impulse response?

Labo exercises

- 3 exercices
  - l01_ecg_enhance.py
  - l02_breathing_estimation
  - l03_hand_washing_detection.py
- Groups of 3 pax (2 or 4 if mod(num. people,3)$\neq 0$)
  - One report for the group
    - Names and surnames of group's participants
    - one section per exercise
      - discuss results
      - answer questions
    - naming: **name1_name2_name3_labo1.pdf**
    - *optional: at the end of the document free comment about curse and exercises*
- upload the **same report for each person individually** (delay:1 week)

Questions →

Figure →

```
# import numerical processing library
"""
    The objective of this exercise is that you analyse the code provided and
    make the link with the curse. You have to provide a short report that
    comments and analyse the results. You can use directly the results or adapt
    them to you needs.

"""

# import the numerical library
import numpy as np
# import signal processing library
import scipy.signal as sp
# import ploting library
import pylab as py
py.ion()
py.close('all')

# load the ecg signal
x = np.genfromtxt('respiration.dat')
# sampling frequency of the signal is 500 Hz
fs = 2
# generate correponding time vector
t = np.arange(len(x))/fs

"""
    The signal is a measurement of the breathing obtained by inductance
    plethysmography.

    The objective is to estimate the breathing frequency.
"""

"""
    The Hilbert transforms permits to estimate the instaneous amplitude and
    phase of a narrow band signal.

    Q: Comment the figures.
    Q: Why the envelope does no follow the maxima of the signal

"""

# compute the analytical signal of x (Hilbert transform)
xa = sp.hilbert(x)

# plot the signal
py.figure(1, figsize=[5,5])
py.clf()
py.plot(t, x, label='breathing signal')
py.plot(t, np.abs(xa), label='envelop')
py.xlabel('time (s)')
py.ylabel('amplitude (a.u.)')
py.legend(loc='upper right')
py.title('Breathing signal')

"""
    The raw breathing signal does not fullfil the requirement of narrow band.
    The normal range of frequency for the breathing is within 0.1 to 0.25 Hz.
    The signal is first filtered for this interval.

    Q: Comment the figures
    Q: How is the estimation of the amplitude envelope.

"""


# Analogic limit of the passband frequency
f_pass = np.array([0.1, 0.25])
# Analogic limit of the stopband frequency
f_stop = np.array([0, 0.6])
# Convertion into Nyquist frequency
f_pass_N = f_pass/fs*2
f_stop_N = f_stop/fs*2
# Max attenutation in passband (dB)
```


EPFL csem

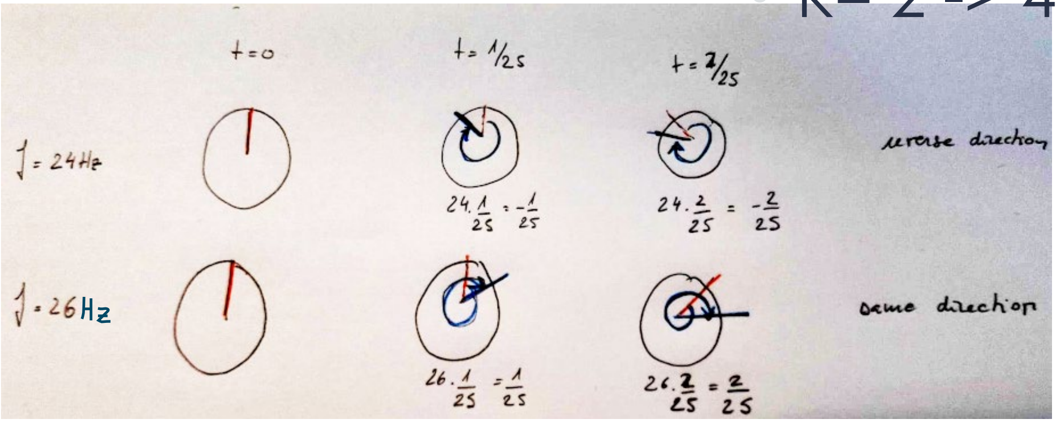
Typical exam questions (answer)

- An unknown signal is sampled at 6 Hz and its digital frequency is 1Hz. What are the four frequencies that can be aliased to this frequency?

- $\mathbf{mod}(|\boldsymbol{f} - \boldsymbol{k} \cdot \boldsymbol{f}_{\boldsymbol{s}}|, {}^{f_s}/_2) = |\boldsymbol{f} - \boldsymbol{k} \cdot \boldsymbol{f}_{\boldsymbol{s}}|, \boldsymbol{k} \in \mathbb{N}^*$
- K = 1 -> 5Hz, 7Hz
- K = 2 -> 11Hz, 13Hz

- In a movie, with 25 frame per second, the wheel of a car is observed as rotating in reverse-to-norm direction at 1rps? What are the two first real rotation speeds that can produce this effect?

- $\boldsymbol{f} - \boldsymbol{k} \cdot \boldsymbol{f}_{\boldsymbol{s}}$ = -1
- K= 1 -> 24 rps
- K= 2 -> 49 rps



Typical exam question

- In an ECG signal, sampled at 100 Hz, you want to estimate the heart rate with a resolution of ½ bpm using a DFT. What is the minimum length of the DFT that ensures this resolution?

- ½ bpm = 0.5/60 = 1/120 Hz
- NFFT = 100/(1/120) = 12'000 samples

- A jogger is using a smart watch to measure his heart rate that is 126 bpm. He is running at a cadence of 2 Hz. The optical signal is sampled at 10 Hz. With a DFT computed on 80 samples are you able to differentiate the peaks of the heart rate and the peak of the motion?

- 126 bpm = 126/60 = 2.1 Hz
- Frequency difference = 2.1 – 2.0 = 0.1Hz
- DFT resolution = 10/80 = 0.125 Hz
- The DFT resolution is not suficcient

EPFL csem

Typical exam question

- A digital filter is given by

$$H(z) = \frac{1-z^{-2}}{1+1/4\cdot z^{-2}}$$

- What are the poles and zeros of the filter?
  - $1 - z^{-2} = 0 \rightarrow zeros = \{-1, 1\}$
  - $1 + \frac{1}{4} \cdot z^{-2} = 0 \rightarrow poles = \{-\frac{1}{2}i, \frac{1}{2}i\}$
- What kind of filter is it?
  - The filter has 1 zero at 1 -> high-pass
  - The filter has 1 zero at -1 -> low-pass
  - The filter is a band-pass filter

- What are the 5 first terms of the impulse response?

$$1 - z^{-2} \quad \Big| \quad 1 + 1/4 \cdot z^{-2}$$

$$-1 - 1/4 \cdot z^{-2}$$

$$-5/4 \cdot z^{-2}$$

$$5/4 \cdot z^{-2} + 5/16 \cdot z^{-4}$$

$$5/16 \cdot z^{-4}$$

$$\ldots$$

$$1 + 0 \cdot z^{-1} - 5/4 \cdot z^{-2} + 0 \cdot z^{-3} + 5/16 \cdot z^{-4} \ldots$$

The first terms of the impulse response are
[1, 0, -5/4, 0, 5/16]