Solutions: Session 10

Exercise 1

Answers:

1) In the diagram representing the pressure insole (Fig. 2), we can see 3 different plates that effectively function as two capacitors. When plate 1 is displaced, it brings about a change in the area "seen" by the other capacitors. We already saw in the course that this double-capacitor system can be conditioned with the circuit below (Fig. 3).

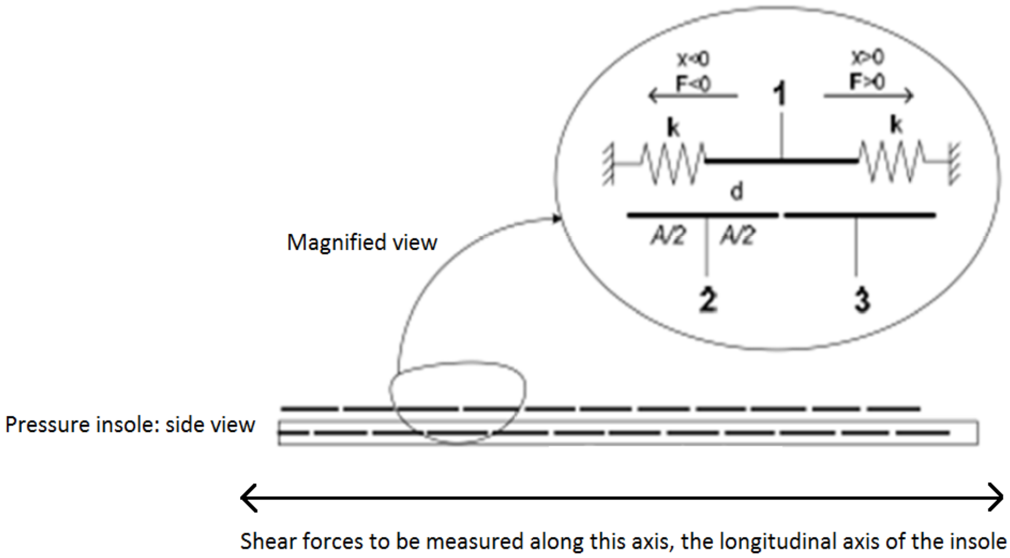


Figure 2: representation of the pressure insole

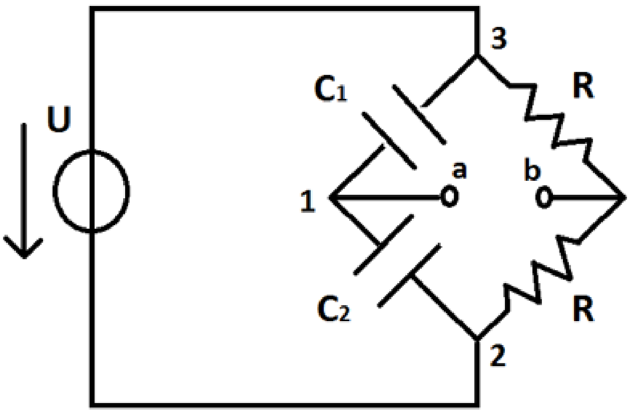


Figure 3: Conditioning bridge

Note that plate 1 in Fig. 2 has been represented as two plates in Fig. 3 for clarity.

2) Let us start by analysing what happens when the insole experiences a shear force F along its longitudinal axis. The effect of this force on the two springs attached to plate 1 in Fig. 1 is shown in Fig. 4 below (where the plate, being a rigid object, has been removed for clarity).

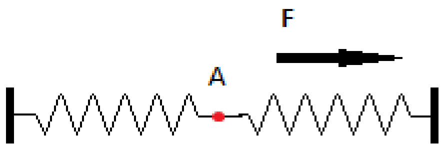


Figure 4: Springs in series

When we apply a force F on point A, we will get a displacement:

$$F_{k1} + F_{k2} = F$$
$$k_1 x_1 + k_2 x_2 = F$$
$$Given\ that\ k_1 = k_2 = k$$
$$\therefore\ x_1 = x_2 = x = \frac{F}{2k}$$

This will enable us to get the capacitance of each sensor. Let us recall the definition of capacitance:

$$Capacitance = C = \varepsilon \frac{Surface\ area\ common\ to\ both\ plates}{Distance\ between\ plates}$$

Here, $\varepsilon = \varepsilon_r \varepsilon_o$, with ε_o being the permittivity of vacuum. Let the side length of each square-shaped capacitor be a. Thus, $A = a \times a$. Let plate 1 be displaced to the left (refer Fig. 2) by an amount $x = \frac{F}{2k}$. Then, due to a change in the surface area of plate 1 “seen” by the plates 3 and 2, the capacitance values C_1(formed by plates 1 and 3) and C_2 (formed by plates 1 and 2) will change in the following way:

$$C_1 = \varepsilon * \frac{a * \left(\frac{a}{2} - x\right)}{d} = \varepsilon \frac{a^2}{2d} - \frac{\varepsilon a F}{2dk}$$
$$C_2 = \varepsilon * \frac{a * \left(\frac{a}{2} + x\right)}{d} = \ \varepsilon \frac{a^2}{2d} + \frac{\varepsilon a F}{2dk}$$

We can then obtain the measured voltage, U_o from Fig. 3 as:

$$U_0 = U_a - U_b$$
$$U_a = \frac{\frac{1}{j\omega C_2}}{\frac{1}{j\omega C_1} + \frac{1}{j\omega C_2}} U = \frac{\frac{1}{C_2}}{\frac{C_2 + C_1}{C_2 C_1}} U$$
$$U_b = \frac{R}{R + R} U = \frac{1}{2} U$$
$$U_0 = \left(\frac{\frac{1}{C_2}}{\frac{C_2 + C_1}{C_2 C_1}} - \frac{1}{2} \right) U$$

$$U_0 = \frac{1}{2} * \frac{C_1 - C_2}{C_2 + C_1} U$$

$$U_0 = -\frac{1}{2} * \frac{\frac{\varepsilon a F}{dk}}{\frac{\varepsilon a^2}{d}} U = -\frac{FU}{2ka}$$

The sensitivity in terms of the supply voltage U can then be written as:

$$S' = \frac{U_0}{F} = -\frac{U}{2ka}$$

Expressed in units of $V/V_{supply}/N$, the sensitivity S becomes:

$$S = \frac{S'}{U} = -\frac{1}{2ka} = \boldsymbol{-5\ mV/V_{supply}/N}$$

Exercise 2

Answers

a) When $\Delta P = 0$, both capacitors have the same capacitance, given by:

$$C_o = \frac{A\epsilon}{x_o}$$

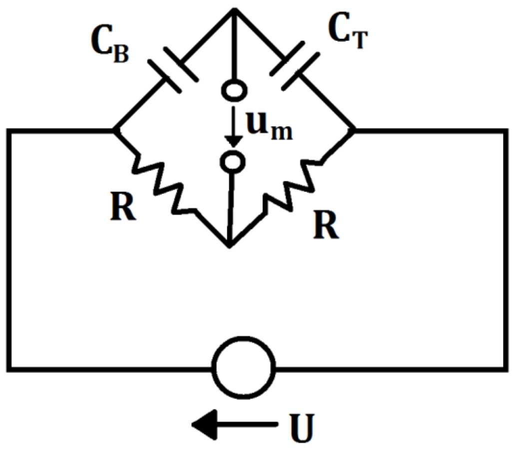
Consider that the mobile frame is displaced upwards by Δx so that the distance between the top fixed plate and the central movable plate reduces to $x_o - \Delta x$. The capacitance C_T between the top fixed plate and the central movable plate becomes:

$$\boldsymbol{C_T} = \frac{A\epsilon}{x_o - \Delta x} = \frac{A\epsilon(x_o + \Delta x)}{(x_o - \Delta x)(x_o + \Delta x)} = \frac{A\epsilon}{x_o}\left(1 + \frac{\Delta x}{x_o}\right) = \boldsymbol{C_o\left(1 + \frac{\Delta x}{x_o}\right)}$$

At the same time, this means that the distance between the bottom fixed plate and the central movable plate increases to $x_o + \Delta x$. Thus, the capacitance C_B between the bottom fixed plate and the central movable plate becomes:

$$\boldsymbol{C_B} = \frac{A\epsilon}{x_o + \Delta x} = \frac{A\epsilon(x_o - \Delta x)}{(x_o + \Delta x)(x_o - \Delta x)} = \frac{A\epsilon}{x_o}\left(1 - \frac{\Delta x}{x_o}\right) = \boldsymbol{C_o\left(1 - \frac{\Delta x}{x_o}\right)}$$

b) The scenario described in part (a) above can be represented schematically as shown in the figure below:



From this, we can write an expression for u_m as follows:

$$\boldsymbol{u_m} = u\left(\frac{C_T}{C_T + C_B} - \frac{R}{2R}\right) = u\left(\frac{C_o\left(1 + \frac{\Delta x}{x_o}\right)}{C_o\left(1 + \frac{\Delta x}{x_o} + 1 - \frac{\Delta x}{x_o}\right)} - \frac{1}{2}\right) = \frac{\text{u}\Delta x}{2x_o} = \frac{\text{u}\Delta P}{4x_o} = \frac{\boldsymbol{uQ}}{\boldsymbol{40x_o}}$$

From this, we find the sensitivity S to be:

$$\boldsymbol{S} = \frac{u_m}{uQ} = \frac{\mathbf{1}}{\boldsymbol{40x_o}} = \mathbf{2.5\ V_{rms}/(V_{rms}.\,m^3/s)}$$

c) From the expression found for u_m in part (b), we obtain an expression for Q and calculate it when $u_m = 10\ mV_{rms}$ as follows:

$$Q = \frac{40u_m x_o}{u} = \frac{40 \times 10^{-2} \times 10^{-2}}{1} = 4 \times 10^{-3}\ m^3/s$$

From the flow rate, we may calculate the mean cross-sectional velocity in the tube v_m as:

$$v_m = \frac{4Q}{\pi D^2}$$

where D is the diameter of the tube given to be 20 mm.

To check whether flow is laminar or not, we need to compute the Reynolds number, which is given by:

$$\boldsymbol{Nr} = \frac{\rho_{air} v_m D}{\eta_{air}} = \frac{\left(\rho_{air} \times \frac{4Q}{\pi D^2} \times D\right)}{\eta_{air}} = \frac{4\rho_{air} Q}{\pi \eta_{air} D} = \mathbf{17234}$$

The criterion for flow to be laminar is $Nr < 2000$. Since $Nr = 17234$ in our case, the flow is **not laminar**.

Exercise 3

The total amount of inert gas is the same at the beginning and end of the measurement, but its concentration has changed from C1=10% to C2=3.7%. At the beginning, it is confined to the inspired volume (V1) measured by pneumotachometer and at the end to the patient's TLC:

$$C_1 . V_1 = C_2 . TLC$$

$$V_1 = 10\ s \times 0.2\ l/s = 2\ l$$

$$TLC = 2\ l \times \left(\frac{10\ \%}{3.7\ \%}\right) = 5.4\ l$$