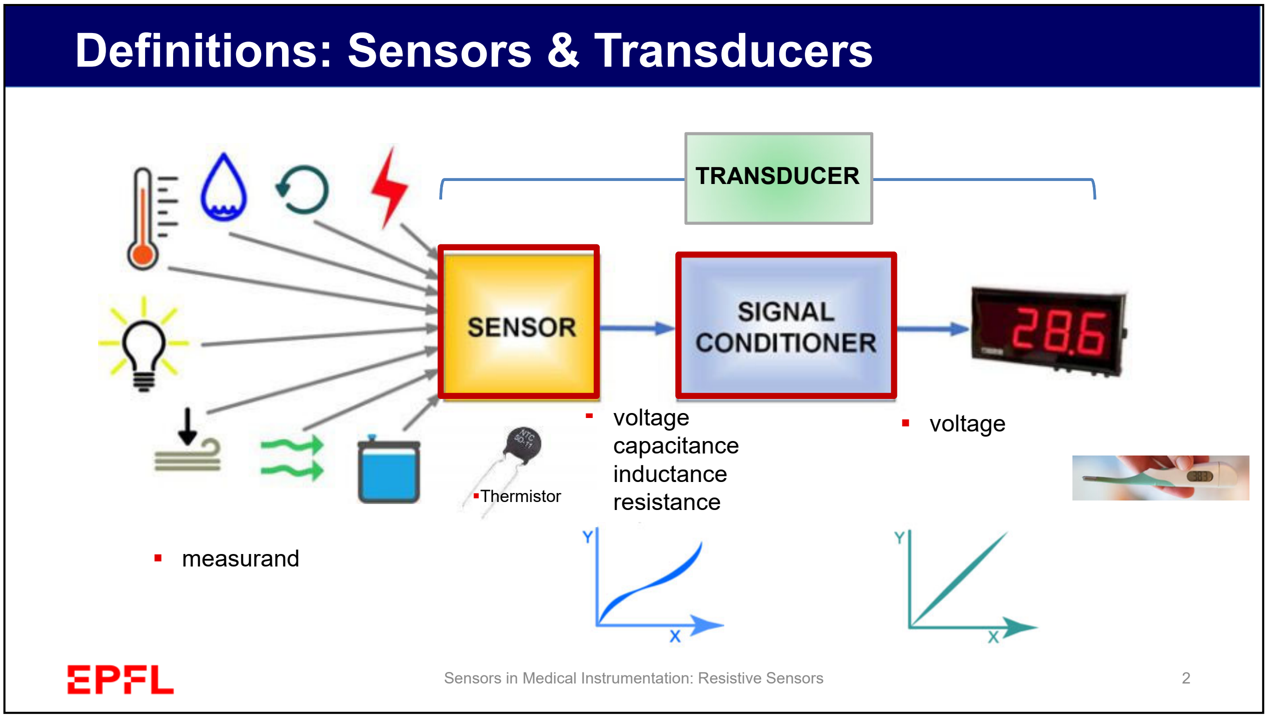
GENERAL INTRODUCTION

SENSORS

Definitions and Classification



Sensors detect physical properties of their environment, e.g., temperature, pressure, speed, called **measurands,** and translate them into electrical signal. This conversion typically involves changes in properties of electrical components - resistance, capacitance, inductance, or directly in current/voltage. For instance, a thermistor's resistance varies with temperature (R=f(T)).

Sensor output often requires **signal conditioning** (e.g., conversion, filtering, amplification, linearization), specific to the sensor type.

When integrated with conditioning and interface for display, the sensor becomes part of a **transducer.** For example, a standalone thermistor is a sensor, but within a larger circuit or device, it becomes a component of a transducer, such as a thermometer.

Classification based on the power requirement

- **Active sensors**: Convert a form of energy into another form (electrical) **without using an external source of power.**
  Examples: Electrodes, Piezoelectric

- **Passive sensors:** Convert a form of energy into another form (electrical) by **making use of an external source of power**.
  Examples: Resistive, Inductive, Capacitive

Sensors are often classified as **active or passive** based on whether they need an external power source to operate. **Active sensors** do **not** require an outside power supply to produce a signal; they directly convert the measured phenomenon's energy into an electrical output. For example, a thermocouple generates a voltage from a temperature difference on its own (no external excitation needed). **Passive sensors**, on the other hand, **do** require external energy or excitation to function. They typically modulate or respond to an applied electrical signal rather than self-generating one. For instance, a resistance temperature detector (RTD) needs a current passed through it to detect temperature via changes in resistance. **The key distinction is in energy dependency: active sensors are self-powered by the input they measure, whereas passive sensors depend on additional external power to operate.**

Active sensors

Measured variable	Physical effect	Output
temperature	thermoelectricity (Peltier-Seebeck effect)	voltage
biopotential, pH	redox	voltage
force, pressure acceleration, vibrations, sound	piezoelectricity	charge
speed, flow	magnetic induction	voltage
optical radiation flux	photovoltaic	voltage

Active sensors operate based on various physical phenomena, including:

- **Thermoelectricity (Peltier–Seebeck effect):** Enables the direct conversion of heat into electricity through the Seebeck effect, and electricity into heat via the Peltier effect, relying on two closely related thermoelectric phenomena.

  Note: Peltier modules (also called thermoelectric devices) are widely used commercially as heaters and coolers by utilizing the Peltier effect, in which applying an electric current creates a temperature difference between their two sides. Depending on their use, these modules can function either as active sensors or actuators. As actuators (Peltier effect), they require external electrical power to heat or cool and thus do not function as sensors. However, when used as temperature sensors (Seebeck effect), they spontaneously generate a voltage from a temperature gradient, functioning as active sensors without external power.

- **Redox reactions:** Involve converting ionic currents directly into electrical currents through electrochemical processes.
- **Piezoelectricity:** Generates electrical charges in response to mechanical stress or strain.
- **Magnetic induction:** Produces an electromotive force (voltage) across a conductor placed within a changing magnetic field, commonly known as Faraday’s law of induction.
- **Photovoltaic effect:** Generates voltage when a material absorbs light energy, exciting electrons and creating an electrical potential.

Passive sensors

Impedance

Measured variable	Sensitive characteristic	Comment
temperature	resistance	semiconductor: thermistor, metal: Pt, Ni
Strain- deformation	resistance	Strain gage: metal, semiconductor
force, pressure, acceleration, vibrations, sound, displacement	resistance, capacitance, inductance	potentiometer, microphone LVDT
humidity	resistance, capacitance	

<u>Passive sensors:</u> are mostly **impedances**, so need some power supply (voltage, current) to read the value proportional to the measurand. This table shows examples of physical variables that can change the properties of basic passive electrical components.

The <u>resistance</u> of a material can change in response to temperature, deformation, pressure, and force variations, so these variations can be measured by designing <u>resistive sensors</u>

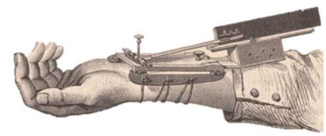
RESISTIVE SENSORS
& Applications

Introduction

Part I- Temperature sensors
Part II- Thermal mass flowmeter

Part III- Strain gage
Part IV- Direct measurement of arterial pressure
Part V- Indirect measurement of arterial pressure

Part VI- Force measurement: instrumented implant
Part VII- Force plate: application in biomechanics
Part VIII- Wearable force measurement: gait analysis



Sphygmograph de Marey, 1878

EPFL

This lecture covers various **resistive sensor technologies** applied in biomedical and biomechanical contexts. Part I introduces resistive **temperature sensors** (thermistors and RTDs) for monitoring temperature, and Part III focuses on **strain gauges** for measuring mechanical deformation. Part VI examines **force measurement using instrumented implants**, while Parts VII and VIII explore **force plates and wearable force sensors** in biomechanics. Together, these topics illustrate how resistive sensing elements (e.g., thermistors, RTDs, strain gauges) are used to measure temperature, strain, and force in different medical instrumentation devices/sytems.

The picture shows the **Marey Sphygmograph**, invented by **Dr. Étienne Jules Marey** in **1860**, a groundbreaking machine that allowed the **graphical recording of blood pressure**.

Introduction: Theory of operation

The resistance of a material depends on four factors:

- Composition
- Temperature
- Length
- Cross-sectional Area

$$R = \rho \frac{l}{A}$$

Possibility to measure:

- Temperature
- Deformation (due to Pressure, Force)

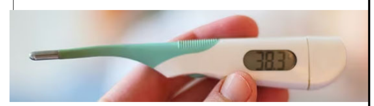
- When the length l of a conductor changes, the resistance varies directly. Conversely, modifying the cross-sectional area A leads to an inverse change in resistance.
- But, **alterations in material composition or temperature affect resistivity ρ in a more complex manner.**

Resistance quantifies how difficult it is for an electric current to pass through a wire or component. It is directly related to the ***resistivity*** of the material.

Resistivity is a characteristic of the material, whereas the resistance is a characteristic of the wire or component depending on both, the material of the component as well as the geometrical dimensions (length, cross sectional area). Changing any of these quantities will change the electrical resistance *R*.

RESISTIVE SENSORS

Part I- Temperature sensors



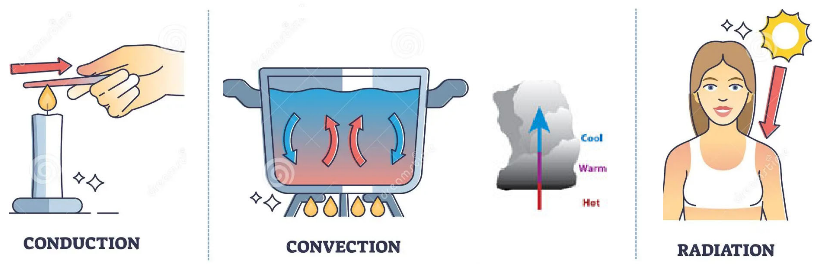
EPFL

There are 2 main applications in medical instrumentation:

1) Direct measurement of temperature for example in the classical thermometer.

2) Integrated as a temperature sensor in medical systems (measurement of various parameters such as fluid velocity in thermal flowmeter, temperature compensation in signal conditioning electronic circuits)

Heat transfer



- Heat is transferred via solid material contact (conduction), liquids and gases (convection), and electromagnetic waves (radiation)
- Heat will flow from one object (milieu) to the other if the two objects are at **different temperature** .

Heat naturally flow from a hotter to a colder object (2nd Law of Thermodynamics)

There are essentially three ways to transfer the heat: **Conduction**; **Convection**; & **Radiation**

Temperature measurement

$R = \rho \frac{l}{A}$

- Relation resistance ← temperature

  $R(T) = R(T_0) f(T - T_0)$, with $f(T - T_o) = 1$ for $T = T_o$

  f is generally a non-linear function depending on the thermosensitive element

- **Metallic resistance** (RTD: resistance temperature detector)

  Platinum: $\boxed{R(T) = R(T_o) \cdot (1 + a(T - T_o)} + b(T - T_o)^2 + c(T - T_o)^3)$

  Linear approximation

- **Oxide semiconductor**

  Thermistor:

  $T : °K$

  $T_o = 273°K$

  $\beta \cong 4000°K$

  $R(T) = R(T_0) e^{\beta(\frac{1}{T} - \frac{1}{T_o})}$

  - *R(T)* is the resistance at temperature *T* (K)
  - *R(T_0)* is the resistance at temperature T_0 (K)
  - T_0 is the **reference** temperature (normally 273K or 25ºC, but can differ according to the application)
  - β is a constant, its value depends on the characteristics of the material.

EPFL

The resistive temperature sensors operate on the principle that the **resistance of a material (metal, ceramic, semiconductor) changes due to variation in resistivity with temperature.**

There are two main types of materials used as resistive temperature sensors:

- **Noble metals** : for example **Platinum**. The relationship resistance - temperature includes nonlinear terms. For small variations around a working temperature (To) this equation can be approximated linear (by retaining only the 1st order term)

Physical principle: Electrons flowing through a metallic conductor are impeded by atoms and molecules. The more these atoms and molecules bounce around due to increased temperature , the harder it is for the electrons to get by. Thus, **resistance generally increases with temperature (especially for metallic conductors).**

For small temperature changes the *resistivity* varies linearly with temperature:

$\rho = \rho_o (1 + \text{alpha}^* \text{ delta}T)$, where α is the temperature coefficient of resistivity.

We often write this in terms of resistance instead: $R = R_o (1 + \text{alpha}^* \text{ delta}T)$ which means we're assuming that dimension (*l* and *A)* don't change as temperature changes.

- **Oxide semiconductors**, are used for fabrication of the **Thermistor –** a device with a highly nonlinear relationship resistance-temperature (exp).

In some materials (like silicon) the temperature coefficient of resistivity is negative, meaning the resistance decreases as temperature increases. In such materials an increase in temperature can free more charge carriers, which would be associated with an increase in current.

(A parenthesis here – please note that this property can be exploited for **temperature compensation** in electronic circuits, to make a resistor with a resistance that is almost independent of temperature (especially when the resistor is used for another measurand, like pressure/deformation, to avoid the error due to temperature influence). The resistor is made from two resistors placed in series, one with a positive temperature coefficient, and the other with a negative temperature coefficient. The resistance values are chosen so that when the temperature changes, the increase in resistance experienced by one resistor is cancelled by the decrease in resistance experienced by the other.

Metallic resistance (RTD)

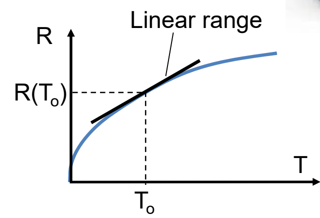
$$R(T) = R(T_o) \cdot (1 + \alpha_R \Delta T)$$

$$\alpha_R = \frac{1}{R(T_0)} \frac{\Delta R}{\Delta T}$$ Temperature coefficient

- Example
  - ***Platinum***

    $\alpha_R = 3.9 \cdot 10^{-3} / °C$

    Pt100 sensor :
    - Linear
    - Accurate
    - Reproducible (precise)
    - Large measurement span



$R(0°C) = 100\Omega$

$R(37°C) = 114.4\Omega$

$R(37°C) = R(0°C)(1+3.9\times10^{-3}\times37) = 100(1.14) = 114\Omega$

The temperature coefficient α_R is a measure of how much the resistance of the sensor changes with respect to changes in temperature.

A higher value of α_R means that the resistance will change more for a given change in temperature, and vice-versa. It is an important parameter to consider when selecting and using resistive sensors, as it can affect the **accuracy** (i.e., how closely a sensor's measured value matches the true or actual temperature) and **reliability** (i.e., the sensor's ability to consistently provide stable and repeatable measurements over time under given conditions).

Platinum is particularly useful for an RTD because it does not oxidize easily and its resistance varies smoothly—nearly linearly—over a wide range of temperatures. Properly ***calibrated***—meaning adjusted or compared against known, accurate reference standards—a platinum resistance thermometer can achieve reproducible (precise) and accurate readings.

Advantages & limitations:

- High ***accuracy***, low drift, wide operating range, suitability for ***precision*** applications
- Compared to thermistors, platinum is less sensitive to small temperature changes and has slower response time

Note:

Accuracy refers to how closely a sensor's measured values align with the true or actual value.

Precision refers to the consistency or repeatability of a sensor's measurements, indicating how close multiple measurements are to each other, regardless of their accuracy.

Reliability: consistency and stability of sensor performance over a longer period, ensuring that precision and accuracy remain stable and dependable (long-term consistency).

While closely related (both emphasizing consistency), **precision** usually addresses measurement repeatability, and **reliability** addresses long-term operational stability and robustness.

Oxide semiconductor

$$R(T) = R(T_0)e^{\beta(\frac{1}{T}-\frac{1}{T_o})}$$

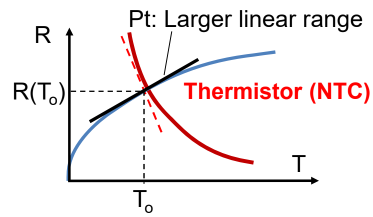
$$\alpha_R = \frac{1}{R}\frac{dR}{dT} = -\frac{\beta}{T^2}$$

- Example
  - ***Thermistor***

    For $T_o = 310°K$

    $\alpha_R = -0.03 \text{ to } -0.05/°C \quad \text{(negative)}$

    - Small volume
    - High sensitivity
    - Non-linear
    - Low measurement span



NTC: Negative Temperature Coefficient

For Positive temperature coefficient (**PTC**) thermistors when temperature increases, the resistance increases

There are two main types of thermistors: **negative temperature coefficient (NTC)** thermistors and **positive temperature coefficient (PTC)** thermistors.

NTC thermistors have resistance that **decreases** as temperature increases, as illustrated by the red curve on the graph, compared to the platinum RTD characteristic. Due to their sensitivity and accuracy, NTC thermistors are typically preferred for precise temperature sensing and measurement applications.

PTC thermistors, whose resistance **increases** as temperature rises, are generally used for safety, circuit protection, and self-regulating heater applications.

Advantages and Limitations:

- Thermistors can be very compact and are therefore widely embedded in devices for internal temperature sensing and correction.
- They typically operate over a narrower temperature range than other temperature sensors (e.g., RTDs), but they can offer high precision and accuracy within this limited range.
- Compared to RTDs, thermistors generally have a smaller usable temperature range and lower long-term stability.

RESISTIVE SENSORS

Part IIa- Thermal mass flowmeter:
Constant current method

EPFL

Measuring respiration flow is important in medical and physiological applications, as it can help to diagnose respiratory diseases and monitor respiratory function, related for example to asthma.

Thermal mass flowmeters are used to measure how fast you can push air out of your lungs when you blow out as hard and as fast as you can. This is called peak flow.

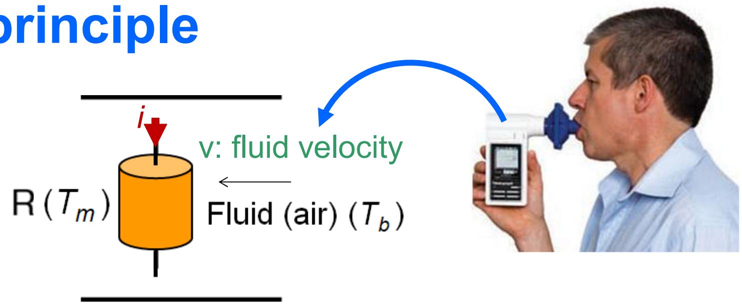
Peak flows measure how open the airways are in the lungs (lung capacity).

There are 2 types of thermal mass flowmeters, according to electronic implementation:

1. constant current
2. constant temperature

Measuring the flow

Basic principle



- R is heated to temperature T_m slightly *higher* than the temperature of the fluid T_b: $T_m > T_b$
- T_m varies (decreases) when the air is circulating into tube by breathing

The subject breathes in and out through a tube containing a resistor (**R**), which is heated via the Joule effect by passing a current (**i**) through it. The resistor is maintained at a temperature (**Tm**) higher than the fluid's expected baseline temperature (**Tb**). When air moves through the tube during respiration, the resistor is cooled down due to convective heat transfer. This cooling effect is directly proportional to the airflow speed (**v**) through the tube.

Thus, resistor **R** acts simultaneously as both an actuator (heated electrically by the Joule effect) and a sensor (detecting changes in temperature due to airflow).

Measuring the flow with *constant current supply*

Principle

1. The resistance is *heated* by *Joule effect* with constant current, *i* → thermal power:

$$P = R \cdot i^2$$

2. The resistance is *cooled* by the fluid circulation → heat transfer by *convection:*

$$P = A \cdot h \cdot (T_m - T_b)$$

A : sensor's lateral surface
h : heat transfer coefficient, *function of velocity v of fluid*
T_m: temperature of the resistance
T_b: temperature of the fluid

Two physical phenomena involved:

- **Resistance heating**: Thermal power dissipation by **Joule heating effect** → electric energy is converted into thermal energy as the electric current i flows through resistor.
- **Resistance cooling**: Thermal power transferred by convection (known as the **Newton's Law of Cooling)**

When the fluid circulate into the tube around the resistance R, the power transferred by convection depends on the diff in temperature (Tm-Tb), the lateral surface A of R, and the **heat transfer coefficient h**, **which expresses the effect of convection;**

- h is related to fluid velocity v (and other parameters such as density, viscosity negligible here) **indicating the fact that the heat transfer increases when the velocity of air increases, so h=f(v).**
- **as the measurand is v, we want to extract from these equations the relation between h=f(v) and variation of R (to note that variation of R is 'measurable' by an electrical signal conditioner) - see next slides**

Measuring the flow with *constant current supply*

Principle (cont.)

- At equilibrium (i.e., no heat transfer):

$$A \cdot h \cdot (T_m - T_b) = R \cdot i^2$$

$$h = f(v) = \frac{R(T_m) \cdot i^2}{A \cdot (T_m - T_b)} \qquad (T_m \text{ variable, } T_b \text{ fixe})$$

Temperature sensor

$$T_m = f(v)$$

$$\Delta R = f(v)$$

  - h varies mainly in a non-linear fashion as a function of velocity v (e.g. $h = a + b \cdot \log v$)

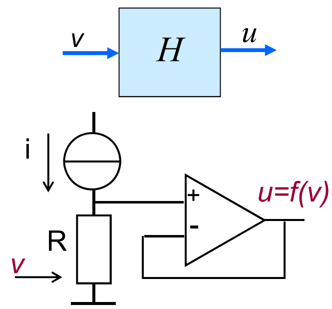
- At instants when R is cooled to the point that Tm reaches Tb (temperature of air during breathing, may depend from person to person, but can be considered constant for the same person during measurement) we can write the equilibrium equation.
- As T_b is constant, a decrease of T_m depends on the velocity v of air, Tm=f(v)
- By measuring variation of Tm using temperature resistive sensors (e.g., thermistor or RTD), we obtain a relationship between the change of resistance, ΔR and the velocity of the fluid v, ΔR=f(v)
- **So R which changes with T_m will depend also on v.**

The next step is to devise a signal conditioner circuit allowing to convert the variations of R=f(v) into electrical signal (voltage)

Measuring the flow with *constant current supply*

Signal conditioner

- If $v\uparrow \;\Rightarrow\; T_m\downarrow \;\Rightarrow\; R\uparrow$ (e.g. NTC thermistor)



- Dynamic situation (when *t* varies):

$$i^2(R+\Delta R) - Ah(T_m - T_b) = K\frac{dT_m}{dt}$$

K: thermal capacity

Assuming linear sensitivity around equilibrium:

$h = h_o + K_v \cdot \Delta v$ $\quad K_v$: constant

$T_m = K_T(R+\Delta R)$ $\quad K_T$: constant

- A simpler and efficient conditioner could be an op amplifier (OA) used as voltage follower. The resistor R should be used as temperature sensor, and the optimal solution is a NTC thermistor (more accurate than PTC)
- **To note**: Without the buffering provided by the voltage follower—characterized by very high input impedance, low output impedance, and excellent precision and stability due to negative feedback—the thermistor's output voltage could be adversely influenced by the input impedance of subsequent measurement devices, such as analog-to-digital converters. This would negatively affect measurement accuracy and stability. By isolating the thermistor from downstream circuits, the voltage follower ensures that the thermistor delivers precise, stable voltage signals unaffected by external circuit conditions.
- In order to characterize the sensor system we have to consider the **dynamic model,** necessary because the voltage *u* should follow the respiration rate of the subject, so un important parameter to analyze is the system **response time.** For this, we have to deduce the transfer function. For variation of the temperature in a narrow range (small change of fluid velocity v), the resistive sensor can be approximated as a **1st order system.**
- The equation describing the 'dynamic situation' states that the sum of thermal power exchanged should be equal to the thermal capacity K multiplied by derivative of temperature (minus indicate exchange of thermal power..)
- If we assume a linear sensitivity around equilibrium, the heat transfer coeff. *h* can be approximated by linear eq where *ho* is the value without air velocity, plus a term which depends on the variation of velocity (first order system by assuming small variations of the velocity of the fluid).
- The second assumption is that the variation of the resistance depend on the temperature Tm=f(R)

Measuring the flow with *constant current supply*

Signal conditioner (cont.)

$$h = h_o + K_v \cdot \Delta v$$

$$T_m = K_T(R + \Delta R)$$

$$i^2(R + \Delta R) - Ah(T_m - T_b) = K\frac{dT_m}{dt}$$

$$i^2(R + \Delta R) - A(h_o + K_v\Delta v)(K_TR + K_T\Delta R - T_b) = KK_T\frac{d\Delta R}{dt}$$

$$\cancel{i^2R} + i^2\Delta R - \cancel{Ah_oK_TR} - Ah_oK_T\Delta R + \cancel{Ah_oT_b} - AK_v\Delta vK_TR - AK_vK_T\cancel{\Delta v\Delta R} + AK_v\Delta vT_b = KK_T\frac{d\Delta R}{dt}$$

$i^2R - Ah_oK_TR + Ah_oT_b = 0$ (static condition)

$\Delta v\Delta R$: second order (can be negligible):

$$(i^2 - Ah_oK_T)\Delta R - K \cdot K_T\frac{d\Delta R}{dt} = AK_v(K_TR - T_b)\Delta v$$

- By expanding this equation, we can observe that the terms in red correspond to the static condition: $i^2(R) - Ah(T_m - T_b) = K\frac{dT_m}{dt}$ =0 at equilibrium (no temperature variation), in this condition we have also h=ho and ΔR=0, so $T_m = K_TR$
- If we neglect the small second order term (slow variation of velocity and R), we obtain a 1st order differential equation describing the dynamic behavior of the sensor system
- We can observe that we have the relationship between ΔR and Δv (the output and the input of our 1st order system)

Measuring the flow with *constant current supply*

Signal conditioner (cont.)

- First order system:

$$(i^2 - Ah_o K_T)\Delta R - K \cdot K_T \frac{d\Delta R}{dt} = AK_v(K_T R - T_b)\Delta v$$

Laplace transform:

$$((Ah_o K_T - i^2) + sK \cdot K_T) \cdot \Delta R = AK_v(T_b - K_T R)\Delta v$$

$$(\left(Ah_o - \frac{i^2}{K_T}\right) + sK) \cdot \Delta R = AK_v(T_b - K_T R)\Delta v = k\Delta v$$

$$\frac{\Delta R}{\Delta v} = \frac{k}{1+\tau_i s}$$

- Slow response !

$$\tau_i = \frac{K}{Ah_o - \dfrac{i^2}{K_T}}$$

To characterize the dynamic response we need to find the time constant. For this we **use the Laplace transform**, we **re-arrange the equation to correspond to the 'characteristic equation' of a 1st order system and we identify the time constant from equation coefficients**

What you can see in the expression of τ_i is that the difference at the denominator could be very low when close to equilibrium, therefore **the time constant is high and the response of the system is very slow.**

The respiration rate is approximately 1 Hz; thus, the system design should ensure a sufficiently fast response time to accurately capture respiratory variations. Possible design strategies to improve response speed include reducing the current (**i**) or increasing the resistor's surface area (**A**). However, these approaches can conflict, since a larger surface area typically requires higher current levels to maintain the resistor at the desired temperature (**Tm > Tb**). Therefore, an optimal balance between surface area and heating current must be established for efficient and accurate respiratory monitoring.

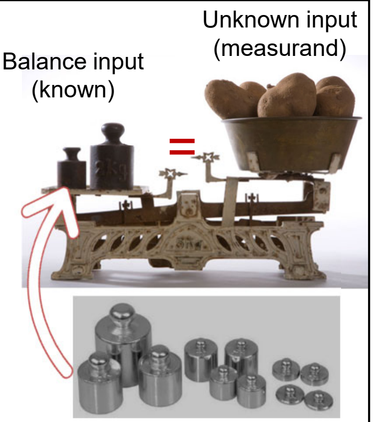
Another solution to optimize the response type is the '*constant temperature method*' described in the next slides.

RESISTIVE SENSORS

Part IIb- Thermal mass flowmeter:
Constant temperature method

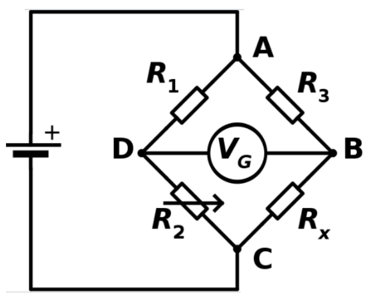
EPFL

Null method: Principle



- Exerts an influence on the measured system so as to oppose the effect of the measurand
- The influence and the measurand are balanced

Null method measurement circuit: Wheatstone bridge



Example circuit:

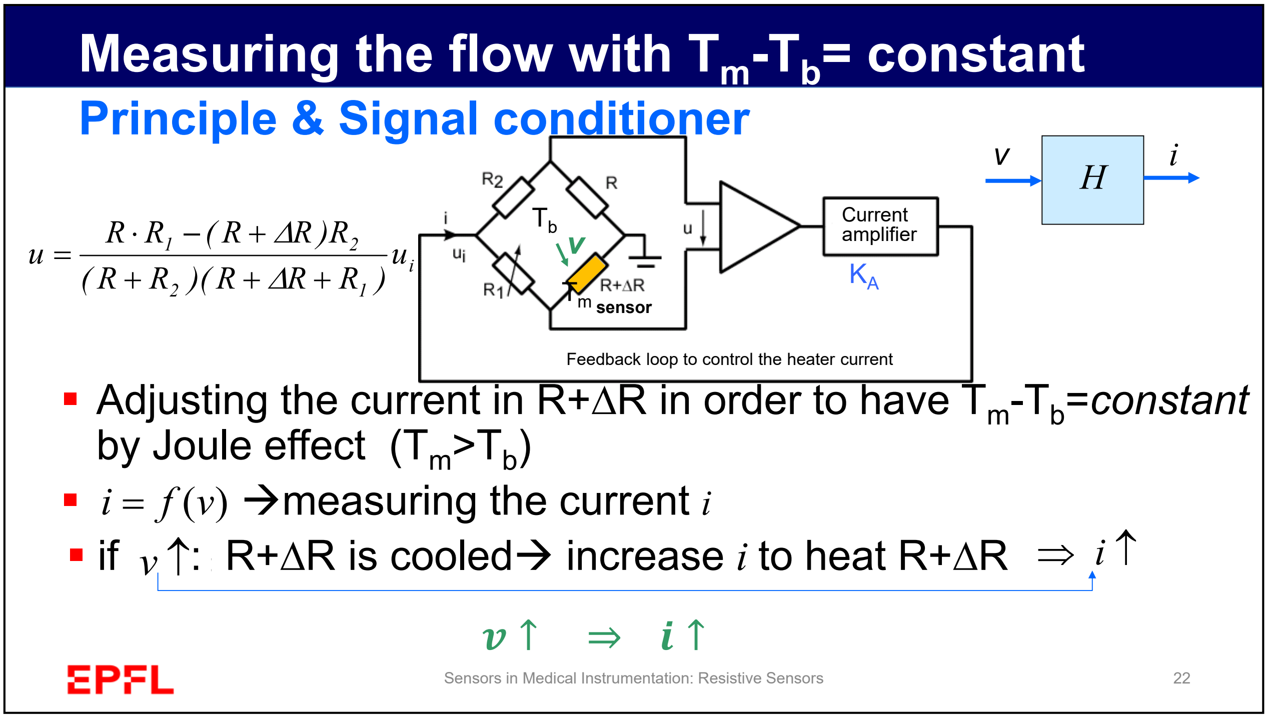
- Rx unknown (to be measured)
- R_1, R_2 and R_3 known, R_2 adjustable until . $V_G = 0$
- When the measured voltage $V_G = 0$, both legs have equal voltage ratios: $R_2/R_1=Rx/R_3$, → $Rx=R_3R_2/R_1$

The constant temperature thermal flow meter is based on the concept of 'null method' measurement.

The null-method is a technique used in measurement instrumentation to determine the value of an unknown quantity by balancing it with a known reference value. This is done by adjusting a variable component in the instrument until the output of the instrument is zero or null.

For example, suppose you want to measure the resistance of an unknown resistor. You can use a null instrument, such as a Wheatstone bridge, which is designed to balance the unknown resistance with a known resistance, for example a variable resistor (potentiometer) adjusted until the output of the Wheatstone bridge is zero.

By knowing the values of the other components in the Wheatstone bridge and the value of the variable resistor, you can calculate the value of the unknown resistance using simple mathematical equations.



This method can be implemented using a Wheatstone bridge combined with a feedback loop. Within the Wheatstone bridge, the sensor is represented as a resistance R+ΔR, and resistor R1 is typically a potentiometer adjusted to set the resistor temperature Tm above the fluid temperature Tb.

When an input voltage ui is applied to the bridge, the unknown resistance R+ΔR is determined by comparing the voltage drop across the sensor resistor with those across the other bridge resistors. To maintain a constant temperature difference (Tm−Tb=constant), the bridge output voltage u (indicating imbalance) is amplified to generate an appropriate current i, which heats the resistor. If increased fluid flow cools the resistor, decreasing its temperature and thus increasing voltage u, the circuit responds by injecting additional current to restore equilibrium.

This relationship can be described mathematically by the bridge's transfer function, effectively linking the current i to the fluid velocity v: when v increases, the sensor resistance cools, lowering T. The feedback mechanism compensates by increasing current i, restoring Tm and stabilizing the system.

In short, the null method balances the convective heat loss (due to fluid flow) against electrical power input (Joule heating from current i). The method achieves equilibrium (the "null" condition) when the heat loss exactly matches the supplied Joule heating, resulting in a stable temperature difference Tm−Tb. The required current at this equilibrium point directly correlates with fluid flow velocity, allowing precise determination of airflow rate.

Measuring the flow with T_m-T_b= constant

Principle (cont.)

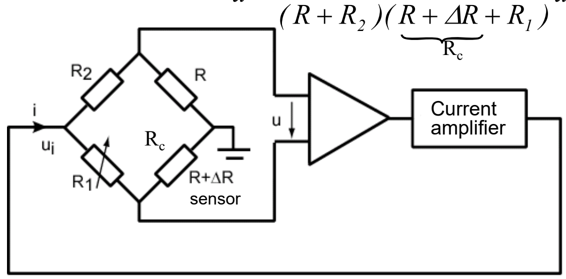
- Balance the bridge in the absence of fluid circulation (Δv=0):
  - Slightly increase the potentiometer value R_1

    $\Rightarrow u\uparrow \Rightarrow i\uparrow \Rightarrow T_m\uparrow \Rightarrow R_c\uparrow$ (Platinum)
  - Bridge balanced again with $T_m=T_b+T_o$ and $u=0$
- $\Delta v \neq 0$

$$v\uparrow \Rightarrow T_m\downarrow \Rightarrow R_c\downarrow$$

$$\Rightarrow u\uparrow \Rightarrow i\uparrow \Rightarrow T_m\uparrow$$

$$u = \frac{R \cdot R_1 - (R+\Delta R)R_2}{(R+R_2)(\underbrace{R+\Delta R}_{R_c}+R_1)}u_i$$



We assume the use of a platinum or PTC thermistor, whose resistance increases with rising temperature. The measurement principle relies on controlling the current (**i**) to maintain a constant sensor temperature (**Tm**). The airflow velocity (**v**) is determined from the value of this current.

Measurement Procedure:

Initially, the sensor temperature (**Tm**) must be slightly higher than the ambient fluid temperature (**Tb**) to detect cooling due to airflow. With no airflow (Δ**v** = 0), the Wheatstone bridge is balanced by adjusting the potentiometer (**R_1**) to slightly raise its resistance. This adjustment increases the bridge voltage (**u**), driving a higher current (**i**) through the sensor, raising its temperature and resistance until equilibrium is restored at the desired temperature differential (**Tm - Tb = constant**).

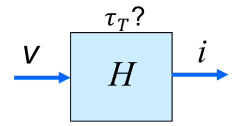
When airflow begins (Δ**v** ≠ 0), the increased fluid velocity cools the resistor, lowering **Tm** and consequently its resistance. This reduction unbalances the bridge, increasing voltage (**u**) and driving more current (**i**) through the resistor. The higher current restores the resistor temperature (**Tm**) to maintain the constant temperature difference (**Tm - Tb = constant**).

In practice, the circuitry is more complex than this simplified description, but the underlying principle remains: airflow velocity (**v**) is accurately determined from the controlled current (**i**) required to maintain a stable sensor temperature.

Measuring the flow with T_m-T_b= constant

Principle (cont.)

- The resistance varies with i and v : $\Delta R = f(\Delta i, \Delta v)$: $\Delta R = \frac{K_i}{1+\tau_i s}\Delta i - \frac{K_v}{1+\tau_i s}\Delta v$
- Wheatstone bridge of gain **K_B**: $\Delta R = -\frac{\Delta u}{K_B}$
- Current amplifier of gain (transconductance) **K_A**:$\Delta i = K_A \cdot \Delta u$
- $\Delta R = -\frac{\Delta i}{K_A \cdot K_B}$
- Time constant of the measurement system i=f(v):



$$\left(\frac{1}{K_A \cdot K_B} + \frac{K_i}{1+\tau_i s}\right)\Delta i = \frac{K_v}{1+\tau_i s}\Delta v$$

$$\boxed{\tau_T = \frac{\tau_i}{1 + K_i \cdot K_A \cdot K_B}} \quad << \tau_i \ !$$

- Demonstration in exercise session 7

→**System is much faster than the circuit with constant current**

For the calculation of the new time constant see exercise session 7

Measuring the flow

- Example: Flowmeters to measure respiration (spirometer)

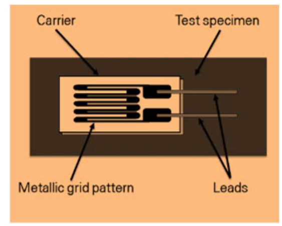
Spirometers are used to diagnose and assess a number of conditions and diseases, for example:

- **Asthma** – an obstructive lung disease in which the airways become periodically swollen and narrowed.
- **Chronic Obstructive Pulmonary Disease (COPD)-** lung conditions that narrow the airway and make it difficult to breathe.
- **Cystic fibrosis**– a degenerative condition in which the lungs and digestive system become clogged with thick, sticky mucus.
- **Pulmonary fibrosis**– scarring of the lungs caused by pollutants, medications and interstitial lung disease.

RESISTIVE SENSORS

Part III- Strain gage (gauge)

Strain gage



- In this section, we will see another type of resistive sensor known as the **strain gage (gauge)**, which is used in various applications.
- A strain gage (SG) is a type of sensor that detects variations in resistance when exposed to an applied **force**, such as **pressure**, **tension, weight**. It transforms the force into a change in electrical resistance that can be measured. According to the material used for resistance there are 2 types: metallic and semiconductor SG
- The picture illustrates a metallic SG that typically consists of a metal foil insulated by a flexible substrate. The SG is connected to two leads that pass a current through it. As the surface of the object being measured stretches or contracts, the change in resistance of gage is detected. This change in resistance is proportional to the change of the surface of the object being tested.

Strain gage: resistive property

- Converts a mechanical elongation/displacement produced by a force in its corresponding change in resistance
- Conductor of length l and cross-section surface A:

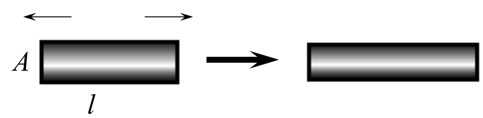
R : resistance

ρ : resistivity

l : length

A : section area

$$R = \rho \frac{l}{A}$$



$$R_2 = R_1 + \Delta R$$
$$R_1 < R_2$$

$$R_1 = \rho \frac{l}{A} \qquad R_2 = \rho \frac{l + \Delta l}{A - \Delta A}$$

The working principle of the resistance strain gage is the '**strain effect**' which means resistance value changes with mechanical deformation of elastic elements.

As shown in Fig, a metallic resistance wire will elongate along the axial direction (length *l*) and shorten along the radial direction (section area *A*) when subjected to force in its elastic range.

If R1 is the resistance corresponding to this conductor, when the conductor is subject to extension, due to elasticity, the value of resistance change: *l* increases, *A* decreases -> R2>R1, so we have a change in resistance by ΔR

Strain gage: mechanical property

- Stress: $\sigma = \frac{F}{A}$ F: force A: area
- Strain: $\varepsilon = \frac{\Delta l}{l}$

Stress measures the internal pressure experienced by a material when subjected to a force. A larger force or smaller area increases the likelihood of deformation (shape change)

Strain is the deformation resulting from stress. When a material experiences stress, it typically lengthens if pulled or shortens if compressed.

- Strain in direction of the stress:

$$\varepsilon_{\parallel} = \frac{\sigma}{Y}$$

Y: Young's modulus

- Strain perpendicular to the stress:

$$\varepsilon_{\perp} = -\nu\varepsilon_{\parallel}$$

ν: Poisson's ratio

An important characteristic of the strain gage is its mechanical property, which is based on the relationship between **stress** and **strain**.

When a stress, defined as F/A, acts on a material, it results in a deformation of the material, represented by Δl/l (relative change), known as strain.

The strain can be defined in two directions:

1. In direction (parallel) to the stress.
2. Perpendicular to the stress, which is negative because the area decreases in the direction perpendicular to the stress.

Strain gage response

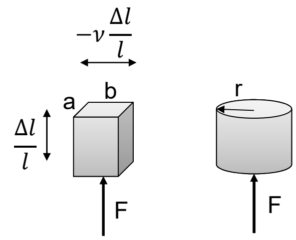
- $R = \rho \frac{l}{A}$

$$\frac{\Delta R}{R} = \frac{\Delta\rho}{\rho} + \frac{\Delta l}{l} - \frac{\Delta A}{A}$$

- The surface is mainly rectangular or circular

$$A = a \cdot b \text{ or } A = \pi r^2$$

- $\frac{\Delta A}{A} = \frac{\Delta a}{a} + \frac{\Delta b}{b} \text{ or } \frac{\Delta A}{A} = 2\frac{\Delta r}{r} \Rightarrow \frac{\Delta A}{A} = -2\nu\frac{\Delta l}{l}$



$$\boldsymbol{\frac{\Delta R}{R} = (1+2\nu)\frac{\Delta l}{l} + \frac{\Delta\rho}{\rho}}$$

Computation of the relative variation of the resistance:

Expand resistance R into a Taylor series

- Ignore higher order terms

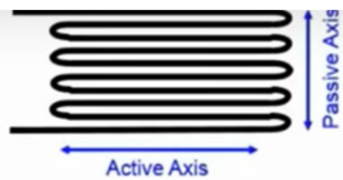
$$R = R_0 + \Delta R$$

$$\Delta R = \frac{\partial R}{\partial L}\Delta L + \frac{\partial R}{\partial A}\Delta A + \frac{\partial R}{\partial \rho}\Delta\rho$$

- Taking partials

$$\Delta R = \frac{\rho}{A}\Delta L - \frac{\rho L}{A^2}\Delta A + \frac{L}{A}\Delta\rho$$

$$\boxed{\frac{\Delta R}{R}} = \frac{\Delta L}{L} - \frac{\Delta A}{A} + \frac{\Delta\rho}{\rho} = (1+2\nu)\varepsilon + \boxed{\frac{\Delta\rho}{\rho}}$$



Resistance of wire:

$$R = \frac{\rho L}{A}$$

- ρ: Resistivity
- L: Length of wire
- A: Cross-sectional area

V: POISSONS RATIO
ε: STRAIN cm/cm

The relative variation of resistance in a strain gauge is influenced by either length or resistivity, giving rise to two types of effects:

1. When strain affects the length of the gauge, it is referred to as a resistive strain gauge.
2. When strain affects the resistivity of the gauge, it is referred to as a piezoresistive strain gauge.

Metallic conductor

(1) $$\frac{\Delta\rho}{\rho} = C\frac{\Delta V}{V} = C(1-2\nu)\frac{\Delta l}{l}$$

V: volume ($V = A \cdot l$)

C: Bridgman correction factor

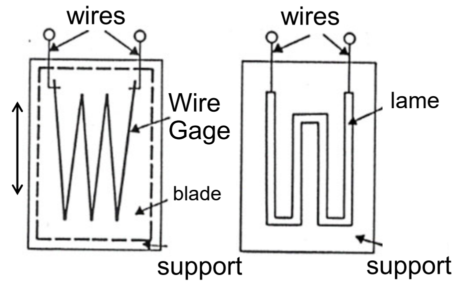
- The relative variation of the gage resistance is:

(2) $$\frac{\Delta R}{R} = \{(1+2\nu)+(1-2\nu)C\}\frac{\Delta l}{l} = K\frac{\Delta l}{l}$$

$$K = (1+2\nu)+(1-2\nu)C$$

  - K : gage factor
    (for metals $C \cong 1$ and $\nu \cong 0.3$ and $K \cong 2$)

$$\frac{\Delta R}{R} = (1+2\nu)\frac{\Delta l}{l} + \frac{\Delta\rho}{\rho}$$



$$\frac{\Delta R}{R} = K\frac{\Delta l}{l}$$

A metallic strain gauge consists of wires on a support made of either plastic or paper. This gauge is fixed onto the object that needs to be measured for deformation.

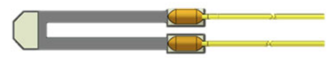
In the case of a metallic strain gauge, the change in resistivity can be expressed as a function of change of the volume or length, where C is the Bridgman correction factor.

This allows us to define the relative change of resistance of as function of thee relative change of length multiplied by the gage factor K.

Semiconductor

- Resistivity changes with doping: p-type (deficit of electron) or n-type (excess of electron)
- $$\frac{\Delta\rho}{\rho} = \pi \cdot \sigma = \pi \cdot Y \frac{\Delta l}{l}$$
  π : piezoresistivity coefficient

$$\frac{\Delta R}{R} = (1+2\nu)\frac{\Delta l}{l} + \frac{\Delta\rho}{\rho}$$



- The relative variation of the gage resistance is :

$$\frac{\Delta R}{R} = \{(1+2\nu) + \pi \cdot Y\} \cdot \frac{\Delta l}{l}$$
$$K = (1+2\nu) + \pi \cdot Y \approx \pi \cdot Y$$

$$\frac{\Delta R}{R} = K \frac{\Delta l}{l}$$

- $K \approx 100 - 200$ for a semiconductor gage

In the case of a semiconductor strain gage, the resistance is a type of semiconductor material whose resistivity changes with the type of doping. This change in resistivity, represented by Δρ/ρ, is much higher than in the case of metals due to the higher Young's modulus of semiconductors.

If we replace Δρ/ρ and determine the gage factor K, we find that its value is higher (compared to metallic) due to the higher Young's modulus. This type of strain gauge is called a piezoresistive strain gauge because it utilizes the resistivity change with force and deformation.

The prefix "piezo" refers to the state of pressing or being pressed, or the exertion of force by one body on the surface of another.

In summary

$$\frac{\Delta R}{R} = K \frac{\Delta l}{l}$$

- Metallic strain gage
  - More sensitive to mechanical strain
  - K=2 to 4

- Semiconductor strain gage
  - More sensitive to resistivity (piezoresistif effect)
  - K=100 to 200

Temperature effect

$R(T_1)=R(T_o)(1+\alpha_R(T-T_o))$

- Thermal expansion of the gage: $l_j=l_{jo}(1+\lambda_j(T-T_o))$
- Thermal expansion of the support: $l_s=l_{so}(1+\lambda_s(T-T_o))$
- At T_o, $l_{jo}=l_{so}=l_o$; At T, $l_s-l_j=l_o(\lambda_s-\lambda_j)\Delta T$ with $\Delta T=T-T_o$

$$\left.\frac{\Delta R}{R}\right|_{expansion}=K\frac{l_s-l_j}{l_o}=K(\lambda_s-\lambda_j)\cdot\Delta T$$

- $$\left.\frac{\Delta R}{R}\right|_{T}=\{\alpha_R+K(\lambda_s-\lambda_j)\}\cdot\Delta T=\beta\cdot\Delta T$$
  - α_R : temperature coefficient , ° C^{-1}
  - λ_S : expansion coefficient of support, ° C^{-1}
  - λ_j : expansion coefficient of gage, ° C^{-1}

EPFL Sensors in Medical Instrumentation: Resistive Sensors 33

Temperature variations can affect the accuracy of strain gauge measurement.

- Specifically, thermal expansion of both the gauge and its supporting structure can change the gauge's resistance even when no mechanical strain or stress is applied. This thermal expansion acts as an interfering input, introducing errors into the strain measurement.
- Additionally, the electrical resistance of the strain gauge itself changes directly with temperature (another interfering input). Therefore, the gauge's temperature coefficient (**α_R**) must also be taken into account.

The highlighted equation represents the strain gauge resistance variation due to temperature effects. When such temperature-induced errors become significant, the measurement circuitry requires appropriate temperature compensation strategies.

Temperature effect – Comparison

$$\left.\frac{\Delta R}{R}\right|_T = \left\{\alpha_R + K(\lambda_s - \lambda_j)\right\} \cdot \Delta T = \beta \cdot \Delta T$$

Metallic strain gages	**Semiconductor strain gages (piezoresistive)**
$\alpha_R = 0.01 \text{ to } 0.04\%/°C$ relatively low If $\lambda_S - \lambda_j$ low, β stays low Compensation for temperature for $\beta > 1.5 \cdot 10^{-6}/°C$	$-\alpha_R$ high - K varies with temperature Compensation for temperature mainly necessary $\lambda_j = 3.2 \cdot 10^{-6}/°C << \lambda_S$ - Sensors integrated in a silicon support $\lambda_S - \lambda_j = 0$

- For piezoresistive sensors, temperature compensation is essential due to the high temperature sensitivity (large temperature coefficient) inherent in semiconductor materials.
- Typically, temperature compensation is achieved by incorporating an additional semiconductor sensor whose resistance changes exclusively with temperature. This compensating element adjusts the current through the primary sensor, thereby minimizing temperature-induced errors in the measurement.

Comparison

Strain Gage	Metal	Semi-conductor
Linearity	High	Medium
Sensitivity	Low	High
Thermal sensitivity	Low	High

RESISTIVE SENSORS

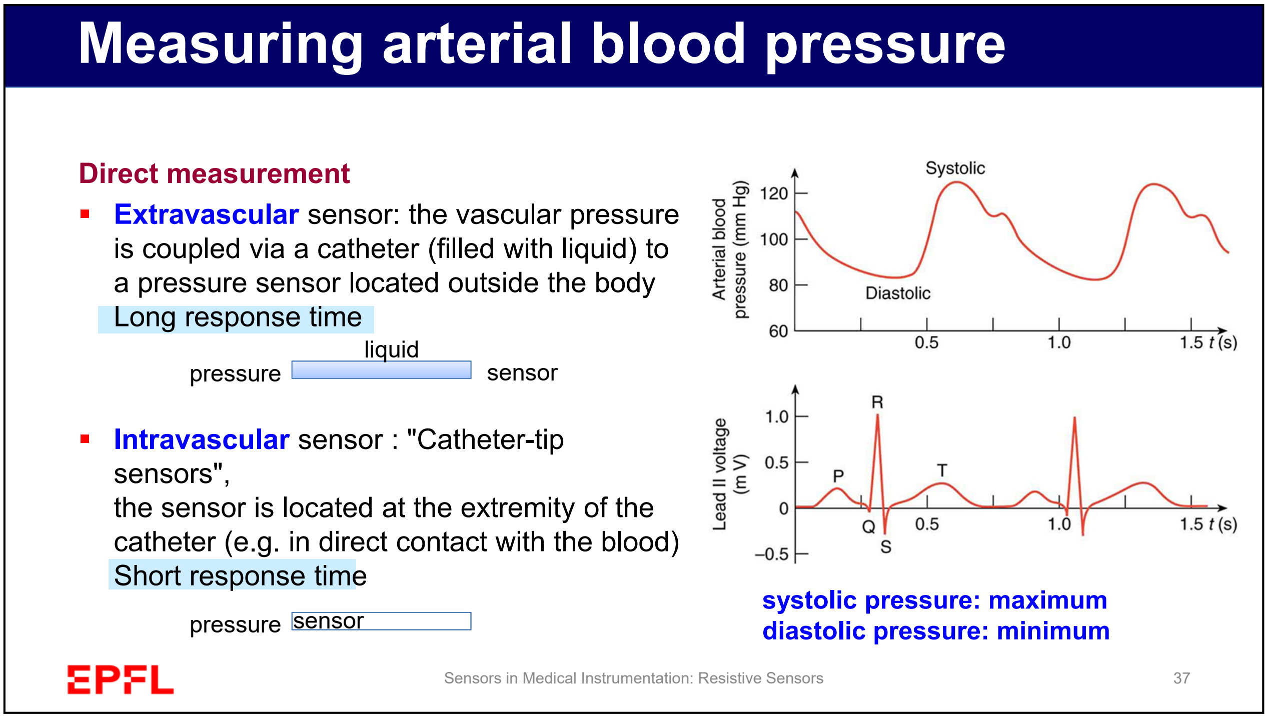
Part IV- Direct measurement of arterial pressure

(Strain gage application)

Blood pressure is the measurement of the pressure of blood inside arteries. At each beat, the heart pumps blood into arteries that carry blood throughout your body.

Clinical importance of measuring blood pressure (BP):

Direct methods (invasive):
Direct arterial measurement, typically via arterial catheterization, provides continuous, accurate, real-time blood pressure readings. Clinically important in critical care settings, surgeries, and hemodynamic instability where precise monitoring is essential to guide immediate therapeutic interventions.



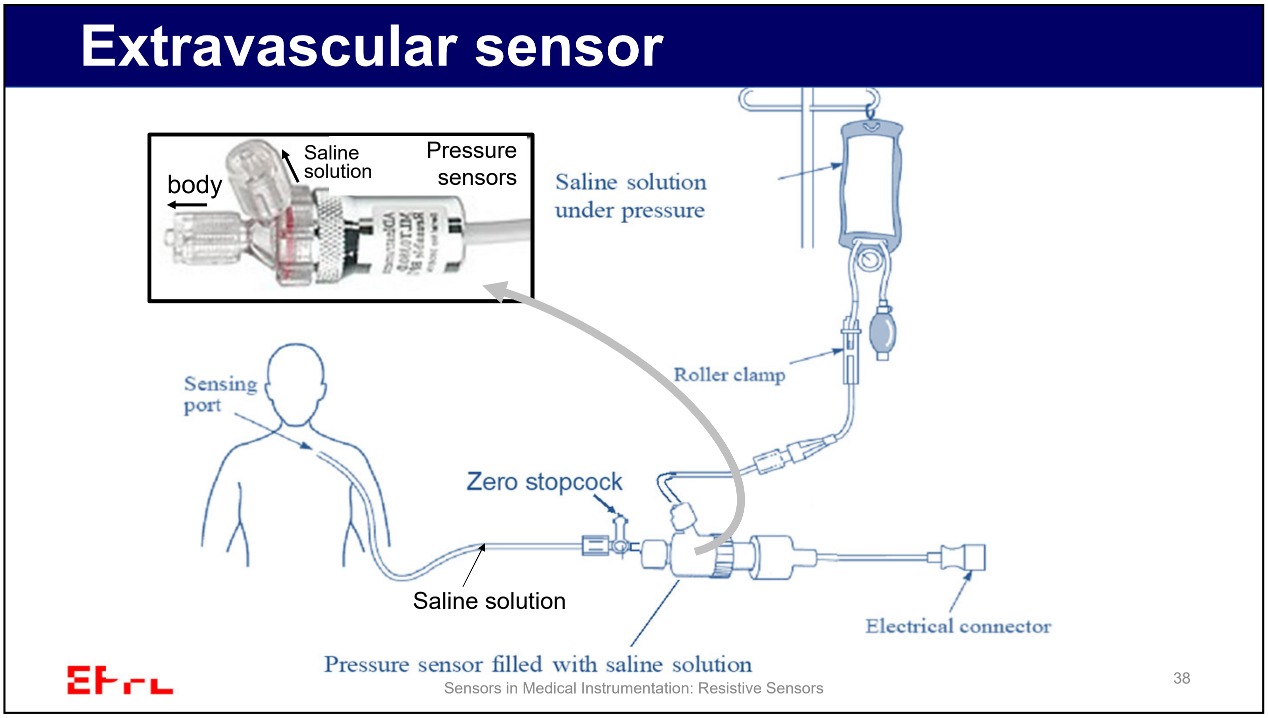
Arterial blood pressure is defined as the pressure exerted by circulating blood against the inner walls of arterial vessels. It reflects the combined effects of cardiac output (the amount of blood pumped by the heart per minute) and peripheral vascular resistance (resistance within arteries). It is typically expressed in millimeters of mercury (mmHg) as two values:

- **Systolic pressure:** Peak arterial pressure during ventricular contraction (reflects cardiac output, arterial elasticity).
- **Diastolic pressure:** Lowest arterial pressure during ventricular relaxation (indicates vascular resistance and arterial tone).

The delay between the ECG R-wave and systolic blood pressure is due to the time it takes for the electrical activation of the ventricles to be converted into mechanical contraction and the blood to be pumped into the aorta. This delay is known as the *electromechanical delay* and lasts between 40-60 milliseconds.

There are two direct ways to measure the arterial blood pressure:

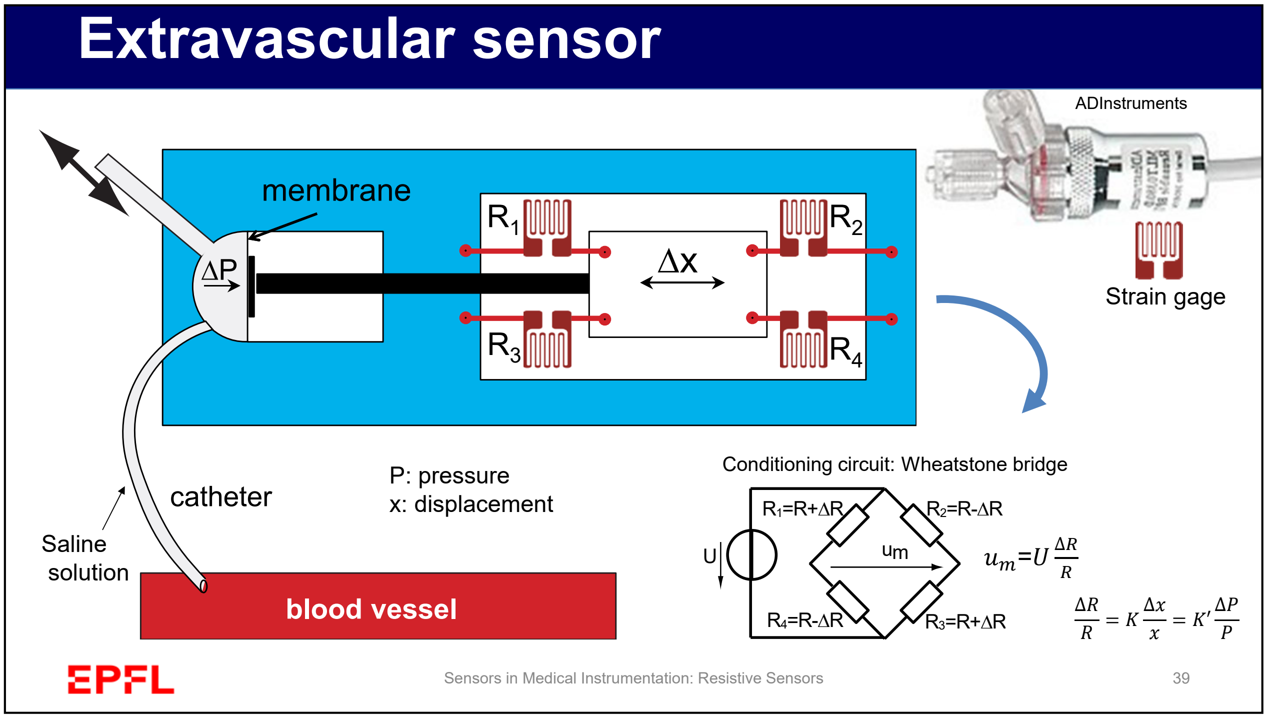
1. Using **extravascular sensor** where a catheter filled with a liquid (e.g., saline solution), is used to couple the blood pressure to a sensor outside the body. Given that the sensor and the pressure being measured are separated by the catheter, the consequence is a high inertia between the pressure and the sensor and a long response time, which limits the frequency response.
2. Using **an intravascular sensor** located at the extremity of the catheter, at the same location as the vascular pressure. This eliminates the need for a catheter filled with saline water, resulting in a lower time constant and a faster response time.



- This is the example of monitoring BP using an extravascular sensor where a catheter filled with saline solution is inserted into an artery and coupled to an elastic membrane. Strain gages are placed around the membrane to measure deformation related to the change in pressure, which is then converted into an electrical signal through electronics. This signal is then connected to a measurement system for processing and display.
- **One of the main concerns with this system is to prevent any air bubbles from entering the vascular system.** Air bubbles in an intravascular catheter can be dangerous for several reasons:
  - *Air Embolism*: If air bubbles enter the bloodstream through the catheter, they can travel to the heart or lungs and cause an air embolism. This occurs when the air bubbles block blood flow in the vessels, potentially leading to tissue damage, organ dysfunction, or even death.
  - *Cardiovascular Complications*: Air bubbles in the bloodstream can interfere with the normal circulation of blood, leading to cardiovascular complications such as arrhythmias, decreased cardiac output, or even cardiac arrest.
  - *Pulmonary Complications*: Large air emboli reaching the lungs can obstruct blood flow through the pulmonary arteries, causing pulmonary embolism, which can be life-threatening.

To minimize the risk of air embolism, healthcare providers take precautions during catheter insertion and use techniques to remove air from the system before and during catheterization. Additionally, proper monitoring and immediate intervention are crucial if air embolism is suspected.

- This technique is commonly used during surgery as it allows for continuous monitoring of blood pressure, which is crucial in ensuring patient safety and detecting any changes in blood pressure that may require prompt intervention.



The catheter is filled with saline water and is in contact with the blood. The blood pressure is transmitted through the catheter to a cavity (the probe) filled also with saline solution and realized with an elastic membrane. The displacement or deformation of the membrane during systole/diastole is measured using four strain gages placed in the Wheatstone bridge (conditioning circuit).

To achieve maximum sensitivity for the Wheatstone bridge, it's important to configure it correctly. When the membrane pushes the bridge on the right, resistors R2 and R4 are compressed, while resistors R1 and R3 are extended. This corresponds to the relative change of each resistor, ΔR, as shown in the circuit.

In order to achieve maximal sensitivity for the bridge, resistors with the same sign of deformation (ΔR) should be on opposite arms, while resistors with opposite deformation should be on adjacent arms.

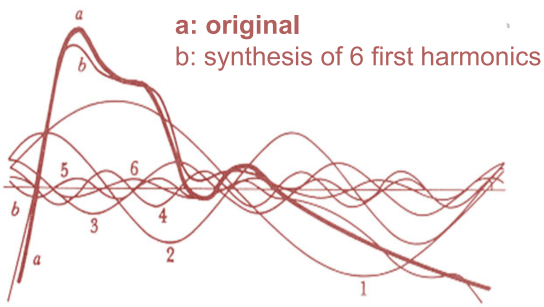
As the deformation depends on the strain being measured (Δ x/x), and the strain is dependent on the pressure, the output voltage of the bridge, um, will depend on the pressure being measured.

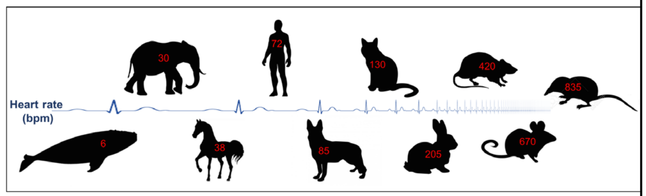
The catheter-liquid-membrane system is essentially a hydraulic system that should transfer the blood pressure without distortion. It's important to characterize its transfer function to ensure that the frequency band of the blood pressure is not affected by this transfer function.

For this purpose, we must model the catheter to ensure that its inertia and response time are suitable for accurately measuring the temporal dynamics (time variations) of blood pressure.

Dynamic properties of pressure measurement systems

- Arterial pressure bandwidth in humans: 0 to 50 Hz





- Need to estimate the transfer function of the system

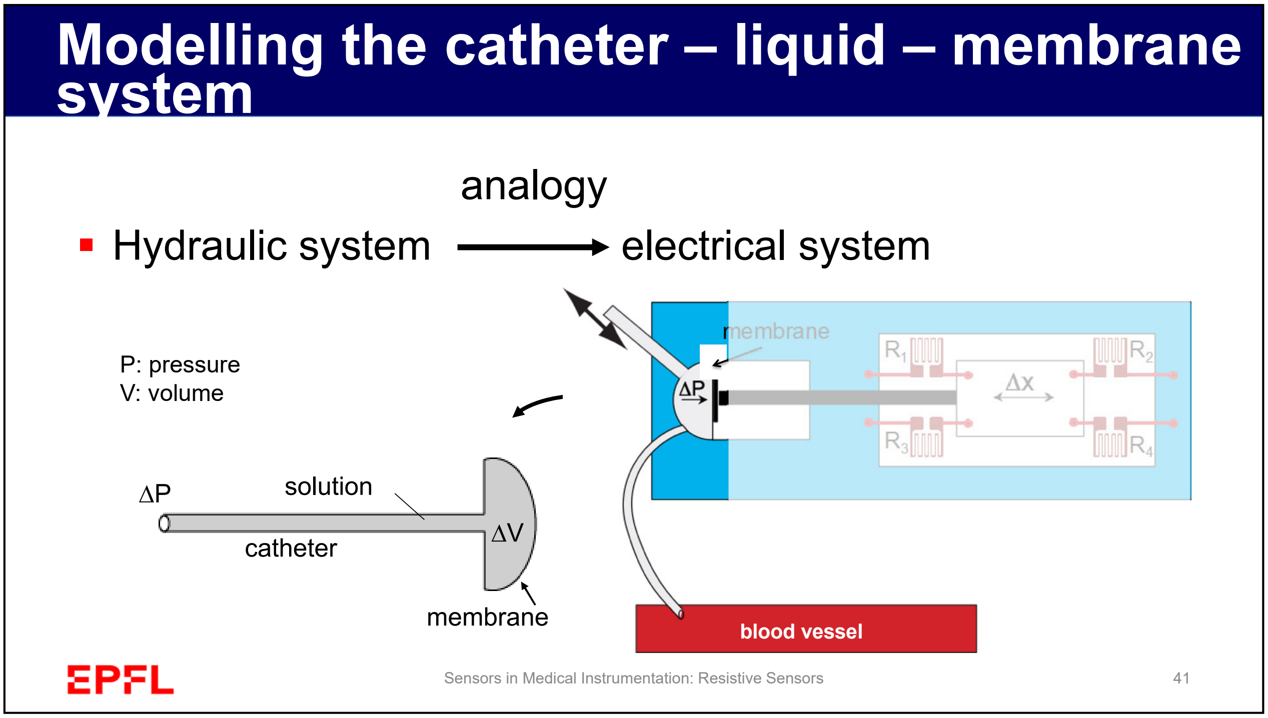
EPFL Sensors in Medical Instrumentation: Resistive Sensors 40

For accurate blood pressure (BP) monitoring, the system should be able to track the variations in the BP signal. The signal, shown as (a), contains different harmonics that can be analyzed using a power spectrum analysis (by FFT, for instance). Taking the first six harmonics, as an example, results in signal (b), which is close but not exactly the same as the original signal (a).

To accurately measure the BP signal for humans (1 Hz =60 pulses/min) the bandwidth of the measurement system should cover the range of 0 to 50 Hz, which is the norm for human BP.

It's crucial that the catheter used in the measurement system has enough bandwidth to measure the BP signal without distortion. Additionally, the bandwidth of the measurement system should be adapted to the specific dynamic of BP for the species being monitored. For instance, monitoring small animals may require an increased bandwidth, as the heart rate increases when the tail of the animal decreases.

We need a dynamic model of the catheter to verify in which conditions the system can follow the dynamic (temporal variations) of BP signal (need to model the transfer function)



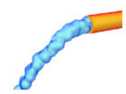
To evaluate the response and understand the potential limitation factors, it's crucial to model the hydraulic part of the system. **One useful technique for this is by using an analogy between the hydraulic and electrical systems**. This is because electrical systems are easier to model using basic components, such as resistive, capacitive, and inductive elements, as well as current and voltage.

To make this analogy, we **need to consider the geometry of the hydraulic circuit**. For example, if we focus on the catheter, the change in pressure ΔP leads to a change in volume ΔV due to the movement/expansion of the membrane.

Therefore, we need to accurately model this situation in order to achieve an accurate response time for the BP measurement system.

Analogous Variables: Electrical - Hydraulic

power ← ×

Quantity	Electrical System	Pressure System
Flowing quantity	**Current** (Amps = Coul/s)	**Flow** (mL/s)
Volume quantity	**Charge** (Coul)	**Volume** (mL)
Energy Density	**Voltage** (Joule/Coul)	**Pressure** (kPa) (Pa=F/Area = J/Vol)
Energy Density to maintain Flow	**Resistance** (Ohms = Volt/Amps)	**Resistance** kPa / (mL/s) 
Volume maintained by Energy Density	**Capacitance** (Farads = Coul/Volt)	**Compliance** mL / kPa 
Energy Density to maintain d/dt (Flow)	**Inductance** (Henry = Volt/(Amps/s))	**Inertance** (kPa / (ml/s^2)) 

General Idea of the Analogy:

In electrical circuits, voltage, current, charges and electrical components – resistance, capacitance, inductance - are fundamental variables. A similar logic applies to hydraulic systems (like catheters filled with liquid), where the analogies are those indicated in the table.

Detailed Explanation of Each Parameter:

Flow Rate ↔ Current (Q ↔ I)

- **Hydraulic:** Volumetric flow rate (volume per unit time, e.g., m^3/s or ml/s).
- **Electrical:** Current (charge per unit time, e.g., Coulomb/s = Ampere).

Pressure ↔ Voltage (ΔP ↔ ΔV)

- **Hydraulic:** Pressure difference drives liquid flow.
- **Electrical:** Voltage difference drives electrical current.

Volume ↔ Charge (Vol ↔ Q)

- **Hydraulic:** Volume is integral of the flow
- **Electrical:** Charge is integral of the current

Hydraulic Resistance ↔ Electrical Resistance (R ↔ R)

- **Hydraulic**: Resistance is the frictional opposition to flow, depending on viscosity, length, and diameter of the catheter.
- **Electrical**: Resistance limits current flow

Hydraulic Compliance ↔ Electrical Capacitance (C ↔ C)

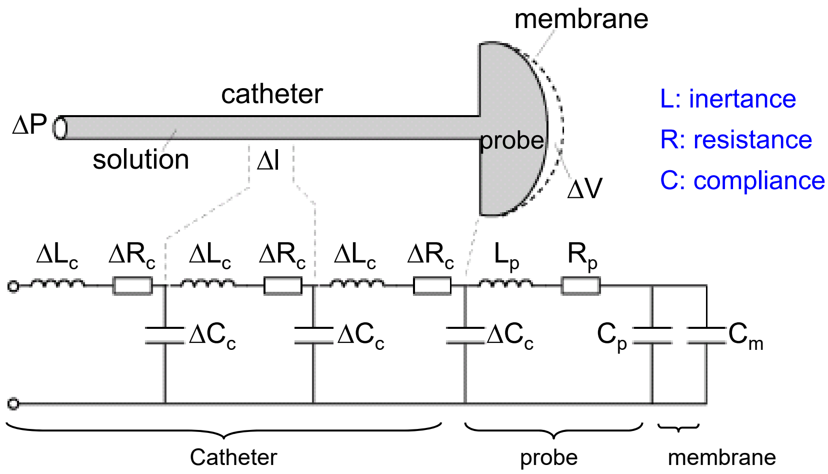
- **Hydraulic:** Compliance represents how much a container can deform or stretch under pressure. In a catheter system, compliance often comes from flexible tubing or surrounding elastic structures.
- **Electrical:** Capacitance represents the ability to store electrical charge under a voltage difference

Fluid Inertia (Inertance) ↔ Electrical Inductance (L ↔ L)

- **Hydraulic**: The inertia of the liquid corresponds to the fluid's mass and its tendency to resist changes in velocity
- **Electrical**: Inductance (coil) resists changes in electrical current

Modelling the catheter – liquid – membrane system

- Hydraulic system $\xrightarrow{\text{analogy}}$ electrical system



Why Resistance and Inertia are Series Components:

In a hydraulic catheter (or pipe), fluid flows sequentially through the length of the tubing. Both friction (**resistance**) and fluid mass (**inertia**) act along the same continuous fluid pathway.

Why Compliance in Parallel:

Compliance is placed **in parallel** because the compliant chamber (probe) and the tubing/catheter share the **same pressure** at their connection point. Thus, at any junction where fluid can either continue flowing down the main tube or enter an elastic (compliant) space, the hydraulic compliance naturally appears in parallel.

Why the electrical equivalent circuit of a catheter is piecewise:

The electrical equivalent circuit of a catheter is usually represented as a **piecewise model** because the catheter can be viewed as a transmission line segmented into multiple small sections. Each segment has its own distinct parameters, including resistance, inductance, and capacitance. As the catheter is inserted into a biological environment (e.g., a blood vessel or artery), the conditions along its length—such as geometry, surrounding tissue properties, and fluid conductivity—can vary significantly. Therefore, to accurately capture these varying electrical properties and impedance characteristics, the equivalent circuit is represented as discrete segments or pieces, each modeled individually.

When can the circuit be approximated as uniform (the same for the entire length)?

The catheter's electrical equivalent circuit can be approximated as uniform (identical parameters throughout its length) under the following conditions:

- The catheter is relatively short or significantly shorter than the wavelength of signals used (low-frequency signals), meaning transmission-line effects (such as reflections or standing waves) are negligible.
- The surrounding medium and physical environment (tissue, blood, fluid) is homogeneous, resulting in constant electrical properties along the entire length of the catheter.
- The geometry and materials of the catheter are uniform and constant throughout its length.

Under these simplified conditions, variations along the catheter length become insignificant, allowing the use of a single, uniform equivalent circuit model instead of a piecewise approach.

Modelling the catheter – liquid – membrane system

- Measure of energy density: Pressure (ΔP)→voltage
- Measure of flow quantity: Flow (Q)→current
- The resistance of the catheter liquid is:

$$R_c = \frac{\Delta P}{Q} = \frac{\Delta P}{v \cdot A} \quad Pa \cdot s / m^3 \qquad (Q = v \cdot A)$$

  - v : average speed of liquid
  - A : cross-sectional area of catheter

- Laminar flow, the Poiseuille equation*:
  - η : viscosity of the liquid
  - l_c : length of the catheter
  - r_c : radius of the catheter

$$R_c = \frac{8\eta l_c}{\pi \cdot r_c^4}$$

*See Moodle: Annex_HydraulicResistanceCalculation

- In analogy with electrical circuit, the pressure (measure of energy density) corresponds to the voltage, and flow to current

→ we can calculate the resistance of catheter Rc as the ratio $\Delta P/Q$

- In laminar flow we assume no transfer of mass in radial direction. This is particularly true in the presence of low flow.

Modelling the catheter – liquid – membrane system

- Inertance of the liquid

Electrical circ:
L=u/(di/dt)

$$L_c = \frac{\Delta P}{\frac{dQ}{dt}} \quad \text{with} \quad \frac{dQ}{dt} = \frac{dv}{dt} \cdot A = a \cdot A \qquad \text{and } \Delta P = \frac{Force}{surface} = \frac{m \cdot a}{A}$$

$$m = \rho \cdot V = \rho \cdot A.l$$

$$L_c = \frac{m \cdot a}{A.a \cdot A} = L_c = \frac{m}{A^2}$$

a: acceleration
m: mass of the liquid
ρ: density
A: Section
l: length

- Compliance of catheter liquid:
  - E_c : Volumetric elastic modulus
  (measure of material rigidity/stiffness)

$$C_c = \frac{\Delta V}{\Delta P} = \frac{1}{E_c}$$

Electrical circ:
C=Q/V (Q=charge)

EPFL Sensors in Medical Instrumentation: Resistive Sensors 45

Inertance: In electrical circuit u=Ldi/dt →L=u/(di/dt) so, in the hydraulic circuit L can be deduced by considering the analogy

Compliance: In electrical circuit, the capacitance is the ratio charge/voltage, so the compliance can be deduced by considering the analogy.

To note that a high compliance correspond to a flexible material (low E_c)

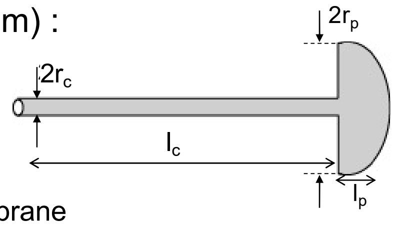
(Elastic modulus is the material ability to resist elastic deformation when stress is applied to it. It is a measure of a material's rigidity or stiffness.)

Modelling the catheter – liquid – membrane system

- For the probe(p), catheter (c) and membrane (m) :

$$C_{p,c,m} = \frac{1}{E_{p,c,m}} \quad R_{p,c} = \frac{8\eta l_{p,c}}{\pi \cdot r_{p,c}^2} \quad L_{p,c} = \frac{m_{p,c}}{A_{p,c}^2}$$

  - $L_{p,c}$: length of probe or catheter
  - $R_{p,c}$: radius of probe or catheter
  - $E_{p,c,m}$: elastic modulus of the probe, catheter or membrane

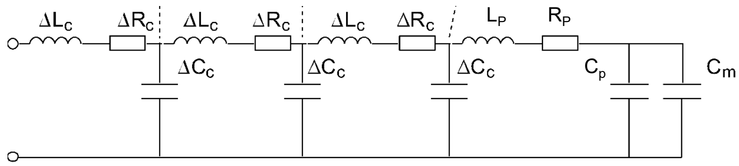


- For l_P<<l_c and r_P>>r_c: R_c>>R_P and L_c>>L_P
- For E_m<<E_c and E_m<<E_P: C_m>>C_P and C_m>>C_c

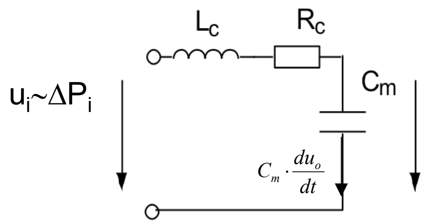
$$C_m = \frac{1}{E_m} \quad R_c = \frac{8\eta l_c}{\pi \cdot r_c^4} \quad L_c = \frac{m_c}{A_c^2}$$

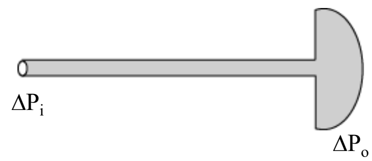
We assume: long, thin and hard catheter (pipe), while the probe part is thick and short with a flexible membrane. Based on these, we can make approximations to simplify the equivalent circuit

Modelling the catheter – liquid – membrane system



R_c>>R_P and L_c>>L_P C_m>>C_P and C_m>>C_c



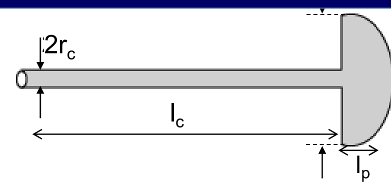


$$u_i(t) = L_c C_m \frac{d^2 u_o(t)}{dt^2} + R_c C_m \frac{du_o(t)}{dt} + u_o(t)$$

- ΔC_c is very low (rigid catheter) so there is no flow going through, we replace all ΔC_c by open circuit.
- The final equivalent circuit corresponds to a 2nd order system.
- We can find the relationship between the input voltage ui, analogous to the applied pressure, and the output voltage uo, analogous to the pressure at the diaphragm , by using Kirchoff's voltage law in the RLC circuit.

Modelling the catheter – liquid – membrane system

- Second order system



$$u_i(t) = L_c C_m \frac{d^2 u_o(t)}{dt^2} + R_c C_m \frac{du_o(t)}{dt} + u_o(t)$$

Characteristic equation

$$u_i(t) = \frac{1}{\omega_o^{\ 2}} \frac{d^2 u_o(t)}{dt^2} + \frac{2\zeta}{\omega_o} \frac{du_o(t)}{dt} + u_o(t)$$

- $\omega_o = \frac{1}{\sqrt{L_c C_m}}$ $\qquad \omega_o = \pi \cdot r_c \sqrt{\frac{E_m}{\pi \rho l_c}}$
- $\zeta = \frac{R_c}{2} \sqrt{\frac{C_m}{L_c}}$ $\qquad \zeta = \frac{4\eta}{r_c^3} \sqrt{\frac{l_c}{\pi \rho E_m}}$

$$C_m = \frac{1}{E_m} \qquad R_c = \frac{8\eta l_c}{\pi \cdot r_c^4}$$

$$L_c = \frac{m_c}{A_c^2} = \frac{\rho A_c l_c}{A_c^2} = \frac{\rho l_c}{\pi r_c^2}$$

ρ: density

EPFL Sensors in Medical Instrumentation: Resistive Sensors 48

Considering the characteristic equation we can identify the natural or resonance frequency and the damping in terms of geometry and materials properties of the catheter-liquid-membrane

- Natural frequency (pulsation): is the frequency or rate at which an object /system vibrates/oscillates naturally when disturbed
- Damping: is the process of dissipating energy to prevent vibratory motion (mechanical oscillations)
- For the calculation of natural frequency and damping using the theoretical equations we need to know the dimension and properties of catheter-liquid-membrane system

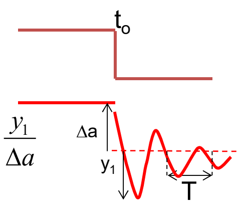
In practice, we can study (measure) the **transient response** and the **frequency response** of the catheter-sensor system experimentally, as shown in the next slides

Modelling the catheter – liquid – membrane system

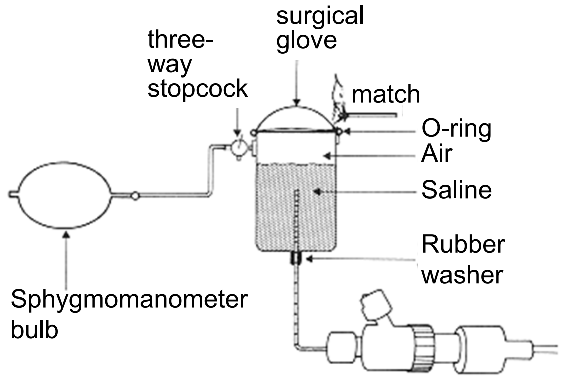
- Step response

$$\zeta = \frac{1}{\sqrt{\left(\frac{\pi}{\ln D}\right)^2 + 1}}$$

$$\omega = \frac{2\pi}{T\sqrt{1-\zeta^2}}$$



$$D = \frac{y_1}{\Delta a}$$



At time t_o the glove is burst

The natural frequency and damping are measured experimentally by using step response (explosion of an air balloon or surgical glove).

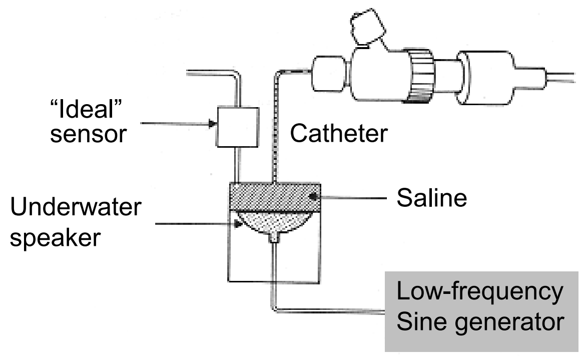
The basis of the transient steps response method (called also ‘pop’ technique) is to apply a sudden step input to the pressure catheter (e.g., explosion an air balloon or surgical glove) and record the resultant damped oscillation of the system.

Method described in the figure:

- Usage of surgical-glove material (elastic) to cover the cavity and allow increasing the pressure
- System pressurized by squeezing the sphygmomanometer bulb
- Balloon punctured with a burning match /hot iron
- Response observed using a storage oscilloscope or data acquisition system
- Measure the amplitude of successive peaks and determine the logarithmic decrement D
- Measure the time between successive positive peaks
- Calculate the damping ratio ζ and natural pulsation ω

Modelling the catheter – liquid – membrane system

- Response to a sinusoidal signal



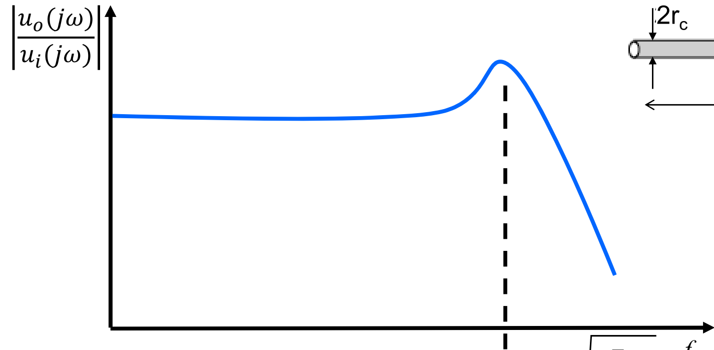
The natural frequency and damping are measured experimentally by exciting the catheter with a sine wave pressure actuator with changing frequency.

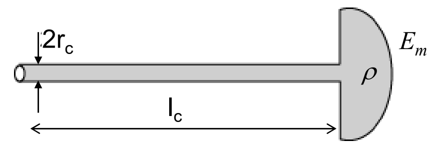
- This method is more complex compared to the step response

Method:

- A low-frequency sine generator drives an underwater-speaker system that is coupled to the catheter of the pressure sensor under test.
- An ‘ideal’ pressure sensor (freq response 0-100Hz) is connected directly to the test chamber housing and monitors input pressure
- The model of the catheter –sensor system is obtained by determining the amplitude and the phase of the output as a function of frequency

Frequency range of extravascular sensor





$$f_o = r_c\sqrt{\frac{E_m}{4\pi\rho l_c}}$$

The frequency range is limited by the size of catheter (l,r), the membrane elasticity (E_m) and density (ρ) of the liquid
→ low pass behavior

Once we have determined the frequency range of the catheter, we need to test if it is wide enough to encompass the frequency band of the blood pressure.

To broaden the frequency band, we must mitigate the inertia of the liquid in the catheter. One approach is to place a membrane at the tip of the catheter, ensuring direct contact with the blood.

Intravascular sensor

- Move the sensor at the tip of catheter and avoid the RLC circuit which reduces the frequency band
- Example: "Gaeltec"
  - Structure: metallic diaphragm with deposed gauges
  - Excitation: 5V AC r.m.s. maximum or 1V DC maximum
  - Bridge resistance: 1.5kΩ nominal
  - Sensitivity: 5μV/V/mmHg
  - Range: 0 - 150mmHg for urology
  - Range with temperature compensation:15-40° C
  - Temperature offset: < 0.05%FS/° C
  - Temperature sensitivity: < 0.2%/° C
  - Linearity error and hysteresis: < ±1%FS BSL
  - Surpressure: 600mmHg

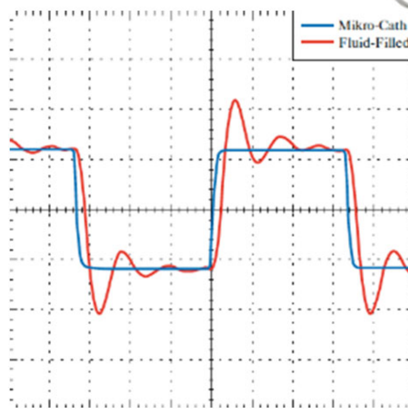
- Example catheter diameter: F5
  - F: French scale (0.33 *mm*)
  - F5=1.65mm

This sensor is also used for measuring pressure inside and outside of the bladder

Millar intravascular sensors

Specification

Part Number	825-0101
Catheter Material	Nylon
Effective Length	120 cm
French Size (Sensor)	3.5F (1.2 mm o.d.)
French Size (Catheter Body)	2.3F (0.9 mm o.d.)
Tip Characteristics	Straight
Connector Type	Low Profile
Reusable	No
Ship Sterile	Yes



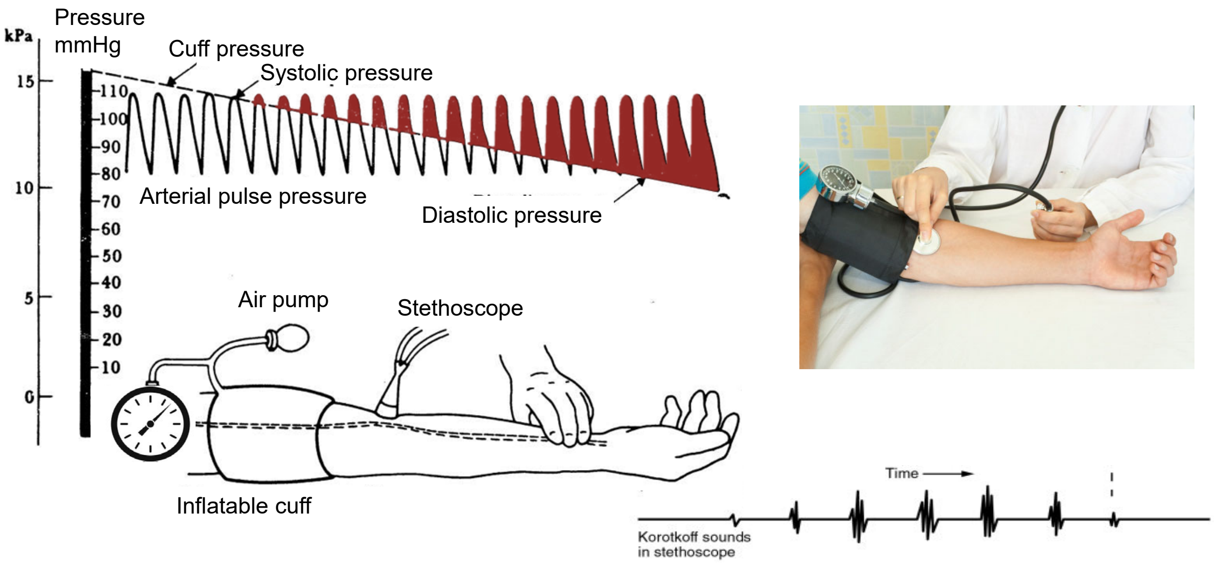
In the right figure we can see the faster response of the intravascular sensor (blue signal) as compared to the response of fluid-filled catheter (red signal).

RESISTIVE SENSORS

Part V- Indirect measurement of arterial pressure

Indirect methods (non-invasive):
Techniques such as auscultatory or oscillometric methods are clinically significant for routine screening, diagnosis, and management of hypertension and cardiovascular risks. They are safer, simpler, and suitable for widespread use, facilitating early detection and ongoing management of cardiovascular diseases.

Measuring arterial pressure: Indirect measurement



At the beginning, using air pump, the cuff exerts a pressure higher than systolic value (e.g. 180mmHg or higher to block blood flow in artery).The stethoscope allows to listen to sound produced by circulation of blood (Korotkoff sounds). A valve in the the air pump is open to decrease the cuff pressure linearly. The user notes the value of pressure displayed by the manometer when the sound appears (first sound) corresponding to systolic pressure and when the sound disappears (last sound) corresponding to diastolic pressure. Only these two values of pressure profile can be measured.

Korotkoff method

- The cuff pressure is raised above the systolic pressure e.g. 180mmHg: The artery is blocked preventing all circulation of blood.
- The cuff pressure is decreased by 2-3mmHg/s.
- When the systolic pressure = occlusive pressure, the blood begins to flow again
- The stethoscope detects an audible sound (Korotkoff). The pressure value corresponds to the systolic pressure (around 120 mmHg).
- When the cuff pressure reaches the diastolic pressure, the Korotkoff sound disappears (~ 80 mmHg)

Korotkoff method

- The Korotkoff sounds goes through 5 phases:
  - phase 1 : the initial sound (cuff pressure = systolic pressure)
  - phase 2 : the intensity of the sound increases
  - phase 3 : the sound reaches its maximum intensity
  - phase 4 : the sound is muffled and muted
    (cuff pressure = diastolic pressure)
  - phase 5 : the sound disappears
- **Origin**
  - Turbulences, vibration of the arterial walls
- **Accuracy**
  - underestimates the systolic pressure by 1 to 13 mmHg
  - overestimates the diastolic pressure by 8 to 18 mmHg

Why Korotkoff method underestimate systolic blood pressure and overestimate diastolic blood pressure?

1. **Underestimation of Systolic Pressure**:

- *Cuff Deflation Rate*: If the cuff is deflated too rapidly, the Korotkoff sounds indicating systolic pressure may be missed or not heard clearly. This can result in underestimation of systolic blood pressure.
- *Arterial Wall Characteristics*: In some individuals, especially those with stiff arteries (common in older adults), the Korotkoff sounds may be muffled or difficult to hear. This can lead to underestimation of systolic blood pressure.
- *Pressure Sensitivity*: In certain conditions such as hypotension or shock, the point at which Korotkoff sounds appear can be difficult to discern due to low pressure levels, resulting in underestimation of systolic blood pressure.

2. **Overestimation of Diastolic Pressure:**

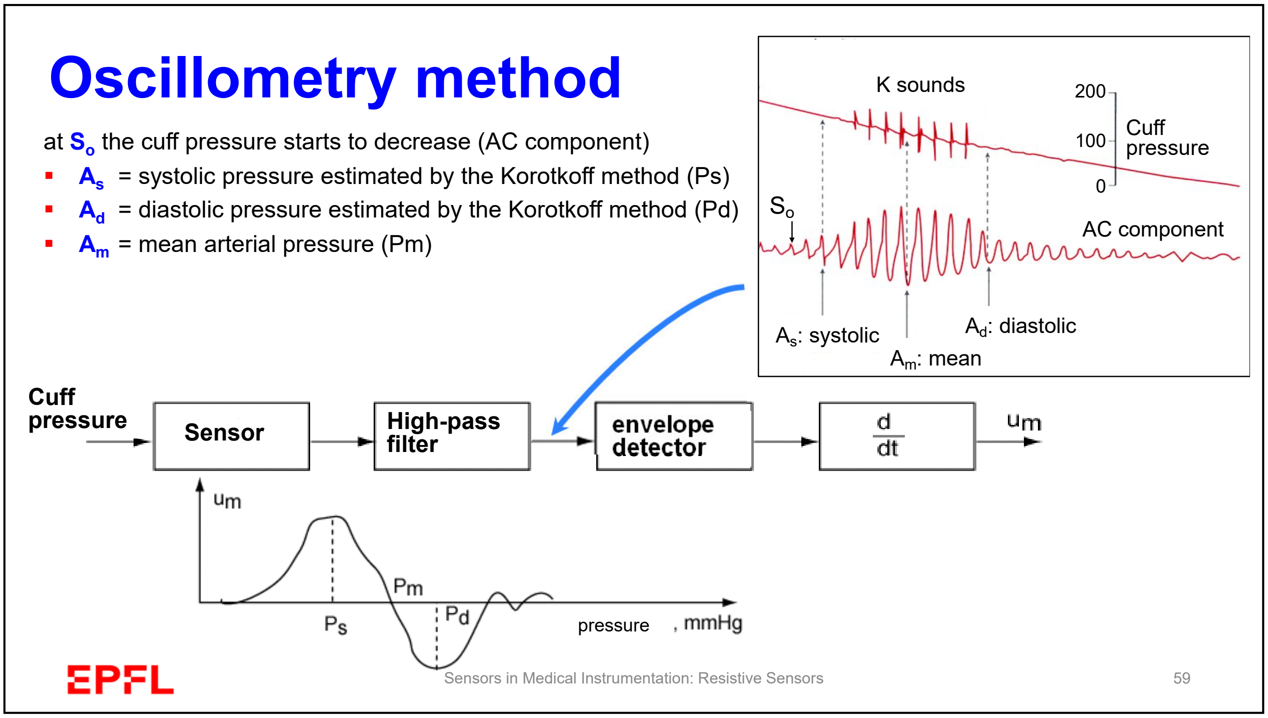
- *Incorrect Technique*: If the practitioner fails to properly position the stethoscope or accurately determine the point at which Korotkoff sounds disappear, it can result in overestimation of diastolic blood pressure.

Oscillometry method

- Identical to the Korotkoff method; however the measurement of systolic and diastolic pressures is done directly by reading the cuff pressure and not by the stethoscope
- Change of volume under the cuff at each pulse
- Change of air volume inside the cuff
- Change in cuff pressure
- Sensors measure the pressure value inside the cuff

Most automatic blood pressure monitors use an oscillometric technique. The cuff is inflated and then released gradually just like the manual method (Korotkoff method), but instead of listening for sounds, **a pressure sensor monitors the small oscillations in cuff pressure caused by the pulsing of blood under the cuff.**

By analyzing the amplitude and pattern of these oscillations during deflation, the systolic, diastolic, and mean arterial pressures are determined electronically and/or using signal processing methods, without needing a stethoscope.



With each heartbeat, even while the artery is partially compressed, a tiny fluctuation (an **oscillation**) in the cuff pressure occurs due to the arterial wall vibrating. The device filters and analyzes these pressure oscillations. Typically, the amplitude of oscillations starts small when the cuff is above systolic, then grows and reaches a maximum around mean arterial pressure, and then decreases again as the cuff pressure drops below diastolic. The algorithm identifies key points in the oscillation amplitude curve to estimate systolic and diastolic pressures.

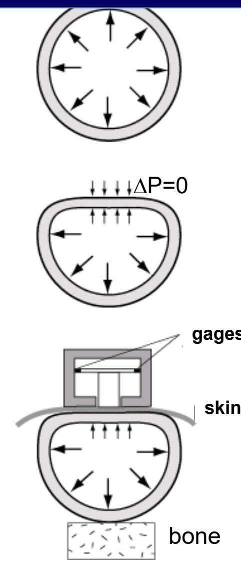
Advantages: Ease of use (only an arm cuff and a microprocessor are needed), and it can even be used in noisy environments or on patients where Korotkoff sounds are hard to hear.

Disadvantages: It can be less accurate in patients with arrhythmias, very low blood pressure, or other conditions, and it provides no auscultatory information (like detection of arrhythmias that a human might hear while listening).

Tonometry

- Vessel wall partially flattened
- Pressure gradient vanishes through flattened part
- Pressure measured outside vessel = arterial pressure

- **Applications**
  - Measuring heart rate
  - Measuring ocular pressure



Another non-invasive technique involves pressing a sensor against an artery to partially flatten it (**applanation tonometry**). When the artery is flattened just enough that the pressure inside equals the pressure applied outside (no net pressure gradient across the wall), the sensor reading on the surface equals the arterial pressure.

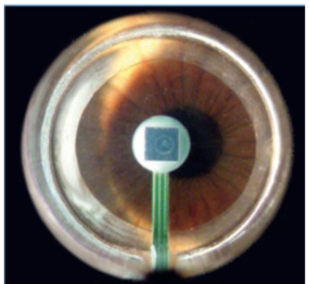
In practice, a flat pad with strain gauges (a force sensor) is pressed over a superficial artery (like the radial artery at the wrist). When calibrated and held at the correct pressure, it can capture the pulse pressure waveform continuously without needing an internal catheter.

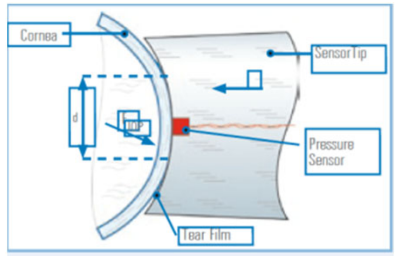
Advantages: Provides a continuous arterial pressure waveform non-invasively and can be used for beat-to-beat monitoring.

Disadvantages: It requires precise positioning and constant contact; motion or slight shifts can cause error. It’s also highly dependent on calibration and usually needs individual calibration with a conventional cuff method

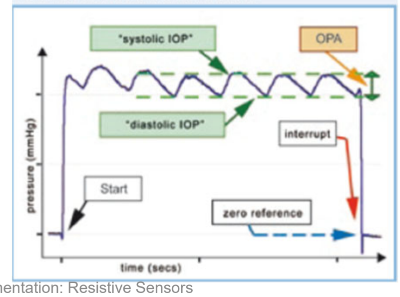
Example: Ziemer tonometer

SensorTip with contour-matched contact surface





This is what the PASCAL tonometer measures:



In ophthalmology, a similar concept is used for intraocular pressure measurement: **applanation tonometry** on the eye flattens the cornea until internal eye pressure equals external force – the force measured at that point (via a strain-gauge based force sensor) indicates the intraocular pressure.

This is critical for **glaucoma diagnosis**.

RESISTIVE SENSORS

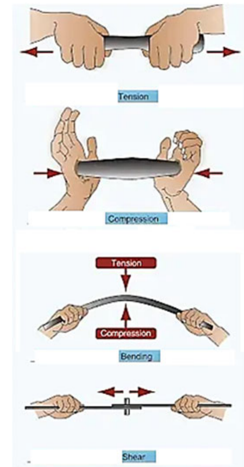
Part VI- Force measurement: instrumented implant

Resistive sensors (especially strain gauges and force-sensing resistors) are widely used to measure forces in biomedical applications, from internal implantable devices to external platforms and wearables.

Kinetic measurement

Force sensors

- The force is transformed to deformation via a test specimen
- Testing in tension, compression, flexion or shear
- The deformation is measured by a metallic or semiconductor strain gage mounted on a Wheatstone bridge.

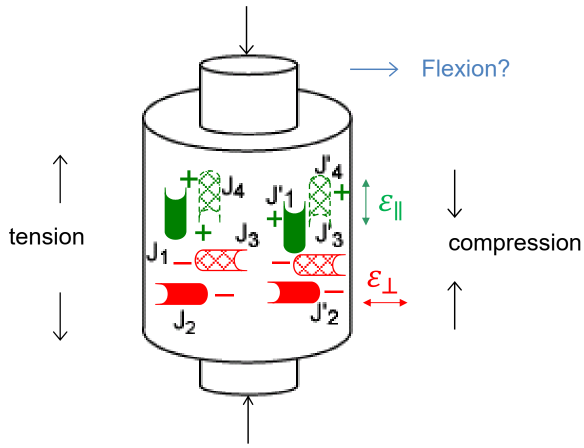


The measurement of the force requires a mechanical component, such as a beam or bar, to convert force into strain. For instance, when force is applied to a beam in flexion, the superior part of the beam elongates. Various forces, including tension, compression, flexion, and shear force, can cause different deformations in the test specimen.

To measure these deformations, metallic or semiconductor strain gauges can be mounted on a Wheatstone bridge. This allows for accurate detection and quantification of the resulting strain caused by the force applied to the test specimen.

Kinetic measurement

- Example of a test specimen in tension – compression mode

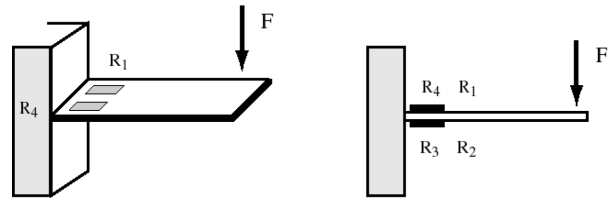


Here is an example of test specimen - the metallic cylinder - where we want that the sensor measures the vertical force and not be sensitive to other components of force (i.e. shear force). **So, it is important to place the sensors (strain gage) correctly and understand the effect of the various forces on sensor deformation.**

For a force providing an elongation (resp. compression) of cylinder, the green gages measure a tension (resp. compression) while the red ones measure compression (resp. tension depending Poisson coefficient). **Therefore red (resp. green) gages should be placed in opposite arms (two in each arm), to increase the sensitivity of the bridge.**

Note that a shear force (horizontal component of force) will provide a flexion (e.g. tension in J1 and J4 and compression in J'1 J'4) which should not affect the measurement since green gages are in opposite arms. Similar is the case of red gages.

Example flexion mode



$$u_o = u_a - u_b = \frac{R_2}{R_1 + R_2}u - \frac{R_4}{R_3 + R_4}u$$

$$= \frac{R_2 R_3 - R_1 R_4}{(R_1 + R_2).(R_3 + R_4)}u$$

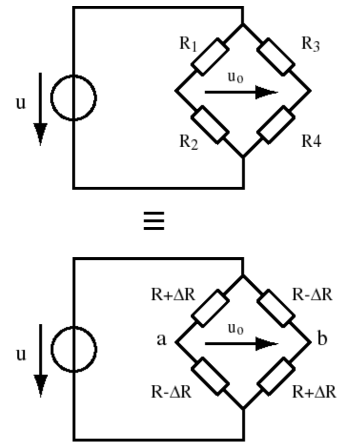
For $R_1 = R_2 = R_3 = R_4 = R$

$$u_o = \frac{(R - \Delta R)^2 - (R + \Delta R)^2}{(2R)(2R)}u = -\frac{\Delta R}{R}u$$

$$\frac{\Delta R}{R} = K\frac{\Delta l}{l} = K\varepsilon$$

Relative variation of resistance for metallic strain gage

$$\Rightarrow \quad u_o = -K\frac{\Delta l}{l}u = -K'uF$$



EPFL

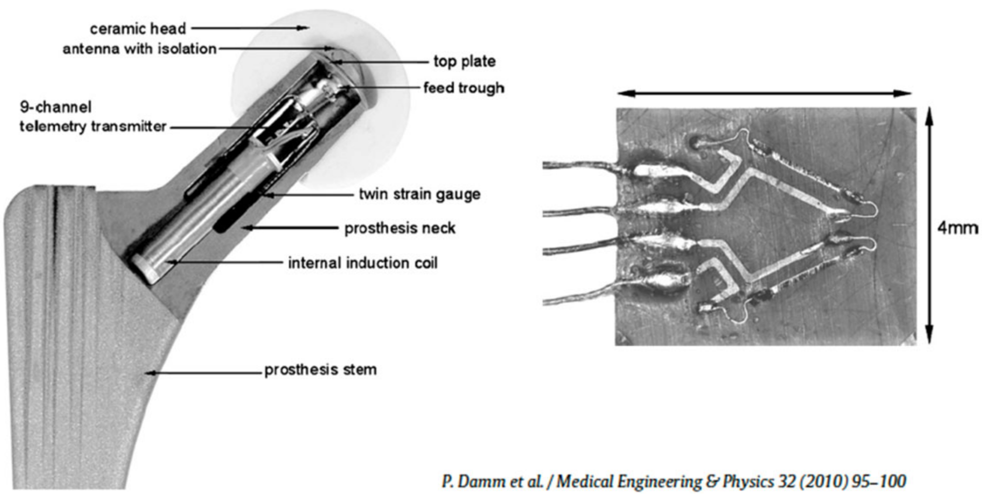
Here is an example of the beam in flexion mode, and the goal is again to know how to place the strain gages in order to have a maximal sensitivity. The force shown in the figure corresponds to a flexion - that means the superior part of the beam will be elongated and the inferior part compressed.

Imagine that you have 4 strain gages, we will place 2 on the superior part (R1, R4), and 2 on the inferior part (R2, R3). **To obtain the maximal sensitivity of Wheatstone bridge, we place R1 and R4 on opposite arms, and R2 and R3 on the other 2 opposite arms**

The sensitivity of the force sensor is K'u, where K' includes the gage factor as well as dimensions and properties of the test specimen (relationship between strain and force, according to dimensions and shape of the test specimen).

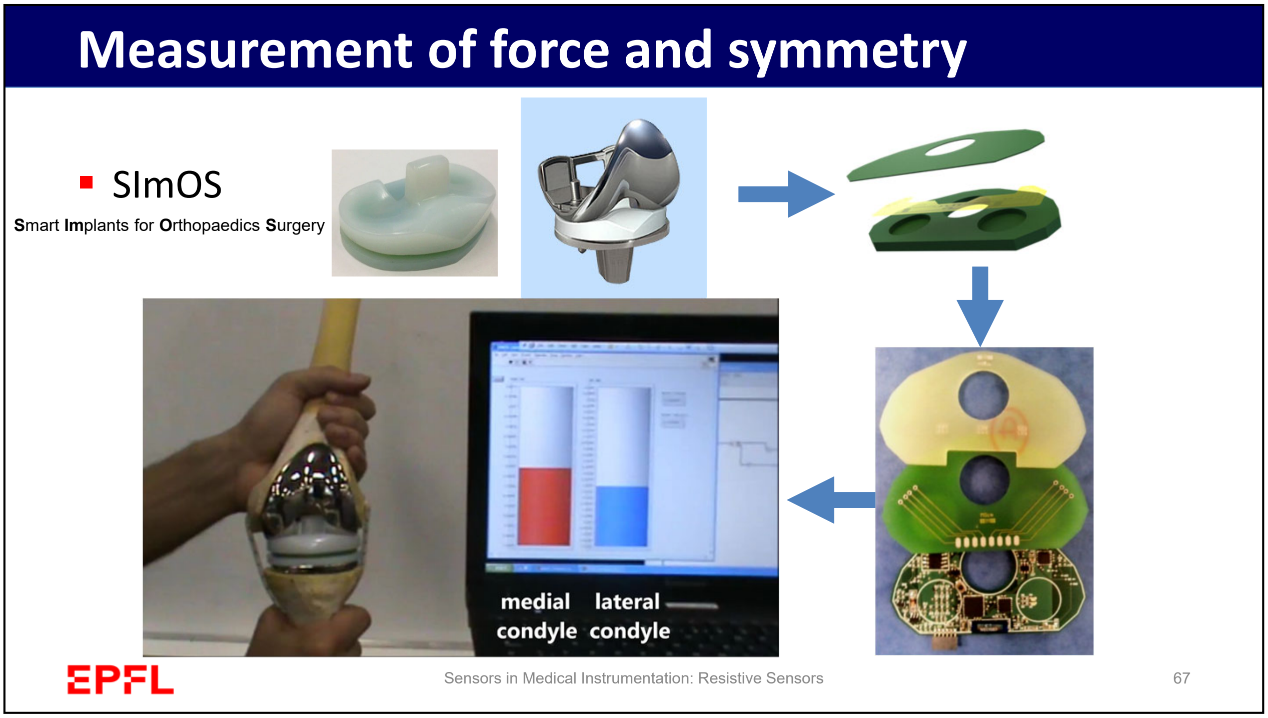
Important note: A Wheatstone bridge's output is **ratiometric with respect to its excitation voltage**. This means the output voltage scales linearly with the supply (excitation) voltage. The bridge output is often specified in terms of **millivolts per volt (mV/V)** of excitation. For example, suppose our full-bridge strain-gage sensor has a rated sensitivity of **2 mV/V** under a certain bending strain. This indicates that for every 1 V of excitation, the bridge will output 2 mV of differential voltage (at full strain). If you excite the bridge with a 10 V supply, the output would be about **20 mV** at that same strain level. Likewise, using a 5 V excitation would yield a 10 mV output, and so on. This ratiometric behavior is useful because it allows easy scaling of the sensor's output and ensures that any variation in excitation voltage (within the sensor's linear range) proportionally affects both the output and the measured reference, maintaining accuracy.

Instrumented prosthesis



P. Damm et al. / Medical Engineering & Physics 32 (2010) 95–100

An interesting application of strain gages in medical settings is in instrumented prostheses. In the image shown here, strain gage (shown on the right) are inserted into the prosthesis, and a coil is used to provide power the Wheatstone bridge inside through magnetic coupling. The instrumentation is integrated into the **hip implant**, and during daily activities such as walking, the force can be measured, and its value evaluated to provide feedback to the subject. This feedback can help the individual adapt or improve their walking or gait. It is also useful in research to determine the contact force in the hip, which is crucial for designing new prostheses.



This is another example developed in EPFL, where instead of instrumenting the metallic part of the prosthesis, which could necessitate changing the design/shape in order to integrate the electronics, here the instrumentation is inserted into the soft polytene part which is placed between femoral and tibial part of the implant (here the example is a knee prosthesis).

Two strain gages are placed on the medial condyle and the lateral condyle of the insert to measure the amount of force applied. This can inform, for example, if the subject has a normal gait (walking pattern) with a normal distribution of force. If for example there is a medical condition – the prothesis is loosen or there is an injury - this can be detected and corrected.

Can be used also during surgery to align the femoral with the tibial part of the knee

The animation shows that his micro structure inside the insert allow measuring force balance of prosthesis.

RESISTIVE SENSORS

Part VII- Force plate:
application in biomechanics

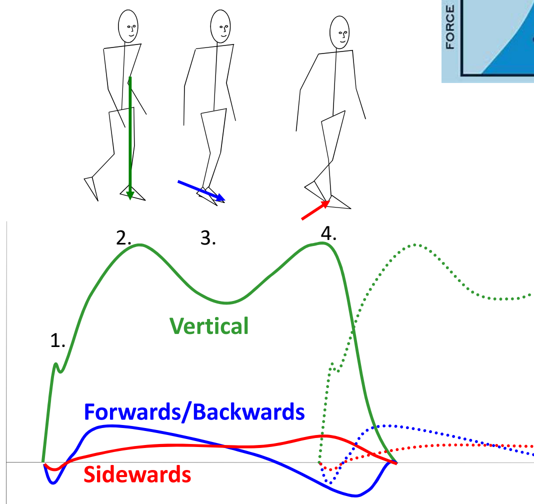
Force plates are mechanical sensing systems designed **to measure the ground reaction forces** and **moments** involved in human movements. A force plate relies on the use of load cells to determine forces. The load cells may contain strain gauges or piezoelectric elements (Chapter Piezoelectric Sensors).

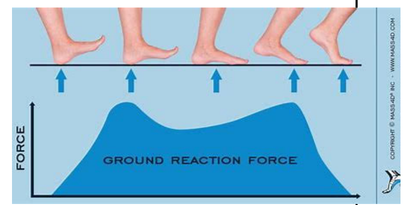
As force is applied to the plate, the sensors distort thereby causing measurable voltage changes that are proportional to the applied force. Placing the sensors in different orientations enables the direction and magnitude of forces in 3D to be obtained.

Force platform

- The force platform allows:
  - the measurement of external forces
  - the estimation of the center of pressure (CoP)
  - the measurement of joint forces and moments

Force platform: ground reaction force





1. Heel Strike
2. Weight Acceptance
3. Mid Stance Phase
4. Push Off

Measurement of External Forces

The force platform (also known as a force plate) measures the external reaction forces exerted by a person or an object upon it. These reaction forces typically include vertical (ground reaction force), horizontal (anteroposterior and mediolateral), and shear forces. Such data are crucial in biomechanics, as they help analyze movement patterns, gait, balance, and performance. For instance, measuring external forces allows engineers and researchers to understand how much force is generated during jumping, walking, or running, enabling evaluations related to performance enhancement or injury prevention.

Typical usage scenarios:

1. Clinical assessment of gait abnormalities.
2. Analysis of athlete performance (jumping, running).

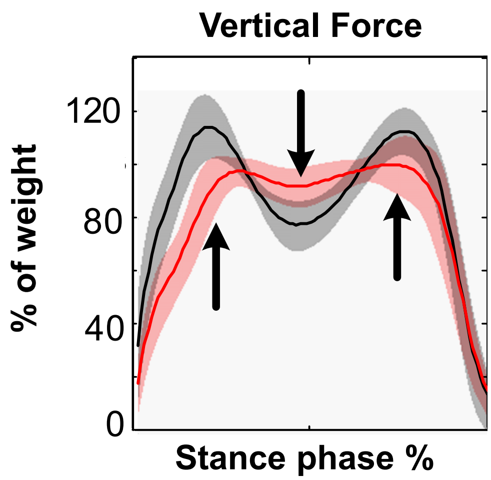
1. Clinical assessment of gait abnormalities: The vertical ground reaction force (GRF) increases when the center of mass is accelerated upwards and decreases when the center of mass is accelerated downwards (deceleration). The GRF is therefore not constant and varies according to the displacement of the center of mass during the different phases of gait.

During walking (see figure), the vertical GRF is characterized by two maximums and a minimum.

- The first maximum is after the initial contact of the heel (heel strike) and exceeds the body weight: the body weight is fully supported on one foot and the body is accelerated upwards.
- Then the body decelerates (acceleration downwards) and the vertical GRF decreases. The minimum corresponds to the late stance (pre-swing phase), when the vertical GRF drops below body weight as the foot prepares to leave the ground.
- The second maximum is associated with the acceleration phase, or pushing and again exceeds the body weight: the body weight is fully supported on one foot and the body is again accelerated upwards.

Finally, the force reaches zero when the foot leaves the platform (terminal contact) while the other foot fully supports the weight of the body.

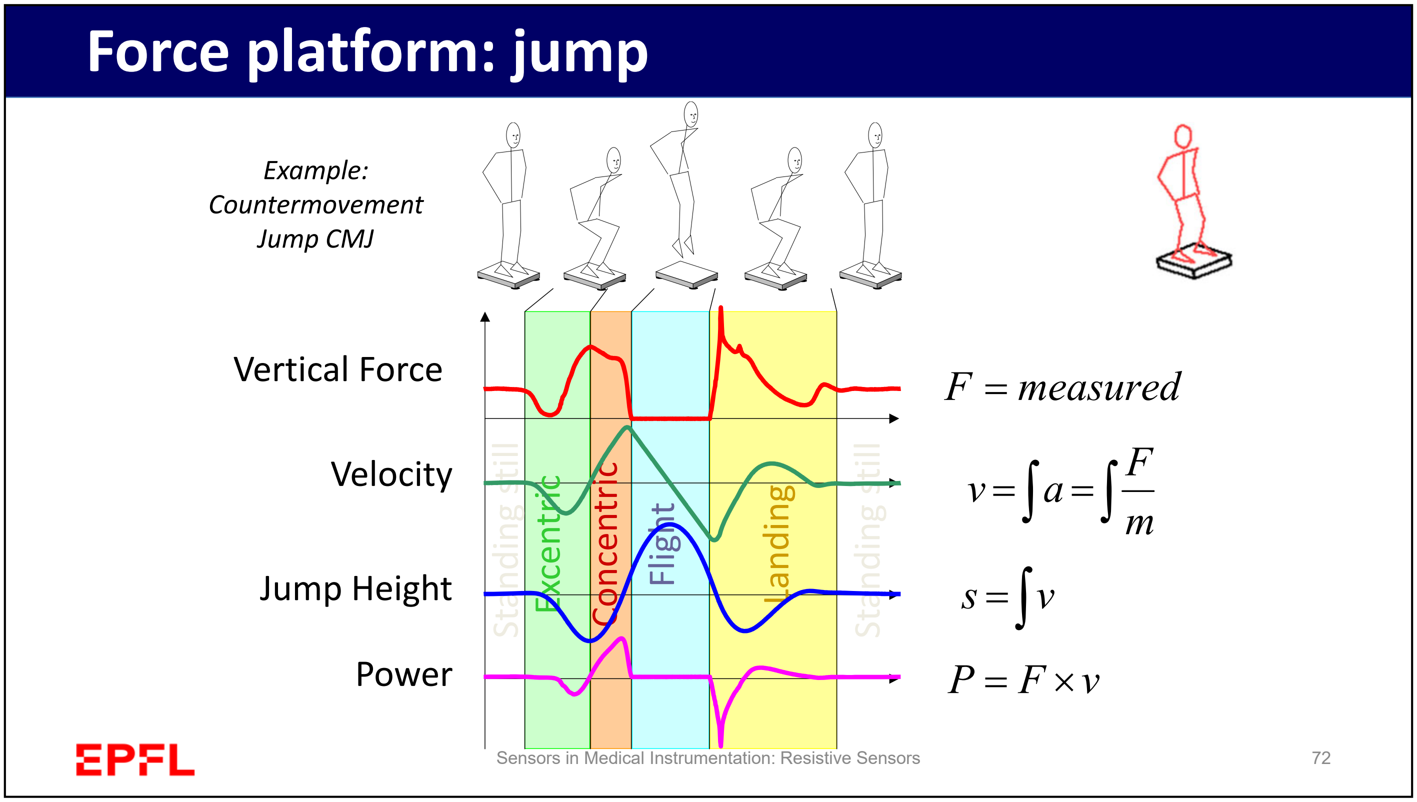
Example: vertical ground reaction force during walking



Healthy subjects
Patients with ankle arthritis

EPFL

The figure shows the vertical reaction force during the stance phase comparatively between healthy individuals and patients with ankle arthritis, represented as a percentage of their body weight. For patients with ankle arthritis, a plateau is visible, indicating that they walk slowly and with a low heel strike impact to avoid joint pain. The graph shows the average value and standard deviation for the group of compared subjects.



Force plates have multiple applications in sports, one of which is to evaluate the performance of athletes by asking them to perform a specific action, such as a countermovement jump (CMJ). During the CMJ, the athlete starts in a standing position, performs a quick downward movement (the countermovement), and then explosively jumps upward.

In the example shown here, the force plate records the profile of the vertical reaction force during the various phases of the jump.

The vertical force can be used to calculate the acceleration (acc) of the athlete using the formula F/mass. By integrating the acceleration, we can obtain velocity, and by integrating the velocity, we can find the height of the jump. Additionally, by multiplying the force by velocity, we can calculate the power of the jump.

These estimations of various parameters, such as power and duration of flight, can be used to evaluate the performance of the athlete. For example, by asking them to perform several jumps, we can identify athletes with high performance or evaluate their rehabilitation progress after injury.

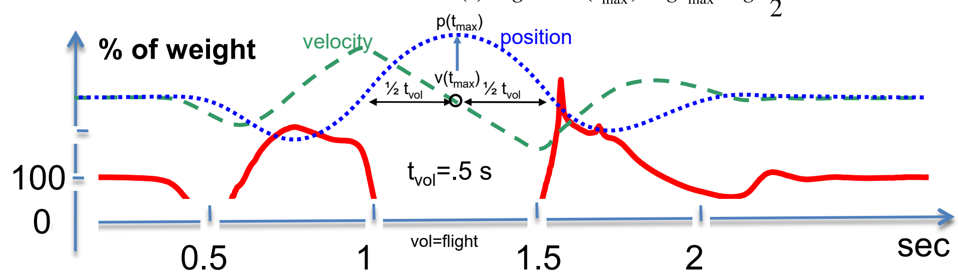
Example: calculating the jump height

Show that: $h_{vol} = \frac{g}{2} \cdot \left(\frac{t_{vol}}{2}\right)^2 = \frac{9.8}{2} \cdot 0.25^2 \approx 30cm$

$$mgh_{\max} = \frac{1}{2} mv^2(t_{\max}) = \frac{1}{2} mg^2\left(\frac{t_{vol}}{2}\right)^2$$

$$v(t) = gt \Rightarrow v(t_{\max}) = gt_{\max} \approx g\frac{t_{vol}}{2}$$

vol=flight

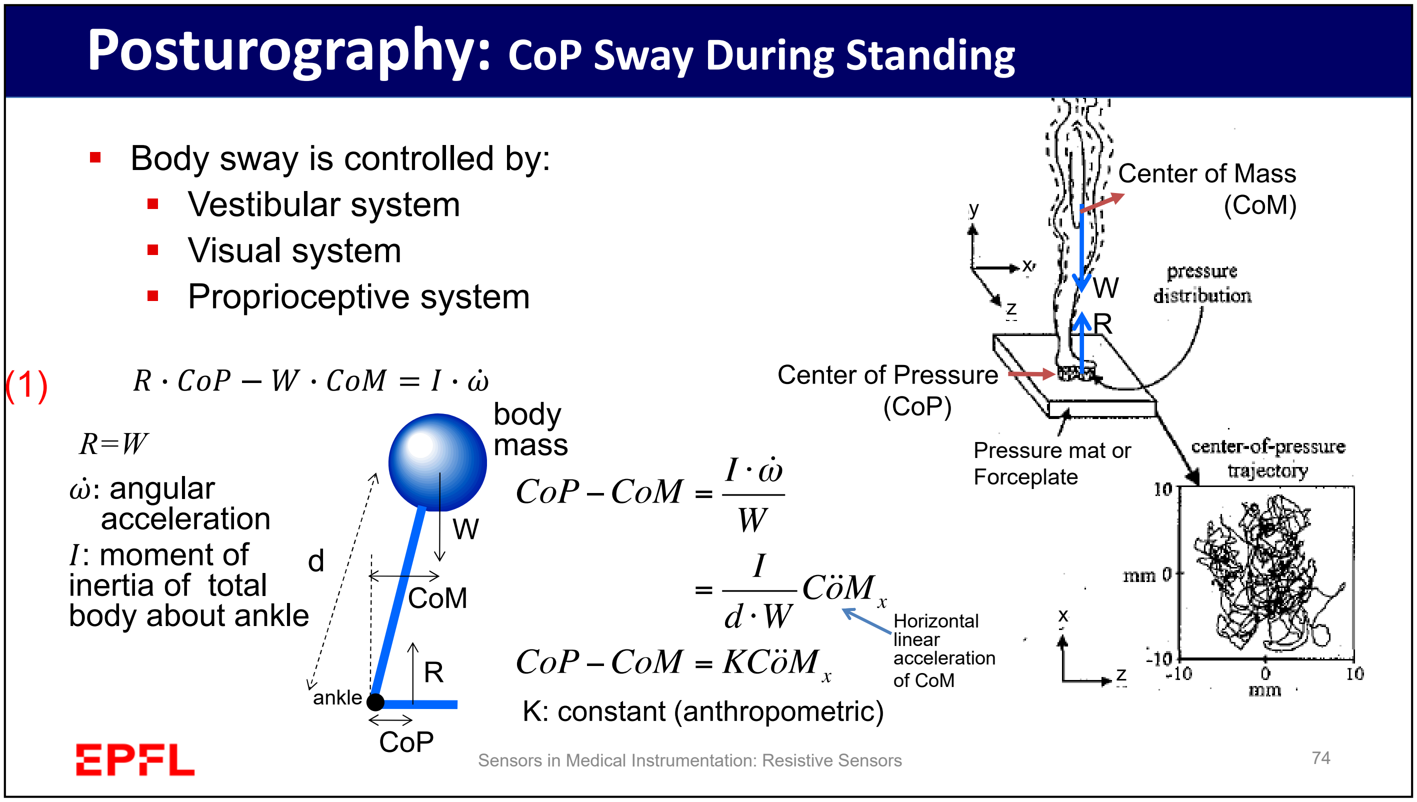


Here is another technique to **measure the jump height based on assumption that during the jump the kinetic energy is transformed entirely to potential energy.**

The maximal height corresponds to the maximal **change** of velocity. From the profile of velocity, we can observe that at the beginning there is a certain velocity, but during the jump this velocity is loosen: at the beginning of jump the velocity is maximal (go upward), at the maximal jump height velocity is close to zero, then during downward velocity is negative because it is in opposite direction at landing compared to takeoff.

So, the maximal change in velocity corresponds to the difference between velocity at takeoff and velocity at the maximal position.

It can be assumed that the maximal position (height) is reached at the middle of flight time (same time for ascending and descending phase)



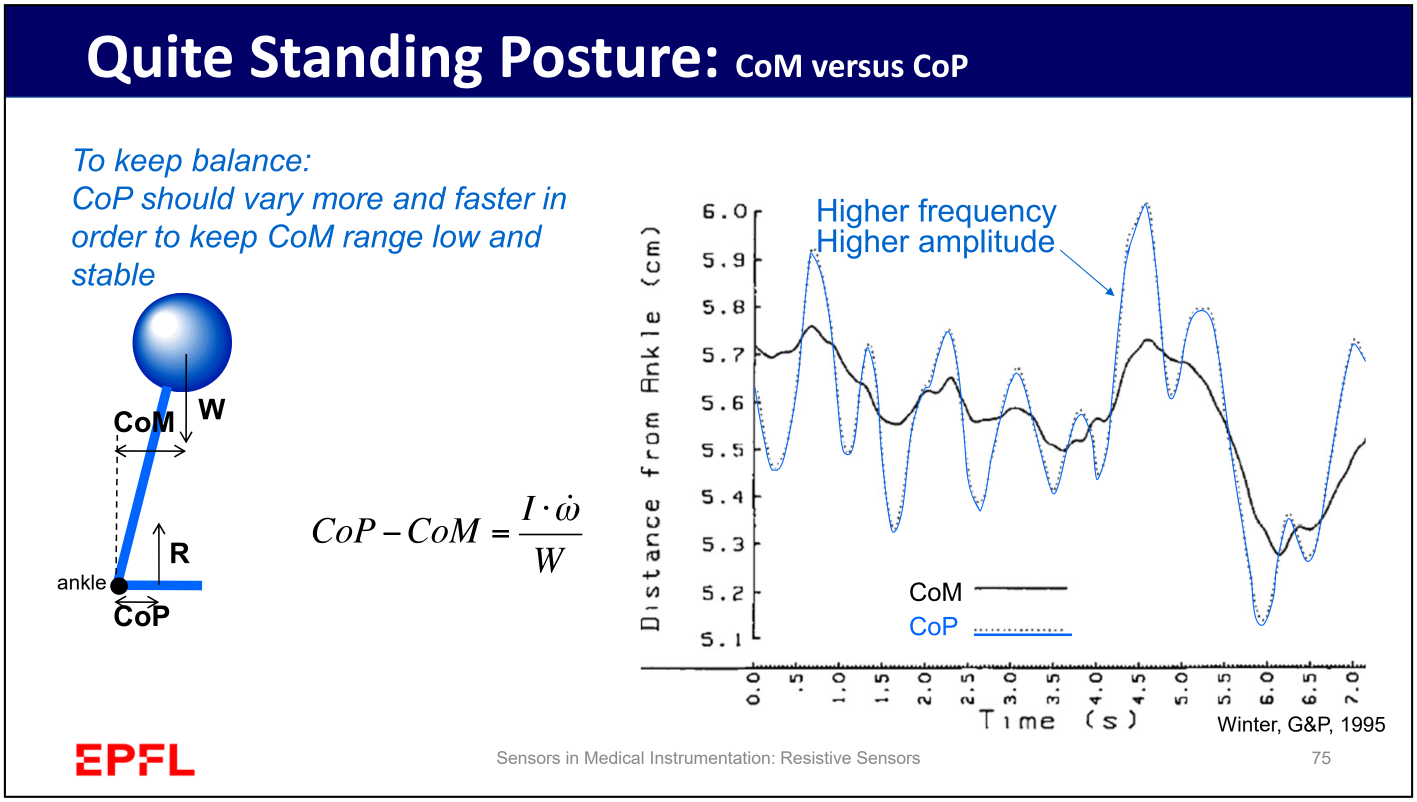
Maintaining balance involves continuous integration of information from the muscular, proprioceptive, and vestibular systems, along with visual input. The central nervous system constantly combines data from these sources to assess stability, predict potential imbalance, and produce timely muscular corrections. If one system is compromised, the others typically compensate to maintain equilibrium, although balance may become more challenging or less stable under demanding conditions.

The center of pressure is the point location on the force platform surface where the resultant ground reaction force vector is applied. This point provides valuable information on balance and postural control. As the body shifts or moves, the CoP moves accordingly, indicating how effectively an individual maintains balance. Analyzing CoP trajectories helps engineers and researchers assess stability, evaluate balance disorders, and athletic performance.

CoP trajectory: When you are in a standing position, you are in an unstable situation because your body's center of mass (CoM) is approximately 1 meter above the ground. It's like to an inverted pendulum that requires you to maintain balance and equilibrium at all times. This is done by the muscular force applied on your lower limbs and your trunk in order to keep your CoM stable; at any time, your muscles are stimulated in order to keep your balance. So, in order to keep your body mass in stable position, you have to apply a certain **force** on your foot (feet) that can be measured by the force plate.

Relationship Force-Pressure: In purely physics terms, force (F) represents a vector quantity whereas pressure (p) represents a scalar quantity defined by the ratio between the normal force acting on a given surface and the area over which this force is applied. **With the force-plate posturography, the postural control assessment is based on the characteristics of the center-of-foot pressure (COP) oscillations which are equivalent to spontaneous COM motions on the level of the base of support (feet).**

Relationship CoP–CoM: Body weight W is equal and opposite to the vertical reaction force R, and this 'parallelogram' of forces' acts at distance CoM and CoP respectively from the ankle joint. Both W and R will remain constant during quiet standing. Assuming the body to be an inverted pendulum (see figure), pivoting about the ankle, a counterclockwise moment equal to R*CoP and a clockwise moment equal to W*CoM will be acting according to eq (1), where I is the moment of inertia of the total body about the ankle joint.



When the CoP moves ahead of the CoM, it generates a backward acceleration to bring the CoM back into alignment. Conversely, when the CoP moves behind the CoM, it accelerates the CoM forward. As a result, the amplitude and frequency of CoP fluctuations are always greater than those of CoM variations, making CoP a more dynamic and responsive measure, while CoM moves relatively less and more smoothly over time.

This explains why CoP variations are higher frequency and amplitude compared to CoM, especially noticeable during quiet standing, where continuous and subtle neuromuscular adjustments maintain balance.

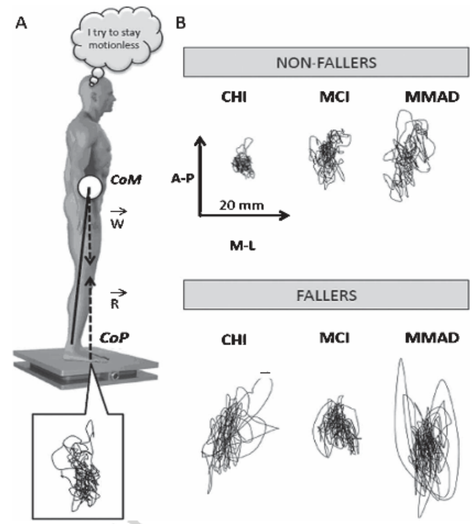
Postural sway in clinical assessment

Objective:
To examine the center-of-pressure (COP) velocity association with cognitive status and history of falls, in cognitively healthy individuals (CHI), patients with mild cognitive impairment (MCI), and with mild-to-moderate Alzheimer's disease (MMAD).

Conclusion:
identifying people with and without cognitive impairment who are at risk of falls risk via the evaluation of the postural control strategies might be a valuable window of opportunity for fall-prevention interventions.

Mignardot, Jean-Baptiste, et al. "Postural sway, falls, and cognitive status: a cross-sectional study among older adults." *Journal of Alzheimer's Disease* 41.2 (2014): 431-439.

J.-B. Mignardot et al. / Postural Sway, Cognitive Status, and Falls



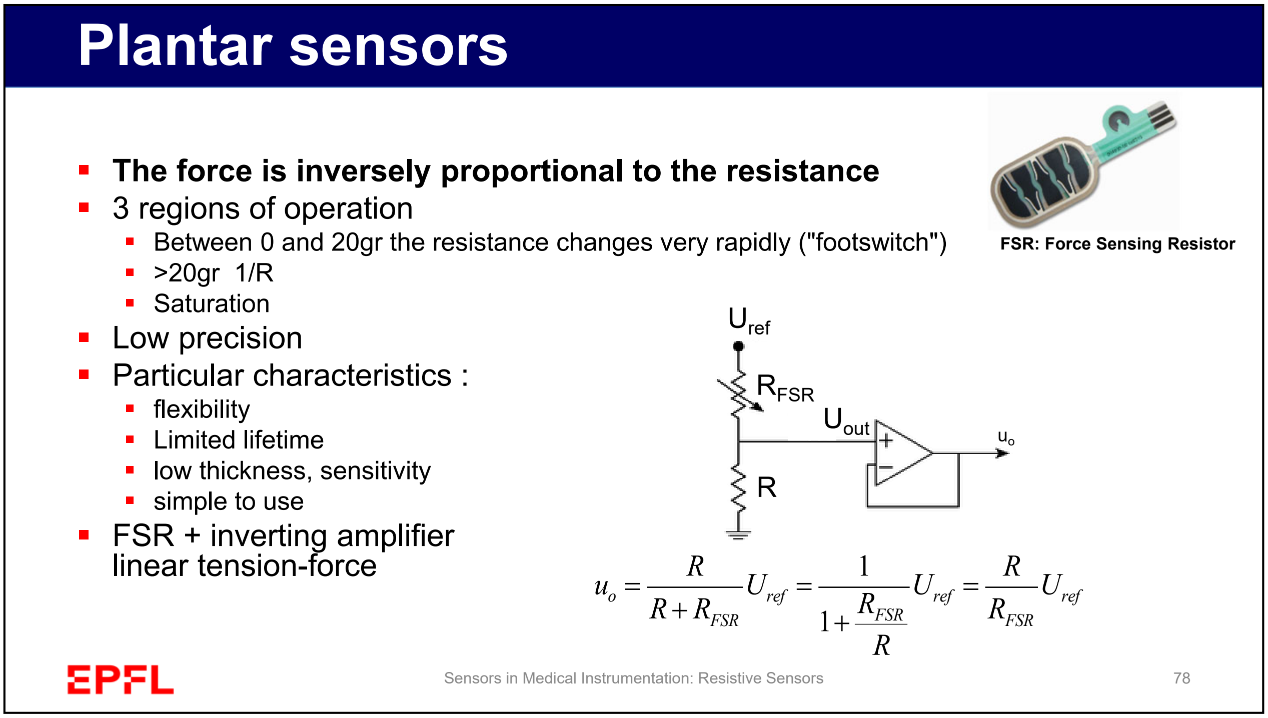
1. Representative examples of center-of-pressure (COP) trajectories recorded using a force platform (A), as a function of the cog us (CHI, MCI, and MMAD) and fall risk (non-fallers versus fallers) (B). CHI, cognitive healthy individual; MCI, mild cognitive impair IAD, mild-to-moderate dementia; AP, anteroposterior axis; ML, medio-lateral axis.

Measurement of postural sway is clinically significant as it helps detect balance impairments in older adults. Increased sway is associated with higher fall risk and cognitive decline, particularly in conditions like mild cognitive impairment (MCI) and Alzheimer's disease. Thus, analyzing postural sway can aid clinicians in early identification of individuals at risk of falls and cognitive deterioration, supporting timely intervention and management strategies to enhance quality of life and reduce injury risk among older populations.

The observed associations between increased postural sway, higher fall risk, and cognitive impairment can be explained by the interconnectedness of motor control and cognitive functions. Cognitive decline—such as mild cognitive impairment or Alzheimer's disease—often affects executive functions like attention, planning, and sensory integration, which are essential for maintaining balance. When cognitive processing is impaired, the ability to quickly integrate sensory information and generate appropriate motor responses diminishes, resulting in increased sway and instability. Consequently, these individuals become more susceptible to falls. Thus, postural sway not only reflects motor control but also serves as an indirect marker of cognitive health.

RESISTIVE SENSORS

Part VIII- Wearable force measurement: gait analysis



Force Sensing Resistor (FSR): is a flexible sensor, designed to measure the force at the location where the sensor is placed, for example under the foot to measure the heel strike or toe-off. The sensor consist of a resistive film, flexible, and placed on a flexible material.

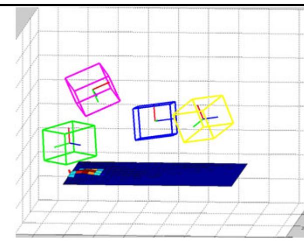
When conditioning a flexible FSR with an operational amplifier (voltage follower), a compensating resistor R is necessary because:

- The resistor forms a stable voltage divider with the FSR, converting its nonlinear resistance into a measurable voltage output.
- It provides a defined DC path for the amplifier's input bias currents, preventing drifting or floating inputs.
- Selecting an appropriate resistor value enhances the sensitivity, linearity, and accuracy of the measurement across the sensor's useful range.

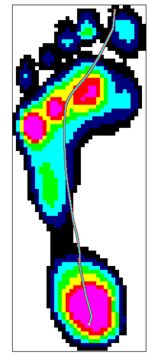
So, the compensating resistor ensures stability, accuracy, and consistent performance of the FSR measurement system.

The following slides illustrates few applications of FSR for analysis of gait. The terms ‘gait’

Plantar sensor



- Applications
  - measure the pressure distribution under the foot
  - Platform with high number of sensors (cells)
  - Integrate the sensors inside an insole
  - Diabetes
  - Functional Electrical Stimulation (FES)
  - Gait analysis



Applications of Planar Force Sensors:

1. Measuring Pressure Distribution Under the Foot
Planar force sensors measure the detailed pressure distribution beneath the foot during standing, walking, or running. This data reveals how pressure is distributed across the plantar surface, identifying abnormal loading patterns that can contribute to discomfort or injury.

2. Platforms with High Number of Sensors (Cells)
Platforms incorporating many closely-spaced sensing cells provide high-resolution pressure maps. This detailed data allows clinicians and researchers to precisely detect pressure peaks and loading patterns, aiding in clinical diagnoses and biomechanical analyses.

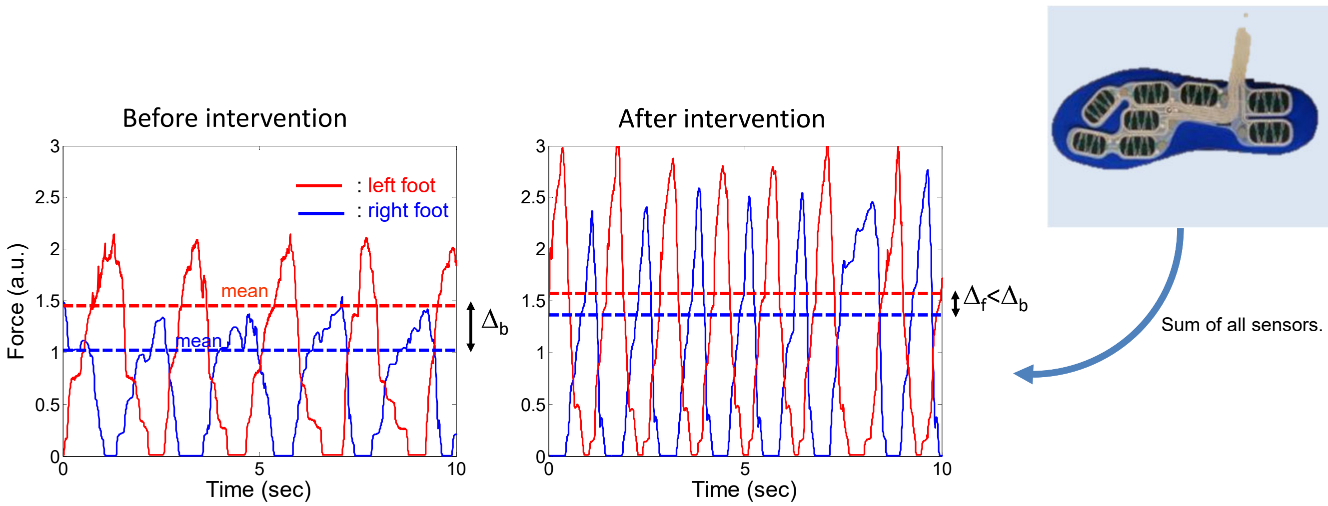
3. Sensor-Integrated Insoles
Sensors integrated into shoe insoles permit continuous, real-world monitoring of pressure distribution within footwear. These wearable systems capture foot pressure during daily activities, providing critical data outside clinical or laboratory settings.

4. Diabetes Management
Patients with diabetes often suffer from peripheral neuropathy and reduced sensation, leading to foot ulcers due to unnoticed excessive pressure. Planar force sensors help detect these critical areas early, guiding interventions such as customized orthotics, therapeutic footwear, and patient education to reduce ulcer risk.

5. Functional Electrical Stimulation (FES)
In FES, planar force sensors provide real-time feedback on pressure distribution to adjust electrical stimulation patterns dynamically. This feedback improves walking performance, gait symmetry, and postural stability in individuals with neurological impairments, such as spinal cord injury or stroke.

6. Gait Analysis
Planar sensors play a fundamental role in gait analysis by quantifying temporal-spatial parameters such as stride length, step width, and walking speed. Pressure distribution data obtained during gait assessments assist in diagnosing gait abnormalities, evaluating treatment effectiveness, and designing rehabilitation interventions to improve locomotion.

Example: Foot loading during walking after hip fracture surgery



Moufawad El Achkar et al., EPFL thesis, 2016

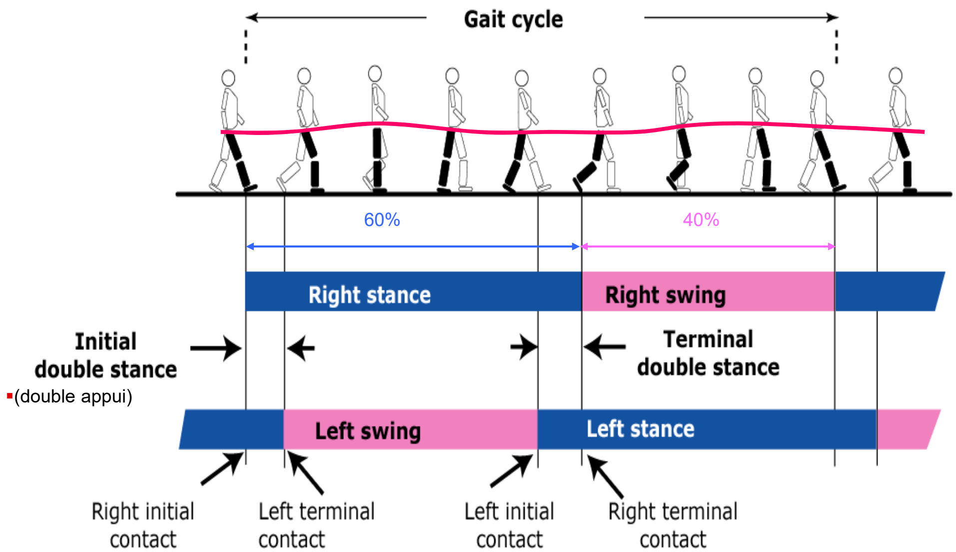
The graph on the left depicts, the total vertical force applied under each foot after surgery, but before starting the rehabilitation. On the right we can see measurements for the same patient after two weeks of rehabilitation.

While $\Delta_f<\Delta_b$ shows less asymmetry, the amplitude of the force has increased in both feet after rehabilitation.

Main raisons:

- No use of walking aids
- Increased walking speed
- Less pain

Temporal gait analysis



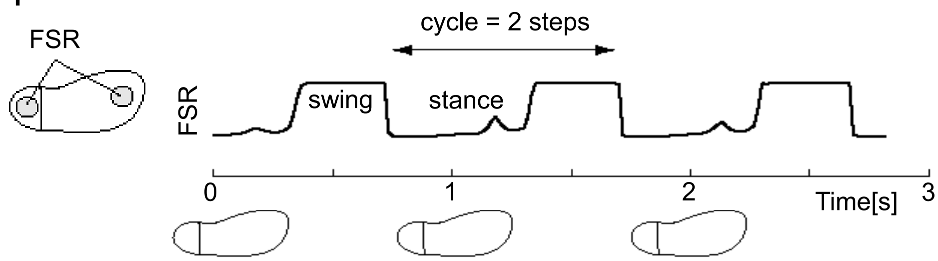
FSR can be used for **gait analysis.** This topic will be analyzed in more detail in the chapters dedicated to capacitive and piezoelectric sensors.

Definition of 'gait': Gait is the pattern or manner in which a person walks, involving coordinated movements of limbs and body segments.

Importance in clinical assessment: Analyzing gait is critical in clinical settings because it helps detect and characterize neurological, musculoskeletal, or functional impairments. Changes in gait patterns often indicate underlying disorders, track disease progression, or inform rehabilitation strategies.

Measuring the temporal gait parameters

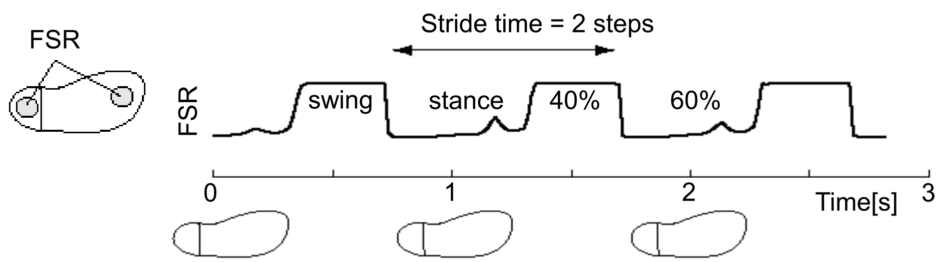
- Stance and swing phases, initial and terminal double stance, ground impact (heel strike), heel and toe off, rolling of the foot.
- Sensors are fixed directly on the sole of the foot or incorporated in an insole.



During the stance (foot on ground) the FSR decrease (more exerted force). So its possible to detection swing (foot on air) and stance phase

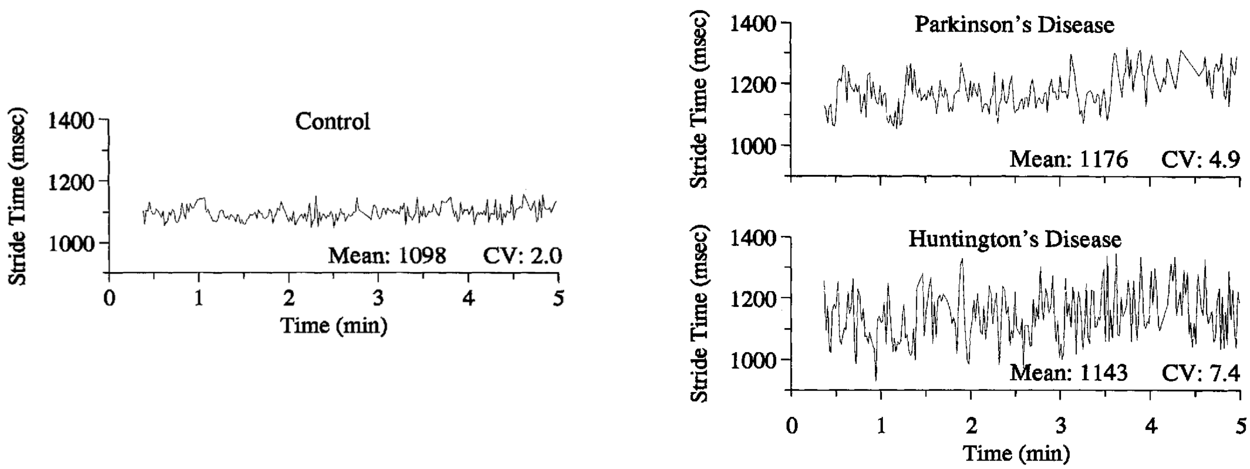
Measuring the temporal gait parameters

- Application
  - Distinguish normal from pathological gait
  - Better understand the muscle activity of the lower limbs



FSR can be completed by EMG measurement of lower limbs to evaluate the muscle activation during different phase of gait.

Example: increased Stride time variability with disease



CV=SD/mean, %

Hausdorff, Jeffrey M. et al. *Movement disorders* 1998

FSR allows to detect stride duration and quantify stride variability.

Stride variability refers to fluctuations in stride duration from one step to the next. It typically increases due to factors such as aging, neurological disorders, balance impairments, or fatigue, due to the complex controlled mechanism.

Gait is primarily controlled by the central nervous system (CNS) through coordinated actions involving:

- **Motor cortex:**
  Initiates and controls voluntary movements, adjusting walking patterns based on environmental demands.
- **Basal ganglia and cerebellum:**
  Refine and coordinate movements by regulating timing, rhythm, and precision of muscle activation.
- **Brainstem and spinal cord:**
  Contain central pattern generators (CPGs) that produce rhythmic stepping patterns automatically, even without conscious control.
- **Sensory feedback:**
  Sensory information from muscles, joints, and skin continuously informs and modifies gait patterns for stability and adaptability.

Disruptions or impairments in these CNS areas can significantly alter gait, making gait analysis crucial for assessing neurological and functional conditions.